# A Tale of Two Risks

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**June 4**-2015, 11:15-12:30

Bldg. 72, room 465

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First Draft: February 17, 2015
This Draft: May 25, 2015

Preliminary - Comments are welcome

**Abstract**

In this paper we further extend the role of jumps in asset prices to liquidity. The positive relation between volatility and liquidity costs has been well documented in the literature. We show that when decomposing total volatility into its jump-driven and diffusion-driven variance components, the former has a substantially stronger influence on illiquidity. These findings identify the exact source of volatility affecting illiquidity. Moreover, it emphasizes how investors demand higher compensation for bearing discontinuous jump risk compared to continuous diffusive risk.

## 1 Introduction

The relationship between asset liquidity and return risk has been studied extensively both theoretically and empirically. Market microstructure theories predict that higher return volatility increases liquidity costs, see for example the pioneering works of Stoll (1978a) and Copeland and Galai (1983). These theoretical works were further supported by a line of empirical studies that confirmed the predicted impact on liquidity costs, among them Stoll (1978b, 2000), Amihud and Mendelson (1989), Pastor and Stambaugh (2003), and Bao and Pan (2013). For a more extensive review see additional references therein.

However, treating the return risk as a uniform measure with a homogeneous impact on liquidity overlooks the more subtle structure that often comprises total volatility. More

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realistic developments in the asset pricing literature treat the return process as a jump-diffusion process, that is, as a combination of a continuous Brownian motion component and a discontinuous jump component. This approach dictates that the total return variance is an aggregate outcome of two separate sources of risk which have very different characteristics. While risk patterns generated by a discontinuous jump process create infrequent, large, isolated "surprise" price changes, a continuous diffusion process generates smooth and more expected small changes. The overall volatility is merely the integration of these two sources of risk.

Given this framework, one question that arises is whether the two types of risks impact liquidity in the same way or, alternatively, the *structure* of risk matters for liquidity in addition to raw *levels* of risk. Put differently, does one unit of jump-based variance have a different impact on liquidity than an equivalent unit of variance that is diffusion-based?

The fundamental role that liquidity plays in financial markets emphasizes the importance of the decomposition of total volatility into its underlying components. Improving the smooth functioning of exchanges, implementing efficient regulatory policies, and adopting suitable investment management strategies, all depend on the ability to identify the correct form of risk and accurate factors that influence liquidity. A better understanding of these determinants would allow for addressing these issues more effectively.

For example, targeting the reduction of jump volatility as opposed to diffusive volatility would require the implementation of a different set of regulatory policies. While jumps are mostly attributed to new information that dramatically affects prices, diffusive variance may be attributed to noise traders. Therefore, if it is primarily the jump-driven volatility component that affects illiquidity, implementing accounting policies that encourage more continuous information disclosure may be recommended in order to increase liquidity, rather than policies targeting noise trading activities, such as restricting short-selling or lending securities. Similarly, if jumps increase illiquidity by influencing investor confidence in financial markets, adopting policies that are relevant for reducing diffusive risk alone will not suffice.

From a theoretical perspective there are several reasons why jump risk could have a more dominant effect on liquidity. First, the central role of jumps might already be an implied property of the traditional approaches for modeling liquidity costs. The market microstructure literature recognized two main channels that affect bid-ask spreads and the price impact cost of liquidity. Inventory risk approaches, pioneered by Stoll (1978a), Amihud and Mendelson (1980), and Ho and Stoll (1981, 1983), emphasized the risk of fundamental price changes to market-makers' stock inventories, which they must maintain to provide immediacy in the market. Bid-ask spreads are therefore set to compensate for bearing those risks. Alternatively, the asymmetric information approach, starting with Glosten and Milgrom (1985) and Kyle (1985), emphasized market-makers' inevitable losses caused by trading with informed traders. Under this approach bid and ask quotes are strategically set so that these losses are offset by gains on trades with noise traders.

Jumps in prices are relevant for both reasonings. On the one hand jumps represent large price changes and therefore increase inventory risk. On the other hand, jumps are typically driven by new information and thus increase the risk of larger losses to market-makers trading with investors who privately hold this information. Hence, it is reasonable to expect that under both approaches market-makers would primarily protect themselves against this class of risk and demand higher compensation for bearing it compared to diffusive risk.

Second, jumps impose a more restrictive set of risk management tools and stopping rules compared to diffusive price changes. Market-makers can control their potential losses, update their inventory portfolios, and adopt "stop-loss" rules in a more flexible and gradual manner in a diffusive environment compared to a trading environment that exhibits infrequent dramatic price changes. See Longstaff (1995, 2014) who models a similar aspect for evaluating the cost of illiquidity.

Last, in order to reduce risk market-makers often hedge their inventories with correlated instruments, such as options and other correlated stocks or ETFs. Therefore it is mainly the non-hedgeable portion of their inventory and trading activity that should drive market-makers' compensation in the form of bid-ask spreads, as the remaining portion can be offset by various risk management techniques. For more on this see Benston and Hagerman (1974), Ho and Stoll (1983), Froot and Stein (1998), and Naik and Yadav (2003a, 2003b). Jump risk, as a discontinuous price change, cannot be hedged away as dynamic replicating strategies become infeasible under incomplete markets. Therefore, as the non-hedgeable portion of total volatility, it is the jump-driven component that market-makers would primarily view as costliest.[1]

For these reasons it is reasonable to expect that not only absolute levels of volatility matter for determining bid-ask spreads, and consequently affecting liquidity costs, but also the particular structure of that volatility should matter.

These theoretical motivations are also complemented by several empirical studies that demonstrated how in other dimensions of asset pricing jump risk plays a more dominant role than

[1] For a similar effect in the context of option pricing see for example Garleanu et al. (2009), Jameson and Wilhelm (1992), Gromb and Vayanos (2002), and Chen et al. (2014).

diffusive risk. For example, jump risk has a larger impact on equity risk premia (Pan, 2002, and Bollerslev and Todorov, 2011), on variance swaps risk premia (Ait-Sahalia et al., 2015, and Todorov, 2010), and on return predictability (Bollerslev et at., 2015). Here we extend the discussion to the realm of liquidity and explore whether various types of risk structures, particularly jump risk, matter for liquidity beyond pure levels of risk.

Notwithstanding, the relation between jump-driven variance and illiquidity might also be attributed to effects taking place in the other direction, from illiquidity to jumps. Thin trading implies infrequent transactions with large price impacts per trade, making any connection between jumps and illiquidity trivial. These two-directional effects between jumps and illiquidity prevent simply treating the correlation as an effect from jumps to liquidity and demand a method to discriminate between the two causal directions before deriving meaningful conclusions. We explicitly address this issue later.

In our analysis we fit a log-normal jump-diffusion process to all stocks listed on the NYSE and NASDAQ between 2002-2012, we estimate the parameters of the jump and diffusive processes, measure total return variance, and disentangle the respective contribution of the jump and diffusion processes to the overall variance. Then, using Fama-MacBeth portfolio and regression analyses we test for the potential impact each class of volatility has on a number of measures for stock liquidity, particularly bid-ask spreads and Amihud (2002) illiquidity measure.

To preview our results we find that the relation between volatility and illiquidity is almost exclusively driven by the jump component. The jump-driven volatility component has a substantially stronger effect and a more statistically significant one. Moreover, its economic significance dwarfs that of the diffusive volatility. An increase of one standard deviation in the jump-driven volatility component increases bid-ask spreads by approximately 50 basis points, while changes to the diffusive volatility component have negligible economic effects.

Finally, we account for the direction of causality by using additional extended return frequencies. As we elaborate later, price changes over longer horizons eliminate local intraday price impacts as they are more affected by fundamentals rather than liquidity providers (see Bao, Pan, and Wang, 2011). This way we isolate and confirm a significant causal effect from jumps to illiquidity. Our findings suggest that approximately half the correlation is explained by the effect jumps have on liquidity, while the other half is explained by short-lived effects from illiquidity to jumps.

The remainder of this paper is organized as follows. In the next section we describe our methodology and empirical approach followed by our data sources and descriptive statistics.

Sections 5 and 6 describe our results, using portfolio analyses and Fama-MacBeth regressions, respectively. Section 7 discusses the direction causality and Section 8 concludes.

# 2 Methodology

Our econometric methodology is a two stage process. In the first stage we fit a log-normal jump-diffusion process to stock returns, and obtain the parameters that characterize the separate sources of risk. Then, based on these parameters, we test for the relations between liquidity and the jump-driven and diffusive -driven volatilities. We dedicate a sub-section to describe each stage.

## 2.1 Model Description and Calibration

Following Merton (1976) we assume a continuous trading market for a stock with price $S_t$ at time $t$, in which there are three sources of uncertainty: a standard Brownian motion $W_t$, an independent Poisson process of jump events $N_t$ with intensity $\lambda$, and random jump size $Z_t$ which is distributed lognormally with mean $\alpha$ and variance $\gamma^2$. The stock return dynamics are described by the following stochastic differential equation

$$\frac{dS_t}{S_t} = (\mu - \lambda \cdot \kappa)\, dt + \sigma \cdot dW_t + dJ_t \tag{1}$$

where $\mu$ and $\sigma$ are constants, $\kappa \equiv E(Z_t - 1)$ is the expected relative jump of $S_t$, and $J_t \equiv (Z_t - 1) \cdot N_t$ denotes the compound Poisson process. Since the Brownian motion and the Poisson process of jump events are independent, the total return variance can be decomposed into

$$V \equiv Var\left(\frac{S_t}{S_0}\right) = Var(\sigma W_t) + Var(J_t) \tag{2}$$

which is the sum of the diffusion-related variance and the jump-related variance. We denote

$$\begin{aligned} V^d &\equiv Var(\sigma W_t) \\ V^j &\equiv Var(J_t) \end{aligned}$$

as the respective variances. Furthermore, following Merton (1976) and Navas (2003) these variances can be expressed in terms of the respective basic process parameters as

$$\begin{aligned} V^d &= \sigma^2 t \\ V^j &= \lambda\left(\alpha^2 + \gamma^2\right) t \end{aligned} \tag{3}$$

which allow for easily calculating these values.

In our estimation process we follow Ait-Sahalia (2002) and apply Maximum Likelihood (ML) methods to historical time-series data on stock returns to calibrate the model and obtain a vector of parameter estimates $\theta_i^t = \left(\mu_i^t, \sigma_i^t, \lambda_i^t, \alpha_i^t, \gamma_i^t\right)$ for each stock $i$ estimated over period $t$. Based on $\theta_i^t$ we can then calculate $V_i^{d,t}$ and $V_i^{j,t}$, that is, the respective components of the diffusive and jump variances out of the total variance.[2]

## 2.2 Empirical Analysis

Our empirical approach for testing the relation between liquidity and the two variance components is divided into two stages. In the first stage we carry out informal tests based on stock portfolios sorted on levels of variance. In the next stage we carry out Fama-MacBeth regressions to formally test those relations.

### 2.2.1 Sorted Portfolios

In the first stage, for each year $t$ we sorted all stocks in our sample on their total level of daily return variance, $V_t^i$. For each year we formed five equally weighted portfolios, where the first quintile portfolio contains stocks with the lowest variance for a given year and the fifth quintile contains stocks with the highest. Then for each ranking $k = 1, ..., 5$ we calculated the corresponding average value of our measures for liquidity across all stocks $i$ and years $t + 1$. We denote these averages by $\overline{Liq}_k$. Based on prior empirical studies we expect to find a positive relation between $\overline{Liq}_k$ and total variance ranking $k$.

In the next stage, for each year $t$ we further sorted each of the five portfolios by their jump-driven variance portion $V^j$ to form additional five equally weighted sub-portfolios per portfolio rank $k$. The first quintile sub-portfolio contains stocks with the lowest $V^j$ and the fifth quintile sub-portfolio contains stocks with the highest $V^j$. This way we created for each year $t$ and portfolio rank $k$ five sub-groups of stocks ranked from 1-5 sorted on $V^j$. We denote these sub-portfolios by $n = 1, ..., 5$.

We repeated the same procedure for the diffusive variance component $V^d$ to obtain equivalent five additional sub-portfolio rankings sorted on the diffusive variance component, per year $t$ and total variance rank $k$.

These sub-portfolios allow for exploring the relative impact the two different variance classes have on liquidity while controlling for total levels of variance. When going up the rankings in the jump-driven variance ($n$) while controlling for total levels of variance ($k$), we necessarily

[2] For a detailed description of our estimation procedure see Appendix.

increase its relative share at the expense of the diffusive share. The opposite holds true when going up the rankings in the diffusion-driven portfolios. If indeed jump variance has a different impact on liquidity than diffusive variance, we expect to see two results. First, that changing the relative shares of variance-types does matter for liquidity, despite the fact that total levels of variance are held fixed. Second, that changes in levels of each variance-type have different effects on liquidity. We elaborate on the specific methods of analysis later when we present our results.

### 2.2.2 Fama-MacBeth Regressions

In addition to portfolio analyses we also ran Fama-MacBeth regressions to formally test for different influences each type of variance has on liquidity costs. We first confirm that indeed total volatility has a positive effect on our measures for illiquidity in our sample, as previous studies have argued. Therefore, we ran the following cross-section regression year-by-year

$$Liq_{i,t+1} = \beta_0 + \beta_{1,t} V_{i,t} + \sum_{j=1}^{J} \beta_{1+j,t} Control_{i,t}^{j} + \varepsilon_{i,t} \tag{4}$$

The explanatory variables include total variance $V_{i,t}$, $J$ control variables $Control_{i,t}^{j}$ for $j = 1, ..., J$, and a random noise $\varepsilon_{i,t}$, all measured for stock $i$ in year $t$. This cross section regression is estimated year-by-year, and then time-series averages are calculated for all coefficients, following the Fama-MacBeth method. Therefore this procedure yields a vector of estimates $\beta = \left(\beta_0, ..., \beta_{1+J}\right)$ that characterizes the variables' effect on liquidity.

Control variables include the log of market-capitalization and average turnover rate for stock $i$ in year $t$. Stoll (1978a,b) and Jameson and Wilhelm (1992) and others showed that bid-ask spreads depend on expected holding duration, as more trading activity decreases the duration of risk exposure. Therefore, we expect to find a negative relation between turnover and illiquidity.

The dependent variable $Liq_{i,t+1}$ is our measure for liquidity costs for stock $i$ in the following year $t+1$. As measures for liquidity costs we used annual averages of bid-ask spreads (in percent), and Amihud (2002) annual illiquidity measures, for each stock $i$ in year $t$. We calculated Amihud (2002) illiquidity measure in the following way

$$A_{i,t} = \frac{1}{D_{i,t}} \sum_{n=1}^{D_{i,t}} \frac{|r_{i,n}|}{Dvol_{i,n}}$$

where $A_{i,t}$ is the Amihud measure for stock $i$ calculated over year $t$; $r_{i,n}$ and $Dvol_{i,n}$ are daily return and daily dollar trading volume for stock $i$ on day $n$; $D_{i,t}$ is the number of days with available ratio in year $t$.

In the next step, we explicitly included in the model the decomposition of total variance into its jump and diffusion driven components. Therefore the new specification is,

$$Liq_{i,t+1} = \beta_0 + \beta_{1,t} V_{i,t}^d + \beta_{2,t} V_{i,t}^j + \sum_{j=1}^{J} \beta_{2+j,t} Control_{i,t}^j + \varepsilon_{i,t} \tag{5}$$

where the explanatory variables $V_{i,t}^d$ and $V_{i,t}^j$, the diffusive and jump driven variance components, respectively, replace the total variance $V_{i,t}$ in Equation (4). Both $V_{i,t}^d$ and $V_{i,t}^j$ are obtained from the ML estimation. All other variables in the new specification remained unchanged.

We used another specification to demonstrate the effects of jump and diffusive driven volatilities while controlling for total levels of variance. That is, we used the following models,

$$Liq_{i,t+1} = \beta_0 + \beta_{1,t} V_{i,t} + \beta_{2,t} V_{i,t}^d + \sum_{j=1}^{J} \beta_{2+j,t} Control_{i,t}^j + \varepsilon_{i,t} \tag{6}$$

$$Liq_{i,t+1} = \beta_0 + \beta_{1,t} V_{i,t} + \beta_{2,t} V_{i,t}^j + \sum_{j=1}^{J} \beta_{2+j,t} Control_{i,t}^j + \varepsilon_{i,t} \tag{7}$$

These specifications demonstrate the effect of substituting one unit of jump-driven variance with an equivalent unit of diffusion-driven variance while controlling for total levels of volatility. If the diffusive variance has a weaker impact on illiquidity, then increasing its share at the expense of jump variance would have a negative effect on illiquidity. In this case we would expect a negative coefficient for the diffusive variance $V_{i,t}^d$ in Equation (6). For the same reasoning, we would expect a positive effect on illiquidity when increasing the jump variance share at the expense of the diffusive variance share. That is, a positive coefficient for the jump variance $V_{i,t}^j$ in Equation (7).

# 3 Data

We downloaded from CRSP daily stock prices, volume, shares outstanding and market-capitalization for all stocks listed on the NYSE and NASDAQ between 2002-2012. For these stocks and years we also downloaded TAQ historical data for bid-ask quotes and calculated their average annual percentage spreads. Average annual turnover rates were calculated using volume and shares outstanding data for each stock.

In our final sample we eliminated all firm-years that had less than 245 observations per year, and whose bid-ask spreads (percent) were larger than 50% or negative. We also eliminated securities that did not have data on market capitalization for year $t$ in the CRSP database, this excludes non-stock securities listed on exchanges. Overall we ended up with 9,088 different stocks between 2002-2012, and 61,299 stock-year observations.

## 4 Descriptive Statistics

We calibrated the return process model specified in Equation (1) for daily returns and obtained for each stock $i$ and year $t$ a vector of parameters $\theta_i^t = \left(\mu_i^t, \sigma_i^t, \lambda_i^t, \alpha_i^t, \gamma_i^t\right)$ that characterizes the jump-diffusion return process. In order to gauge the consistency of our calibration to the realized historical data we compared our model-implied daily return variance ($V_i^t$ as specified in Equation (2)) with the realized daily return variance, measured over the corresponding year $t$. We denoted the realized variance by $\widetilde{V}_i^t$. For more than 90% of our sample the ratio $\frac{\widetilde{V}_i^t}{V_i^t}$ fell between 0.8 and 1.2, implying that there was a good fit between our predicted variance and the actual variance, i.e., no more than 20% deviation.

Finalizing our sample we eliminated all estimates with extreme values, that is, the highest and lowest 1% of the vector $\theta_i^t$ and for the illiquidity measure $A_{i,t}$. We also eliminated all observations that did not satisfy the condition $\frac{\widetilde{V}_i^t}{V_i^t} \in [0.8, 1.2]$. After applying these additional filtering our final sample contained 55,558 stock-years observations.

Tables 1-2 report descriptive statistics for the vector of parameters $\theta_i^t$. Table 1 reports overall average and quantile values for the parameter estimates. Stocks experience on average 49 jumps per year ($\lambda$), and range from no jumps to 241 jumps per year. The average expected jump size ($\alpha$) is 1%, but with a large standard deviation of 4% across all stocks and years.

Table 2 provides a more detailed breakdown of average values per year. A clear difference in patterns exists between crisis and non-crisis years. The number of jumps ($\lambda$) is particularly high for crisis years (2002 and 2008-2009) where it reached levels of 60-70 jumps a year, on average. For non-crisis years $\lambda$ is around half this magnitude, in the range of 30-40 jumps per year on average. The diffusive standard deviation ($\sigma$) is also somewhat higher during crisis years and reached levels of over 30%, compared to around 20% during regular times. Similarly, the diffusive trend ($\mu$) had negative values for years 2007, 2008 and 2011. Average jump sizes are consistently between 0% -2% across all years.

Finally, Table 3 reports overall average and quintile values for our main variables of interest: total volatility, jump and diffusive volatility, and average bid-ask spreads. Average total return volatility across all years and stocks is around 29%. Average values for the diffusive and jump components are of the same order of magnitude, 18% and 20%, respectively, and their medians are around 17%. Their similar orders of magnitude are also maintained throughout their quintile distribution. Finally, average bid-ask spreads are around 1.8% across our sample, with a standard deviation of 2.5%. The median bid-ask spread is around 85 basis points.

# 5 Sorted Portfolios - Results

## 5.1 Total Volatility and Liquidity

We start by addressing the basic case of the relation between total volatility and liquidity costs. Table 4 presents average values for portfolios sorted on total variance. We reported average total standard deviations per portfolio rank $k = 1, ..., 5$ along with the respective averages for our liquidity measures $\overline{Liq}_k$ per portfolio, as defined in Section 2.2.1. Liquidity is measured by both bid-ask spreads and Amihud (2002) illiquidity measure.

As seen, both measures increase with total variance ranking. The lowest ranked portfolio has an average standard-deviation of 1.2% and exhibits an average bid-ask spread of 1.3%; the highest ranked portfolio has an average standard deviation of 5.2% and exhibits an average bid-ask spread of 2.9%. Similarly the respective average Amihud (2002) measures are 0.085 and 1.180. The differences between high and low portfolio means are 1.6% for bid-ask spreads, and 1.095 for the Amihud measure. Formal $t$-tests for the difference in means confirm that these differences are highly statistically significant, with $t$-statistics of 42 and 26 for the respective illiquidity measures.

These findings are consistent with prior studies which found a positive relation between volatility and illiquidity costs, see for example Stoll (1978b, 2000) and Pastor and Stambaugh (2003).

## 5.2 Volatility Components and Liquidity

In the next step, we addressed our main goal of decomposing total volatility into its jump and diffusive components and explore their individual impacts on illiquidity. We did so by further sorting each of the five portfolios of total volatility on the jump-driven variance component. This way we formed $n = 1, .., 5$ additional sub-portfolios per total variance portfolio, see Section 2.2.1 for details. This allows for exploring the impact of the jump driven variance while controlling for total level of variance.

To control for total level of variance we used two methods. In the first method we tested the effect of the jump-driven variance component for each of the five total-variance portfolio ranking separately. Therefore, for each of the jump-driven volatility ranks $n$, we calculated average values of liquidity costs $\overline{Liq}_k$ across all years for each total-volatility level $k$ separately. That is, we constructed five-by-five portfolios that allow for exploring the impact of five jump-variance levels on illiquidity while holding the five total variance rankings fixed. We repeated an equivalent analysis for sorting on the diffusion driven variance component and its impact on $\overline{Liq}_k$. We report these results in Tables 5 and 6 for bid-ask spreads and Amihud (2002) illiquidity measure, respectively. Panels A and B of each table are dedicated to the jump and diffusive variances, respectively.

As seen in Table 5 Panel A, average bid-ask spreads increase as the total share of the jump-driven variance increases at the expense of the diffusion-variance share. This holds true for all levels of total-variance portfolios. For example, focusing on the lowest total-variance portfolio: the bid-ask spread for the lowest jump portfolio is 1.15% compared to 1.46% for the highest. Similarly, for the fourth total variance portfolio: the bid-ask spread for the lowest jump portfolio is 1.25% compared to 2.54% for the highest. The differences in means between high and low portfolios range from .31% to 2.54%. Formal $t$-tests for these difference reject the null that the corresponding average bid-ask spreads are identical within a given total variance portfolio, with $t$-statistics ranging from 5 to 21.

The opposite results were obtained when sorting on the diffusion-driven variance component. As seen in Table 5 Panel B, overall bid-ask spreads decrease with the relative share of the diffusion-based variance component for each portfolio level of total volatility. Formal $t$-tests for the difference in means between high and low portfolios reject the null that the corresponding average bid-ask spreads are identical within a given total variance portfolio, with $t$-statistics ranging from -2.88 to -19.11, confirming the negative relation.

Our results remained qualitatively the same when using the Amihud (2002) illiquidity measure instead of bid-ask spreads, as reported in Table 6 Panels A and B.

In the second method we did not hold total volatility levels fixed at rank $k$. Instead, for each jump-volatility rank $n$ we calculated average values of the liquidity measures $\overline{Liq}$ across all years and across all total variance portfolios. This way, we controlled for total volatility by averaging across all total volatility ranks $k = 1, ..., 5$. We repeated an equivalent analysis for the diffusion driven variance component.

Average bid-ask spreads for each portfolio level are reported Table 7 Panel A. As seen, illiquidity increases with the share of the jump-driven variance component, and decreases for the diffusive component. Bid-ask spreads increase from 1.24% for the lowest jump portfolio to 2.41% for the highest portfolio. On the other hand, bid-ask spreads decrease from 2.5% for the lowest diffusive portfolio to 1.36% for the highest one. The difference in means between the high and low portfolios are 1.17% and -1.14% for the jump and diffusive portfolios, respectively. Formal $t$-tests for these difference reject the null that the corresponding average bid-ask spreads are identical, with $t$-statistics ranging of 31 and -29, respectively. All results remained qualitatively the same using the Amihud (2002) illiquidity measure, as reported in Panel B.

In summary, both methods lead to the same conclusion. Illiquidity increases when the jump volatility component constitutes a larger share of total volatility. The opposite holds true when the diffusive volatility component constitutes a larger share.

## 5.3 Marginal Impact of Volatility Components

In contrast to the previous tests, we also addressed the marginal effect of jump volatility while controlling for total levels of diffusive volatility, and vice versa. That is, instead of increasing the *share* of one volatility component at the expense of the other, this time we tested for the effects of increasing total levels of volatility by adding jump volatility or diffusive volatility, and compared their marginal effects.

We did so by sorting first on the diffusive component and forming five portfolio levels. We then further sorted each diffusive portfolio on the jump volatility component and formed five additional sub-portfolios. Following the second method mentioned above, for each jump portfolio level we took averages of bid-ask spreads across all five diffusive portfolios to control for levels of diffusive volatility.

We repeated an equivalent analysis for adding diffusive driven volatility while controlling for total levels of the jump volatility. Table 7 Panel A reports our results for bid-ask spreads, and Panel B for Amihud (2002) illiquidity measure.

As seen in Table 7 Panel A, when controlling for diffusive volatility, bid-ask spreads increase with levels of jump volatility. The difference in means between the high and low jump portfolios is 1.95%, and highly significant. On the other hand, when controlling for jump-volatility, bid-ask spreads slightly decrease with levels of diffusive volatility. The difference in means between the high and low diffusive portfolios is -0.65%, and highly significant.

These findings imply two important phenomena. First, adding jump volatility to total volatility increases bid-ask spreads, whereas adding diffusive volatility slightly decreases bid-ask spreads. Second, jump volatility has a much larger impact on bid-ask spreads in absolute terms compared to diffusive volatility.

Similar results were obtained for the Amihud (2002) illiquidity measure, as reported in Panel B. Additionally, we repeated the same analysis using the first method mentioned above. That is, we tested the effect of increasing the jump volatility level for each diffusive portfolio level separately, and vice versa. All our results remained qualitatively unaltered. See Tables 8 and 9 for these results, for bid-ask spreads and Amihud (2002) illiquidity measure, respectively.

## 5.4 Other Control Variables

Finally, we further refined our analysis by controlling for additional three variables: market capitalization, volume of trade, and turnover. For each of these variables for each year we sorted all stocks into five different portfolios, from low to high. Then, for each portfolio level we repeated our second method of double sorting on total variance and jump-driven variance and then averaging across all years and all total volatility ranks $k$. This process was carried out for each of the five control variable portfolios. Therefore, for each control variable we have a five-by-five portfolio ranking sorted on control variable level and jump-driven volatility level. Table 7 reports the results in Panel A, B, and C for each of the three control variables.

As seen in all three panels, the general pattern is maintained: higher jump-driven portfolios always exhibit higher average bid-ask spreads, per control variable portfolio. This holds true for all five portfolio rankings for all three variables. Moreover, formal $t$-tests for the difference between high and low jump-portfolios all reject the null hypothesis that the corresponding average bid-ask spreads are identical, per control variable portfolio. $t$-statistics are highly significant and range from 2.62-10.69.

In summary, our results indicate two phenomena: the structure of volatility matters for bid-ask spreads beyond raw levels of volatility. Moreover, the jump-driven variance component has a stronger impact on liquidity compared to the diffusion component. This result was obtained for when substituting jump volatility with diffusive volatility (i.e., holding total volatility fixed), and for when adding jump volatility to diffusive volatility (i.e., increasing total volatility). Last, these results remained robust after controlling for other variables relevant to determining liquidity costs. In the next section we formally test these finding.

# 6 Fama-MacBeth Regressions - Results

In the first step we replicated the results from previous studies to confirm that indeed total variance has a positive impact on illiquidity in our sample. Table 11 reports Fama-MacBeth regression results based on the model specified in Equation (4) using bid-ask spreads and Amihud (2002) illiquidity measure as the dependent variable in Panels A and B, respectively.

As reported in Panel A, total variance indeed has a positive and significant impact on bid-ask spreads, with a coefficient estimate of 2.98 and $t$-statistic of 7.5. A similar result is obtained when using the Amihud (2002) illiquidity measure as the dependent variable, as reported in Panel B. In both cases, the turnover coefficient is negative, as expected, consistent with prior studies that argue that higher trading activity decreases illiquidity. Market capitalization also has a negative and statistically significant effect on both illiquidity measures as expected, since larger firms tend to have lower trading costs.

In the next step, we decomposed total volatility into its jump and diffusive driven components. Table 12 reports Fama-MacBeth regression results for the regression specified in Equation (5) which explicitly models separate effects for each component. Panel A reports the results using bid-ask spreads. As expected, both variance components have a positive effect on bid-ask spreads. However, the size of the jump-driven variance coefficient dwarfs that of the diffusion-driven variance coefficient, 5.15 compared to 0.02, respectively, indicating a substantially stronger effect. Similarly, the jump component coefficient has substantially higher statistical significance, with a $t$-statistic of 8.30 compared to 0.04 for the diffusion component coefficient. Finally, turnover and size maintain a very similar effect compared to those obtained in Table 11. The Fama-MacBeth average $\overline{R}^2$ is 50% indicating a strong explanatory power for our model.

Interestingly, the coefficient for total variance reported in Table 11 falls between the values of the diffusive and jump driven coefficients reported in Table 12, indicating that the total effect of the variance is indeed an aggregate outcome of a combined dominant jump and marginal diffusive effect.

Panel B reports the results for the Amihud (2002) illiquidity measure, which remained qualitatively the same. Under this specification, the coefficient for the jump-driven variance is approximately four times larger that of the diffusion-driven variance, 387 compared to 98, respectively. Again, the jump component coefficient has substantially higher statistical significance, with a $t$-statistic of 7.64 compared to 3.85 for the diffusion component coefficient.

Last, the jump and diffusive coefficients are again higher and lower, respectively, than the effect that total variance has on the Amihud (2002) illiquidity measure, as reported in Table 11. This indicates that the effect of total variance is an aggregate outcome of these two separate components.

We used two additional models to demonstrate the effect of increasing the share of one volatility component at the expense of the other, while holding total levels of volatility fixed. These models are specified in Equations (6) and (7), and their results are reported in Table 13 Panels A and B, respectively.

Overall these results further support the dominance of the jump component. As seen in Panel A, when holding total variance fixed the diffusive variance has a negative coefficient. Moreover increasing the diffusive volatility share of total variance (i.e., at the expense of jump volatility) almost entirely offsets the effect of total variance, as indicated by their coefficient values of -5.13 and 5.15, respectively. This implies that out of total volatility it is only the jump volatility share that affects illiquidity, consistent with our previous results.

The same result was obtained for Equation (7), as reported in Panel B. Jump variance share has a positive and significant coefficient, while the total variance coefficient is marginal and non-significant. This implies that only the jump volatility share out of total volatility affects illiquidity, consistent with our previous results.

In summary, these results confirm the findings previously obtained in the portfolio analysis. The structure of volatility matters for the cost of illiquidity beyond raw levels of volatility, as jump and diffusive variance coefficients have distinct effects on illiquidity. Moreover, the jump-driven variance component has a dominant and almost exclusive effect on illiquidity.

# 7 Direction of Causality

As mentioned in the introduction, it may be argued that the correlations we found between the jump-driven component and our measures for liquidity do not capture causal effects from jumps to illiquidity but rather in the other direction, from illiquidity to jumps. By definition, illiquid assets are subject to greater jump risk: thin trading means infrequent transactions where each transaction is more likely to generate large price impacts. Put differently, "technical jumps" can be generated through prices that bounce between bid and ask quotes for wide bid-ask spreads. This makes the connection between jumps and illiquidity trivial.

However, the impact of thin trading or intraday price bounces becomes negligible as we lower the frequency of price-quotes. Price changes over longer horizons are more affected by fundamentals and new information than by liquidity providers (see Bao, Pan, and Wang, 2011). Therefore, in order to control for the direction of causality we ran our regressions again, this time increasing the return frequency from daily to weekly and biweekly return-frequencies.

In order to carry out these additional regressions we re-calibrated our price-process model in Equation (1) this time using weekly and biweekly return frequencies. For each return frequency we obtained a new set of parameters $\theta_i^t = \left(\mu_i^t, \sigma_i^t, \lambda_i^t, \alpha_i^t, \gamma_i^t\right)$, and it was then used in the regression model specified Equation (5). All other variables in the regression model remained unchanged.[3]

Table 14 reports Fama-MacBeth regression results for weekly and biweekly return frequencies in Panel A and Panel B, respectively, where the dependent variable is annual average of bid-ask spreads. The results in both cases did not change qualitatively compared to the original estimates obtained for daily return frequencies. As reported in Panel A, the diffusive variance coefficient is negative and non-significant, compared to a positive and highly significant jump variance coefficient. The size of the diffusive coefficient is -0.83 compared to 2.16 for the jump coefficient, with $t$-statistics of -1.59 and 5.99, respectively.

The same pattern is maintained for biweekly returns as reported in Panel B. The diffusive coefficient is -1.2 compared to 1.58 for the jump coefficient. This time both coefficients are statistically significant, but the $t$-statistic for the jump coefficient is still larger, 4.82 compared to -3.26, respectively.

These results imply that the jump-driven volatility has a larger and more significant effect on illiquidity even for lower return frequencies, supporting the existence of a causal effect in the direction from jumps to illiquidity.

[3] Similar to our estimation of daily returns, we compared the analytical model-implied daily return variance ($V_i^t$ as specified in Equation (2)) with the realized daily return variance, measured over the corresponding year $t$. For approximately 90% of our sample the ratio $\frac{V_i^t}{\widetilde{V}_i^t}$ fell between 0.8 and 1.2, implying that there was a good fit between our predicted variance and the actual variance for the weekly and biweekly return frequencies as well. That is, no more than 20% deviation.

Finalizing our sample we eliminated all firm-year observations that had less than 245 quotes a year or that had at least three trading weeks with less than 4 quotes a week. All other elimination criteria remained identical to those of daily returns: we eliminated all estimates with extreme values and all observations that did not satisfy the condition $\frac{V_i^t}{\widetilde{V}_i^t} \in [0.8, 1.2]$. See Sections 3 and 4 for more details.

We repeated all regressions using the Amihud (2002) illiquidity measure as the dependent variable. As presented in Table 15, all findings remained qualitatively the same for this specification as well. For both weekly and biweekly return frequencies, the diffusive coefficient is non-significant and much smaller in magnitude compared to a large and significant jump coefficient.

When comparing the estimates for daily, weekly and biweekly return frequencies an interesting pattern can be detected. The coefficients for the jump volatility component decrease in size from the daily through the weekly and biweekly return frequencies. The jump-variance coefficients decreased from 5.15 to 2.16 and 1.58, respectively. The coefficients for the diffusive volatility remain marginal and non-significant on the whole (see Tables 12 and 14).

A similar pattern is detected when comparing average values for the parameter estimates $\theta_i^t = \left(\mu_i^t, \sigma_i^t, \lambda_i^t, \alpha_i^t, \gamma_i^t\right)$ obtained from the ML procedure for the three return frequencies, as reported in Table 16. Average estimates for all three frequencies remain stable for all three return frequencies, with the exception of $\lambda$. The average number of jumps $\lambda$ declines as frequency declines: from 49 to 9 and 5 jumps per year on average for daily, weekly, and biweekly return frequencies, respectively. Moreover, notice that the weekly and biweekly coefficients and $\lambda$s are quite close to each other in magnitude compared to their daily frequency values.

The gradual decline in coefficients and number of jumps and their stabilization may be attributed to the declining impact bid-ask spreads have on the variance. As mentioned before, the positive correlation between jumps and bid-ask spreads may arise through two channels: from jumps to illiquidity and from illiquidity to jumps. However, as we argued before, the latter impact disappears in longer trading horizons as local illiquidity and price-bounces play a more negligible role over longer time horizons.

Hence, for low frequencies, such as weekly and biweekly horizons, the remaining correlation represents a cleaner impact that the variance has on illiquidity. This implies that the majority of the impact that thin trading has on the jump component disappears already at the weekly frequency and stabilizes thereafter. Based on the ratio of coefficients' size, it can be evaluated that between half to two-thirds of the correlation between jumps and bid-ask spreads is the impact illiquidity has on jumps due to thin trading, while the remaining half to one-third is the impact that "true" jumps have on illiquidity. The latter impact survives in longer horizons while the former disappears.

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Table 1
Descriptive Statistics
Average and Quantile Parameter Values - Overall

| Parameter | **Mean** | **S.D.** | **Min** | **.25** | **Mdn** | **.75** | **Max** | **N** |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | 0.14 | 0.44 | -1.29 | -0.07 | 0.14 | 0.35 | 1.99 | 55,558 |
| $\sigma$ | 0.29 | 0.15 | 0.00 | 0.17 | 0.27 | 0.38 | 0.93 | 55,558 |
| $\lambda$ | 49.31 | 47.76 | 0.00 | 9.35 | 35.65 | 77.57 | 241.72 | 55,558 |
| $\alpha$ | 0.01 | 0.04 | -0.18 | 0.00 | 0.00 | 0.03 | 0.17 | 55,558 |
| $\gamma$ | 0.06 | 0.06 | 0.00 | 0.02 | 0.04 | 0.08 | 0.42 | 55,558 |

Table 2
Average Annual Values of Parameters

| **Year** | $\boldsymbol{\mu}$ | $\boldsymbol{\sigma}$ | $\boldsymbol{\lambda}$ | $\boldsymbol{\alpha}$ | $\boldsymbol{\gamma}$ | **N** |
|---|---|---|---|---|---|---|
| 2002 | 0.02 | 0.34 | 60.52 | 0.01 | 0.06 | 5,418 |
| 2003 | 0.48 | 0.28 | 46.07 | 0.02 | 0.06 | 5,389 |
| 2004 | 0.21 | 0.27 | 41.60 | 0.02 | 0.06 | 5,470 |
| 2005 | 0.09 | 0.25 | 37.53 | 0.02 | 0.06 | 5,379 |
| 2006 | 0.17 | 0.25 | 36.69 | 0.02 | 0.06 | 5,389 |
| 2007 | -0.01 | 0.26 | 49.20 | 0.01 | 0.05 | 5,311 |
| 2008 | -0.29 | 0.38 | 72.28 | 0.00 | 0.07 | 4,789 |
| 2009 | 0.49 | 0.38 | 67.00 | 0.01 | 0.06 | 4,604 |
| 2010 | 0.24 | 0.30 | 44.63 | 0.02 | 0.05 | 4,702 |
| 2011 | -0.01 | 0.30 | 56.39 | 0.00 | 0.05 | 4,684 |
| 2012 | 0.17 | 0.26 | 33.36 | 0.02 | 0.06 | 4,459 |

Table 3
Summary Statistics:
Bid-Ask Spreads & Standard-Deviations of Daily Returns

| | **Mean** | **S.D.** | **Min** | **.25** | **Mdn** | **.75** | **Max** |
|---|---|---|---|---|---|---|---|
| Return Std - Total | .0292 | .0168 | .0050 | .0169 | .0255 | .0376 | .1129 |
| Return Std - Diffusion | .0186 | .0100 | 0 | .0111 | .0170 | .0243 | .0591 |
| Return Std - Jump | .0203 | .0159 | 0 | .0090 | .0173 | .0286 | .1067 |
| Bid-Ask Spread | .0187 | .0252 | .0003 | .0030 | .0085 | .0235 | .2525 |

Table 4
Average Bid-Ask Spreads and Standard Deviations:
for portfolios sorted on Total Variance

| | Low $Var$ | 2 | 3 | 4 | High $Var$ | *High-Low* | *t-stat* |
|---|---|---|---|---|---|---|---|
| Avg Standard Deviation | .012 | .020 | .027 | .035 | .052 | | |
| Avg Bid-Ask Spread | .013 | .011 | .013 | .017 | .029 | *.016* | *42.49* |
| Avg Amihud Measure | .085 | .234 | .333 | .458 | 1.180 | *1.095* | *26.30* |

Table 5
Average Bid-Ask Spreads for Sorted Portfolios:
Sorted by Total-Variance, Jump-Variance, and Diffusion Variance
Per Total-Variance Portfolio

| Portfolio | Low $Var$ | 2 | 3 | 4 | High $Var$ |
|---|---|---|---|---|---|
| | | | Panel A: Jump-Var Ranking | | |
| Low Jump | .0115 | .0094 | .0090 | .0125 | .0203 |
| 2 | .0149 | .0085 | .0099 | .0130 | .0238 |
| 3 | .0125 | .0098 | .0109 | .0158 | .0296 |
| 4 | .0132 | .0124 | .0155 | .0203 | .0347 |
| High Jump | .0146 | .0194 | .0222 | .0254 | .0454 |
| *High-Low* | *.0031* | *.0100* | *.0132* | *.0129* | *.0251* |
| *t-stat* | *5.43* | *14.50* | *18.01* | *15.27* | *21.09* |
| | | | Panel B: Diffusive-Var Ranking | | |
| Low Diffusive | .0184 | .0203 | .0227 | .0273 | .0377 |
| 2 | .0171 | .0110 | .0148 | .0189 | .0267 |
| 3 | .0134 | .0096 | .0108 | .0147 | .0254 |
| 4 | .0094 | .0094 | .0098 | .0137 | .0265 |
| High Diffusive | .0081 | .0091 | .0094 | .0124 | .0340 |
| *High-Low* | *-.0103* | *-.0104* | *-.0133* | *-.0149* | *-.0037* |
| *t-stat* | *-19.11* | *-16.46* | *-18.24* | *-17.52* | *-2.88* |

Table 6
Average Amihud (2002) Illiquidity Measure for Sorted Portfolios:
Sorted by Total-Variance, Jump-Variance, and Diffusion Variance
Per Total-Variance Portfolio

| Portfolio | Low $Var$ | 2 | 3 | 4 | High $Var$ |
|---|---|---|---|---|---|
| Panel A: Jump-Var Ranking | | | | | |
| Low Jump | .06 | .16 | .23 | .36 | .59 |
| 2 | .02 | .11 | .20 | .23 | .70 |
| 3 | .04 | .20 | .17 | .28 | .88 |
| 4 | .09 | .22 | .33 | .57 | 1.45 |
| High Jump | .18 | .47 | .72 | .85 | 2.65 |
| *High-Low* | *.12* | *.31* | *.49* | *.49* | *2.06* |
| *t-stat* | *5.28* | *6.64* | *6.05* | *4.54* | *11.23* |
| Panel B: Diffusive-Var Ranking | | | | | |
| Low Diffusive | .13 | .50 | .78 | 1.00 | 1.59 |
| 2 | .08 | .15 | .27 | .43 | .81 |
| 3 | .05 | .14 | .17 | .29 | .82 |
| 4 | .05 | .15 | .20 | .24 | 1.01 |
| High Diffusive | .09 | .21 | .25 | .35 | 1.74 |
| *High-Low* | *-.04* | *-.29* | *-.53* | *-.65* | *-.15* |
| *t-stat* | *-1.28* | *-5.76* | *-6.22* | *-5.83* | *0.85* |

Table 7

Variance Components Across Control Variables

| Control for: | Rank by: | Low | 2 | 3 | 4 | High | *High-Low* | $t$-stat |
|---|---|---|---|---|---|---|---|---|
| **Panel A: Avg Bid-Ask Spreads** | | | | | | | | |
| Total Var | Jump Var | .0124 | .0138 | .0154 | .0186 | .0241 | *.0117* | *31.45* |
| Total Var | Diffusive Var | .0250 | .0174 | .0145 | .0134 | .0136 | *-.0114* | *-29.37* |
| Diffusive Var | Jump Var | .0110 | .0117 | .0136 | .0187 | .0305 | *.0195* | *50.31* |
| Jump Var | Diffusive Var | .0228 | .0165 | .0140 | .0138 | .0165 | *-.0063* | *-15.99* |
| **Panel B: Avg Amihud Illiquidity Measure** | | | | | | | | |
| Total Var | Jump-Var | .2731 | .2583 | .3053 | .5020 | .8737 | *.6006* | *14.31* |
| Total Var | Diffusive-Var | .7845 | .3396 | .2860 | .3156 | .4718 | *-.3127* | *-7.13* |
| Jump Var | Diffusive Var | .1791 | .1967 | .2364 | .4329 | 1.2383 | *1.059* | *22.79* |
| Diffusive Var | Jump Var | .6167 | .3263 | .3091 | .3619 | .5779 | *-.0388* | *-0.90* |

Table 8
Average Bid-Ask Spreads for Sorted Portfolios:
Sorted by Jump-Variance and Diffusion Variance

| Portfolio | Low | 2 | 3 | 4 | High |
|---|---|---|---|---|---|
| Panel A: Diffusion Sorted on Jumps | | | | | |
| Low Diffusive | .0166 | .0169 | .0204 | .0253 | .0364 |
| 2 | .0141 | .0106 | .0133 | .0190 | .0270 |
| 3 | .0086 | .0088 | .0113 | .0166 | .0262 |
| 4 | .0092 | .0089 | .0102 | .0139 | .0294 |
| High Diffusive | .0093 | .0102 | .0139 | .0184 | .0361 |
| *High-Low* | *-.0073* | *-.0067* | *-.0065* | *-.0068* | *-.0003* |
| *t-stat* | *-13.10* | *-10.95* | *-8.42* | *-7.76* | *-0.20* |
| Panel B: Jumps Sorted on Diffusion | | | | | |
| Low Jump | .0162 | .0085 | .0097 | .0090 | .0115 |
| 2 | .0149 | .0090 | .0090 | .0099 | .0160 |
| 3 | .0147 | .0103 | .0103 | .0125 | .0208 |
| 4 | .0183 | .0157 | .0151 | .0169 | .0284 |
| High Jump | .0339 | .0266 | .0261 | .0285 | .0393 |
| *High-Low* | *.0176* | *.0181* | *.0164* | *.0194* | *.0278* |
| *t-stat* | *18.94* | *22.90* | *20.86* | 24.*80* | *27.57* |

Table 9

Average Amihud (2002) Illiquidity Measure for Sorted Portfolios: Sorted by Jump-Variance and Diffusion Variance

| Portfolio | Low | 2 | 3 | 4 | High |
|---|---|---|---|---|---|
| **Panel A: Diffusion Sorted on Jumps** | | | | | |
| Low Diffusive | .0277 | .1579 | .4682 | .8719 | 1.6753 |
| 2 | .0403 | .1266 | .2712 | .5236 | .7290 |
| 3 | .0704 | .1096 | .1706 | .3615 | .9000 |
| 4 | .1552 | .2046 | .1914 | .2460 | 1.1258 |
| High Diffusive | .2491 | .2680 | .4166 | .4621 | 1.8031 |
| *High-Low* | *.2213* | *.1101* | *-.0515* | *-.4098* | *.1277* |
| *t-stat* | *5.61* | *2.18* | *-0.76* | *-4.68* | *0.66* |
| **Panel B: Jumps Sorted on Diffusion** | | | | | |
| Low Jump | .0327 | .0589 | .1757 | .2234 | .3976 |
| 2 | .0396 | .1014 | .1779 | .2272 | .4443 |
| 3 | .1245 | .1353 | .1708 | .2010 | .5790 |
| 4 | .3781 | .3313 | .2358 | .3840 | .8826 |
| High Jump | 1.5867 | .9435 | .8002 | .8500 | 2.2166 |
| *High-Low* | *1.5540* | *.8845* | *.6245* | *.6266* | *1.8190* |
| *t-stat* | *13.90* | *9.66* | *9.81* | *7.63* | *11.57* |

Table 10

Average Bid-Ask Spreads for Sorted Portfolios:

Sorted on Total-Variance and Jump-Variance

by Control Variables

| | Low Jump | 2 | 3 | 4 | High Jump | *High-Low* | $t$-stat |
|---|---|---|---|---|---|---|---|
| Panel A: Market Capitalization | | | | | | | |
| Low Cap | .0479 | .0466 | .0491 | .0520 | .0550 | *.0071* | *5.91* |
| 2 | .0203 | .0205 | .0216 | .0220 | .0236 | *.0033* | *5.27* |
| 3 | .0098 | .0099 | .0101 | .0099 | .0107 | *.0009* | *2.64* |
| 4 | .0048 | .0047 | .0048 | .0051 | .0055 | *.0007* | *3.19* |
| High Cap | .0024 | .0023 | .0024 | .0023 | .0025 | *.0001* | *1.08* |
| Panel B: Volume | | | | | | | |
| Low Vol | .0497 | .0469 | .0494 | .0525 | .0559 | *.0062* | *5.28* |
| 2 | .0187 | .0195 | .0208 | .0213 | .0224 | *.0037* | *6.90* |
| 3 | .0087 | .0093 | .0096 | .0102 | .0107 | *.0020* | *6.94* |
| 4 | .0043 | .0042 | .0042 | .0044 | .0050 | *.0007* | *3.79* |
| High Vol | .0020 | .0020 | .0021 | .0021 | .0023 | *.0003* | *3.22* |
| Panel C: Turnover | | | | | | | |
| Low Turnover | .0354 | .0384 | .0389 | .0424 | .0479 | *.0125* | *10.73* |
| 2 | .0174 | .0201 | .0213 | .0227 | .0248 | *.0074* | *10.08* |
| 3 | .0083 | .0103 | .0107 | .0126 | .0151 | *.0068* | *13.19* |
| 4 | .0050 | .0058 | .0059 | .0071 | .0088 | *.0038* | *10.76* |
| High Turnover | .0040 | .0044 | .0044 | .0052 | .0066 | *.0026* | *8.26* |

Table 11
Fama-MacBeth Regression Results
Total Variance

| **Variable** | **Coefficient** | **FMB-SE** | ***t*-stat** | ***p*-value** |
|---|---|---|---|---|
| | Panel A: Bid-Ask Spreads | | | |
| Total Var | 3.4452 | .5723 | 6.02 | .000 |
| Turnover | -0.0015 | .0003 | -5.50 | .000 |
| $ln$(size) | -0.0076 | .0006 | -11.93 | .000 |
| Constant | 0.1145 | .0088 | 12.90 | .000 |
| Average-$\overline{R}^2$ | 50% | | | |
| Observations | 44,171 | | | |
| | Panel B: Amihud Illiquidity Measure | | | |
| Total Var | 288.87 | 34.41 | 8.39 | .000 |
| Turnover | -0.07 | 0.01 | -6.64 | .000 |
| $ln$(size) | -0.29 | 0.03 | -9.31 | .000 |
| Constant | 4.02 | 0.43 | 9.32 | .000 |
| Average-$\overline{R}^2$ | 14% | | | |
| Observations | 44,171 | | | |

Table 12
Fama-MacBeth Regression Results
Jump and Diffusive Components

| **Variable** | **Coefficient** | **FMB-SE** | ***t*-stat** | ***p*-value** |
|---|---|---|---|---|
| Panel A: Bid-Ask Spreads | | | | |
| Diffusive-var | 0.0225 | .6039 | 0.04 | .971 |
| Jump-var | 5.1547 | .6213 | 8.30 | .000 |
| Turnover | -0.0015 | .0003 | -5.44 | .000 |
| $ln$(size) | -0.0075 | .0006 | -11.55 | .000 |
| Constant | 0.1136 | .0090 | 12.57 | .000 |
| Average-$\overline{R}^2$ | 50% | | | |
| Observations | 44,171 | | | |
| Panel B: Amihud Illiquidity Measure | | | | |
| Diffusive-var | 98.12 | 25.51 | 3.85 | .004 |
| Jump-var | 387.74 | 50.75 | 7.64 | .000 |
| Turnover | -0.07 | 0.01 | -6.57 | .000 |
| $ln$(size) | -0.28 | 0.03 | -9.44 | .000 |
| Constant | 3.96 | 0.42 | 9.41 | .000 |
| Average-$\overline{R}^2$ | 14% | | | |
| Observations | 44,171 | | | |

Table 13
Fama-MacBeth Regression Results
Variance Components and Total Var
Dependent Variable: Annual Average Bid-Ask Spreads

| Variable | Coefficient | FMB-SE | $t$-stat | $p$-value |
|---|---|---|---|---|
| Panel A: Diffusive Var | | | | |
| Total-var | 5.1547 | .6213 | 8.30 | .000 |
| Diffusive-var | -5.1321 | .4420 | -11.61 | .000 |
| Turnover | -0.0015 | .0003 | -5.44 | .000 |
| $ln$(size) | -0.0075 | .0006 | -11.55 | .000 |
| Constant | 0.1136 | .0090 | 12.57 | .000 |
| Average-$\overline{R}^2$ | 50% | | | |
| Observations | 44,171 | | | |
| Panel B: Jump Var | | | | |
| Total-var | 0.0225 | .6039 | 0.04 | .971 |
| Jump-var | 5.1321 | .4420 | 11.61 | .000 |
| Turnover | -0.0015 | .0003 | -5.44 | .000 |
| $ln$(size) | -0.0075 | .0006 | -11.55 | .000 |
| Constant | 0.1136 | .0090 | 12.57 | .000 |
| Average-$\overline{R}^2$ | 50% | | | |
| Observations | 44,171 | | | |

Table 14
Fama-MacBeth Regression Results
Dependent Variable: Annual Average Bid-Ask Spreads
(for Weekly & Biweekly Returns)

| **Variable** | **Coefficient** | **FMB-SE** | ***t*-stat** | ***p*-value** |
|---|---|---|---|---|
| Panel A - Based on Weekly Returns | | | | |
| Diffusive-var | -0.8347 | .5245 | -1.59 | 0.146 |
| Jump-var | 2.1615 | .3607 | 5.99 | 0.000 |
| Turnover | -0.0013 | .0003 | -4.80 | 0.001 |
| $ln$(size) | -0.0080 | .0008 | -9.31 | 0.000 |
| Constant | 0.1217 | .0123 | 9.87 | 0.000 |
| Average-$\overline{R}^2$ | 49% | | | |
| Observations | 39,209 | | | |
| Panel B - Based on Biweekly Returns | | | | |
| Diffusive-var | -1.2302 | .3768 | -3.26 | .010 |
| Jump-var | 1.5803 | .3280 | 4.82 | .001 |
| Turnover | -0.0014 | .0004 | -3.87 | .004 |
| $ln$(size) | -0.0083 | .0010 | -7.92 | .000 |
| Constant | 0.1270 | .0154 | 8.25 | .000 |
| Average-$\overline{R}^2$ | 49% | | | |
| Observations | 23,616 | | | |

Table 15
Fama-MacBeth Regression Results
Dependent Variable: Amihud Illiquidity Measure
(for Weekly & Biweekly Returns)

| **Variable** | **Coefficient** | **FMB-SE** | ***t*-stat** | ***p*-value** |
|---|---|---|---|---|
| Panel A - Based on Weekly Returns | | | | |
| Diffusive-var | -3.44 | 30.428 | -0.11 | .91 |
| Jump-var | 31.05 | 14.468 | 2.15 | .06 |
| Turnover | -0.03 | .005 | -5.44 | .00 |
| Market-cap | -0.21 | .039 | -5.35 | .00 |
| Constant | 3.00 | .567 | 5.30 | .00 |
| Average-$\overline{R}^2$ | 13% | | | |
| Observations | 39,209 | | | |
| Panel B - Based on Biweekly Returns | | | | |
| Diffusive-var | -9.58 | 32.754 | -0.29 | .77 |
| Jump-var | 77.41 | 20.071 | 3.86 | .00 |
| Turnover | -0.03 | .005 | -7.09 | .00 |
| Market-cap | -0.27 | .022 | -12.26 | .00 |
| Constant | 3.92 | .321 | 12.19 | .00 |
| Average-$\overline{R}^2$ | 16% | | | |
| Observations | 23,616 | | | |

Table 16
Average Parameter Estimates
by Return Frequency

| **Frequency** | $\boldsymbol{\mu}$ | $\boldsymbol{\sigma}$ | $\boldsymbol{\lambda}$ | $\boldsymbol{\alpha}$ | $\gamma$ | **Obs.** |
|---|---|---|---|---|---|---|
| Daily | 0.143 | 0.295 | 49.317 | 0.014 | 0.059 | 55,558 |
| Weekly | 0.123 | 0.272 | 9.524 | 0.025 | 0.089 | 52,141 |
| Biweekly | 0.128 | 0.235 | 5.445 | 0.026 | 0.087 | 40,208 |

# Appendix: Model and Estimation Method

Following Merton (1976), let $S_t$ denote a stock price at time $t$ on a filtered probability space $(\Omega, F, (F_t), P)$, which is assumed to satisfy the following stochastic differential equation

$$\frac{dS_t}{S_t} = (\mu - \lambda \cdot E(Z-1))\, dt + \sigma dW_t + (Z-1)\, dN_t,$$

where $\mu$ and $\sigma^2$ denote the instantaneous mean and variance of the stock return in the absence of jumps, and $W_t$ is a Wiener process. Furthermore, $N_t$ is a Poisson process with intensity $\lambda > 0$, and $Z$ is the log-normal jump amplitude with $\ln Z \backsim N(\alpha; \gamma^2)$ such that

$$E(Z-1) = \exp(\alpha + \frac{\gamma^2}{2}) - 1.$$

We postulate that $W_t, N_t$ and $Z_t$ are mutually independent. The parameter vector $\theta$ is $\theta = (\mu, \sigma^2, \lambda, \alpha, \gamma^2)'$, where $\alpha$ and $\gamma^2$ represent the mean and variance of the jump size of stock returns.

Following Ait-Sahalia (2002), under these assumptions the transition density $f_{\Delta \ln S}$ of $\ln S_t$ can be expressed by

$$f_{\Delta \ln S}(x;\theta) = (1 - \lambda \cdot \Delta t) \cdot f_{\Delta \ln S|\Delta N_t=0}(x|\Delta N_t = 0;\theta) + \lambda \cdot \Delta t \cdot f_{\Delta \ln S|\Delta N_t=1}(x|\Delta N_t = 1;\theta),$$

where $f_{\Delta \ln S|\Delta N_t=0}$ and $f_{\Delta \ln S|\Delta N_t=1}$ represent the transition densities of $\ln S_t$ conditioning on $\Delta N_t = 0$ and $\Delta N_t = 1$ jumps between two sampling points, respectively, and $\Delta t > 0$ denotes the time distance between sampling points. Since

$$\begin{aligned} P(\Delta N_t &= 0) = 1 - \lambda \cdot \Delta t + o(\Delta t) \\ P(\Delta N_t &= 1) = \lambda \cdot \Delta t + o(\Delta t) \\ P(\Delta N_t &> 0) = o(\Delta t) \end{aligned}$$

additional jumps between two sampling points are neglected. Closed form expressions for the conditional densities are given by

$$f_{\Delta \ln S|\Delta N=k}(x|\Delta N_t = k;\theta) = \frac{1}{\sqrt{2 \cdot \pi \cdot v(k)}} \cdot \exp\left(-\frac{(x - m(k))^2}{2 \cdot v(k)}\right)$$

where

$$\begin{aligned} m(k) &= \left(\mu - \sigma^2/2 - \lambda \cdot E(Z-1)\right) \cdot \Delta t + k \cdot a \\ v(k) &= \sigma^2 \cdot \Delta t + k \cdot \gamma^2 \end{aligned}$$

with $k \in \{0,1\}$. Based on a sample of $n$ stock returns $\Delta \ln s_1, ..., \Delta \ln s_n$ the resulting likelihood estimate $\widehat{\theta}$ of $\theta$ is computed numerically as

$$\hat{\theta} = \arg\max_{\theta} \left( \sum_{i=1}^{n} \ln f_{\Delta \ln S}(\Delta \ln s_i; \theta) \right).$$