The Tried-Stone Scheme and a Million-Dollar Bet

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The Tried-Stone Scheme and a Million-Dollar Bet*

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1 Introduction

One's ability to identify and harness top-level experts becomes a crucial requirement in an environment of growing complexity. The gravity of this problem is even more significant when entire markets and economies are at stake. The 2007–2009 financial crisis offers some insights on how ill-motivated experts can produce poorly-designed, risky contracts that, in return, unleash a dramatic chain reaction throughout the global economy.[1] In this paper, we tackle the problem of experts testing in such a set-up, i.e., in the delegated portfolio-managers market.

For several decades, the portfolio-managers problem has been the focus of many empirical and theoretical studies.[2] A significant part of these studies derive impossibility results, showing that an investor cannot separate the skilled from the unskilled managers. The reason is clear: it is quite difficult to detect low-level managers, or even charlatans, in a noisy, risky market with little to no prior information. Nevertheless, we prove that no matter how difficult it may seem, the separation is still possible.

Before exploring further into our results, we wish to dwell on the paper by Foster and Young (2010). This paper accurately explains the portfolio-managers problem by giving one of the strongest impossibility result so far.[3] Foster and Young (2010) explore the profitable ability of low-level managers to mimic the investment strategies of high-level managers, in an undetectable manner, to a point where the investor's fund is lost entirely. The fact that personal collaterals are more problematic for high-level managers, as they can produce excess expected returns for every dollar held in escrow, plays a significant part in the impossibility result. That is, they show that penalties either deter all managers, or deter none.

Our model captures a similar situation. We consider a *single-period* investment where an investor, faced with managers of different abilities, can only observe their realized returns. In this framework we design incentives schemes such that *only the best* portfolio managers, in terms of expected payoffs, agree to participate in the single-period investment.

This goal is ambitious is several respects. First, experts testing typically involves some form of repetition. Namely, the consecutive sampling enables the investor to gain additional information about the managers' subjective capabilities. However, such learning cannot take place in a single-stage model as ours. Moreover, we do not settle for an ex-post detection, but aim to deter the suboptimal managers

[1] See, e.g., Hansen (2009); Fligstein and Goldstein (2010); and Simpson (2011). For a general survey on the wrong incentives that led to the financial crisis of 2007–2009, see Fligstein and Roehrkasse (2013).

[2] See, among many others, Sharpe (1981); Barry and Starks (1984); Starks (1987); Scharfstein and Stein (1990); Chevalier and Ellison (1997); Carpenter (2000); Lo (2001); Hodder and Jackwerth (2007); Goetzmann et al. (2007); Van Binsbergen et al. (2008); Dasgupta et al. (2011); Chassang (2013).

[3] A simple example to illustrate their result follows in Subsection 2.1.

from entering the market completely. Second, our model follows the strict assumptions of previous impossibility studies: it does not assume that the investor has superior monitoring capabilities, nor that managers have a liquidation boundary as in, e.g., He et al. (2015). Thus, we do not restrict low-level managers from using various mimicking strategies. Third, we assume that the investor cannot use biased schemes, i.e., she must treat all managers equally. Based on all of the above, we conclude that the ability to separate good investors from poor ones applies in general financial markets, where an uninformed investor can only observe the managers' one-shot realized returns.

Our work is motivated by one specific element that previous works did not take into account, and that is the ability the run a constant-sum competition between the managers. In previous studies, the endowments and returns of one manager do not balance off with the ones of the other managers. We, in contrast, consider schemes where all the managers' collaterals are pooled together, and later redistributed based on their performances.[4] In other words, we design a (partially) constant-sum game where the deposits of suboptimal managers are used to compensate superior managers for a-priori depositing funds. Along with the basic assets-under-management fees, our endowments-based trade-off incentivizes high-performance managers to participate in the investment, and the others to refrain.

The simplicity of our proposed scheme, entitled the *Tried-Stone scheme*,[5] is an important part of this paper, related to the policy implications. By requiring few conditions and little knowledge on the part of the investor, the designed scheme is implementable in small and large scales alike. That is, the optimal Tried-Stone scheme, described in Sections 3 and 4, is implementable in general markets and results in situations where only managers above a certain level agree to participate.[6]

Our analysis also includes an impossibility result. When the single-period potential-returns exceed 100%, we show that two managers of different abilities that interact with each other through risky binary options cannot be separated by any feasible contract. That is, we prove that in case one of the managers can more-than-double his funds, while the other losses everything, then no contract, based solely on their realized returns, can distinguish between the two.[7] This double-or-nothing condition is not a mere technical issue: it goes to the core of our solution, and our understanding of the screening problem. It stems from the inability to compensate a manager's potential loss due to the personal

[4]The redistribution of funds among financial managers was the focus of Lagziel and Lehrer (2016).

[5]The term 'Tried Stone' is taken from Isaiah (28:16): "Therefore thus said the Lord GOD, Behold, I lay in Zion for a foundation a stone, *a tried stone*, a precious corner stone, a sure foundation: he that believes shall not make haste." The stone symbolises a test, a benchmark, upon which one can get a truthful assessment.

[6]This result is motivated by the work of Bhattacharya and Pfleiderer (1985), and discussed in Section 1.1.

[7]The interaction between managers carries some resemblance to the more recent trend of studies concerning optimal contracting jointly with asset prices. See, e.g., Qiu (2009); Buffa et al. (2014); Malamud and Petrov (2014); and Garleanu and Pedersen (2015).

collateral, with another deposit of the same amount. Therefore, the same condition is also crucial for our positive results.

One could find an interesting (and rather entertaining) precedent to our Tried-Stone scheme in Warren Buffett's million-dollar bet with the money management firm, Protégé Partners.[8] The "Oracle of Omaha" made a 10-year bet, expiring on the last day of 2017, that the market performance of an index fund of his choice can beat the average performance of five hedge funds carefully chosen by Protégé. With slightly over a year ahead, Buffett is leading by more than 40 points! (65.67% versus 21.87%.) Though it is merely a friendly wager, the bet holds the same basic idea of running a competition between two funds, and granting the entire deposit of a suboptimal contender to the optimal one (and, in this case, his charity of choice). This example also shows that our solution is not limited to the portfolio-managers setting. In fact, every competition that includes an entering fee, ranging from best-photography contests to submitting academic papers to top journals, could employ our reward scheme to attract only high level competitors, and thereby set a high-level competition.

1.1 Related literature

Studies of the delegated portfolio-managers problem range from the vast economic literature[9] to the financial one. As both fields are well-studied,[10] we can only address several key papers that strongly relate to our work, starting with the work by Bhattacharya and Pfleiderer (1985).

Similarly to our work, Bhattacharya and Pfleiderer (1985) consider a one-period setting with no-learning involved. They showed that it is quite difficult, yet sometimes possible, to screen managers below a certain level. However, their results depend greatly on normally distributed payoffs (i.e., constant absolute risk aversion utilities) of both the investor and the managers, and on the common-knowledge of the managers' deterministic alternative-employment wages. Though their work could be extended beyond the CARA-utility conditions, some form of risk-aversion is needed, as well as additional restrictions over the distributions of the available assets. Foster and Young (2010) argue that such risk-aversion is somewhat limiting in the screening problem. Namely, an investor can use various schemes, where managers invest their own private funds, to deter low-level risk-averse managers. These schemes do not apply otherwise.

The normally distributed payoffs is also a basic requirement in the work of Admati and Pfleiderer (1997), which studies the design of a benchmark-adjusted schemes to infer the managers' skills from ex-post returns. Their analysis shows that the use of exogenous benchmarks in linear contracts is,

[8] Fortune, "Why Buffett's Million-Dollar Bet Against Hedge Funds Was a Slam Dunk" on May 11, 2016.

[9] The manipulability abilities of low-level agents were proven to exist in many economic papers, such as Lehrer (2001); Sandroni et al. (2003); Sandroni (2003); Shmaya (2008); and Olszewski and Sandroni (2008).

[10] See Stracca (2006) for a comprehensive survey of the field.

generally, suboptimal with respect to risk sharing, exerting effort, and choosing the optimal portfolio for the investor. Our solution for this benchmark problem is another important aspect of the Tried-Stone scheme. The Tried-Stone scheme generates a competition between managers such that each manager is compared to all the other managers. In other words, our benchmark is endogenously derived from the induced-competition equilibrium, thus enabling optimality.

Bhattacharya and Pfleiderer (1985) and Admati and Pfleiderer (1997) also share a key feature regarding the managers' available assets. Specifically, they assume that managers can invest either in a risk free asset, or in a unique asset of normally-distributed returns. In such a set-up, the difference between managers depends solely on their private signals, while mimicking strategies are not considered. We, on the other hand, do not limit the available assets in such a way, and allow the use of different mimicking strategies.

In a more recent work Dasgupta and Prat (2006) show that managers' career concerns might limit the ability to screen out uninformed managers. They use a two-stage model, where all sides are risk-neutral, to prove that low-level managers might invest randomly, since a no-trade policy signals for a lack of information. Their analysis leads to a situation where an average return (with respect to the market) is as bad as a negative return, since both outcomes signal for poor performance. This is fundamentally different from our results. Our goal is to screen out all sub-optimal managers. Therefore, by definition, all the post-screening participating managers generate similar optimal expected returns. More importantly, Dasgupta and Prat (2006) assume that the mangers' payoff are a linear function of the returns. Though this assumption is feasible when returns are positive, it is unclear how to generate negative payments when returns are negative. This understanding is one of the main contributions of Foster and Young (2010).

Foster and Young (2010) present one of the broadest impossibility result so far, arguing that low-level managers can always use options-trading strategies to mimic expert managers. They also consider risk-neutral parties and derive specific strategies for unskilled managers to use in the presence of skilled ones. In many respects, our positive result responds to their impossibility result with a single condition that returns are bounded by the double-or-nothing range. That is, we assume that managers cannot produce returns that exceeds 100% in a single time period. Though this assumption could be weakened and is broadly discussed in Section 2.2, we stress that the length of a single time period is not limited in any way. Thus, for practical purposes, this restriction is quite weak.

The bounded assets are also vital for the positive result of He et al. (2015). Their recent work assumes that the investor can prevent the managers from losing the entire fund, thus basically ignoring mimicking strategies á la Foster and Young (2010). This restriction is based on an investor with superior monitoring and regulating capabilities, that are redundant in our set-up.

Note that we do not remain naive to the possible implications of our proposed contracts. Personal deposits fix an opportunity cost with potentially significant repercussions. Therefore, the results throughout this work should not be considered as a call-for-action. The accumulated practical and theoretical experience teaches us that many suggested reforms, such as postponing bonuses and instituting clawback provisions, do not effectively limit the gaming ability of financial managers. Therefore, our theoretical analysis is aimed to what could be done, rather than what should be done.

1.2 Structure of the paper

The paper is organized as follows. In Section 2 we described a two-managers model. We discuss the main assumptions, along with a simple example to illustrate the complexity of the problem. Section 3 presents the Tried-Stone scheme and shows how the investor could fix the endowments such that only the superior manager is willing to participate in the investment (Theorem 1 and Corollary 1). In Section 4 we extend the model to numerous portfolio managers and generalize previous results (Lemma 1, Theorem 2, and Theorem 3). Our impossibility result (Lemma 2), with several concluding remarks and further comments are given in Section 5.

2 The model - a two-manager framework

Consider a risk-neutral investor with an initial endowment of $2w$ that she wishes to invest. Her main goal is to maximize her expected earnings using the abilities of two risk-neutral fund managers. The first manager, denoted Manager 1, has superior trading abilities than the second manager, denoted Manager 2. Specifically, every Manager i can invest in any portfolio from a set A_i, and has an *optimal* portfolio Y_i (a portfolio that maximizes expected returns). Assume that the optimal portfolio of Manager 1 produces excess expected returns relative to that produced by the optimal portfolio of Manager 2. That is, $\mathbf{E}[Y_1] > \mathbf{E}[Y_2]$.

The investor introduces a *scheme* f meant to incentivize managers to invest in her best interest. The scheme f induces a competition between the managers, referred to as an *investment game* G_f. In addition, the investor may require an endowment $C \geqslant 0$ as an entry fee for participating in G_f. Given the investment game and entry fee, the managers decide whether to participate in the investment. Ideally, the investor produces an *optimal* scheme that lures only Manager 1 to invest her funds and only in Y_1. In other words, a scheme is *optimal* if it induces, in every equilibrium, the maximal expected gross return of $2w\mathbf{E}[Y_1]$ for the investor.

Formally, the problem begins when the investor publicly commits to a scheme f. Let $f : \mathbb{R}^2_+ \to \mathbb{R}^2_+$ be the investor's proposed scheme where $f_i(r_1, r_2)$ is the payoff of an active Manager i based on the realized-returns vector $(r_1, r_2) \in \mathbb{R}^2_+$. The scheme f induces an investment game, denoted G_f, in which every Manager i decides first whether to participate or decline, considering the *opportunity cost*[11] $C\mathbf{E}[Y_i]$. A manager who is willing to participate in G_f is called an *active* manager, otherwise we refer to him as *inactive*.

In case both managers decide to be active, they get an equal share, w, and individually decide on how to invest it. Let the profile $\sigma = (\sigma_1, \sigma_2)$ be an equilibrium in the investment game G_f, where σ_i is the portfolio of Manager i. The expected value of the investor's portfolio is then $w\mathbf{E}[\sigma_1 + \sigma_2]$, and the expected profit of Manager i is $\pi_i = \mathbf{E}[f_i(\sigma_1, \sigma_2)] - C\mathbf{E}[Y_i]$.

However, if only a single manager is active, say Manager i, then he receives the entire amount of $2w$ to invest in a portfolio that he chooses. In this case, no endowment is needed and Manager i's payoff is fixed to be a share $\lambda \in (0, 1)$ of the overall funds after the investment is realized. Namely, if Manager i is the only active manager, then his optimal action is the optimal portfolio Y_i and his expected profit is $\pi_i = \lambda\mathbf{E}[2wY_i]$, whereas the investor's expected net profit is $(1 - \lambda)\mathbf{E}[2wY_i]$.

We require that every scheme f satisfy two conditions: *symmetry and feasibility*. The symmetry of f is considered in the normal sense of $f_1(r_1, r_2) = f_2(r_2, r_1)$ for every realized rates $r_1, r_2 \in \mathbb{R}_+$. It follows from the investor's inability to, a priori, distinguish between the two managers. The feasibility condition states that a scheme f must be non-negative, implying that any penalty for bad performance is limited to the managers' personal uninvested funds (endowment C), previously held in escrow in some risk-free asset. This requirement is presumably the toughest obstacle in designing an optimal scheme, since it significantly limits the investor's ability to impose penalties on under-preforming managers. The funds held in escrow generate a significant loss to high-performance managers, relative to low-performance managers, thus resulting in a difficulty to effectively deter low-level managers without deterring the better ones. Using invested funds to penalize managers is not considered feasible, since such funds are possibly lost due to bad investments.

2.1 An illustrative example

Before discussing the main assumptions and results, we consider a concrete example. Assume that Manager 1's optimal portfolio produces a return of 5% with probability 1, whereas Manager 2's optimal portfolio produces the same rate with probability $\frac{1}{1.05}$, and otherwise loses the entire amount. The investor decides on a scheme f where an active manager needs to deposit an amount $C > 0$ that will be returned to him by the end of the period, unless he generate a negative return (i.e., below 1). In addition, every active manager will get a share λ out of the funds he manages by the end of the

[11]The entry fee C is given in nominal values, whereas the opportunity cost $C\mathbf{E}[Y_i]$ is given in real values, adjusted to Manager i's investment capabilities.

investment, including the returns. Specifically, when Manager j invests in σ_j, the expected profit of an active Manager i induced by the scheme f is

$$\begin{aligned}\pi_i &= E[f_i(\sigma_1,\sigma_2)] - C\mathbf{E}[Y_i] \\ &= \lambda w\mathbf{E}[\sigma_i] + C\Pr(\sigma_i \geqslant 1) - C\mathbf{E}[Y_i].\end{aligned}$$

The first term is the manager's fees out of the managed funds; the second term follows from the potential loss of endowment C; and the last term is the manager's opportunity cost for not personally investing the amount C.

The individual decision of each manger to participate depends on C. Specifically, the expected profit of the two managers in equilibrium[12] (Y_1, Y_2) are

$$\pi_1 = \lambda w 1.05 - 0 - C[1.05 - 1] \;\; ; \;\; \pi_2 = \lambda w - C\left[1 - \tfrac{1}{1.05}\right] - 0.$$

On the one hand, for every $C \geqslant \lambda w\frac{1.05}{0.05}$, the profit of both managers is non-positive. On the other hand, if $C < \lambda w\frac{1.05}{0.05}$, then both profit are positive. Therefore, there exists no deposit that enables the investor to distinguish between the two managers. This result is not surprising since the proposed scheme lacks the key element of our Tried-Stone solution, which is a competition between managers. In this case, the individual profit of one manager is independent of the other's realized return, generating no competition whatsoever.

2.2 Main assumptions

There are three main assumptions concerning our portfolio-managers model that we wish to clarify: (i) the different trading capabilities, (ii) the distributions of available portfolios, and (iii) risk neutrality.

The excess abilities of Manager 1 over those of Manager 2 may stem from several sources, such as a superior understanding of the market, lower transaction costs, a technological trading hedge, and so on. Specifically, the difference could also follow from Manager 2's willingness to exercise a mimicking strategy. As in many studies of this field, one can assume that Manager 2 mimics Manager 1 to generate the same realized values, but with inferior probabilities. These types of mimicking strategies play a crucial rule in previous impossibility results, and are also considered in the current work. However, to maintain generality and avoid the complexity of different information structures and signalling systems (as in, e.g., Bhattacharya and Pfleiderer (1985) and Berk and Green (2004)), we simply assume that Manager 1 has the ability to invest in an optimal portfolio Y_1, whereas Manager 2 cannot match the expected return of Y_1.

[12] To simplify the analysis, we assume that all other assets in A_i fall short of Y_i.

The second assumption relates to the distributions of the available portfolios. We assume that both managers can produce returns ranging from losing the entire fund to doubling it in a single stage. That is, $\Pr(0 \leqslant Y \leqslant 2) = 1$ for every portfolio $Y \in A_1 \cup A_2$. The reason for these bounds lays in the feasibility condition and the rebalanced endowments. Namely, a potential loss of more than 100% due to an endowment of C, cannot be matched by another endowment of the same amount. From a practical perspective, this restriction is quite weak, as one can always shorten the basic time frame such that the probability of more than doubling the fund is sufficiently close to zero.[13] The restriction also teaches us that the rebalancing should be limited to a specific time frame such that the double-or-nothing range is met.

We could weaken this assumption to some extent, since the investor can use her private funds to offset the managers' potential losses. Otherwise, we make no additional assumptions over the distributions of the portfolios. Our results are robust in the sense that we do not condition neither on the types of distributions, such as a geometric Brownian motion, nor on possible correlations or dependence. For the sake of simplicity, we fix the risk-free return rate to zero. However, one could assume that the rates generated by the portfolios in A_i are relative to some non-zero risk-free rate.

The last assumption we wish to clarify is the managers' risk neutrality. When managers are risk averse, the investor can impose some simple restrictions to eliminate low-level managers. For example, the investor may require that active managers invest their own private funds in the same portfolio, and by doing so he can eliminate managers that do not generate excess returns relative to a risk-free asset. For this reason, the assumption that managers are risk neutral complicates the problem compared to risk-averse managers. For the same reason we also assume that the expected optimal returns of both managers is non-negative (i.e., $\mathbf{E}[Y_i] \geqslant 1$ for every $i = 1, 2$). Omitting these two assumptions may involve additional technical difficulties, yet will not significantly affect our results.

3 Optimal schemes

The design of an optimal scheme must be a based on a combinations of penalties and positive rewards, carrots and sticks. Any scheme based solely on penalties will not attract Manager 1, just as a strictly positive scheme will not deter Manager 2. Thus, our optimal scheme, referred to as *the Tried-Stone* scheme, is a two-stage plan: it begins when every active manager secures a deposit $C > 0$ beforehand, an ends when payoffs are distributed according to realized returns. Though the value of C is later determined, we should point out that the use of this deposit is one of the main new elements of our proposed solution. Our optimal scheme uses the deposit of Manager i to compensate the other active

[13]For example, eight years into the Buffett-Protégé wager and neither party has yet exceeded this threshold.

manager for cases where Manager i under preforms.

Assume that Manager 1 and Manager 2 produce the realized returns of r_1 and r_2, respectively. The Tried-Stone scheme f states that

$$f_1(r_1, r_2) = \lambda w \left[1 + \frac{r_1 - r_2}{2}\right] + C\left[1 + \frac{r_1 - r_2}{2}\right], \quad (1)$$

$$f_2(r_1, r_2) = \lambda w \left[1 + \frac{r_2 - r_1}{2}\right] + C\left[1 + \frac{r_2 - r_1}{2}\right]. \quad (2)$$

In words, the fees and deposits are divided such that every active manager starts with the same basic amount of $\lambda w + C$. That is, every manger i gets a fee of λw and gets his own deposit C. Once the returns (r_1, r_2) are observed, Manager i's gains and losses are proportional to the difference between the realized returns. Note that the realized returns (r_1, r_2) are public, while the opportunity cost $C\mathbf{E}[Y_i]$ is Manager i's (stochastic) private information, therefore f is well-defined. In addition, f is symmetric, and feasible as the investor only divides the overall available fees and deposits.[14]

The implementation of the Tried-Stone scheme is simple. First, the investor secures the amount of $2\lambda w$ that will eventually be divided as fees. Next, every manager, who is willing to participate in the investment, secures an amount of C. These endowments could be handled by a third party and invested in a risk-free asset until investments are realized. After returns are observed, the fees and deposits are allocated according to the normalized differences of realized returns.

An important property of the Tried-Stone scheme is the ability to divide a fixed predetermined sum of $2w\lambda + 2C$ between the managers based on their results. This possibility is similar to the General Reward Scheme presented in Lagziel and Lehrer (2016), where a problem of different incentives with homogeneous managers is considered. Using this ability, the investor secures an endowment of C from each active manager, where the potential loss of Manager 1, due to the opportunity cost, is matched to the loss of Manager 2, by the expected reimbursement. The reimbursement bypasses the main problem presented in previously mentioned studies. The scheme's linearity implies that the unique dominant-strategy equilibrium of the investment game induced by the Tried-Stone scheme is $\sigma = (Y_1, Y_2)$. Thus, the profits are given by

$$\pi_1 = \lambda w \left[1 + \frac{Y_1 - Y_2}{2}\right] + C\left[1 - \frac{Y_1 + Y_2}{2}\right],$$

$$\pi_2 = \lambda w \left[1 + \frac{Y_2 - Y_1}{2}\right] + C\left[1 - \frac{Y_2 + Y_1}{2}\right],$$

and the second terms of both profits are identical.

[14]The feasibility requirement follows from the condition that returns are bounded by the double-or-nothing range. In case a higher bound is needed, one can take a larger denominator to normalize $\pm(r_1 - r_2)$.

Denote $e_+ = \mathbf{E}[Y_1+Y_2]$ and $e_- = \mathbf{E}[Y_1-Y_2]$, where $e_+ > 2$ and $e_- > 0$ follow from the assumptions given in Subsection 2.2. The following theorem specifics the values of C such that The Tried-Stone scheme is optimal.

Theorem 1. *If* $\frac{C}{\lambda w} \in \left(\frac{2-e_-}{e_+-2}, \frac{2+e_-}{e_+-2}\right)$, *then the Tried-Stone scheme is optimal.*

Note that the deposit of every active manager is proportional to his basic fee of size λw. This relation is necessary, since an optimal scheme must ensure that the expected profit of a low-level manager is negative, while maintaining a positive expected profit for high-level managers.

Proof. When both managers are active, the linearity of f_i w.r.t. the return of Manager i implies that the dominant-strategy equilibrium is $\sigma = (Y_1, Y_2)$. Hence, the expected profits of the participating managers are

$$\begin{aligned}
\pi_2 &= \mathbf{E}[f_2(\sigma)] - C\mathbf{E}[Y_2] \\
&= \lambda w \left[1 - \frac{e_-}{2}\right] + C\left[1 - \frac{e_+}{2}\right] \\
&< \lambda w \left[\frac{2-e_-}{2}\right] + \lambda w \left(\frac{2-e_-}{e_+ - 2}\right)\left[1 - \frac{e_+}{2}\right] \\
&= 0 \\
&= \lambda w \left[\frac{2+e_-}{2}\right] + \lambda w \left(\frac{2+e_+}{e_+ - 2}\right)\left[1 - \frac{e_+}{2}\right] \\
&< \lambda w \left[1 + \frac{e_-}{2}\right] + C\left[1 - \frac{e_+}{2}\right] \\
&= \mathbf{E}[f_1(\sigma)] - C\mathbf{E}[Y_1] \\
&= \pi_1,
\end{aligned}$$

as needed. Thus, Manager 2 cannot gain from participating in the fund along with Manager 1, who becomes the only active portfolio manager. In this case, no endowment is required (as no penalties are involved) and Manager 1 invests the entire amount of $2w$ in Y_1 in order to maximize his expected λ-share of the portfolio. ■

Before extending Theorem 1, let us reconsider the same concrete environment previously described in Section 2. Fix the fee λ to 0.5% and assume that the optimal portfolio of Manager 1 produces 5% w.p. 1, whereas Manager 2's optimal portfolio produces the same rate w.p. $\frac{1}{1.05}$, and -100% otherwise. That is, $Y_1 = 1.05$ and

$$Y_2 = \begin{cases} 1.05, & \text{w.p. } \frac{1}{1.05}, \\ 0, & \text{w.p. } \frac{0.05}{1.05}. \end{cases}$$

Using the Tried-Stone scheme along with the condition of Theorem 1, take $\frac{C}{\lambda w} = 40$, or equivalently $C = 0.2w$. Hence, each active manager needs to deposit 20¢ for every dollar bestowed in his hands to invest. When doing so, Theorem 1 shows that Manager 2 will decline, while Manager 1 becomes the only active portfolio manager.

A priori, a 20¢ on-the-dollar deposit may seem excessive compared, e.g., to the Basel III regulatory framework that requires less than roughly 12% in high-quality liquid assets.[15] Nevertheless, this requirement is based on the risky nature of the portfolio-management market, and it is also proportional to the managers' share of the profits. Namely, in case an active Manager i produces a 5% profit, then the ratio between the 0.5% manager's fee (of the basic fund) and the total profit is 0.1, which means that the manager gets 10% of the profits, while his 20%-deposit is kept in a risk-free asset. Such a deposit might even be aligned with the manager's personal investment strategy, thus resulting with no potential loss.

These deposits are not the only univested funds, according to the Tried-Stone scheme. The investor also keeps a share λ of her funds in a risk-free asset, to be distributed as fees after returns are observed. One could solve this inefficiency by updating the scheme such that it will consist of the basic share-the-profits fees. Specifically, consider an updated scheme where Eqs. (1) and (2) are altered such that

$$\begin{aligned} f_1(r_1, r_2) &= \lambda w r_1 + C\left[1 + \frac{r_1 - r_2}{2}\right], \\ f_2(r_1, r_2) &= \lambda w r_2 + C\left[1 + \frac{r_2 - r_1}{2}\right]. \end{aligned}$$

In simple terms, each manager gets a share λ of his fund's realized value, with the same offset of deposits as in the original Tried-Stone scheme. The use of the common assets-under-management fees is important, since it enables to transform the currently-used contracts into optimal scheme just by introducing the endowments condition.

Note that the unique equilibrium, given that both managers are active, is still $\sigma = (Y_1, Y_2)$. This outcome holds throughout the current work. All the schemes we produce are linear w.r.t. the managers' returns, therefore the *dominant strategy* of every active manager is to invest in an optimal portfolio. One can even generalize this result to non risk-neutral cases by replacing r_i with $U(r_i)$ for some utility function U, as in Lagziel and Lehrer (2016).

The following corollary shows that one can fix C such that Manager 2 cannot gain from participating in the investment. The proof is a straightforward computation following the same arguments as the proof of Theorem 1, and therefore omitted.

Corollary 1. *If* $\frac{C}{\lambda w} \in \left(\frac{2\boldsymbol{E}[Y_2]}{e_+ - 2}, \frac{2\boldsymbol{E}[Y_1]}{e_+ - 2}\right)$*, then the updated Tried-Stone scheme is optimal.*

[15] See Atkinson and Blundell-Wignall (2010); Slovik and Cournède (2011); Financial Times, "Europes banks face tougher demands" on July 15, 2012; Wall Street Journal, "Basel III Is Simpler and Stronger" on October 14, 2012.

One problem arising from Theorem 1 and Corollary 1 is the difficulty to accurately determine C with no information on the managers' available portfolios. However, the implementation of the Tried-Stone scheme (either the original, or the updated version) requires only the average of optimal expected returns, since one can fix $C = \frac{2\lambda w}{e_+ - 2}$ or $C = \frac{\lambda w e_+}{e_+ - 2}$, depending on the chosen scheme. Therefore, the investor need not be aware of all the available assets in the market. In addition, both results hold for a range of values, hence some error in C is acceptable. Nevertheless, we believe that the exogenous dependency calls for further research of the proposed schemes and model, such that C becomes an indigenous element.

Remark 1. *A possible extension of our model to generate an indigenous entry fee C is to use a* Dutch auction*. Starting with a sufficiently high value, the fee is lowered until one manager agrees to pay the required price. Next, the entry fee is fixed accordingly and both managers are offered to become active. Though Manager* 2 *can overbid to eliminate Manager* 1 *from the competition, there exists an equilibrium where an always-active Manager* 1 *bids within the relevant range of* Theorem 1*, or* Corollary 1*. If indeed Manager* 1 *chooses the always-active strategy, Manager* 2 *cannot profit from overbidding and will never become active, maintaining optimality. However, the always-active strategy is also relevant for Manager* 2*. Therefore, as previously stated, a complete examination of this problem is left for future research.*

Another possible extension relates to the managers' ability to independently fix their subjective fees, i.e., their share of the managed funds. In an actual market, the managers can privately and freely choose their fees based on their personal abilities and the investors' preferences. On the one hand, low-level managers might reduce their fees to balance their relatively suboptimal turnouts. On the other hand, any reduction might also signal for poor performance, resulting in a low ability to raise public funds. Ultimately, an investor could (and probably, would) condition on these privately-determined fees when designing the competition. In Section 5 we present one way of dealing with individual fees.

This extension also enables managers to adjust their deposits. According to Theorem 1 and Corollary 1, the collaterals depend linearly on the share λ of the managers, such that a decrease in λ reduces the needed endowment. Our model lays the groundwork for such analysis, by enabling the investors to strategically manipulate fund flows between managers, thus opening the possibility for a combined fee- and return-based competition. This understanding becomes even more significant in the following section where a market with more than two managers is considered.

4 A managers market

Studies of the portfolio-manager problem are not easy to apply. First, many results in this field are impossibility results stating that managers cannot be separated based on different trading capabilities. Second, positive results usually involve complicated contracts that cannot be applied easily by either retail clients, or even by a regulatory agency in a large scale environment with numerous investors and managers. Our model and specifically, the symmetry and feasibility conditions, along with the Tried-Stone scheme solution are of a different nature. Our solution is designed such that every investor, from small private ones to large corporations, can apply it. In this section, we show how our Tried-Stone scheme could be used as a policy tool.

Though the theoretical results are unequivocal, we emphasize that our suggested schemes should be considered with caution to the full extent. The use of personal transferable collaterals is unprecedented in the portfolio-managers market. Therefore, the practical implications of such regulatory restrictions remain relatively vague. Nonetheless, previously-stated impossibility results along with our model and proposed schemes suggest that transferable collaterals are the only way to deal with sophisticated strategies in a stochastic ill-monitored environment. Therefore, the following results primarily facilitate the screening of sub-optimal managers in a wide range of scenarios, rather than call for such implementations.

Before exploring further into this issue, we wish to discuss the concept of a regulator from a theoretical perspective. Both in real life and in the current research, a regulator is, in fact, a game designer. The regulator takes a known game or a known mechanism and considers ways of improving it for her or others' benefit. The regulator can either make small adjustments or she can change the mechanism completely. In our set-up, the regulator uses the managers' deposits in order to screen out low-level managers,[16] while allowing other managers to operate freely. By doing so, she designs a competition, a game, where the managers act as players, and their behaviour is affected by the rules that the regulator dictates.

4.1 The Regulatory Tried-Stone scheme

Formally, fix $k \geqslant 2$ portfolio managers and recall that the optimal portfolio of Manager i is Y_i where $\mathbf{E}[Y_i] \geqslant 1$. Denote $v_i = \mathbf{E}[Y_i]$ and assume, as we previously did, that the managers' indices are aligned according to their trading capabilities, such that a lower index implies a higher optimal expected return (i.e., $v_i > v_j$ for every $i < j$). In this subsection we assume that the yields of all portfolios are bounded

[16]In the current section, the reader can refer to the managers as investment firms or large financial institutions, rather individuals.

by a $\pm 100R\%$ return rate, where $0 < R \leqslant 0.5$. That is, $\Pr(1 - R \leqslant Y \leqslant 1 + R) = 1$ for every portfolio Y. Though this assumption is stricter than the assumption given in the previous section, we still do not limit the time frame by which the managers are assessed. Therefore, one can take a sufficiently-short time frame such that this assumption holds for practical purposes.[17] In any case, a broad discussion over the choice of R and its importance is given at the end of the current section.

The multi-managers market evolves as follows. Every manager, who is willing to actively participate in the market, gets an initial amount of w to invest.[18] We assume that all managers share the same fee $\lambda \in (0, 1)$ taken out of their managed-portfolios' realized value. The regulator, as a market designer, determines the endowment $C > 0$ that each manager needs to deposit before entering the market. In case only a single manager chooses to be active, we use the assumptions from Section 2, where no endowment is needed as the manager gets a share of the portfolio as fees.

A scheme designed for k managers is a function defined for every set of active managers with a vector of returns r. That is, for every set of active managers of size $2 \leqslant l \leqslant k$, the vector $r \in \mathbb{R}^l_+$ states their returns, and $f_i(r)$, where $f : \mathbb{R}^l_+ \to \mathbb{R}^l$, defines the payoff of an active Manager i.

Define the *Regulatory Tried-Stone scheme* (RTS) for every active Manager i with a realized return r_i by

$$f_i(r) = \lambda w r_i + C\left[1 + \frac{r_i - \tilde{r}_{-i}}{2R}\right],$$

where $\tilde{r}_{-i}$ is the average realized return of all active managers excluding Manager i. In words, every active Manager i, who produces a realized return of r_i, gets a fee of $\lambda w r_i$ from the investor, along with a compensation $C\left[1 + \frac{r_i - \tilde{r}_{-i}}{2R}\right]$ based on Manager i's return, relative to the average of the market (i.e., the other managers). This offset is managed by the regulator using the initial endowments of all active managers. As before, the RTS is symmetric and feasible by the normalization of the term $\frac{r_i - \tilde{r}_{-i}}{2R} \in [-1, 1]$.

Observation 1. *The payoff of an active Manager i is linearly increasing in the realized return r_i. Thus, similarly to the original* Tried-Stone scheme*, the dominant strategy of every active Manager i is to invest in the optimal portfolio Y_i. Specifically, assuming that the realization of Y_i is r_i, then the profit of an active Manager i is*

$$\pi_i = f_i(r) - Cr_i = \lambda w r_i + C\left[1 + \frac{r_i(1 - 2R) - \tilde{r}_{-i}}{2R}\right].$$

[17]This assumption could be weakened as well, at a cost of limiting the results of this section to specific cases.

[18]To simplify the computation and without loss of generality, we assume that w is uniform among active managers. Since all payoffs are linear in w, one could change this assumption by fixing w according to the number of active managers, as in the 2-manager model.

Another important observation regarding the RTS is the relation between the decision to actively participate in the investment and the managers' optimal expected returns. Specifically, if C is fixed such that a low-level manager gains from entering the market, the same decision holds for any high-level manager as well. This observation is proved in the following lemma.

Lemma 1. *In every equilibrium induced by the* RTS *with more than one active Manager j, every superior Manager $i < j$ is also active.*

The more-than-one condition of Lemma 1 prevents an absurdly high C from being fixed, such that no manager can profit from entering the market along with another manager. Otherwise, the only equilibria include a single (arbitrary) active manager with no endowment and no competition. Alternatively, the more-than-one condition could be replaced by an upper bound on C such that $C < \lambda w \alpha_1$.

Proof. Assume, by contradiction, there exists an equilibrium σ where Manager j participates, but Manager $i < j$ does not. Without loss of generality, assume that Manager j is the lowest-level active manager (w.r.t. expected optimal returns). The RTS is linearly increasing in an active manager's return and linearly decreasing in the other active-managers' average return. Thus, if Manager i decides to participate, his expected profit will be higher then Manager j's profit for two reasons. First, Manager i produces excess expected return relative to Manager j as $v_i > v_j$. Second, Manager i also gains from the reduced benchmark, which consists of the returns of other active managers (including Manager j), compared to Manager j's benchmark in the assumed equilibrium σ (that does not include Manager j). Since Manager j's profit is positive given σ, Manager i must enter the market as well, contradicting the equilibrium assumption. ■

The monotonicity effect presented in Lemma 1 has a complementary aspect: high-level managers can completely drive out low-level managers from the market. Namely, when high-level mangers become active, they increase the market's average, thus increasing the benchmark of all other active managers. The increase leads to higher losses for below-average managers, that could result in an elimination from the market. In some respects, the RTS is designed to create an inverse effect to *the lemon market* problem of Akerlof (1970). Akerlof's lemon market shows how traders of low-value cars use information asymmetry to decrease the expected value of a traded car to a point where high-value cars are eliminated from the market, resulting in a no-trade theorem. In contrast, we prove (in Theorem 2) that a regulator can use the RTS's inverse effect to fix C such that, *in every equilibrium*, every manager below a certain level remains inactive.

4.2 Relatively superior managers

Before we present the main result of this section, a few notations are needed. For every Manager i, let $S_i = \{1, \ldots, i\}$ be the set of managers that are at least as good as Manager i in terms of optimal expected returns. For every $i > 1$, denote $\tilde{v}_{-i} = \frac{1}{i-1}\sum_{j=1}^{i-1} v_j$ to be the average expected return of managers superior to Manager i and define

$$\alpha_i = \frac{v_i}{\frac{1}{2R}[\tilde{v}_{-i} - v_i(1-2R) - 2R]}$$

to be the ratio between the expected return of Manager i and the expected loss for depositing C (including the induced compensation) in case only higher-level managers are active. Since $\tilde{v}_{-1}$ is not defined, fix $\alpha_1 = \frac{v_1}{v_1 - 1}$. These α_i-ratios play a crucial part in the ability to screen out inferior traders.

We say that Manager i is *relatively superior* (RS) if $\alpha_i > \alpha_j$ for every $j > i$. That is, Manager i is RS if its expected optimal return, relative to the potential loss, due to superior active managers, is higher than the same ratio of every inferior Manager j, where $j > i$. A straightforward examination shows that the RS condition is equivalent to the monotonicity of $v_i/(\tilde{v}_{-i} - 2R)$ w.r.t. i. Specifically, Manager i is RS if

$$\frac{v_i}{\tilde{v}_{-i} - 2R} > \frac{v_j}{\tilde{v}_{-j} - 2R}, \tag{3}$$

for every $j > i$.

One can verify that the condition given by Ineq.(3) becomes weaker as R decrease. In other words, a manager remains RS when limiting the support to $R' < R$ instead of R. In addition, Ineq. (3) is sensitive to changes in R. For example, taking the maximal $R = 0.5$, meaning that the portfolios realizations range between -50% and $+50\%$, implies that the condition does not hold whenever the expected optimal returns change linearly. That is, assuming that $v_i = a - (i-1)b$ for some $a > 1$ and $b > 0$, there exists no relatively-superior manager. However, if $R = 0.25$ and given the same linearity, then every manager is RS. In fact, if $R = 0.25$ and the expected returns are decreasing according to some concave function, then every manager is RS (see Lemma 3 in the appendix).[19]

A sufficient condition leading a RS Manager i is

$$ia_i \geqslant \frac{2}{i-1}\sum_{j=1}^{i-1} ja_j.$$

where $a_j = v_j - v_{j+1}$. Every term ja_j could be viewed as the marginal contribution of Manager j relative to the adjacent-level Manager $j+1$, and the inequality suggests that Manager i's marginal contribution is at least twice as high as the average marginal contribution of all superior managers.

[19] One could even extend the last statement beyond concave functions. See the comment following Lemma 3.

The necessity of the RS condition follows from the ability of low-level managers to enter the market and decrease the average expected returns, thus reducing the penalty for poor performance. Once the penalties are reduced, the endowment C must be sufficiently high to deter low-level managers. However, if low-level managers choose not to participate, then the average expected return remains high, and combined with a high endowment, might also deter high-level managers from actively investing. Hence, the expected optimal return of inactive managers must be bounded away from the lowest-active manager's return, such that the regulator has a sufficient margin to keep the endowment C as low as possible, while deterring low-level managers.

From a technical preceptive, the decision of a low-level manager to enter the market causes a discontinuity in $\tilde{r}_{-i}$, and the margin produced by the relatively-superior condition enables the regulator to deal with such discontinuity. The following theorem shows that the relative superiority is a necessary and sufficient condition to screen out managers below a certain level.

Theorem 2. *Given the* RTS*, there exists $C < \lambda w \alpha_i$ such that all active managers have an expected return rate of at least v_i if and only if Manager i is relatively-superior.*

The proof is given in the Appendix. We wish to clarify though that the result of Theorem 2 holds in every equilibrium. To be specific, the existence of a RS Manager i ensures that one can fix C, so that there exists no equilibrium with a below-v_i expected-return active manager.

While Theorem 2 states that managers $i+1, \ldots, k$ are inactive, it does not specify conditions for the existence of an equilibrium with an active set S_i of managers. For that purpose, we introduce the following theorem, that exhibits a necessary and sufficient condition for such an equilibrium.

Theorem 3. *Given the* RTS*, there exists an equilibrium where the set of active managers is S_i if and only if Manager i is* RS *w.r.t. Manager $i+1$ (i.e., if $\alpha_i > \alpha_{i+1}$).*

Note that Theorem 3 requires Manager i to be RS only compared to Manager $i+1$, whereas Theorem 2 requires relative superiority over every manger j, where $j > i$. In addition, in case the number of managers is very large and the influence of every manager over the market is negligible, the condition of Theorem 3 becomes redundant. Another scenario that may lead to the same redundancy is when managers are allowed to enter the market with different volumes of trade. However, such extensions go beyond our analysis and requires more complicated schemes, and therefore postponed to future research.

In Theorem 2 and 3 we did not specify the possible range for the nominal entry fee C. These values could be derived directly from the expected payoff of the lowest-level active manager. However and more importantly, the requirement to fix C might even be redundant in the more-than-one manager set-up. Namely, when facing numerous managers, there is a simple method to transform C into an endogenous variable of the problem. The following subsection explains this solution.

4.3 Endogenizing C

The transition from two managers into numerous managers caries some advantages concerning the investor's available information. The screening of a suboptimal manager among two possible candidates leaves the investor with no benchmark to asses the performance of the, allegedly, optimal manager. Thus, the investor cannot re-evaluate the performance of such manager and reward or penalize him, in accordance. This lack of information also affects the investor's ability to accurately, and a priori, fix the entry fee C. On the other hand, the possibility to maintain more than one manager (post screening) enables the investor to use the different active managers to evaluate each other in a competitive set-up. In addition, it also enables the investor to endogenize C to the point where he needs no information over the possible expected profits in the market.

Specifically, consider a mechanism similar to the one described in Remark 1 where the entry fee C is not fixed by the investor, but auctioned among the managers. To simplify the auction, assume that C is proportional to λw, i.e., proportional to the active managers' share of the managed funds. Now, assume that the investor runs a first-price auction for C when the highest value defines the entry fee for all managers. Contrarily to the Dutch auction mentioned in Remark 1, the investor also establishes a *competition-only* rule where no funds are distributed when only a single manager agrees to pay the entry fee. In other words, the investment is annulled when only one manager agrees to participate.

The competition-only rule eliminates the possibility of an over-bidding always-active manager, as described in Remark 1. That is, no manager can gain from overbidding C and eliminating all other managers from the market. Thus, the conditions of Lemma 1 are met and top-level managers can fix C (through their bidding strategy) to eliminate sub-optimal managers such that at least two managers remain in the competition. In return, the investor needs no prior information about the ability of the available managers, other than the R-bound which could be manipulated based on the time frame in question. For practical purposes, top-level managers have a strong incentive to eliminate some of the competition, as their expected profit is proportional to the amount of funds they manage. Therefore, we conclude that the investor can use the market to induce a self-sustainable competition, that inherently generate the required information for its existence.

5 Discussion

5.1 Critical bounds

In Section 4 and throughout this paper, we try to minimize restrictions over possible portfolios. In fact, the R-bound on available-portfolios yields is the only restriction we impose on the managers' investment strategies. It is one of the two critical values mentioned in the paper, the other being the endowment C. Thus, we designate some concluding remarks for its resolution.

5.1.1 The elimination of superstar managers

There are two ways to incorrectly determine R such that the equilibrium results of Section 4 change dramatically. On the one hand, fixing a too-low R may lead to realized returns that violate the $1 \pm R$ bounds, resulting in an impossibility to exercise the RTS. On the other hand, trying to avoid the possibility of an ill-defined mechanism by fixing R above the 50% barrier, may deter high-level managers from entering the market altogether.

Consider the first problem of underestimating realized returns. The offset of endowments is based on the ability to deliver non-negative compensations that, in return, sum up to the overall amount bestowed by active managers. Once realized returns exceed the fixed bounds, the declared scheme is not necessarily well-defined. To be specific, these bounds ensure that the allocation rule is normalized and the scheme is feasible. Once they are breached, the scheme may assign negative returns, and therefore cannot be used.

To solve this issue, we can truncate the RTS using the piecewise-linear function $\phi : \mathbb{R} \to \mathbb{R}$ defined by

$$\phi(x) = \begin{cases} 1 - R, & \text{if } x < 1 - R, \\ x, & \text{if } 1 - R \leqslant x \leqslant 1 + R, \\ 1 + R, & \text{if } x > 1 + R. \end{cases}$$

Instead of evaluating the performance of Manager i by his realized return r_i, we use $\phi(r_i)$ to ensure that the allocation rule is well defined. However, such truncation leads to another problem, involving the second concern of deterring high-level managers from becoming active.

Observation 1 shows how taking $R > 0.5$ can lead to a linearly-decreasing scheme with respect to managers' returns. That is, the compensation term is reduced by the enlarged normalization factor R, whereas the opportunity cost does not change. The same problem arises when combing the feasibility condition with a truncated scheme. Since the opportunity cost is unaffected by ϕ, the truncated RTS can inflict heavy losses on high-level managers with no real pacifications.

The following example illustrates how the truncated scheme can produce an interesting phenomenon where, in equilibrium, only mediocre-level managers remain active, while both extremes (either high- or low-level managers) do not participate in the investment.

Example 1. Consider the truncated RTS with $R = 0.5$, where all managers are RS, and their available investment strategies agree with the $1 \pm R$ range. Clearly, the results of Section 4 still hold given $C = \lambda w \alpha_i$ for some $1 < i < k$.

Now consider an additional manager, denoted Manager 0, with an optimal portfolio Y_0 that exceeds the R-bound, such that $\mathbf{E}[Y_0] = \frac{1.5\alpha_i}{\alpha_i - 1}$. Though the conditions of Lemma 1 and Theorem 2 may still hold, the expected profit π_0 of an active Manager 0, given a profile of strategies $\sigma = (Y_0, Y_1, \ldots, Y_i)$, is

$$\begin{aligned}
\pi_0 &= \lambda w \boldsymbol{E}[Y_0] + C\left[1 + \frac{\boldsymbol{E}[\phi(Y_0)] - v_{-0}}{2R}\right] - C\boldsymbol{E}[Y_0] \\
&= \lambda w \boldsymbol{E}[Y_0](1 - \alpha_i) + \lambda w \alpha_i \left[1 + \boldsymbol{E}[\phi(Y_0)] - v_{-0}\right] \\
&< \lambda w \boldsymbol{E}[Y_0](1 - \alpha_i) + \lambda w \alpha_i \boldsymbol{E}[\phi(Y_0)] \\
&< \lambda w \boldsymbol{E}[Y_0](1 - \alpha_i) + 1.5\lambda w \alpha_i \\
&= \frac{1.5\lambda w \alpha_i}{\alpha_i - 1}(1 - \alpha_i) + 1.5\lambda w \alpha_i = 0,
\end{aligned}$$

where the first inequality follows from the positive expected returns of all managers (above 1), and the second inequality follows from the bounds on ϕ. Therefore, the set of active managers include only Managers $1, \ldots, i$, while the optimal Manager 0 and suboptimal Managers $i+1, \ldots, k$ remain inactive.

5.1.2 The impossible competition

Another natural question regarding the R bound and arising from Theorem 1 and Corollary 1, is whether these results are extendible beyond the double-or-nothing condition. Though the answer might be positive in some cases, we can also find situations where the answer is negative for any symmetric and feasible scheme. We describe here one possible scenario where two managers of different capabilities interact in a risky binary investment, making them inseparable from an uninformed investor's perspective.

Consider the two managers case described in Section 2, and assume that both managers interact with each other only via the options market, similarly to the cash-or-nothing puts-based strategies of Foster and Young (2010). Specifically, let (Y_1, Y_2) agree with the joint distribution given in Table 1. where $a > 2$ and $\mathbf{E}[Y_1] > \mathbf{E}[Y_2] \geqslant 1$. That is, the two managers are entangled in an investment where one of the two managers produces a return of $100a\%$ (with probability $p > 0.5$ in favor of Manager

$Y_1 \backslash Y_2$	0	a
0	0	$1-p$
a	p	0

Table 1: The joint distribution of (Y_1, Y_2).

1), whereas the other's return is -100%. The following lemma shows that a symmetric and feasible scheme f cannot be optimal given this distribution.

Lemma 2. *Let f be a symmetric and feasible scheme. In case both managers are active, either the expected profits of both managers is positive, or that of Manager 1 is non-positive.*

In other words, in case both managers choose to be active, the only impossible outcome is the one given in the proof of Theorem 1, where Manager 1's expected profit is positive and Manager 2's expected profit is negative.

Proof. Fix a symmetric and feasible scheme f with an endowment $C \geqslant 0$. The symmetry and feasibility conditions imply that $f_1(r_1, r_2) = H(r_1, r_2)$ and $f_2(r_1, r_2) = H(r_2, r_1)$ where $H : \mathbb{R}^2_+ \to \mathbb{R}_+$ is some non-negative function. The managers' expected profits are

$$\begin{aligned} \pi_1 = \mathbf{E}[f_1(Y_1, Y_2)] - C\mathbf{E}[Y_1] &= pH(a, 0) + (1-p)H(0, a) - Cap, \\ \pi_2 = \mathbf{E}[f_2(Y_1, Y_2)] - C\mathbf{E}[Y_2] &= (1-p)H(a, 0) + pH(0, a) - Ca(1-p). \end{aligned}$$

Denote $C_+ = \frac{(1-p)H(a,0)+pH(0,a)}{(1-p)a}$ and $C_- = \frac{pH(a,0)+(1-p)H(0,a)}{pa}$. A straightforward examination shows that $p > 0.5$ yields $C_- \leqslant C_+$. One can also verify that a deposit $C > C_+$ produces negative expected profits for both managers, while a deposit of $C < C_-$ generates positive expected profits for both managers. Otherwise, $C_- \leqslant C \leqslant C_+$ leads to $\pi_1 \leqslant 0 \leqslant \pi_2$ and concludes the proof. ■

Lemma 2 extends the results of Foster and Young (2010) to the competition framework, by stating that the expected profits of both managers are either jointly-positive or jointly-negative. However, Lemma 2 also shows that there exists an additional possibility where only the suboptimal Manager 2 gains in expectation. This last possibility bares some clear resemblance to Example 1.

We could further extend Lemma 2 to cases where both managers lose their funds at the same time with positive probability, i.e., to cases where $\Pr(Y_1 = Y_2 = 0) > 0$. However, the fact that both managers cannot produce profitable investments simultaneously is a necessary condition for Lemma 2 (i.e., $\Pr(Y_1 = Y_2 = a) = 0$). In other words, for every arbitrarily small, yet positive, probability $\Pr(Y_1 = Y_2 = a)$ there exists a scheme that screens out only Manager 2.[20] The reason for this zero-probability condition lays in the feasibility condition. Once this probability is positive, the opportunity

[20] In addition, it also implies that the set of distributions enabling an optimal screening is not (topologically) closed.

cost of both managers increase by the same term of $aC\Pr(Y_1 = Y_2 = a)$, thus allowing the investor to enlarge both the required deposit and the compensation for the highest earning manager. Though this increase affects both managers, it is (relativity) more significant for Manager 2 since his opportunity cost, as well as his probability of being the highest earning manager, are originally lower than the ones of Manager 1.

In general, Example 1 and Lemma 2 are mainly motivated by the feasibility condition. The inability to truncate the opportunity cost in Example 1 and the inability to compensate the high-level manager in Lemma 2 generate a significant loss for extremely talented managers, preventing them from participating in the investment. For this reason and due to its importance, we consider the possibility of alternating the opportunity cost in the following subsection.

5.2 The opportunity cost - real versus nominal

Though the opportunity cost and feasibility condition are the driving force of the current research, they do not necessarily represent every portfolio-managers setting. One can think of scenarios where a risk-averse Manager i outweighs the nominal gains from the rebalanced endowments and fees, over the potential personal loss of CY_i. Moreover, managers' personal preferences may also dictate that a portion of their personal funds should be held in a relatively risk-free asset. In such cases, a low-level manager focusing on the nominal losses of C, due to relatively-poor performance, is more easily deterred from the market.

To exemplify the possible magnitude of such changes, we give a concise review of previous results while considering the opportunity cost in nominal values, rather than real values adjusted to the managers' abilities. In general, this annulment improves previous results, by weakening the basic conditions leading to these statements. Namely, consider the same investment problem as in Section 4.1, while ignoring the (subjective) opportunity cost for depositing C. Hence, every active Manager i with a realized return r_i has a profit of

$$\pi_i = f_i(r) - C = \lambda w r_i + C\left[1 + \frac{r_i - \tilde{r}_{-i}}{2R}\right] - C.$$

The first significant change we observe relates to the bound R, discussed in the previous subsection. Given the new scheme, all previous results hold independently of the support of the portfolios determined by R. To be exact, one could choose any bound R, and determine C proportionally to R to fit previous conditions. In addition, the upper bound on C in Theorem 1 is redundant, since Manager 1 can never lose, in expectation, when facing Manager 2, independently of either C or R. Therefore, one can fix absurdly high values for both parameters without affecting the result of Theorem 1. By the same reasons, the result of Lemma 1 remains valid, but without any conditions whatsoever (neither on C, nor on the number of active managers). We can also reformulate Theorems 2 and 3 such that the parameter R is eliminated from the RS condition, and Ineq. (3) becomes

$$\frac{v_i}{\tilde{v}_{-i}} > \frac{v_j}{\tilde{v}_{-j}}.$$

Clearly, the last inequality produces a weaker condition than Ineq. (3), thus improving both results.

To be clear, we do not suggest to ignore the actual opportunity cost. The real-values analysis goes to the core of our model, and ignoring it altogether seems unrealistic. We believe that some modifications of it (based on further theoretical, experimental, and mostly empirical investigations) are in order, so that it will better fit the managers' actual strategic positions.

5.3 Individual fees and the index-fund competition

Extending our multiple-investor model to a set-up where managers can strategically fix their fees is not easy. The main problem follows from the inability to coordinate the preliminary distribution of funds among managers. Hence, one should consider an exogenous distribution of funds that changes according to the managers' reputation and predetermined fees. However, when a single large investor is concerned, e.g., a pension fund, the problem is straightforward. In this subsection we briefly present a possible solution for this issue, and leave the comprehensive analysis for future research.

Consider the k-manager problem given in Section 4, with a single investor and an initial endowment of w. Assume that every Manager i can fix a publicly-observed individual fee $\lambda_i \in (0,1)$. This stage occurs prior to the initial distribution of funds among the managers. In return, the investor can condition on the vector of fees $(\lambda_1, \ldots, \lambda_k)$ when distributing her funds among active managers.

Now assume that an investor, using the RTS, decides to entrust every active Manager i with an initial endowment w_i such that $\lambda_i w_i$ is constant across all active managers. That is, given a vector $(\lambda_1, \ldots, \lambda_l)$ of active managers' fees, every Manager i gets

$$w_i = \frac{w\lambda_i^{-1}}{\sum_{i=1}^{l} \lambda_i^{-1}}.$$

Note that the distribution is well-defined (non-negative and sums up to w), and $\lambda_i w_i$ is uniform among all active managers. One can also verify that, given this preliminary distribution, previous results (namely, Theorems 1, 2, and 3, along with Lemma 1 and Corollary 1) still hold. Therefore, the extension of previous results to the case of a single investor and individual fees is practical.

However, once the investor employs this uniform-$\lambda_i w_i$ policy, the normalizing factor $\sum_{i=1}^{l} \lambda_i^{-1}$ in the denominator incentivizes active managers to increase their fees. In response, the investor may wish to cap all fees at some level $\overline{\lambda} < 1$. Nevertheless, the first-order conditions suggest that, in equilibrium, all active managers will fix a fee of $\overline{\lambda}$. Hence, although the uniform-$\lambda_i w_i$ policy maintains the return-based competition, it is not necessarily optimal in terms of reducing fees.

This extension is significant to cases where portfolio managers cannot outperform simple index funds.[21] In most cases, index funds provide a portfolio with a broad market exposure and low operating expenses, compared to hedge-fund managers. Probably the most famous competition between two such funds is the previously-described million-dollar bet between Warren Buffett and Protégé Partners. When comparing the net returns of the two sides, one should consider the 0.05% charges of Buffett's fund, with the typical $2\% - 20\%$ of the hedge-fund managers (2% a year of the basic fund and 20% of the profits). Though Buffett's lead remains substantial when considering gross returns, it does shrink considerably relative to his current 40 points net advantage.

5.4 Concluding remarks

In this paper we presented a method for, a-priori, screening low-level managers, using liability contracts. Our design shows that personal collaterals could be effective, once used to compensate high-level managers for their potential losses. These contracts apply in either small, or large scale environments, while using traditional share-the-profits incentives.

However, the practical decision to use endowments in the portfolio-managers market should not be taken in a light-headed manner. The implications of such restrictions (in a relatively open market) bears a lot of risk towards the managers and financial institutions that may surpass the advantages. Therefore, our theoretical analysis should be taken with the utmost discretion and prudence.

Nevertheless, among previously-described advantages, these restrictions also carry a surplus in the form of limiting the manipulation power of scam artists, like Bernard L. ("Bernie") Madoff.[22] The basic requirement to balance publicly-raised funds with personal endowments limits the ability of fraudulent entities to produce "Ponzi schemes", where a manager pays returns to his investors from newly-raised capital rather than from actual profits. On the other hand, this will also limit the ability of portfolio managers to expand quickly in a high-risk market. We leave this notion, along with other possible extensions, for future research.

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[21] Index funds are mutual fund aimed to track a specific market index, such as the S&P 500.

[22] See New York Times, 'Madoff Is Sentenced to 150 Years for Ponzi Scheme" on June 29, 2009, and Business Insider, "5 Years Ago Bernie Madoff Was Sentenced to 150 Years In Prison Here's How His Scheme Worked" on July 1, 2014.

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6 Appendix

Theorem 2. *Given the* RTS*, there exists $C < \lambda w \alpha_i$ such that all active managers have an expected return rate of at least v_i if and only if Manager i is relatively-superior.*

Proof. Fix a relatively-superior Manager i. In case $i = k$, then one can fix $C = 0$ to show that all managers are active. Otherwise, consider $i^* = \text{argmax}_{j>i}\alpha_j$ and fix $\frac{C}{\lambda w} \in (\alpha_{i*}, \alpha_i)$. We start by proving the existence of an equilibrium with active managers S_i.

Consider the strategy profile $\sigma = (Y_1, \ldots, Y_i)$. For every active Manager $j \leqslant i$, the RTS f_j is linearly increasing in r_j, thus Y_j is still a dominant strategy of Manager j. If $i = 1$, then the expected profit of Manager i given σ is strictly positive as no endowment is needed. Otherwise, the expected profit of Manager $1 < i < k$ given σ is

$$\begin{aligned} \pi_i &= \mathbf{E}[f_i(\sigma)] - C\mathbf{E}[Y_i] \\ &= \lambda w v_i + C\left[1 + \frac{v_i}{2R} - \frac{1}{2R(i-1)}\sum_{l<i} v_l\right] - Cv_i \\ &= \lambda w v_i + C\left[1 + \frac{v_i(1-2R) - \tilde{v}_{-i}}{2R}\right] \\ &= \lambda w v_i - C\frac{v_i}{\alpha_i} > 0, \end{aligned} \tag{4}$$

where the inequality follows from the chosen C. By the proof of Lemma 1, the expected profit of every active Manager $j < i$ given σ is higher than $\mathbf{E}[f_i(\sigma)]$ due to the reduced benchmark and the excess expected return relative to Manager i. Thus, no active Manager $j \in S_i$ can gain from becoming inactive.

On the other hand, in case an inactive Manager $j \notin S_i$ becomes active and invests in Y_j, then

$$\begin{aligned} \pi_j &= \mathbf{E}[f_j(\sigma, Y_j)] - C\mathbf{E}[Y_j] \\ &= \lambda w v_j + C\left[1 + \frac{v_j}{2R} - \frac{1}{2Ri}\sum_{l \in S_i} v_l\right] - Cv_j \\ &\leqslant \lambda w v_j + C\left[1 + \frac{v_j(1-2R) - \tilde{v}_{-j}}{2R}\right] \\ &= \lambda w v_j - C\frac{v_j}{\alpha_j} < 0, \end{aligned} \tag{5}$$

where the first inequality follows from the reduced benchmark, and the second inequality follows from the chosen C and the relatively-superior condition. Hence, no Manager has a profitable deviation from σ, establishing the existence of an equilibrium.

The last inequality also provides uniqueness w.r.t. to set of possibly active managers. Lemma 1 proves that, in every equilibrium with an active Manager $j > i$,[23] all the managers of higher ability than Manager j must be active as well. If j is potentially the highest-index active manager, then his expected profit would be $\lambda w v_j - C\frac{v_j}{\alpha_j} < 0$, and therefore actively investing in Y_j cannot be an equilibrium strategy. This concludes the first part of the proof.

For the second part of the proof, fix $C < \lambda w \alpha_i$ such that the stated condition holds. Assume, by contradiction, that Manager i is not relatively-superior. Hence, there exists $j^* > i$ such that $\alpha_{j*} \geqslant \alpha_i > \frac{C}{\lambda w}$. Fix the profile of optimal portfolios $\sigma' = (Y_1, \ldots, Y_{j*})$, and note that

$$\mathbf{E}[f_{j*}(\sigma')] - C\mathbf{E}[Y_{j*}] = \lambda w v_{j*} - C\frac{v_{j*}}{\alpha_{j*}} > 0,$$

where the inequality follows from $\alpha_{j*} > \frac{C}{\lambda w}$. By the proof of Lemma 1 and given σ', we know that the expected profit of every manager superior to j^* is also positive. Therefore, there exists an equilibrium with managers not only from S_i.

Note that σ' is not necessarily an equilibrium as additional managers of lower abilities may join the investment. However, as already mentioned, once low-level managers become active, superior managers have a stronger incentive to invest, thus establishing an equilibrium with active managers beyond S_i. Contradiction. This concludes the proof of Theorem 2. ■

[23] The trivial case of a single active Manager $j > i$ is impossible, since C is bounded by $\lambda w \alpha_i$.

Theorem 3. *Given the* RTS*, there exists an equilibrium where the set of active managers is S_i if and only if Manager i is* RS *w.r.t. Manager $i+1$ (i.e., if $\alpha_i > \alpha_{i+1}$).*

Proof. We prove the equivalence between $\alpha_i > \alpha_{i+1}$ and the existence of an equilibrium $\sigma = (Y_1, \ldots, Y_i)$.

Assume $\alpha_i > \alpha_{i+1}$ and fix $\frac{C}{\lambda w} \in (\alpha_{i+1}, \alpha_i)$. Relying on the proof of Theorem 2 and given σ, we know that no active manager can profit from deviating and becoming inactive. Also, Ineq. (5) still holds when taking $j = i+1$. Thus, we only need to prove that every inactive Manager $j > i+1$ cannot profit from deviating. In case an inactive Manager $j > i+1$ becomes active,

$$\begin{aligned}
\mathbf{E}[f_j(\sigma, Y_j)] - C\mathbf{E}[Y_j] &= \lambda w v_j + C\left[1 + \frac{v_j}{2R} - \frac{1}{2Ri}\sum_{l \in S_i} v_l\right] - Cv_j \\
&= \lambda w v_j + C\left[1 + \frac{v_j(1-2R) - \tilde{v}_{i+1}}{2R}\right] \\
&< \lambda w v_{i+1} + C\left[1 + \frac{v_{i+1}(1-2R) - \tilde{v}_{i+1}}{2R}\right] \\
&= \mathbf{E}[f_{i+1}(\sigma, Y_{i+1})] < 0,
\end{aligned}$$

where the first inequality follows from $v_j < v_{i+1}$ as $j > i+1$. Therefore, σ is indeed an equilibrium.

Now assume, by contradiction, that σ is an equilibrium and $\alpha_{i+1} > \alpha_i$. By Ineq. (4) we know that $\alpha_{i+1} \geqslant \alpha_i > \frac{C}{\lambda w}$. Hence, if Manager $i+1$ becomes active, then by a similar computation to Ineq. (4) we get

$$\mathbf{E}[f_{i+1}(\sigma, Y_{i+1})] - C\mathbf{E}[Y_{i+1}] = \lambda w v_{i+1} - C\frac{v_{i+1}}{\alpha_{i+1}} > 0.$$

Thus establishing that σ is not an equilibrium and concluding the proof. ■

Lemma 3. *Fix $R = 0.25$ and assume that the expected values $\{v_i, i \geqslant 1\}$ are determined according to some concave function. That is, assume that*

$$v_i - v_{i+1} \leqslant v_{i+2} - v_{i+1}, \tag{6}$$

for every $i \geqslant 1$. Then every Manager i is RS.

Proof. The concavity assumption given by Ineq. (6) suggests that $v_i - v_{i+1} \geqslant \frac{v_j - v_i}{i-j}$, for every

$j < i$. Note that $\sum_{j=1}^{i-1}(i-j) = \frac{i(i-1)}{2}$. Thus, an $(i-j)$-weighted average yields

$$\begin{aligned}
v_i - v_{i+1} &\geqslant \frac{2}{i(i-1)} \sum_{j=1}^{i-1} \left[(i-j) \left(\frac{v_j - v_i}{i-j} \right) \right] \\
&= \frac{2}{i(i-1)} \sum_{j=1}^{i-1} (v_j - v_i) \\
&= \frac{2}{i} (v_{-i} - v_i) \\
&= 2 \left[v_{-i} - \left(\frac{i-1}{i} v_{-i} + \frac{1}{i} v_i \right) \right] \\
&= 2 \left[v_{-i} - v_{-(i+1)} \right].
\end{aligned}$$

Hence,

$$\begin{aligned}
(v_i - v_{i+1})(v_{-i} - 2R) &\geqslant 2 \left[v_{-i} - v_{-(i+1)} \right] (v_{-i} - 2R) \\
&= \left[v_{-i} - v_{-(i+1)} \right] (2v_{-i} - 1) \\
&> \left[v_{-i} - v_{-(i+1)} \right] v_i,
\end{aligned}$$

where the first equality follows from $R = 0.25$, the second inequality follows from $v_{-i} > v_i \geqslant 1$. Therefore,

$$-2Rv_i - v_{i+1}v_{-i} + 2Rv_{i+1} > -v_i v_{-(i+1)},$$

or, equivalently,

$$v_i(v_{-(i+1)} - 2R) > v_{i+1}(v_{-i} - 2R),$$

and the RS condition, given by Ineq. (3), holds for every Manager i. ■

Observation 2. *Note that* Lemma 3 *could be extended beyond concave functions. Formally, fix* $\varepsilon > 0$ *and assume that the values* $\{v_i, i \geqslant 1\}$ *sustain the following condition*

$$v_i - v_{i+1} \geqslant \frac{v_j - v_i}{j - i} - \epsilon,$$

for every $j > i$. *For a sufficiently small* $\varepsilon > 0$, *the result of* Lemma 3 *still holds.*