

# Simultaneity in perception of knocking

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*Abstract*— “Knock, knock – who’s there?” Here, we do not address this question but rather the underlying mechanism behind the perception of knocking impacts. When one knocks on a surface, her hand makes a forward-backward motion and, at the point of reversal, the knuckles collide with the rigid surface. How does one perceive the unity, or simultaneity, of the sensory events associated with this impact? Does this binding derive from a temporal estimate of simultaneity, or does the brain use some other mechanism?

In this study, we ask whether the tap and the reversal of the hand are perceived as happening together, since both took place at the same time or at a particular state of motion. The aim of this research is to find out whether a tactile event and the flow of proprioceptive information regarding the state of the arm are matched within the central nervous system according to time or state. We tested this experimentally with subjects who actively moved one arm, as well as subjects who were served by a robotic device. Our results suggest that time is the mechanism used for judging the unity of the modalities for both active and passive movements.

Taken together, these results provide a useful cue for neuroscientists as to the structure and function of the perceptual and motor systems and essential engineering knowledge for the development of effective and realistic augmented reality systems with haptics for the telerobotics, telesurgery, and telepresence applications of the future.

***Index Terms* — haptic, motor system, motor learning, perception, active guidance**

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## I. INTRODUCTION

A VASE crashing to the floor generates visual and audio events, which are perceived as happening concurrently. If the vase falls on one’s foot, a tactile event would also be elicited. This tactile event would then be attributed to the former two, thus, creating a three modalities event (audio, visual, and tactile). If on the other hand, the tactile stimulus arrived later in time or at a different location on the body (e.g., a kick from the frustrated owner of the vase), the nervous system would not attribute the tactile event to the actual crashing of the vase. The difference between the physical event, which in this case is the actual crashing, and the perception of such an event should be stressed. The physical event takes place in the real world, and for simplicity, it is assumed that all physical attributes happen at the exact same time and place (sound, vision, etc.). Those attributes might flow into the central nervous system (CNS) at different velocities due to different transmissions and processing durations and, therefore, will be perceived at different times. It is then the task of the CNS to restore the first event in order to match it with the second. This time difference or window between two events perceived as happening together is considered the “width of now” [1-3]. The magnitude of such a window will define which two events are considered as happening together and which are regarded as two separate events. It has been shown that such a window is adaptable and can be modified by training [4, 5].

Stone et al., as well as others, describe the notion of the “When is Now” in their work [1], where they tested the point of subjective stimulus between visual and auditory stimuli. This point was the relative time delay between the sound and light at which subjects, on the average, perceived the two as simultaneous. The two stimuli used in that work essentially involved no motion since both light and sound, were confined to a single location and the time difference between their occurrences was judged. The correspondence between color change in movement within the visual system has been extensively studied [6, 7]. Mouthoussis et al., claim the difference in perception of the two events (reversal and color change) is due to different neural paths and consequently, different delays. Nishida et al., claim that the mismatch between the two is due to a faulty correspondence match between color transition and position transition (detection of reversal). In a dynamic system in which motion is involved, time and state are closely coupled through  $X = V \cdot T$ , where X

is the position,  $V$  is the velocity, and  $T$  is time. It is therefore not guaranteed that the metric which is used to judge simultaneity in such a system is time as opposed to, for instance, the location or velocity of the hand. Previous studies suggest the motor system adapts to state-dependent force perturbations but that it does not adapt to time-dependent forces that are not in a fixed relation with the state of motion of the arm [8] [9]. Diedrichsen et. al. [10] make a distinction between timing and coordination and suggest these are behaviorally distinct modes of motor control. Additionally, they see the anterior cerebellum as a crucial node in state-dependent motor control. On the other hand, numerous earlier studies suggest the existence of explicit timing structures in the CNS (see [11]). Witney et al. [12], show that temporal delays can be learned by the motor system in a grip force task.

When the nervous system has to match two events, it can either match the time which has elapsed between the two events, or alternatively, due to the dynamic involved, it can match the location or velocity (or both) at the instant the events take place. Assuming we refer to judgment in the time domain as simultaneity and in state domain (referred to as position and velocity) as co-occurrence, one question which might be asked is whether we judge simultaneity or co-occurrence in a dynamic system. A concrete example would be the event of tapping a rigid surface, such as knocking on a door, during which matching of two events within the motor system takes place. Such action involves a) the tactile event of the hand touching the surface (tapping) and b) the point in which the hand reverses at the point of contact (slicing motion). In the context of event matching, one might ask whether subjects judge the two events, tap and reversal, as happening at the same time (simultaneity) or at the same location/velocity (co-occurrence) of the hand.

Haptic exploration is classified into active and passive types [13]. During active exploration, we dynamically roam the environment and gather information. In contrast, during passive exploration, we do not initiate action but monitor the inflow of data emerging from external actions [14]. Symmons et al. [15] characterize active and passive through the *control* of the movement and not by the activation of the muscles. In this framework active exploration would indicate that the subject is controlling her movement, while passive exploration would imply the subject is tracking some other entity (i.e., person or robotic device). In both cases, muscle activation would be evident, since the subject must generate motions. Active exploration has been shown to be more suitable in inducing an effect on the delay between tactile and visual effects [16]. Study of the motor system has revealed a difference in perceived information between active and passive exploration [17]. Symmons et al., [15] have shown an increase occurs in the ability to categorize virtual shapes when exploration of the space was active as opposed to passive. The notion of active and passive control of movement can be considered in the framework of the tapping task mentioned earlier. Such a task can either be carried out when the subject

is actively moving her arm or alternatively guided by a robotic device throughout the motion.

In this study, we have explored the use of state and time information in reporting the temporal order of two events during active and passive arm slicing movements (Fig 1, Panel A). We asked whether subjects use time or state, namely, simultaneity or co-occurrence in reporting these events. Fig 1, panel B displays the main question of the manuscript. Plotted are the location of the arm (red), velocity of the arm (green), and the force applied to the arm (blue). The reversal point is marked by black dots in the top and middle panels and by the vertical dashed line. The time of the tap is also marked in the top and middle panels by black dots and by a vertical dashed line. The key question is whether subjects match the reversal and tap using *time* ( $\Delta t$ ) or *state of the hand* ( $\Delta S = [\Delta x, \Delta v]$ ).

Such a question might be relevant both for the neuroscientist, as well as the engineer. It has been shown that the motor system does not adapt time to varying force fields [8, 9]. Might this be true for the perceptual system as well? Consider the engineer's task of developing effective and realistic augmented reality systems with haptics in fields such as telemanipulation, telesurgery, and telerehabilitation [18-20]. In such systems, a remote operator is interacting with an environment and for every motion the operator generates, the remote system should return adequate (assuming the environment is modeled as impedance) force feedback. Ensuring the quality of such a system would be critical and, therefore, the design engineer must decide whether the most important aspect of the signal to be captured would be its temporal or state aspects. The first aspect would imply the use of a wideband channel that would minimize the delay and communication protocols with minimum packet jitter [21, 22]. On the other hand, if state information is the more important aspect, this would call for the use of highly accurate movement encoders.

## II. METHODS

### A. Setup

Eleven subjects participated in the first experiment (Robot-activated, mean age  $24 \pm 4$  7M/4F), and another group of thirteen subjects participated in the second experiment (Subject activated, mean age  $23 \pm 5$  9M/4F) after signing the informed consent form approved by Northwestern's Institutional Review Board. All subjects in both groups were right handed and none were excluded from the study. During the first experiment, seated subjects held the handle of a two-degree of freedom robotic manipulandum with their dominant (right) hand, and looked at a screen, placed horizontally above their hand that displayed the instructions regarding the flow of the experiment (Fig. 1, Panel C). For further details about the robotic manipulandum, see [23, 24].

A forced choice paradigm was implemented as follows: in each trial subjects held the handle of a robotic manipulandum

that was performing a protraction-retraction movement in the horizontal plane. Subjects were instructed to try to follow the motion the best that they could and they practiced in a short session prior to the actual beginning of the experiment. A visual cue (the word “GO”) was presented prior to movement initialization of the robotic device aimed at assisting the subjects in preparing for the motion. During these movements, the manipulandum delivered a perturbation (“tap” - a sharp burst of force in a direction perpendicular to the movement) at a level which was significantly higher than the sensation threshold. This force pulse lasted 10 ms, equivalent to one sample interval. (The robot’s sampling rate was 100 samples per second.) The subject was then asked whether the tap was delivered before (B) or after (A) the reversal point of the movement (the point at which the velocity changes sign – see Fig. 1, panel A). The answer was indicated by the subject pressing one of two buttons labeled B and A, accordingly. After each trial, there was no feedback provided regarding the correctness of the response.

The actual time difference between the tap and the reversal point was drawn randomly and uniformly from the interval  $[-250, 250]$  ms with increments of 10 ms. Following a single protraction-retraction motion, the subject was asked if the tap had occurred before or after the reversal point. In order to minimize the chance of the subject foreseeing the impending tap, these perturbations were delivered only in roughly two-thirds of the trials, the trials with a tap being randomly distributed. If no tap was delivered, the subject was asked to randomly press one of the buttons. Two types of data were acquired:

1. The responses of the subjects (B or A, for each trial).
2. The position of the hand along the X axis, sampled at a rate of 100 samples per second.

The position of the arm was not visible to the subject. As the robotic device allowed measurements in the horizontal plane, only the position and velocity in the direction of movement in this plane are reported here. Prior to starting the actual experiment, subjects could practice for several tens of trials until they felt comfortable with the experiment’s instructions. A Kalman filter [25] was used to estimate the first 3 derivatives and using a regression of the three, it was possible to estimate the forthcoming reversal point. This estimate was used to create a perturbation before the actual reversal. The information allowed us to create perturbations before the reversal in both conditions.

The primary experiment consisted of two blocks in which the limb of the subject was actuated by the robotic device and the subject was instructed to try and follow the motion of the manipulandum. In each block, the arm was moved at either a fast or a slow pace. The order of the blocks was randomly interchanged across subjects (6 began with high velocity, 5 began with a low velocity block). On a given visual cue (“GO” sign), the manipulandum moved the subject’s hand in a predefined forth-and-back trajectory (See Fig. 1, panel C). In Fig 1, panel A, the position during the two trajectories as a

function of time is shown. It can be seen that the fast movement lasted for approximately one second and the slow movement for two seconds. Subjects performed roughly 250 trials at each velocity, and the entire experiment lasted for about one hour.

Subjects were asked to use earplugs and wear a pair of head phones which sounded a static hum, in order to cancel out any acoustical information coming from the motors of the manipulandum as it delivered the tap.

During a second experiment, 13 subjects were asked to move one hand in a self initiated forth-and-back trajectory at either a slow or fast pace (i.e., the manipulandum was only delivering the tap and not moving the limb). Data was collected in the same fashion as described above (i.e., forced-choice responses and data logging of hand motion).

### B. Psychometric Curves

Psychometric curves were generated by estimating the frequency with which subjects indicated that the tap was delivered after the reversal, as a function of the actual time difference between the two events. Fig 2 illustrates possible psychometric curves describing subject replies as a function of the time gap between the two events.

Assuming the two events occur at the same time, we would expect the answers to reach chance level when the two events are perceived as happening at the same time. In this case a sigmoid shaped curve is expected, as answers are likely to be correct when the time difference is substantially large. For a time difference which is extremely positive, the time of the tap is much bigger than the time of the reversal, and the chances a subject will report it as correct (i.e., report that the tap occurred after the reversal – A) is very high. The psychometric function would get the value of one in this case. On the other hand, if the difference is highly negative, the chances that the subject will report the tap occurring after the reversal are low and, therefore, the function would have the value of zero (or almost zero). These probabilities are estimated by applying the same differences numerous times and then taking the ratio of the number of times the subject reported A and the total number of times the difference was encountered.

Specifically, let  $T_t$  be the time the tap was delivered,  $T_r$  the time when the reversal occurred, and  $\Delta$  the time difference between these two values. Each point on the psychometric curve in Fig. 2 is an estimate of the probability of the subject reporting that the tap was delivered after the reversal, as a function of the actual  $\Delta$ :

$$\hat{P}(\text{reporting that } T_t > T_r | \Delta) = \frac{\sum_{j=1}^{R(\Delta)} N^j}{R(\Delta)} \quad (1)$$

where

$$N^j = \begin{cases} 1 & \text{reply: } T^j_t > T^j_r \\ 0 & \text{reply: } T^j_t < T^j_r \end{cases} \quad (2)$$

$j$  is an index over the trials with time difference  $\Delta_{R(\Delta)}$ , the number of times this time difference was encountered during the entire experiment.

The time at the crossover point (where  $\hat{P} \approx 0.5$ ) is evaluated using a maximum likelihood fit of a sigmoid function to the data points. The bootstrap method was used in order to estimate the goodness of fit [26, 27].

### C. Reversal Duration Estimation

Reversal duration (RD) of a probing motion, as a function of time, is highly correlated with the velocity in which the arm is moved. During the experiments, subjects were required to move at two different velocities and report which of the two events, tap or reversal came first. By comparing the mean RD between these two conditions (slow and fast motion), we can verify that there was an actual change in velocity. Other measures can be used such as the maximal velocity during the motion, but RD is more accurate during the reversal part where the velocity is decreased up into a full stop during the actual reversal point.

In order to assess the RD of the motion, duration of each reversal event was measured by estimation of the time it took the subject to twice visit a point distant  $L$  centimeters from the maximal protraction point. More specifically, assuming a subject reached the maximal extension point  $Y(t_m)$  at time  $t_m$ , on the forward portion of the motion and on the backward part, she visited the point  $Y(t_m)-L$  at times  $t_i$  and  $t_o$  correspondently (see Fig 3). The duration on trial  $j$  was estimated by:

$$D^j = t_o - t_i \quad (3)$$

## III. MODELS

We considered a number of computational models and compared their prediction to the subjects' behavior. As input, all models use some or all of the following variables:

- $\Delta_T$ : the difference between the tapping time and the reversal time
- $\Delta_X$ : the difference in hand location at the instance of tapping and reversal.
- $\Delta_V$ : the difference in hand velocity at the instance of tapping and reversal.

The logistic regression is a standard way to match continuous independent quantities with singular dependent responses. Its closed form and rather tractable likelihood estimation makes it a rather comfortable candidate for our

modeling scheme. Due to the dichotomous/binary nature of the responses reported by the subjects, this method was used to model the expected response by the subject [28], namely:

$$E(Y | d) = \frac{1}{1 + e^{-g(d)}} \quad (4)$$

where  $E$  stands for the expected value of  $Y$  (response) given the data ( $d$ ). The function  $g(d)$  has the following form

$$g(d) = \alpha_n d_n + \dots + \alpha_1 d_1 + \alpha_0 \quad (5)$$

The data variables ( $d_i$ ) are  $\Delta_T$ ,  $\Delta_X$ , and  $\Delta_V$ , depending on the specific model.

In general, an adequate model would generalize across different conditions, and more specifically, across two different velocities in which subjects are moving their hands, as in this experiment. A way to verify the goodness of a model would, therefore, be to fit it to a data set that consists of the two (slow and fast) movements and then verify the goodness of the fit. We regarded the vector  $d_s$  as a measurement made during slow movements. Such a measurement could be the time elapsing between the tap and the reversal or the distance traveled by the hand from the instant of the tap to that of the reversal (see Fig. 1). The vector  $Y_s$  contains ones and zeros which reflects the response of the subject for each trial. Accordingly, the vectors  $d_f$  and  $Y_f$  are the corresponding ones for the fast velocity. The full data set is defined as:

$$Y = \begin{bmatrix} Y_s \\ Y_f \end{bmatrix} \text{ and } d = \begin{bmatrix} d_s \\ d_f \end{bmatrix} \quad (6)$$

Essentially these are the data sets from the two parts of the experiment, combined into an augmented vector.

The parameters  $\alpha = \{\alpha_n, \dots, \alpha_1, \alpha_0\}$  of each model can be estimated using the maximum likelihood algorithm aimed at estimating the parameters vector, which corresponds to the highest likelihood value for the specific data set [29]. The likelihood value which is achieved using a certain model, can serve as a means to compare different models and infer which is the most plausible. Since models might include a different number of parameters, a model with a larger number can essentially fit the data better and, therefore, will show a higher likelihood value. The way to compensate for the different number of parameters and compare these models will be elaborated in section  $D$  below.

### A. Time Representation Model

The time model makes use of the difference between the tapping time and the reversal time. The underlying hypothesis behind this model is that the brain represents the delay between the two events in units of time. This representation is used as the subject judges the order of the two events. The time delay between the two events is represented as  $\Delta_T$ .

$$\Delta_T = T_t - T_r \quad (7)$$

Essentially, the problem the nervous system is encountering is a binary decision in which it has to decide whether the tap

came a) before the reversal or b) after it. Such a decision can be represented by a single binary variable which will take a value of 0 if the tap came before the reversal or 1 if it came after, predicting a step function for the psychometric curve. In such experiments, one usually observes sigmoid function for the psychometric curve, indicating the existence of noise or uncertainty in the decision process. It is, therefore, beneficial to model the output using the logistic regression, rather than to look directly at the location of the transition in the step function.

Assuming time is measured by subjects between the two events, (5) takes the following form:

$$g(\Delta_T) = \alpha_1 \Delta_T + \alpha_0 \quad (8)$$

And the parameters  $\alpha_i$  can be estimated using the maximum likelihood algorithm.

### B. State Representation Model

The state model makes use of the difference between the state variables of the hand at the tapping instance and the reversal instance, where

$$\Delta_X = X(t)|_{t=T_t} - X(t)|_{t=T_r} \quad (9)$$

is the difference in the positions of the tap and reversal. The underlying assumption for this model resembles the one made for the time representation model except for the fact that state information is used in judging the order of the two events.

The second state variable that was used is the velocity, and more precisely, the difference in velocities on the tap and reversal times is:

$$\Delta_V = V(t)|_{t=T_t} - V(t)|_{t=T_r} \quad (10)$$

The formulation of this state model is similar to the time model in section A. In this case, the only difference is the function  $g(\Delta X, \Delta V)$  that has the following form:

$$g(\Delta_V, \Delta_X) = \alpha_2 \Delta_V + \alpha_1 \Delta_X + \alpha_0 \quad (11)$$

The state model has an additional parameter which must be estimated. The rest of the derivation is the same as seen in the previous section for the time model.

### C. Combined Model

The third model combines the two previous models. It could be considered as an alternative hypothesis in which subjects use time and state information together to conclude which came first: the tap or the reversal.

Such a model would have the same structure as the previous ones, except for the function  $g(\Delta_V, \Delta_X, \Delta_T)$  that will have the structure:

$$g(\Delta_V, \Delta_X, \Delta_T) = \alpha_3 \Delta_V + \alpha_2 \Delta_X + \alpha_1 \Delta_T + \alpha_0 \quad (12)$$

Table 1 below shows the three linear models discussed above and the number of parameters that should be estimated for each.

### D. Bayes Factor

A way to compare the performance of the three models would be to compare the likelihood of the model given the data or:

$$L(\alpha) = p(x | M_i) \quad (13)$$

where, in this case, the models  $M_i$  are a collection  $\alpha = \{\alpha_n, \dots, \alpha_1, \alpha_0\}$  for each of the entries in Table 1.

Since the models contain a different number of parameters, a model with a larger number should theoretically fit the data better and, therefore, show a higher likelihood. A way to adjust for these differences in number of parameters is to use the Bayesian Information Criteria (BIC), which adjusts the likelihood value based on the number of samples and the number of parameters that are used in the model [29, 30].

$$BIC = -2 \cdot \ln(L) + k \ln(N) \quad (14)$$

Here  $L$  is the likelihood of the model given the data,  $k$  is the number of parameters, and  $N$  is the number of samples used to estimate the model. A way to compare two models is to look at the difference in their BIC value. A value greater than 10 is considered significant [31, 32].

### E. Alternative Comparison

The following is an alternative for looking at the BIC or likelihood values of each fitted model. First, each of the models in Table 1 is fitted twice, once for the data for the high velocity movement, and once to the data for the slow movements. This fitting procedure is repeated for each subject. We label these two models  $M_{ih}$  and  $M_{il}$ , where  $i$  is the index for the number of subjects ( $i = 1 \dots N$ ,  $N$  – total number of subjects) and the sub index  $h$  and  $l$  refer to the high and low velocities. In essence, each of the models  $M$  is a collection  $\alpha = \{\alpha^v_n, \dots, \alpha^v_1, \alpha^v_0\}$ , where  $v$  can either be  $l$  (low) or  $h$  (high).

A collection of all the  $N$  models across subjects at a single velocity (low or high) can be regarded as a sample of some multivariate distribution, in which each model ( $\alpha = \{\alpha^v_n, \dots, \alpha^v_1, \alpha^v_0\}$ ) is a single point in the  $n+1$  dimension space. For instance, for the time model which appears on the first row of Table 1,  $n$  would be 1 and the space would be of dimension 2. Since each model is fitted twice for each velocity, at hand are two of these distributions where each includes  $N$  points. A statistical test (such as MANOVA) can be used to verify whether the mean value of these two distributions is significantly different. A different value of the mean would imply that the models for the high and low velocities are different, and essentially there is no generalization between the two conditions or two velocities.

## IV. RESULTS

### A. Subjects Able to Move at Two Different Velocities

The very essence of our research question was to verify whether subjects tended to judge the two events of tap and reversal as happening together by means of state or time. Therefore, subjects were either servoed or they actively moved one hand at two different velocities. Comparing the mean reversal time ( $D$ , in (3),  $L = 2$  cm) at the fast and slow conditions shows a significant change for both experiments ( $p < 0.001$  for the first experiment,  $p < 0.005$  for the second experiment). The outcome of significantly different velocities is more obvious for the first experiment in which subjects were actively servoed. In the second experiment, subjects were asked to maintain a fairly high velocity on the fast block to try to maintain a fairly low one on the slow block (randomly assigned as first or second blocks among subjects). Theoretically, subjects may have drifted to a similar velocity in both fast and slow conditions i.e., increasing their servoing speed on the slow block and decreasing it on the fast block. It turned out this was not the case, and evidently, they were able to maintain two different speed profiles. Fig 4 shows the results of the reversal duration for the two experiments.

### B. Shift in Crossover Point

The psychometric curve for a single subject is shown in Fig. 5, bottom panel. Comparing this figure with Fig 2 implies that the subject perceived the reversal as occurring after the tap. In Fig. 5, top panel, the crossover point that corresponds to the value of  $\Delta$  in which (1) reached the value of 0.5 for both velocities is displayed. The figure shows the results for the two groups of subjects for both velocities. On the left is the data retrieved from the group that actively performed the slicing movement and on the right, the data of the group that was passively servoed by the manipulandum. The data on the left side shows a significant shift from zero ( $p < 0.01$ , t-test). The data on the right shows no such tendency ( $p > 0.1$ , t-test). The two scatters are significantly different ( $p = 0.01$ , t-test). A positive shift at the 0.5 point implies that the reversal event is perceived to occur later, which could imply a longer processing time for the reversal event. See Fig. 2 for more details regarding shifts of the psychometric curves. The just noticeable difference (JND) is an estimate of the ability of a person to observe changes. It is estimated by taking the difference between the point in which the psychometric curve reaches the value of 0.75 minus the 0.25 point [33]. We estimated JND for both experiments and found it to be 42 ms for the Robot Servoed experiment and 67 ms for the Active Subjects experiment. We assumed that once two events are within the JND, they are considered to be happening together. This value is smaller than the shift of the curve seen in Fig. 4 and, therefore, the shift can be assumed to be significant. The deviation of the crossover point which was estimated by the bootstrap method (see section II.B) was  $22 \pm 13$  milliseconds.

In Fig. 4, it is evident that the effect for the active

movement exists at both the slow and fast paces. Therefore, though the velocity for the robot in the high speed condition was slightly higher than in the self-paced condition, the low velocity was in-between the high and low velocities in the self-paced condition. It can, therefore, be assumed that the effect of the perceived time of reversal was different for both conditions, as the velocity on average is comparable.

### C. BIC Values for Different Models

The BIC (see (14)) values were estimated for all models elaborated in section 3, for each one of the subjects. Tables 2 and 3 show the BIC value for each of the subjects, along with the mean value for each experiment (Robot-activated and Subject-activated) across subjects. The last row shows the mean of all the BIC values across subjects after subtraction of the BIC value of the time model. If the time model is better than the other models, the mean values of the models should be larger than zero, which is the case here.

In general, a model with a BIC value which is smaller by more than 10 from another model is considered significantly better [31, 32]. It is evident that the time model has a lower value of more than 10 from the state model (XV). The negligible difference in BIC values between the time and combined models for both active and passive groups is supported in the time model. Essentially, the similarity in BIC values for the models suggests that there is no additional information in the state data that is not evident in the time data. Additional modeling work which includes other combinations of state and time is presented in Appendix 1.

### D. Results for the Alternative Methods

Table 4 below shows the P values for each of the models and for each group, as described in section III.E.

The shaded part of the table shows the models that do not include the time information, and it is evident the P values are lower. Low values imply that the two distributions (for the low and high velocities) are significantly different for the state model (XV). Table 5 shows the fitted parameters for the combined model (XVT). Essentially, the only parameter which should be different than zero is the time parameter and not velocity or position. This is true for the Robot activated experiment. In the Subject activated experiment, the velocity term is different from zero as well. This fact does not weaken the time model, as the best candidate *time* is included in the velocity term (X/t). Since correlation between time and velocity exists, we used BIC to quantify this difference.

## V. DISCUSSION

The main result suggests simultaneity, i.e., the time difference between the reversal and the tapping, is much more likely to be the measure that is evaluated by the nervous system compared to other alternatives, such as position and velocity. A second result implies that during active exploration, perception of the reversal event is delayed compared to the tap. This delay was not observed when the

hand was moved (by the robot).

Tap time and reversal time were estimated from each one of the subjects' trajectories. Trajectories in which the tap was not in the  $\pm 250$  ms range around the reversal point were discarded and not included in the analysis presented here. Also discarded were the trials in which no tap was delivered ( $\sim 1/3$  of the trials, see Methods section). We found no effect on subjects' hand trajectories between the trials where a perturbation was delivered and the ones where it was not. This implies that subjects were aware of the impending tap, but it did not affect their behavior.

In this study, subjects performed slow and fast slicing movements under passive and active conditions and were asked to report which event came first, a tactile tap or the reversal of the arm movement. In the active condition, subjects demonstrated a clear bias towards reporting that the tap occurred earlier; such bias was not evident in the passive condition. We fitted computational models using various linear combinations of velocity position and time variables and found that the time model was the best at accounting for the responses made by the subjects.

Numerous previous studies suggest the existence of neural time keepers or clocks within the nervous system (e.g., [11, 12]). These mechanisms are presumably responsible for our ability to accurately perform different motor tasks, such as playing a musical instrument or hitting a tennis ball with a racket. Other studies suggest that the motor system adapts to state-dependent force perturbations but does not adapt to time-dependent forces that are not in a fixed relation with the state of motion of the arm [8, 9]. The results here are supportive of the time representation during the tap/reversal order judgment task. It might well be that within the motor system, the execution of a motion is state-based while the perception of motion or tactile information is time-based.

Tables 2 and 3, as well as Table 4, show that the time model better explains the responses of the subjects regarding the order of the events. It is evident on Tables 3 and 4 that the combined model (XVT) explains the data nearly as well as the time model. This, of course, strengthens our claim that time and not state are used to match the two events, as the addition of state information to the time information does not improve the performance of the model.

Our findings suggest it takes more time to perceive the reversal, on average, than the tap event in active motion (see Fig. 5). Prior to actively moving the hand, it is presumable that the subject's nervous system has encoded the afferent instructions of the upcoming movement. The posterior parietal cortex PPC is a neural structure that might serve as a candidate for holding these instructions [34]. Within this sequence of instructions, the reversal point, which is part of the sequence, should be encoded as well. Hence, prior to starting the motion, the nervous system is aware of the location of the upcoming reversal point. Once completing the motion, the nervous system holds the information regarding the tactile event that was deployed at some point throughout the movement. These two events are compared and a decision

regarding their apparent order is made. One can consider two possible strategies the brain may use to match these two events. In the first possibility, the a priori known reversal information is compared to the sensed tactile event; since the reversal information is already known, the only expected delay is in the tactile information. In the second possibility, both the reversal and the tap information are sensed and then compared. In such a case, delay is expected in both modalities. The current findings, in which the reversal takes longer to process (on active tasks), are in clear contrast to the first possibility and suggest that under the second possibility, the processing of the reversal information takes longer. This fact is supported by previous studies of stiffness estimation [35] where the most plausible model suggested a measurement of the force and position during the action of stiffness estimation (generally a slicing movement into a spring-like surface).

Simultaneity is defined as two events happening at the same time. In practice, due to different propagation paths in the nervous system among other factors, two events can be slightly shifted in time, but perceived as occurring simultaneously. In order to test this perceptual simultaneity, one can follow one of two paradigms. The first is to elicit two events with different time shifts and ask the subject whether the events occurred together or not. The second would be to ask the subject which of the events came first. Both methods can be used to test simultaneity by deliberately presenting two stimuli asynchronously. Moreover, both ways can be used to explore the question at hand and can be used to explain the results (time, state, or both). The work here made use of the second method, and the results support the notion that the difference in perceived simultaneity corresponds to a difference in processing time. In general, it is well known that the nervous system is quite capable of taking such differences into account (e.g., perceiving visual and tactile events as synchronous despite their having very different processing delays). Here, we deliberately separated the tactile and motion stimuli and, therefore, we did not expect the nervous system to calibrate the two events during the experiment.

The difference in temporal perception is known to be different for self vs. actuated motion. Stetson et al., [16] suggest that active exploration of the environment is more suitable to induce a temporal adaptation, which might imply involvement of different neural structures in tasks of temporal order judgment. That specific work concerns visual-haptic matching in which subjects were asked to match a visual stimulus by pressing a button. During the active portion, the subject actively hit a button and judged its simultaneity with a flashing light. In the passive part, the button moved and touched the subject's hand, and once again, subjects were asked to report which event came first. The outcome of that work and the current study are consistent. In both studies, active exploration induced a delay between the action (Button touch/Reversal) and the perception of events (Flash/Tap). Other studies have compared different tasks in which active and passive motions are involved. Dente et al., [17] report difference in tracking haptic patterns between active and

passive exploration. Symmons et al. show a difference in detection performance between active and passive exploration of virtual geometric shapes [15]. On the other hand, Vega-Bermudez et al. did not find any differences between active and passive exploration during letter detection tasks [36]. This gap between active and passive exploration could be a reason for slower guidance of another person across new terrain. Understanding the different mechanisms of active and passive exploration is essential for the design of robotic assistive devices for various human machine interactions.

The work presented here has only considered the two modalities of the motor system; proprioceptions and tactile force. A great deal of effort was taken to exclude all other cues, such as visual or acoustical. It can be assumed that supplying such information to the subject might alter their perception of simultaneity in different modalities [4, 37], but such work is out of the scope of this study.

We have referred to one event as tactile and the other as proprioceptive, however, one should note that these two modalities might share neural pathways. Therefore, the ascending signal might not necessarily travel through different neural paths. Other explanations aside from different processing mechanisms can explain the difference in active versus robot-generated motion. One would be the amount of attention paid by the subject. Another might be muscle activation, which could influence the perceived reversal time. Additionally, this is likely to differ between self- and robot-generated conditions, providing a possible explanation for differences in perceived timing.

Taken together, our findings support a representation of coincidences based on temporal delays rather than on the position and velocity of the hand. Representation of time within the nervous system on the ascending tracks might have an impact on our understanding of various illnesses such as Huntington's or Parkinson's disease. Furthermore, fields such as teleoperation, telerehabilitation, and telesurgery might benefit from understanding how such delays affect the perception of the remote operator during interaction with distant environments.

#### APPENDIX 1.

An important quantity of a logistic regression model is the uniqueness of the independent variables in regard to the response of the subject. For instance, the time model (T) uses the time difference between the tap and the reversal as the independent variable. Therefore, it would show negative values if the tap occurred before the reversal and positive if after. As a consequence, there is a uniqueness of the values. On the other hand, the position variable would always receive positive values, as the reversal point is always greater than the tap point, regardless if the tap occurred before or after the reversal. This fact is apparent in the table below where the position model (X) is much worse than any other model. Actually, it is comparable to regression of this information with random data (see column "Rand").

As for the models reported in this article, all three have this uniqueness property. Time has been explained above. State

[X,V] derives its uniqueness from the velocity variable (positive on the way to the reversal point and negative on the way from it.) The combined model then, of course, demonstrates this uniqueness.

The location along the movement (L) can also be regarded as a position variable and would show the uniqueness property, after shifting by the reversal point. This variable together with the velocity was regarded as another state variable and was also tested. The table below (Table A1) shows the BIC value for all models after subtracting the value for the time model (the same as in Tables 2 and 3).

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achievements are the first measurement of human arm multi-joint impedance, the development of a technique for investigating the mechanisms of motor learning through the application of deterministic force fields, the discovery of a family of integrable generalized inverses for redundant kinematic chains, the discovery of functional modules within the spinal cord that generate a discrete family of force-fields, the development of a theoretical framework for the representation, generation, and learning of limb movements, and the development of the first neurobotic system in which the brainstem of a lamprey controls the behavior of a mobile-robot through a closed-loop interaction.

Dr. Mussa-Ivaldi has 110 full-length publications and 85 abstracts. He is on the editorial boards of the Journal of Neural Engineering and The Journal of Motor Behavior and is member of the Society for Neuroscience and of the Society for the Neural Control of Movement.

Table 1: Model descriptions

Name	Symbol	Equation	Number of Parameters
Time	T	$g(\Delta_T) = \alpha_1 \Delta_T + \alpha_0$	2
State	XV	$g(\Delta_X, \Delta_V) = \alpha_2 \Delta_X + \alpha_1 \Delta_V + \alpha_0$	3
Combined	XVT	$g(\Delta_X, \Delta_V, \Delta_T) = \alpha_3 \Delta_X + \alpha_2 \Delta_V + \alpha_1 \Delta_T$	4

Table 2: BIC values for the Robot-activated group

Robot Act	T	XV	XVT
1	140	173	149
2	323	307	311
3	271	283	275
4	138	171	150
5	235	249	238
6	208	194	196
7	160	191	165
8	234	263	243
9	186	217	197
10	55	104	59
11	115	156	122
Mean	188	210	191
Diff Mean	0	22	3

Table 3: BIC values for the Subject-activated group

Subject Act	T	XV	XVT
1	253	262	262
2	68	96	79
3	133	148	129
4	191	219	202
5	163	182	173
6	332	304	309
7	213	250	220
8	162	216	173
9	149	162	149
10	139	138	139
11	82	109	93
12	223	250	233
13	216	231	215
Mean	179	197	183
Diff Mean	0	18	4

Table 4: *P* values for the both Robot-activated and Subject-activated groups

	T	XV	XVT
Robot Act	0.74	0.01	0.75
Subject Act	0.76	0.01	0.71

Table 5: Statistics of fitted parameters for the XVT model in both experiments. Each entry shows the mean  $\pm$  standard error. In the shaded cells, the mean was significantly different than 0.

Experiment	X [1/m]	V [S/m]	T [1/S]	#
Robot Act	6.0 $\pm$ 92.7	-0.7 $\pm$ 5.6	32.7 $\pm$ 44.8	0.9 $\pm$ 1.3
Subject Act	2.8 $\pm$ 27.5	-0.9 $\pm$ 0.8	19.9 $\pm$ 15.9	-0.4 $\pm$ 0.9

Table A1: BIC value for additional models

Model	T	L	V	LV	LT	VT
Robot act	0	13	22	12	1	1
Subject act	0	19	18	14	2	1
Model	LVT	X	XV	XT	XVT	Rand
Robot act	3	246	22	3	4	253
Subject act	4	169	19	3	4	194

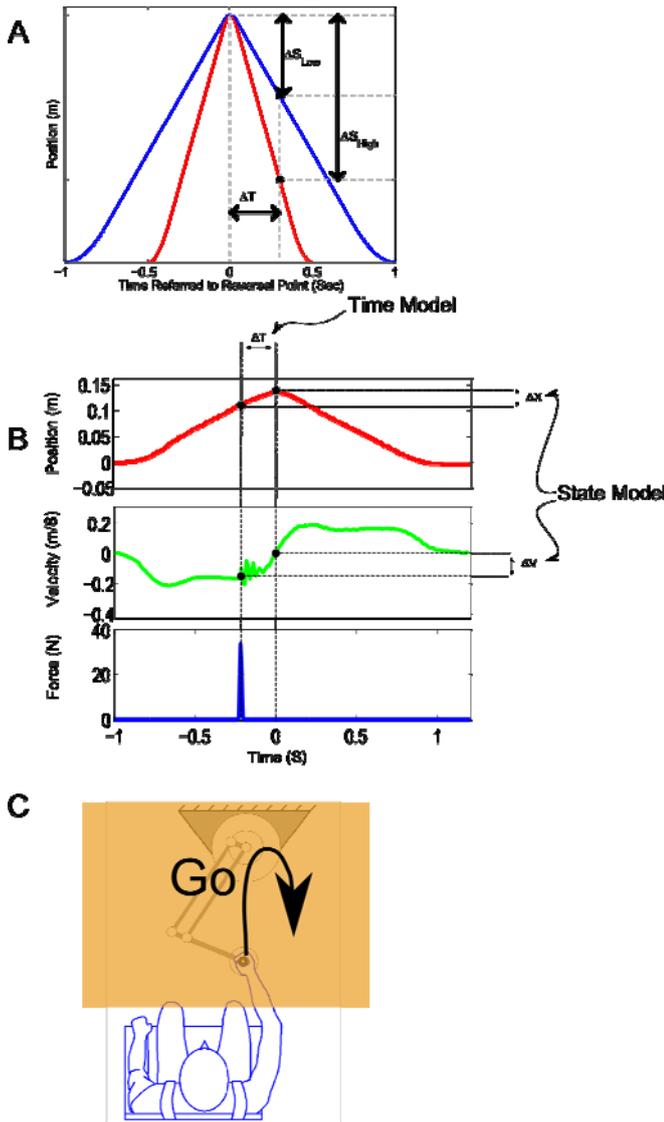


Fig. 1: (Panel A) The two trajectories for the experiment - position as a function of time. The curve describing the slow motion (blue) starts earlier than the fast one (red).  $X_{Low}$  is the distance traveled from the reversal point to the tapping point at low velocity. The same follows for  $X_{High}$ , but at high velocity. (Panel B) Hand position (top), velocity (middle), and applied force (bottom) are shown as a function of time. The question is whether the reversal and the tap are matched using time or state of the hand. Panel C shows the subject holding onto the robotic manipulandum. A horizontal screen obscured the subject's arm and displayed instructions. The black arrow is a cartoon of the slicing motion subjects performed although the front and back motion, in general, followed the same lateral location.

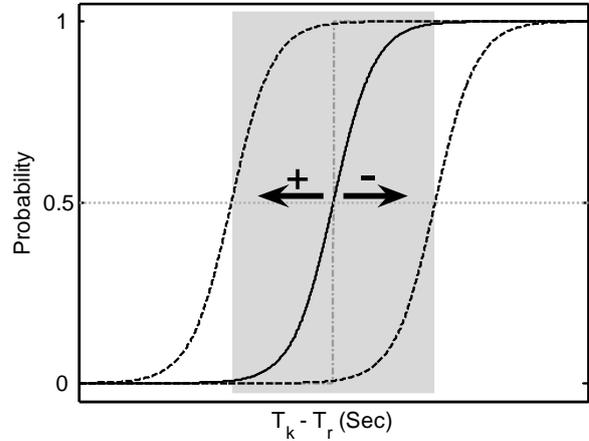


Fig. 2: Possible psychometric curves for the expectation of an answer indicating that the tap arrived after the reversal as a function of the difference in time between the two events. The gray dot-dashed line demonstrates the performance of a "perfect subject" who can accurately estimate whether the tap occurred after the reversal. The black solid line shows a typical subject who would make some mistakes in the transition region (marked as a gray rectangle). A shift in this graph to the left (right), as seen by the dashed black line on the left (right), would suggest the subject perceives the tap as occurring after (before) the reversal, although the tap actually occurred before (after) the reversal.

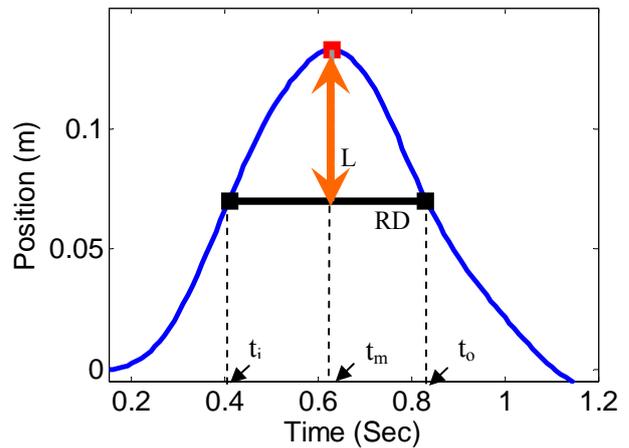


Fig. 3: RD (reversal duration) estimation. The bell-shaped (blue) line shows a typical trajectory in which the top point (red square) is the maximum protraction point. The length of the line between  $t_i$  and  $t_o$  (black line) would serve as proxy to the RD of the motion.

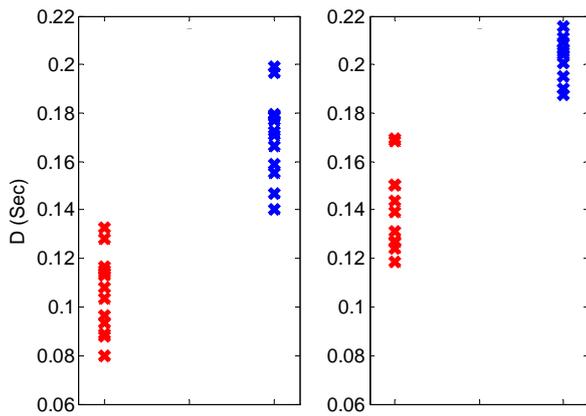


Fig. 4: The reversal duration for each one of the experiments. Each x axis corresponds to the median of the reversal duration of a specific subject. The red x's (left side of each panel) are the values during the fast motions, and the blue (right side of each panel) are for the slow ones. The left panel shows data from the experiment in which the arm was moved by the robot. The right panel shows data of the self-activated motion. The mean of the data in both panels is significantly different ( $p < 0.005$ , t-test).

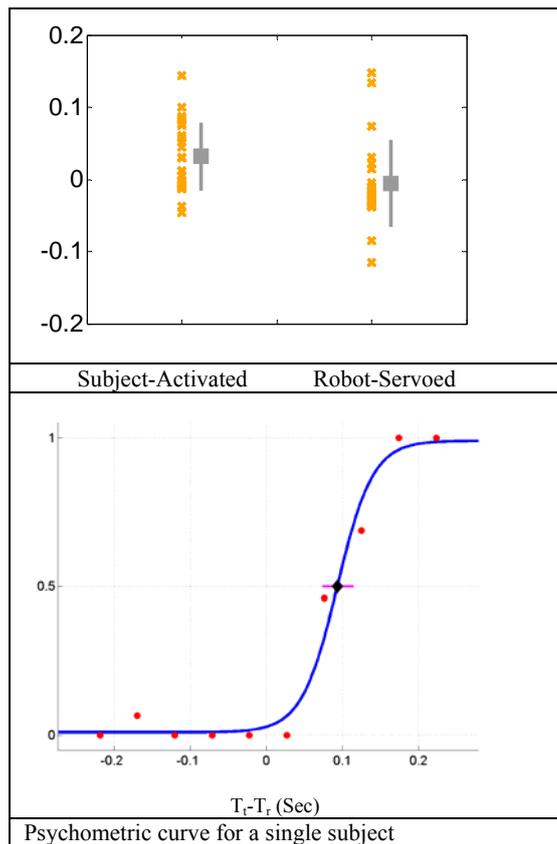


Fig. 5: (Top Panel) The shift of the crossover point in seconds (see part 2.2 in Methods section) for the two experiments as a function of the difference between the tap time and the reversal time ( $T_r - T_f$ ). Each yellow x on the left side of the plot corresponds to the shifts during the self-actuated motion either for the fast or slow conditions for a single subject. On the right are the x's corresponding to the shifts at the 0.5 point during the servoed motion. The active group has a mean significantly different from zero ( $p < 0.01$ , t-test). The gray square is the mean value for each group, and the wings are a single

standard error over all subjects for fast and slow motion. (Bottom Panel): Psychometric curve for a single subject from the active group (left side of top panel). The horizontal axis is the difference between the time of the tap and that of the reversal. The vertical axis is the probability of responding that the tap came after the reversal. Dots (red) are the estimated probability for a particular subject. The sigmoid curve (blue line) is fitted to the data using the maximum likelihood procedure. The black point is the 0.5 probability point, and it is evident it is shifted to the right, which implies the reversal was perceived as occurring after the tap. The horizontal magenta line is the two standard errors confidence interval for the value of the shift.

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