

Dynamics of disordered vortex matter in type II superconductors.

Golan Bel¹ and Baruch Rosenstein^{2,3}

¹*Physics Department, Bar-Ilan University, Ramat-Gan 52900, Israel.*

²*Electrophysics Department, National Chiao Tung University, Hsinchu 30050, Taiwan, R.O.C.*

³*Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel.*

(Dated: September 29, 2005)

Dynamics of homogeneous moving vortex matter is considered beyond the linear response. The framework is the time dependent Ginzburg - Landau equation within the lowest Landau level approximation. Both disorder and thermal fluctuations are included using the Martin-Siggia-Rose formalism. We determine the critical current as function of magnetic field and temperature $J_c(B, T)$. The surface in the $J - B - T$ space defined by the function separates between the dissipative moving vortex matter regime (flux flow) and an amorphous vortex "glass". Both the thermal depinning and the depinning by a driving force are taken into account. The static irreversibility line, determined by $J_c(B, T) = 0$ is compared to experiments in layered HTSC and is consistent with the one obtained using the replica approach. The non-Ohmic I-V curve (in the depinned phase) is obtained and compared with experiment in layered superconductors and thin films.

PACS numbers: 74.40.+k, 74.25.Ha, 74.25.Dw

In type II superconductors the magnetic field penetrates the sample in a form of Abrikosov vortices, which interact strongly creating an elastic "vortex matter". Impurities greatly affect the thermodynamic and especially dynamic properties of the vortex matter. In high T_c superconductors (HTSC) thermal fluctuations also influence the vortex matter, either directly by melting the vortex lattice into a liquid or by reducing the efficiency of the disorder (depinning). As a result of the delicate interplay between disorder, interactions and thermal fluctuations even the static $H - T$ phase diagram of HTSC is very complex and is still far from being reliably determined. Once electric current J is injected into the sample, the dynamical phase diagram should be drawn in the three dimensional $T - H - J$ space, which makes the analysis even more complicated. Generally there are two phases, the pinned phase in which the linear resistivity vanishes and a phase in which vortices can move due to Lorentz force and thus a finite resistivity appears. The surface separating the two phases is determined by the critical current (neglecting very small creep effect) as function of magnetic field H and temperature T . The intersection of the surface with the $H - T$ plane gives the static irreversibility line.

Theoretically two major simplifications are generally made. In London approximation (valid far from H_{c2}) the vortex matter behaves as an array of elastic lines [1]. An alternative simplification to the vortex matter, valid far from H_{c1} , where the magnetic field is nearly homogeneous due to overlaps between fields of the vortices, is the lowest Landau level (LLL) approximation [2, 3]. The original idea of the vortex glass and the continuous glass transition (GT) was studied in the static frustrated XY model [4, 5] using RG, variational methods and numerical simulations [6]. It was found that in this model the conductivity exhibits the glass scaling. In analogy to the theory of spin glasses, in this model the replica

symmetry is broken when crossing the GT line. More realistic model, the elastic medium of interacting line-like objects subject to both the pinning potential and the thermal bath, was treated using the Gaussian approximation [7, 8] and RG [5]. The dynamics in the presence of thermal fluctuations, within the latter model, can be simulated using the thermal bath Langevin force [9].

The irreversibility line (along with other properties) in 2D and 3D disordered vortex matter was found by applying the replica method to the Ginzburg-Landau (GL) model [10, 11] (supersymmetry was also employed to describe the effect of columnar defects in layered HTSC [12]). The irreversibility line of YBCO and a 2D organic superconductor were in good agreement with experiment. Dynamics in the presence of thermal fluctuations and disorder is generally described using the time dependent Ginzburg - Landau (TDGL) model, in which the coefficients have random components [1]. This model was studied by Dorsey, Fisher and Huang [13] in the homogeneous (liquid) phase using the dynamic Martin-Siggia-Rose (MSR) approach [14]. They obtained the irreversibility line and claimed that it is inconsistent with experiments in YBCO.

It is the purpose of this paper to study the dynamics of vortex matter within the TDGL model beyond linear response using the dynamical approach of [13]. We calculate the I-V curve in the homogeneous flux flow phase. The critical surface in the three dimensional $T - H - J$ space, separating the pinned and unpinned phases is obtained. The static GT line (zero current) coincides with the one obtained using the replica method [11]. A relation between the dynamical and the replica methods, which in our mind is crucial for understanding the nature of any glass transition, is discussed. Comparison of the irreversibility line, critical current and resistivity with experimental results in layered superconductors and thin films is made.

Our starting point is the TDGL equation [15] in the presence of thermal fluctuations on the mesoscopic scale represented by a white noise ζ :

$$\frac{\hbar^2\gamma}{4m^*}D_\tau\psi = -\frac{\delta}{\delta\psi^*}F + \zeta, \quad (1)$$

where m^* is the effective mass of Cooper pair and γ is the inverse diffusion constant. The covariant time derivative is $D_\tau \equiv \frac{\partial}{\partial\tau} + \frac{i e^*}{\hbar} \Phi$, where Φ is the scalar potential describing the driving electric force. The variance of the thermal noise ζ determines the temperature via the fluctuation-dissipation relation: $\langle \zeta(x, \tau) \zeta^*(y, \tau') \rangle = \delta(\tau - \tau') \delta(x - y) \frac{\hbar^2\gamma}{8m^*} T$. The static GL free energy including the ΔT_c disorder is:

$$F = \int d^3x \frac{\hbar^2}{2m^*} |\vec{D}\psi|^2 - a'(1+U)|\psi|^2 + \frac{b'}{2} |\psi|^4, \quad (2)$$

where the disorder variance is expressed via dimensionless pinning strength n and the coherence length ξ : $\langle U(x)U(y) \rangle = \delta(x-y)\xi^2 n$. The covariant derivative $\vec{D} \equiv \vec{\nabla} + \frac{i e^*}{\hbar c} \vec{A}$ describes the magnetic field, and the coefficients in Eq.(2) are related to the coherence length and magnetic penetration depth λ , namely $a'(T=0) = \frac{\hbar^2}{2\xi^2 m^*}$ and $b' = \frac{2\pi\hbar^2\lambda^2 e^{*2}}{\xi^2 c^2 m^{*2}}$. The TDGL equation can be written as $\hat{H}\psi = a'U\psi - b'|\psi|^2\psi + \tilde{\zeta}$, where the (non

Hermitian) linear operator is defined by $\hat{H} \equiv \frac{\hbar^2\gamma}{4m^*}D_\tau - \frac{\hbar^2}{2m^*}D^2 - a'$. Several assumptions (identical to those used in [16] and major parts of [3]) are made. In strongly type II superconductors, where $\kappa \gg 1$, magnetic and electric fields are very homogeneous, since fields of vortices overlap. Therefore the Maxwell equations for electromagnetic field are not considered. The Landau gauge with vector potential $\vec{A} = (-By, 0, 0)$ and scalar potential $\Phi = Ey$ is used. Temperature, current and magnetic field should be close enough to the dynamical phase transition line $H_{c2}(T, J)$ in order to apply the GL approach.

In order to perform the averaging over both the thermal fluctuations and disorder in the dynamical situation, we adapt the MSR formalism [14] to the present case. The dynamical "partition function" is defined as a functional integral over the order parameter ψ and an additional "ghost" field ϕ (originating from integral representation of delta function):

$$Z = \int D\psi D\phi \exp(-A_{MSR}[\psi, \phi, U]). \quad (3)$$

Although the formalism can be applied to both the 3D and the 2D cases, to simplify the discussion, we consider a 2D superconductor of thickness L_z . The MSR dimensionless "action" A , with disorder averaged out is:

$$A_{MSR} = -i \frac{L_z}{T} \int_{r,t} \left\{ \left[\phi_{r,t}^* \hat{H} \psi_{r,t} + cc \right] + i \frac{\hbar^2\gamma}{2m^*} |\phi_{r,t}|^2 + b' |\psi_{r,t}|^2 \left[\phi_{r,t}^* \psi_{r,t} + cc \right] + irb' \int_s \left[\phi_{r,t}^* \psi_{r,t} + cc \right] \left[\phi_{r,s}^* \psi_{r,s} + cc \right] \right\},$$

where $r = \frac{n}{2\pi^2\sqrt{2Gi}} \frac{(1-t)^2}{t}$ and $Gi \equiv \frac{1}{2} \left(\frac{2e^{*2}\kappa^2\xi^2 T_c}{\pi c^2 \hbar^2 L_z} \right)^2$ are the disorder dimensionless parameter and the 2D Ginzburg number respectively. In high enough magnetic field, ϕ and ψ can be expanded in a LLL basis (right eigenfunctions of the operator \hat{H} with lowest eigenvalue) $\varphi_{k\omega} = \exp[i(\omega t + kx)] \exp\left[-\frac{b}{2}(y/\xi - k\xi/b + iv/b)^2\right]$. The dimensionless magnetic field and velocity are $b = B/B_{c2}$ and $v = e^*\gamma E \xi^3 / 4\hbar b$ respectively (γ depends on temperature $\gamma = \gamma_0(1-t)^{-\eta}$, for example in BCS $\eta = 1$, $\gamma_0 = 48\pi\kappa^2\sigma_n/c^2$, where σ_n is the normal state conductivity [1]). The LLL basis will be used from now on. The model is still a highly nontrivial field theory and we use the Gaussian effective action approach to treat it. The approximation was applied to the LLL Ginzburg-Landau model in the absence of electric field (using diagram resummation) in Ref. [13], leading to identical gap equation. Two point Green functions are the correlator $C(r, t, r', t') = \langle \psi_{r,t}^* \psi_{r',t'} \rangle$, the response function $R(r, t, r', t') = \langle -i\phi_{r,t}^* \psi_{r',t'} \rangle$ and the auxiliary field cor-

relator $B(r, t, r', t') = \langle \phi_{r,t}^* \phi_{r',t'} \rangle$. In a homogeneous dynamical phase (stationary flow) [17], the correlators time dependence is just on the time difference $t - t'$. Within LLL we find a solution, which does not depend on momentum k : $C(k, t, k', t') = \overline{C}(t - t')$. The variational parameters therefore will be R_ω , C_ω and B_ω , which we write in a matrix form $g = \begin{pmatrix} C & iR \\ -iR^* & B \end{pmatrix}$.

The Gaussian effective action is:

$$A \propto \int_\omega [Tr \log g^{-1} + Tr(g_0^{-1}g)] + \frac{\theta}{2\pi} \int_{\omega,\lambda} C_\lambda (R_\omega - R_\omega^*) + \frac{r\theta}{2} \int_\omega [2B_\omega C_\omega - R_\omega^{*2} - R_\omega^2],$$

where $\theta \equiv 4\pi t b \sqrt{2Gi}$, $a_h = -(1-t-b-v^2)/2$ and

$$g_0^{-1} = \frac{L_z \xi \hbar^2 e^{v^2/b}}{2\pi^{1/2} m^* T b^{3/2}} \begin{pmatrix} 0 & -\omega\gamma\xi^2/2 - 2ia_h \\ \omega\gamma\xi^2/2 - 2ia_h & \gamma\xi^2 \end{pmatrix}.$$

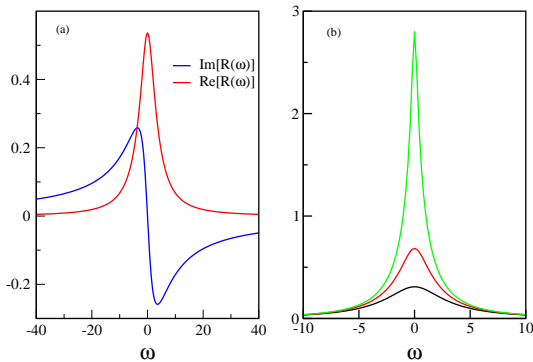


FIG. 1: In (a) the imaginary and real parts of the response function are shown for arbitrary values of the parameters. The correlation function has a peak at $\omega = 0$. In (b) it is shown qualitatively how the correlation function at zero frequency diverges as the parameters approach the critical values.

The corresponding "gap equations" are

$$(2a_h + \theta C(t=0) + i\omega\gamma\xi^2/2) R_\omega - r\theta R_\omega^2 = 1 \quad (4)$$

$$C_\omega = \frac{\gamma\xi^2 |R_\omega|^2}{1 - r\theta |R_\omega|^2}; B_\omega = 0. \quad (5)$$

For $v = 0$ the equations are consistent with those of Ref. [13]. For $\omega = 0$, one obtains

$$R_{\omega=0} = \frac{-a_h + \sqrt{a_h^2 + \theta(1-r)}}{\theta(1-r)} \quad (6)$$

Eqs. (5,6) are consistent with the static ($t = 0$ and $v = 0$) correlator calculated using the replica method [11]. The solution for arbitrary ω is:

$$R_\omega = \frac{a_v + i\omega\gamma\xi^2/2 - \sqrt{(a_v + i\omega\gamma\xi^2/2)^2 - 4r\theta}}{2r\theta}$$

where

$$a_v = \frac{a_h(1-2r) + \sqrt{a_h^2 + \theta(1-r)}}{(1-r)}.$$

The correlator and the response functions versus frequency are plotted in Fig. 1. The correlator decreases as $1/\omega^2$ at large frequencies. The structure of the singularities in the complex ω plane is as follows. In addition to cuts there is a segment of singularities along the positive imaginary axis between $\omega_{\min}^{\max} = 2\gamma^{-1}\xi^{-2} (a_v \pm \sqrt{4r\theta})$. This range of frequencies corresponds to a range of relaxation times. Asymptotically the long time dependence of the correlator is exponential, $C(t-t') \propto \exp[-\frac{t-t'}{\tau_{\max}}]$. The dominant time scale for relaxation is the longest one,

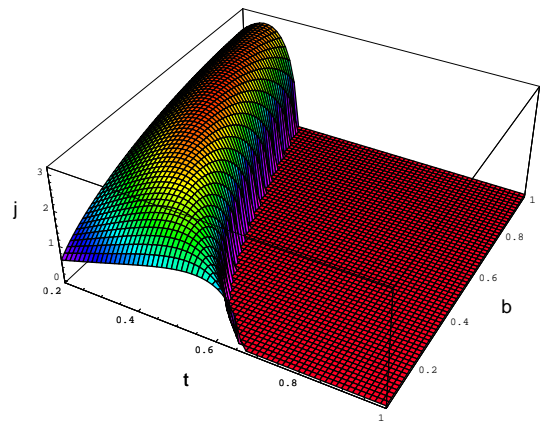


FIG. 2: The Irreversibility surface in the dimensionless j, t, b , parameters space.

given by $\tau_{\max} = \frac{1}{\omega_{\min}}$ which diverges as one approaches the critical surface determined next.

There are two dynamical phases of the system in the tree dimensional external parameter space (T, H, J) . It is more convenient to use instead dimensionless scaled parameters (t, b, v) . The critical surface is defined as a set of values of the parameters for which the correlator C_ω at $\omega = 0$ diverges. We will argue later that below this surface the superconductor acquires certain "glassy" properties. According to Eq. (5), the correlator at zero frequency diverges when the denominator vanishes, namely

$$1 - r\theta |R_{\omega=0}^g|^2 = 0. \quad (7)$$

Using the response function Eq. (6), one obtains the critical surface

$$a_T^g(v) = 4r^{1/2} - 2r^{-1/2}, \quad (8)$$

where the LLL scaled temperature is $a_T(v) \equiv 4a_h\theta^{-1/2}$. In terms of the original dimensionless parameters t, b, v it takes the form:

$$1 - t - b - v^2 = \theta^{1/2} (r^{-1/2} - 2r^{1/2}). \quad (9)$$

The static irreversibility line ($v = 0$), which is the intersection of the dynamical glass transition surface (Fig. 2) with the $H - T$ plane, has been observed in great variety of type II superconductors in magnetic fields. Within the range of applicability of the LLL approximation the static irreversibility line in both 2D and 3D was already obtained using both the replica method [11] and the dynamical approach [13]. The original impression at [13] was that the line is inconsistent with experiments in thin films of YBCO, however recently the irreversibility line in organic superconductor κ type $BEDT - TTF$, was found to be in good agreement with the experiment

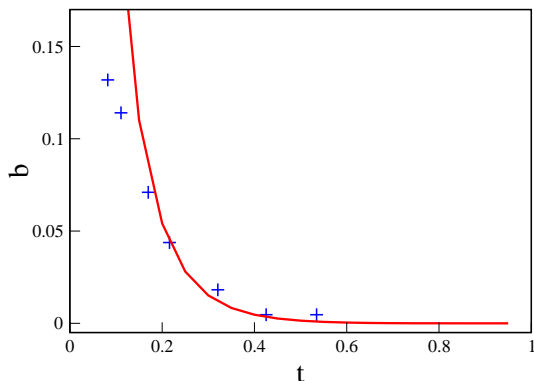


FIG. 3: The glass line in the b - t plane. The pluses correspond to experimental data of [19], and the solid curve corresponds to the analytical formula Eq. (9), where $v = 0$.

[11]. In Fig. 3 we compare the irreversibility line of BSCCO in high magnetic fields [19] with Eq. (9) for $J = v = 0$ and the following values of the material parameters: $H_{c2} = 195T$, $T_c = 93K$, $Gi = 0.00044$, $n = 0.005$. One would like to parametrize the surface in terms of the supercurrent density [16] rather than in terms of the flux velocity v . It provides the nonlinear I-V curve:

$$\begin{aligned} J_S &= \frac{2e^*vTb}{L_z\xi\hbar} R_{\omega=0} \\ &= \frac{\hbar c^2 v}{\xi e^* \lambda^2} \theta^{1/2} \frac{-a_T(v) + \sqrt{a_T^2(v) + 16(1-r)}}{16(1-r)} \end{aligned} \quad (10)$$

Note that Eq. (10) is valid only for $J_S > J_c$. The critical current according to Eq. (9) is:

$$J_c = \frac{\hbar c^2 \theta^{1/2}}{4e^* \lambda^2 \xi r^{1/2}} \left[1 - t - b + 2\theta^{1/2} \left(2r^{1/2} - r^{-1/2} \right) \right]^{1/2}. \quad (11)$$

The critical current of the *MoGe* films [20] as function of temperature compares qualitatively well with Eq. (11) in the region beyond the peak effect (namely in the homogeneous phase).

In the linear response limit Eq. (10) determines the conductivity due to Cooper pairs:

$$\sigma_S = \sigma_n \frac{3\pi^{3/2}}{2} (2Gi)^{1/4} \sqrt{\frac{t a_T(0) - \sqrt{a_T^2(0) + 16(1-r)}}{b(1-r)}}. \quad (12)$$

This expression, valid for non-zero electric field, has a finite limit when one approaches the critical surface. Within the vortex glass theory [4] the conductivity diverges, and in [13] it was argued that higher order corrections to the Gaussian approximation lead to this divergence. We were unable to confirm that and believe that the exponentially small creep may appear when instanton effects are taken into account and higher Landau levels are added. Comparison of the resistivity in BSCCO [19] with the one obtained from Eq. (12) (taking into account

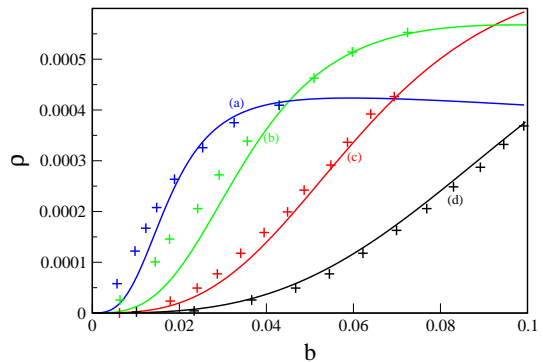


FIG. 4: Resistivity as function of the magnetic field. The solid curves show the analytical results (Eq. (12)), while the pluses correspond to experimental results of [19].

the normal part of the conductivity) is shown in Fig. 4. We conclude by discussion of the transition to the glassy state. When the distance from the critical surface in the parameter space $\Delta = a_T(v) - a_T^g(v)$ approaches zero, certain physical quantities diverge powerwise. For example, the relaxation time diverges as $\tau_{\max} \propto (a_T - a_T^g)^{-2}$ and $C(\omega = 0) \propto (a_T - a_T^g)^{-1}$. Note however that the static correlator $C(t = t')$ (proportional to the magnetization within LLL [16]) does not diverge at the critical surface. Thus it can not serve as an order parameter for the "glass" transition, even in the static limit. The common wisdom is that replica symmetry is broken in the glass (either via steps or via hierarchical continuous process) as in most of the spin glasses theories [21]. The replica method applied to the static LLL model with ΔT_c disorder within Gaussian approximation [11] indicates that there is no replica symmetry breaking in the homogeneous phase. However the Edwards -Anderson parameter vanish above the GT, while is nonzero below it. This is in agreement with the original approach to the glass transition of EA (see [22] for a discussion). The results obtained here demonstrate criticality in this case.

To summarize, using the dynamical approach we obtained the dynamical critical surface separating the liquid and glass phases. In particular the static irreversibility line was obtained, and shown to be in a good agreement with experiment in BSCCO. The resistivity is also found to be in a good agreement with experimental results. I-V curve and critical current are calculated beyond the linear response limit using the dynamical approach at finite electric field.

Acknowledgements

We would like to thank V. Vinokur, A. Koshelev, A. Dorsey and D. P. Li for fruitful discussions. We would also like to thank P. Kes, N. Kokubo and E. Zeldov for very helpful discussions of the experimental point of view on the questions under consideration in this paper. B. R. acknowledges the support of Albert Einstein Minerva

Center for Theoretical Physics at Weizmann Institute, NSC of Taiwan grant 94-2112-M-009-009 and hospitality at Bar Ilan University. G. B. Thanks the NCTS in Taiwan for hospitality at the beginning of the project.

-
- [1] G. Blatter et al, Rev. Mod. Phys. **66**, 1125 (1994).
 [2] D. J. Thouless, Phys. Rev. Lett. **34**, 946 (1975); E. Brezin, D. R. Nelson, A. Thiaville, Phys. Rev. B **31**, 7124 (1985).
 [3] R. S. Thompson and C. R. Hu, Phys. Rev. Lett. **27**, 1352 (1971).
 [4] M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989); D. S. Fisher, M. P. A. Fisher and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
 [5] T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990); T. Natterman and S. Scheindl, Adv. Phys. **49**, 607 (2000).
 [6] C. J. Olson and C. Reichhardt, Phys. Rev. B **61**, R3811 (2000); Y. Nonomura and X. Hu, Phys. Rev. Lett. **86**, 5140 (2001).
 [7] T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. **75**, 3372 (1995); **76**, 3408 (1996); Phys. Rev. B **52**, 1242 (1995); **57**, 11356 (1998).
 [8] S. E. Korshunov, Phys. Rev. B **48**, 3969 (1993).
 [9] A. van Otterlo, R. T. Scalettar, G. T. Zimanyi, Phys. Rev. Lett. **81**, 1497 (1998); C. Reichhardt, A. van Otterlo, G. T. Zimanyi, Phys. Rev. Lett. **84**, 1994 (2000).
 [10] A. V. Lopatin, Europhysics Lett. **51**, 635 (2000).
 [11] D. P. Li and B. Rosenstein, *cond-mat/0411096* (2004).
 [12] Z. Tesanovic and I. F. Herbut, Phys. Rev. B **50**, R10389 (1994).
 [13] A. T. Dorsey, M. Huang, and M. P. A. Fisher, Phys. Rev. B **45**, R523 (1992).
 [14] P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1973); H. Sompolinsky and A. Zippelius, Phys. Rev. B **25**, 6860 (1982).
 [15] A. I. Larkin and A. A. Varlamov, "Handbook on Superconductivity: Conventional and Unconventional Superconductors" edited by K.H. Bennemann and J.B. Ketterson, Springer (2002).
 [16] T. Blum and M. A. Moore, Phys. Rev. B **56**, 372 (1997).
 [17] Recent work on statics using the replica method [11] showed that lateral correlations (which distinguish between a homogeneous and crystalline states) are not important for the location of the irreversibility line, thus we study a simpler homogeneous situation. Note that the irreversibility line is different from the melting line separating the liquid and crystalline phases of the vortex matter [18].
 [18] Y. Paltiel et al, Phys. Rev. Lett. **85**, 3712 (2000).
 [19] Y. Ando et al, Phys. Rev. B **60**, 12475 (1999).
 [20] M. C. Hellerqvist et al, Phys. Rev. Lett **76**, 4022 (1996); M. C. Hellerqvist and A. Kapitulnik, Phys. Rev. B **56**, 5521 (1997); N. Kokubo and P.H. Kes, unpublished results.
 [21] V. Dotsenko, *An Introduction to the Theory of Spin Glasses and Neural Networks (World Scientific Lecture Notes in Physics)* . (1995).
 [22] J. P. Bouchaud, J. De Physique I **2**, 1705 (1992).