Vector vortices in *p*-wave superconductors with arbitrary κ parameter

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(Received 30 July 2002; revised manuscript received 21 November 2002; published 13 June 2003)

We have considered vortices in *p*-wave superconductors with a nonvanishing order parameter in the vortex core for the arbitrary ratio $\kappa=\lambda/\xi$ (here λ and ξ are the magnetic length and the coherence length, correspondently). Vector vortices are characterized by an orientation in the order-parameter space, however, spatial profile of both the order parameter and magnetic field of the vortex demonstrate an essential difference of the vortex structure for different κ parameters and from those calculated for the vortex in liquid ^{3}He. We calculate lower critical field H_{c1} and find that critical $\kappa=\kappa_c=1/\sqrt{2}$ is independent of the detailed form of interactions. Strong asymmetry of some of the components of the order parameter disappears when κ approaches κ_c. The vortex lattice in the multicomponent superconductor resembles a spin system in which the orientation in the order-parameter space plays the role of "spin." Interactions between vortices lead to long-range order. An effective theory for this system is proposed.

DOI: 10.1103/PhysRevB.67.224507 PACS number(s): 74.20.De, 74.20.Rp, 74.25.Op

I. INTRODUCTION

Order parameter describing Cooper pairs in nonconventional and multiband superconductors generally has several components.[1,2] Examples include the description of high-T_c superconductors as a mixture of *d*-wave and *s*-wave components[3] or *p*-wave superconductors like heavy fermion UPt_3[4] newly discovered Sr_2RuO_4,[5] two-gap superconductivity in transition metals Nb,Ta,V,[6] and in superconductors with suprisingly high critical temperature MgB_2.[7] In particular, the symmetry of the order parameter in nonconventional superconductors is related to the crystallographic symmetry group of the material and to the effective attraction mechanism of Cooper pairing, and in multiband systems with several coexisting condensates of Cooper pairs. Although the number of charged fields and their transformation properties under rotations are different, the common feature of theories describing these diverse systems remains the U(1) local gauge invariance. In systems of this kind in the presence of external magnetic field a major role is played by various topological defects which thereby become the most important degrees of freedom in both statics and dynamics. It is well known that, while in the simplest one-component case the Abrikosov vortices (AV's) are the only kind of topological defect, in the multicomponent case other types of defects exist.

There exist at least three distinctive phases (A, B, and C) in the magnetic-field (H)–temperature diagram (T) in the UPt_3 superconductor. In order to describe this phase diagram two scenarios [two-dimensional 2D vs 1D (Ref. 4)] have been proposed. An essential difference between the 1D and 2D scenarios lies in the fact that the degeneracy comes either from the orbital part of the pairing function in the 2D, or from the spin part of the pairing function in the 1D scenario. Machida argued that the 1D scenario which explained the isotropy of the H-T phase diagram was more robust. In the 1D scenario based on the assumption of weak spin-orbit coupling, the triplet pairing function in a matrix form is described in terms of three-dimensional vector [SO(3) symmetry]. In this paper we concentrate on a theory of complex vector field model (describing, in particular, certain superconductors with *p*-wave pairing) that possesses an approximate global SO(3) symmetry. In this case the number of the components of the order parameter $n=3$ provided more sophisticated structure of the topological defects. In this model two topologically distinct "non-Abrikosov" (in this model AV's are simply one-component topological solitons with additional components vanishing) types have been found: coreless magnetic skyrmions,[8] field-carrying topological texture,[9] knot solitons,[2] and the "vector vortices" (VV's) pointed out by some of us recently[10] which have a complex core more typical for superfluid ^{3}He rather than superconductors. The vector vortices are also not related to unconventional vortices in two-component order-parameter models (see Ref. 11 and references therein).

In the recent communication[10] we have considered both structure and dynamical generation of coreless vortices when the ratio $\kappa=\lambda/\xi$ is rather large, $\kappa=10$ (where λ and ξ are the magnetic penetration depth and the coherence length, respectively). We were guided by the $\kappa->\infty$ limit and analogy with superfluid ^{3}He. In real type-II *p*-wave superconductors it varies in a wide range of κ above critical value to be determined below. The large κ case is simpler to investigate both analytically and numerically and it enables a qualitative comparison with a very extensively studied case of the tensorial order parameter in superfluid ^{3}He and a great deal of qualitative features of VV's have been studied. It turned out that the SO(3) symmetry of the order parameter is not respected by the vortex solution. The vortex can be viewed as a nonsymmetric top in the SO(3) space. This fact is very important in enhancing the entropy of the vortex system. An interesting question is how this feature changes when the κ parameter is reduced to experimentally accessible values. An additional question is what is the vortex structure in the opposite limit of critical κ.

In this paper we tackle the more demanding case of the intermediate and small κ. We calculated the structure of the order parameter, field, and of the magnetic field as a function of the distance from the vortex axis and demonstrate the essential difference of the vortex structure for different values of the κ parameter. We predict that, while at large κ one component of the order parameter is nonzero in the vortex core, this component vanishes as a power $|\kappa-\kappa_c|^{\alpha}, \alpha=1/2$ for $\kappa\to\kappa_c=1/\sqrt{2}$. The phase diagram in the $H-\kappa$ space demonstrates the tricritical point at $\kappa=\kappa_c$ where the vector vortex becomes completely symmetric: the vortex symmetry is that of the symmetric top (and entropy is therefore reduced).

II. BASIC EQUATIONS AND SYMMETRIES

The Ginzburg-Landau (GL) Hamiltonian describing a three-component complex vector order parameter (see Ref. 4) $\psi(\mathbf{r})=(\psi^1,\psi^2,\psi^3)$ coupled to magnetic field has a form

$$\mathcal{H}=\int d^2r\left[\frac{K}{2}\left|\left(\nabla-i\frac{2e}{c}\mathbf{A}\right)\psi^a\right|^2-\alpha|\psi^a|^2+\frac{\beta_1}{2}(\psi^a\psi^{*a})^2+\frac{\beta_2}{2}|\psi^a\psi^a|^2+\frac{B^2}{8\pi}\right]. \tag{1}$$

Here $\mathbf{A}$ is vector potential, $\mathbf{B}=\nabla\times\mathbf{A}$ is the magnetic field, and $a=1,2,3$. The theory possesses a "flavor" SO(3) symmetry of rotations $\psi^a->R^{ab}\psi^b$ between different order-parameter components. The two different coupling constants β_1 and β_2 determine the nature of the homogeneous ground state. Stability requires $\beta_1>0$, while the gauge symmetry is broken at $\alpha>0$. We concentrate on the superconducting phase B (which is somewhat reminiscent of the B phase in liquid ^{3}He) for $\beta=\beta_2/\beta_1>0$ in which order parameter is (up to remaining degeneracy)[8]

$$\psi_1=\frac{f_0}{\sqrt{2}};\quad \psi_2=\frac{if_0}{\sqrt{2}};\quad f_0=\sqrt{\frac{\alpha}{\beta_1}}. \tag{2}$$

Since the configuration is assumed to be homogeneous in the direction of the magnetic field, all the fields are considered in two dimensions, $\mathbf{r}=(x,y)$. It is convenient to define the dimensionless variables as follows:

$$r\to\frac{r}{\xi},\quad \xi=\sqrt{\frac{K}{2\alpha}}, \tag{3}$$

$$\psi\to\frac{\psi}{\psi_0},\quad \mathbf{A}\to\frac{2\pi\mathbf{A}}{\xi\Phi_0}. \tag{4}$$

Here ξ, the coherence length, is the unit of length, λ is the penetration depth of the magnetic field, Φ_0 is the flux quantum. The corresponding dimensionless GL Hamiltonian

$$\mathcal{H}=\int d^2r\left[\frac{1}{2}|(\nabla+i\mathbf{A})\psi^a|^2-|\psi^a|^2+\frac{1}{2}(\psi^a\psi^{*a})^2+\frac{\beta}{2}|\psi^a\psi^a|^2+\frac{\kappa^2}{2}\nabla\times\mathbf{A}^2\right] \tag{5}$$

leads to the equations

$$-\psi^a+(\psi^b\psi^{*b})\psi^a+\beta(\psi^b\psi^b)\psi^{*a}+(i\nabla+\mathbf{A})^2\psi^a=0, \tag{6}$$

$$\nabla\times\nabla\times A=\mathbf{J}_s. \tag{7}$$

The superconducting current is defined by

$$\mathbf{J}_s=-\frac{i}{2\kappa^2}(\psi^{*a}\nabla\psi^a-\psi^a\nabla\psi^{*a})-\frac{1}{\kappa^2}|\psi^a|^2\mathbf{A}, \tag{8}$$

while the topological charge in this case has the form

$$P=\int_C\mathbf{A}\cdot dl=\frac{1}{2\pi}\sum_a\int_C|\psi^a|^2\nabla\Phi^a dl, \tag{9}$$

where the phases are defined by $\psi^a=|\psi^a|\exp(i\Phi^a)$.[12]

III. SINGLE VORTEX SOLUTIONS FOR ARBITRARY κ

A vortex is generally a solution invariant under spatial rotations in the xy plane $SO_r(2)$. As we discussed in the proceeding paper,[10] VV in the flavor (by which we denote different components of the order parameter) space are characterized by a "reper." Namely we can define three mutually perpendicular unit vectors $\vec{n},\vec{m},\vec{l}$. We look for a single vortex solution of Eqs. (6)–(8) within the following $SO_r(2)$ symmetric ansatz:

$$\psi^1=C(r)x+D(r)y;\quad \psi^2=D(r)x-C(r)y;\quad \psi^3=f(r), \tag{10}$$

and

$$A_x=b(r)y;\quad A_y=-b(r)x, \tag{11}$$

where $\mathbf{r}=(x,y)$ is a 2D vector. We have chosen here $\vec{l}=(0,0,1),\vec{n}=(1,0,0),\vec{m}=(0,1,0)$, any other orientation in the order parameter (flavor) space being just a flavor rotation. It is naturally a requirement that this ansatz constrain will keep invariance of the Hamiltonian (5) under transformation,

$$x\to y,\quad y\to -x, \tag{12}$$

$$\psi^1\to\psi^2,\quad \psi^2\to-\psi^1,$$

$$A_x\to A_y,\quad A_y\to -A_x.$$

We choose the "third" component as a direction in the flavor space for which the component is rotationally invariant. The two perpendicular components ψ^1 and ψ^2 constitute a vector under rotations in the xy plane. Generally f, C, and D are complex and b is real related to induction by $B_z=-2b-rb'$, where the prime denotes d/dr.

Substituting Eqs. (10) and (11) into Eq. (8) we obtain components of the superconducting current:

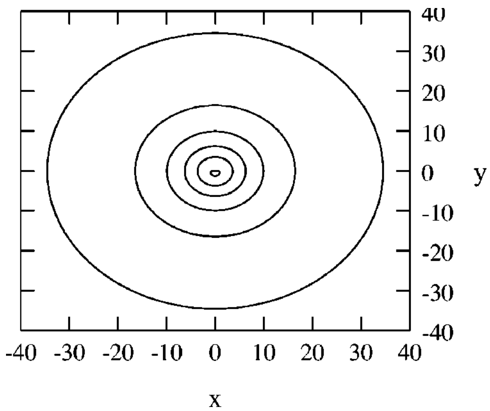

FIG. 1. One of the rotation invariant combinations of the order-parameter components: $C(x,y)=r^{-2}(x\psi^1-y\psi^2)$. It demonstrates that the solution is a vector vortex.

$$J_x=-\frac{i}{2\kappa^2}[xr(D'D^*-DD^{*\prime}+C^*C'-CC^{*\prime})-2(DC^*-D^*C)y]-\frac{2(F^2by)}{\kappa^2} \tag{13}$$

$$J_y=-\frac{i}{2\kappa^2}[yr(D'D^*-DD^{*\prime}+C^*C'-CC^{*\prime})+2(DC^*-D^*C)x]+\frac{2(F^2bx)}{\kappa^2},$$

where $F^2=|f|^2+(|C|^2+|D|^2)r^2$.

The requirement of rotation invariance $J_r=0$ leads to the condition

$$DD^{*\prime}-D'D^*-C^*C'+CC^{*\prime}=0, \tag{14}$$

which is satisfied in particular when

$$C=C^*=c \quad \text{and} \quad D=-D^*=id, \tag{15}$$

where $c(r)$ and $d(r)$ are real. This solution is confirmed by the numerical simulation of Eqs. (6)–(8) (see Fig. 1). We found also that f is real everywhere. Taking into account these properties of the ansatz coefficients, the azimuthal component of the superconducting current simplifies considerably:[10]

$$J_\varphi=\frac{r}{\kappa^2}(2cd+F^2b). \tag{16}$$

Substituting Eqs. (11) and (16) into Eq. (7) and Eq. (10) into Eq. (6) we obtain a set of the equations for coefficients of the ansatz in the form

$$-\frac{\partial^2(br)}{\partial r^2}-\frac{\partial b}{\partial r}=J_\varphi, \tag{17}$$

$$f-F^2f-\beta F_1^2f+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)-br^2f=0, \tag{18}$$

where

$$F_1^2=f^2+(c^2-d^2)r^2,$$

$$c(1-F^2-\beta F_1^2)+c''+\frac{3c'}{r}-b^2r^2c-2bd=0, \tag{19}$$

$$d(1-F^2+\beta F_1^2)+d''+\frac{3d'}{r}-b^2r^2d-2bc=0. \tag{20}$$

This set of equations describes all of the vortex properties in the *p*-wave superconductors.

We solved the equations numerically and checked that the above ansatz indeed solves the GL equations. As an example the following combination:

$$\frac{x\psi^1-y\psi^2}{r^2} \tag{21}$$

is shown in Fig. 1. According to the ansatz it should be a real function of the distance from the vortex center only $c(r)$. This is what is observed. The imaginary part of c vanishes. The same is true for $d(r)$ and $f(r)$.

The functions $c(r),d(r)$ for $\kappa=1$ and $\kappa=10$ are given in Fig. 2(a). One observes that for small κ, $c(r)$ approaches $d(r)$. We therefore plotted the difference $c(r)-d(r)$ in Fig. 2(b). As κ approaches $\kappa_c=1/\sqrt{2}$ the difference vanishes. Figure 2(c) demonstrates that as κ decreases $|\psi^3|\equiv f(r)$ also vanishes. These two quantities allow the determination of critical magnitude of the parameter κ. From numerical simulations we obtain that $\kappa_c=1/\sqrt{2}$ independent of β. We will discuss small κ in the next section.

Profiles of the magnetic field and of the superconducting current are presented in Figs. 3(a) and (b). They resemble those of the Abrikosov vortices. This means that the electromagnetic part of the intervortex interaction is the usual repulsion independent of the ''orientation'' of the vortex in the flavor (component) space. The lower critical field calculated from the vortex line energy is given in the $H-\kappa$ phase diagram, Fig. 4. It is given for $\beta=1$ (dependence on β is minor).

It is easy to find from Eq. (14) that in the vortex core ($r\to 0$) the nonzero third component does not contribute to current density at all. In order to extract the superfluid density n_s in this case one can represent J_s in the form

$$J_\varphi=n_sV_s, \tag{22}$$

where V_s is the superfluid velocity, while $V_s\propto r^{-1}$ at the vortex core [see Eq. (7)]. Comparing $J_\varphi=n_sV_s$ with Eq. (14) we immediately obtain for superfluid density at $r\to 0$

$$n_s\propto r^2 \tag{23}$$

as for *s*-wave superconductor.

It seems *p*-wave symmetry of the superconductivity should not significantly change a well-known de Gennes–

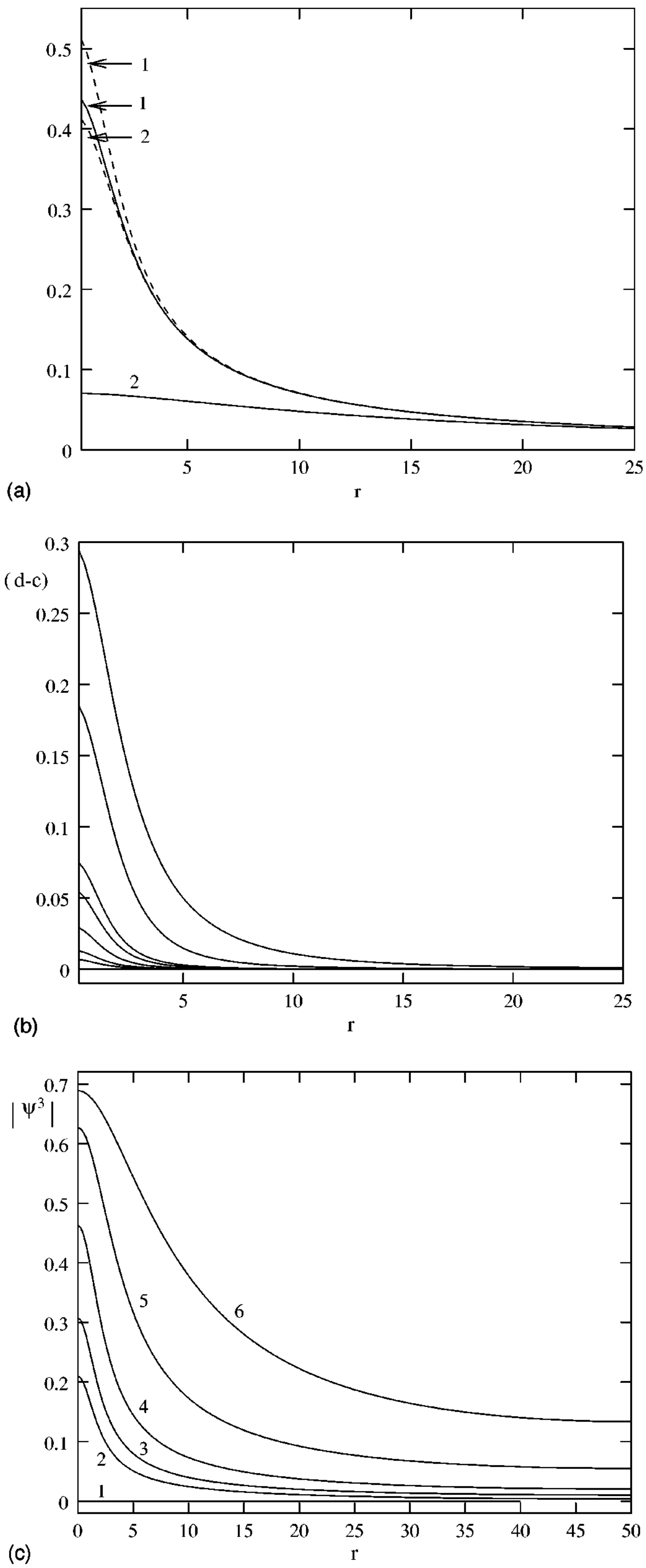

FIG. 2. (a) Spatial distribution of $c(r)$ (solid curves) and $d(r)$ (dashed curves) for $\kappa=1$ (curves 1), $\kappa=10$ (curves 2). For $\kappa=1$, $c(r)$ approaches $d(r)$. (b) Difference between $c(r)$ and $d(r)$ for $\kappa=0.71,0.74,0.75,0.8,0.9,1,2,5$. As κ approaches $\kappa_c=1/\sqrt{2}$ it vanishes. (c) Spatial distribution of $|\psi^3|$ for $\kappa=0.71(1),0.75(2),0.8(3),1(4),2(5),5(6)$. For small $\kappa,|\psi^3|=f$ vanishes.

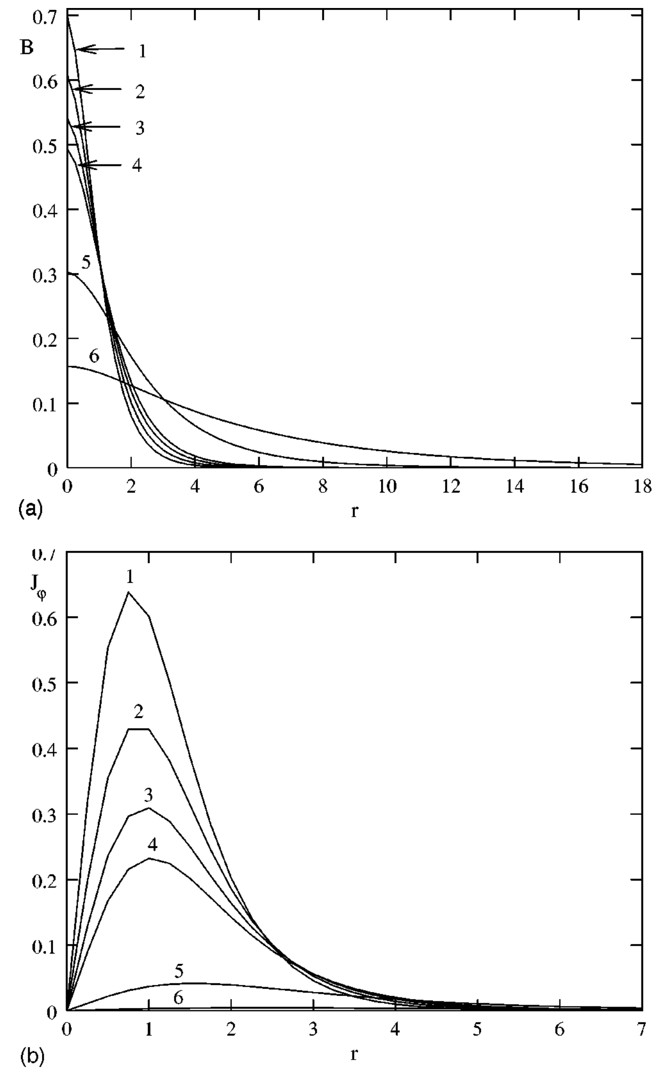

FIG. 3. (a) The magnetic-field distribution inside the vector vortex for $\kappa=0.71(1),0.8(2),0.9(3),1(4),2(5),5(6)$. (b) Azimuthal component of the superconducting current encircling the coreless vortex for $\kappa=0.71(1),0.8(2),0.9(3),1(4),2(5),5(6)$.

Barden−Stephen relation between superfluid density and the order parameter in the vortex core for *s*-wave superconductors. Really the vortex core of the moving *s*-wave vortex is usually in a normal state, and from this point of view very similar to those in *p*-wave superconductors.

IV. SYMMETRIZATION OF THE VECTOR VORTEX AS κ APPROACHES ITS CRITICAL VALUE

The most striking features of the VV's at large κ is a nonvanishing order parameter at the center of the vortex and its asymmetry with respect to rotations in the flavor space. It behaves as an asymmetric top with a superconducting core. It is interesting to ask how the structure of the vortex evolves as the value of κ is reduced and, in particular what is the critical value of κ at which transition into a homogeneous superconducting state occurs. A related question is under what circumstances does the vortex core become essentially normal?

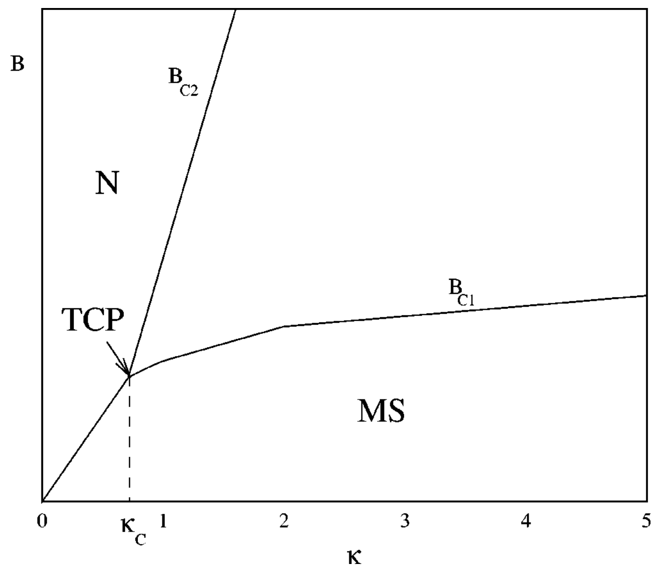

FIG. 4. Phase diagram for vector vortices in the $B-\kappa$ plane. TCP denotes the tricritical point. Although the diagram is calculated for $\beta=1$, the critical value of κ is independent of β. The H_{c1} line is calculated from the free energy of single vortex described in Figs. 2 and 3.

We assume that the set of Eqs. (17)–(20) has a solution at certain critical value $\kappa=\kappa_c$ which is much simpler than the general solution. Motivated by our numerical results, we look for the solution with vanishing $f(\kappa_c,r)$. It easy to see from Eqs. (17)–(20) that in this case one should also demand $c(\kappa_c,r)=d(\kappa_c,r)$. The ansatz (10) is simplified as follows

$$\psi^1=c(x+iy);\quad \psi^2=ic(x+iy),\quad \psi^3=0. \tag{24}$$

Two equations are trivially satisfied and the additional two are identical to the usual one-component Abrikosov vortex solution for the order parameter $\psi_-=(\psi^1-i\psi^2)/\sqrt{2}$ [the orthogonal component $\psi_+=(\psi^1+i\psi^2)/\sqrt{2}\equiv 0$]. Using an additional relation

$$B=\kappa(1-|\psi_-|^2),$$

we obtain the critical value of $\kappa=\kappa_c=1/\sqrt{2}$ independent of parameter β. This solution will be denoted by $\psi_- =c(r,\kappa_c)$ and $b(\kappa,r)=b(r,\kappa_c)$. To understand why κ_c is independent of β and coincides with Abrikosov's one we substitute the ansatz Eq. (24) into the Hamiltonian (5). As a result we obtain

$$\mathcal{H}=\int d^2r\left[\frac{1}{2}|(\boldsymbol{\nabla}+i\mathbf{A})\psi_-|^2-|\psi_-|^2+\frac{1}{2}|\psi_-|^4+\frac{\kappa^2}{2}(\boldsymbol{\nabla}\times\mathbf{A})^2\right] \tag{25}$$

in which the β term vanishes. At the critical κ the Gibbs energy of the vortex vanishes and therefore the translation symmetry is restored (no vortices are stable below κ_c). It is interesting to note that at this point the vortex becomes invariant under SO(2) flavor rotations and becomes a "symmetric top."

Now we turn to the description of the critical phenomena associated with the symmetry breaking at κ_c. As usual the dependence of physical quantities on $\Delta\kappa\equiv|\kappa-\kappa_c|$ is powerwise for all fields larger than H_{c1}. We therefore represent the ansatz coefficient functions $c(\kappa,r),d(\kappa,r),f(\kappa,r),b(\kappa,r)$ in the vicinity of the critical κ in the form

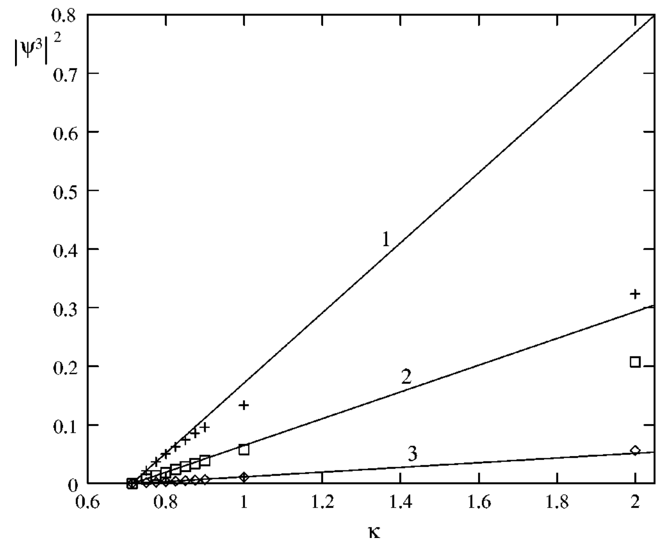

FIG. 5. Critical exponents on the H_{c1} line. $|\psi^3(r)|^2$ for $r^2=1$, 8, and 50 is given as function of κ. As κ approaches κ_c data clearly straight lines indicate that $\alpha=1/2$.

$$c(\kappa,r)=c_c(r)-c_1(r)\Delta\kappa^{\gamma};\quad d(\kappa,r)=c_c(r)-d_1(r)\Delta\kappa^{\gamma},$$
$$b(\kappa,r)=b_c(r)+b_1(r)\Delta\kappa^{\delta};\quad f=f_1(r)\Delta\kappa^{\alpha}. \tag{26}$$

Substituting this into Eqs. (17)–(20) we obtain in the next-to-leading order,

$$\Delta\kappa^{\gamma}\left(\frac{\partial^2(b_1r)}{\partial r^2}+\frac{\partial b_1}{\partial r}\right)$$
$$=\Delta\kappa\frac{4c_c^2r}{\kappa_c^3}(1+r^2b_c)+\Delta\kappa^{\gamma}\frac{2r}{\kappa_c^2}c_c(d_1+c_1)(1+r^2b_c)$$
$$+4\Delta k^{2\alpha}\frac{f_1^2rb_c}{\kappa_c^2}, \tag{27}$$

$$\Big[d_1+2c_c^2r^2d_1+2\beta c_c^2(c_1-d_1)r^2$$
$$+\left(d_1''+\frac{3d_1'}{r}\right)+2b_cc_1-b_cd_1\Delta\kappa^{\gamma}$$
$$+c_c(1-\beta)f_1^2\Delta\kappa^{2\alpha}-b_1c_c(1+2b_cr^2)\Delta\kappa^{\delta}=0. \tag{28}$$

These equations have a solution for $\delta=\gamma=1;\alpha=\gamma/2=1/2$, which are mean-field critical exponents of the phase transition in which κ plays the role of the parameter. The result of numerical simulations which shows approach to criticality of $|\psi^3(r)|^2$ for $r^2=1$, 8, and 50 is shown in Fig. 5. This clearly

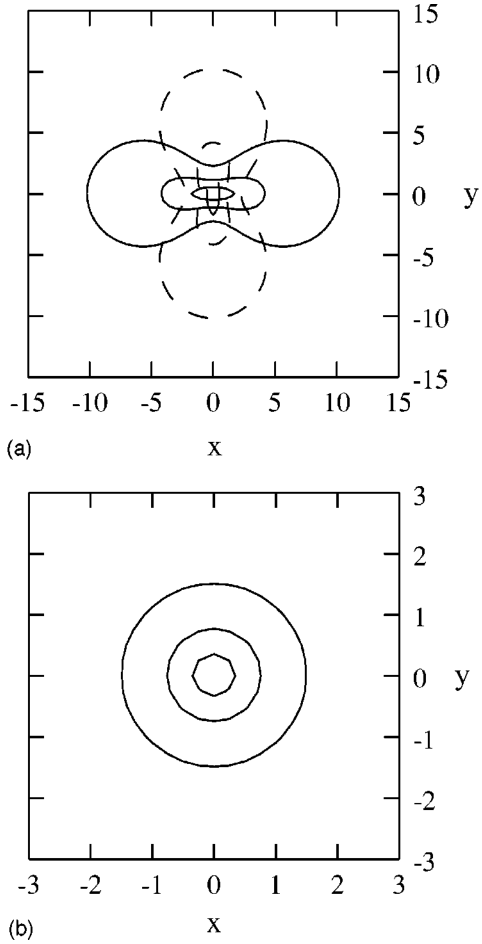

FIG. 6. Strongly anisotropic distribution of $|\psi^1|$ (solid line) and $|\psi^2|$ components of the order parameter for $\kappa=5$ (a) becomes completely isotropic at $\kappa=0.73$ (b).

demonstrates that $\alpha=1/2$. We have checked numerically the rest of critical exponents as well.

V. SUMMARY AND DISCUSSION OF VECTOR VORTEX SYSTEMS

One can conclude that each vortex in *p*-wave superconductor is characterized by its ''orientation'' in flavor (component) space. In the case of the superconducting *B* phase studied here this orientation is determined by a reper of three perpendicular unit vectors $\vec{n},\vec{m},\vec{l}$ in the order-parameter space. In fact it means that in contrast to usual AV's, which is completely isotropic in the plane perpendicular to the external magnetic field the VV's appears to be arbitrarily directed

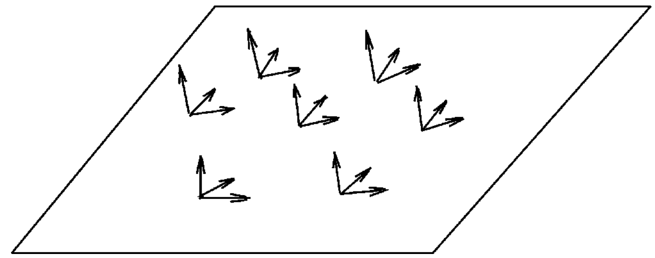

FIG. 7. Schematic picture of the VV lattice. Each vortex has ''internal degree of freedom'' described by a reper $\vec{n},\vec{m},\vec{l}$ or matrix *R*. Interactions between vortices induce long-range order of these ''rotational in order-parameter space'' degrees of freedom.

in the usual coordinate space. On the other hand, in special, experimentally important cases, when the VV appears parallel to the external magnetic field it is still arbitrarily directed in the plane XY perpendicular to the magnetic field. This anisotropy presented in Fig. 6 completely disappears for the *p*-wave superconductors with small $\kappa\rightarrow\kappa_c$. We have found that the critical value of $\kappa=\kappa_c=1/\sqrt{2}$ is universal, independent of the coupling magnitude β while the VV at this κ becomes completely isotropic and very similar to usual AV's. In particular, while at large κ the ψ^3 component of the order parameter is nonzero in the vortex core, this component vanishes as $|\kappa-\kappa_c|^\alpha,\alpha=1/2$ for $\kappa\rightarrow\kappa_c$. Strong asymmetry of ψ^1,ψ^2 of the components of the order parameter disappears in this limit (Fig. 6).

We estimated the line energy of the VV and thereby H_{c1}. Spatial profile of both the order parameter and magnetic field of the vortex demonstrate the essential difference of the vortex structure for different κ parameters and from those calculated for the vortex in liquid ^{3}He. We expect that even for not very large κ basic features of vector vortices are apparent unless κ approaches $\kappa_c=1/\sqrt{2}$.

Now we qualitatively discuss systems of vector vortices since its physics is quite interesting and differs considerably from that of the conventional one-component superconductors. A magnetic field just above H_{c1} generates a sparse hexagonal array of VV's. Since at large distances interactions are mainly electromagnetic, the orientation of each VV vortex is random (degeneracy is lifted by tiny interactions). When the magnetic field grows and the intervortex distance is decreased, the order-parameter orientation dependent part of the interaction between VV's becomes non-negligible and tries to order orientations of VV's ''ferromagnetically'' (short-range order). The orientation of VV's at site *i* is denoted by a matrix R_i, where the matrix elements are the angles between unit reper vectors $\vec{n},\vec{m},\vec{l}$ in the flavor space

TABLE I. Properties of various topological defects in a multicomponent superconductor. Abbreviations: H, hexagonal lattice; N, no SRO; F, ferromagnetic SRO; LRO, long-range order; SRO, short-range order.

Vortex	Screening current	Magnetic field	Order parameter	LRO	SRO
AV	Isotropic	Isotropic	Isotropic, $\psi=0$	H	N
VV with $\kappa\rightarrow 1/\sqrt{2}$	Isotropic	Isotropic	Isotropic, $\psi=0$	H	N
VV with large κ	Isotropic	Isotropic	Anisotropic, $\psi\neq 0$	H	F

and vector $\mathbf{r}=(x,y,z)$ in the real space. The ''ordering'' means $R_i=R$ for all sites i of the lattice (Fig. 7). The system becomes similar to that of classical spins in a 2D hexagonal lattice with two important differences. The direction of the magnetic field does not disappear. If thermal fluctuations are taken into account orientation of spins at different z might be different although stiffness is very large. We ignore it for a moment. Another difference is that the ''spin'' is characterized by a matrix rather than by a vector. The interaction is short range and therefore one can consider only nearest neighbors on the hexagonal lattice as a good approximation. Utilizing symmetries of the system an effective 2D Hamiltinian can be written as

$$H=\frac{1}{2}\sum_{\langle ij\rangle}\mathrm{Tr}(R_iR_j^{-1}).$$

The models of this sort have been extensively studied in statistical physics and even exact results are known.[13] It might be a very promising new system for comparison of a well developed theory with experiment.

The main differences between AV's and the VV's are presented in Table I (external magnetic field is directed parallel to the ψ^3 component of the order parameter).

ACKNOWLEDGMENTS

This work was supported by The Israel Science Foundation Center of Excellence Program ESF Program ''Cosmology in the Laboratory,'' and by the Heinrich Hertz Minerva Center for High Temperature Superconductivity. We are also grateful to the Binational Israel-USA Foundation for support and to the Inter-University Computational Center for providing Cray J932 supercomputer facilities. B.R. is very grateful to the Physics Department at Bar-Ilan University for the warm hospitality and National Science Council of Taiwan for support.

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