Albert Katz International School for Desert Studies
Ben-Gurion University of the Negev, Sede Boqer Campus

Pattern Formation and Spatial Ecology

Course number 001-2-4022
Instructor: Prof. Ehud Meron

3 credits

Self-organization processes leading to pattern formation phenomena are ubiquitous in nature. Intensive theoretical and experimental research efforts during the past few decades have resulted in a mathematical theory of pattern formation whose predictions are well confirmed by controlled laboratory experiments. There is an increasing observational evidence that pattern formation plays a crucial role in the dynamics of spatially extended ecosystems. The purpose of this course is to provide basic knowledge of pattern formation theory and apply it to a few topics in spatial ecology, including vegetation pattern formation in water-limited systems and desertification.

Syllabus

I. Mathematical background
   a. Linear algebra
   b. Linear differential equations

II. Pattern forming systems
   a. Thermal convection
   b. The Belousov-Zhabotinsky reaction
   c. Dryland vegetation

III. Low dimensional dynamical systems
   a. Defining a dynamical system
   b. Basic instabilities
   c. Variational vs. non-variational systems
   d. Two-species population dynamics

IV. Pattern formation analysis
   a. Basic models: Swift-Hohenberg, FitzHugh-Nagumo
   b. Linear stability analysis: symmetry breaking instabilities
   c. Nonlinear analysis: amplitude equations, secondary instabilities
   d. Singular perturbation theory: front solutions, front bifurcations

V. Basic mechanisms of pattern formation
   a. Nonuniform instabilities of uniform states
   b. Multiplicity of stable states and localized structures
   c. Instabilities of localized structures

VI. Applications to spatial ecology:
   a. Dryland vegetation models
   b. Vegetation pattern formation
   c. Regime shifts
   d. Restoration of degraded vegetation

General textbooks:

1. E. Meron. Nonlinear Physics of Ecosystems. Taylor & Francis, 2015
2. S.H. Strogatz, Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry and Engineering. Addison-Wesley, Reading Mass., 1994.
3. M.C. Cross and H. Greenside. Pattern Formation and Dynamics in Nonequilibrium Systems. Cambridge University Press, 2009.

Additional selected readings will be chosen from a variety of books and journals.