

# Syllabus

## Non-Newtonian Fluid Mechanics

001-2-4011

Dr. Roiy Sayag

Year 2021, Spring

3 Credits

### Course Description:

Creeping flow over permeable boundaries, blood flow in small capillaries, the spreading of ice sheets into the ocean, and lava emerging from a volcano, are some examples of flows that are characterised by small inertia compared with viscous forces, may involve moving fronts and interfaces as well as non-linear/non-Newtonian fluids. Understanding such flows is essential to modelling geophysical, physiological and engineering phenomena.

Major topics that will be discussed are dimensional analysis and scaling, vector and tensor calculus, fundamental conservation laws that govern flows and flow of negligible inertia; Non-Newtonian fluids: Generalised Newtonian, Linear and non-Linear Viscoelasticity, viscoplastic fluids; fluid mechanics of rheometry; Buoyancy-driven thin flows of Newtonian and non-Newtonian fluids, Lubrication theory, Similarity solutions; Moving boundaries and interfaces between immiscible fluids, Internal fronts and the transition from shear-dominated to extensional dominated flows; Multiphase flow

### Course objectives:

This course aims to examine and motivate the mathematical modeling of physical processes that govern a range of natural phenomena, and that are also relevant to understanding various environmental and engineering processes. Specific objectives are to develop qualitative and quantitative understanding of general non-Newtonian fluid mechanics and to acquire modelling and solving skills of flows with negligible inertia that involve non-Newtonian fluids and moving interfaces.

### Course requirements:

Graduate student

### Structure of final course grade:

1. Problem sets 40%

2. Final project 60%

—— Total: 100%

A "Pass" requirement regarding final Project: Yes, 65

### Lecturer details:

Reception hours: Day, 16:00-18:00, Building, Room E-mail: roiy@bgu.ac.il

## Description of Meetings:

### Part I: Introduction

Meeting-1: Phenomenology and Preliminaries

1.1. Newtonian vs non-Newtonian phenomenology ((4, Chapter 2))

1.2. Examples: complex fluids in nature: Ice, Lava, Mud ((5, Chapter 9), (2, Chapter 2)) 1.3. An introduction to dimensional analysis

Meeting-2: Fluid Mechanics Preliminaries (notes, (3, Chapter 3)), (10, Chapter 2) 2.1. Introduction to vector and tensor calculus

2.2. reference frames: Eulerian vs Lagrangian

2.3. Conservation laws (mass, linear (Cauchy's equation) and angular momentum, energy)

Meeting-3: Fluid Mechanics Preliminaries (cont)

### Part II: Non-Newtonian fluids basics

Meeting-4: Material functions: Shear and Elongational flows, steady and unsteady ((4, Chapter 3), (11, Chapter 5))

Meeting-5: Generalised-Newtonian fluids ((4, Chapter 4), (11, Chapter 7))

Meeting-6: Linear Viscoelasticity ((4, Chapter 5), (11, Chapter 8))

Meeting-7: Non-linear Viscoelasticity ((4, Chapter 6), (11, Chapter 9))

### Part III: Applications: Buoyancy-driven flows:

Meeting-8: Introduction (8; 7; 15; 13)

8.1. Motivation: Lava spill, grounded ice-sheet, honey on a toast

8.2. Lubrication theory

8.3. Gravity current in 2D: contact line & similarity ((6)) solutions

Meeting-9: Viscoplastic flows (i)

9.1. Introduction: Bingham fluid in between concentric cylinders

9.2. Shallow lava theory ((2, Chapter 7), (8))

Meeting-10: Viscoplastic flows (ii)

10.1. Lava domes

10.2. Blood flow ((5, Chapter 3), (9, Chapter 4, and selected topics))

Meeting-11: Multiphase flow (personal notes; (8; 1; 14))

11.1. Motivation: carbon sequestration

11.2. Flow over a permeable boundary without capillary threshold

Meeting-12: Internal contact lines in buoyant viscous sheets

12.1. Marine ice sheets (personal notes, (12))

## Part IV: Closure

Meeting-13: Fluid Dynamics of Rheometry

13.1. Fluid Dynamics of Rheometry

13.2. Student presentations of personal projects

### Prerequisites

Recommended: Undergraduate courses in fluid/continuum mechanics, ordinary/partial differential equations, basic knowledge of Matlab

### Course Bibliography

#### REFERENCES

- [1] Acton, J. M., Huppert, H. E. & Worster, M. G. 2001 Two-dimensional viscous gravity currents flowing over a deep porous medium. *J. Fluid Mech.* DOI: [10.1017/S0022112001004700](https://doi.org/10.1017/S0022112001004700). URL <https://doi.org/10.1017/S0022112001004700>.
- [2] Balmforth, N.J. & Provenzale, A. 2001 *Geomorphological Fluid Mechanics*, Lecture Notes in Physics, vol. 582. Springer.
- [3] Batchelor, G. K. 1997 *An Introduction to Fluid Dynamics*. Cambridge Univ. Press.
- [4] Bird, R. B., Armstrong, R. C. & Hassager, O. 1987 *Dynamics of polymeric liquids*, volume 1: Fluid mechanics, 2nd edn., , vol. 1. Wiley-Interscience.
- [5] G. Batchelor, K. Moffatt & Worster, G. 2000 *Perspectives in Fluid Dynamics*. Cambridge Univ. Press.
- [6] Huppert, H. E. 1982 Flow and instability of a viscous current down a slope. *Nature* DOI: [10.1038/300427a0](https://doi.org/10.1038/300427a0). URL [http://adsabs.harvard.edu/cgi-bin/nph-data\\_query?bibcode=1982Natur.300..427H&link\\_type=ABSTRACT](http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=1982Natur.300..427H&link_type=ABSTRACT).
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- [8] Huppert, H. E. 2006 Gravity currents: a personal perspective. *J. Fluid Mech.* DOI: [10.1017/S002211200600930X](https://doi.org/10.1017/S002211200600930X). URL [http://adsabs.harvard.edu/cgi-bin/nph-data\\_query?bibcode=2006JFM...554..299H&link\\_type=ABSTRACT](http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=2006JFM...554..299H&link_type=ABSTRACT).
- [9] K. B. Chandran, S. E. Rittgers & Yoganathan, A. P. 2012 *Biofluid Mechanics: The Human Circulation*. CRC.
- [10] Landau, L. D. & Lifshitz, E. M. 1987 *Fluid Mechanics*, 2nd edn., A Course of Theoretical Physics, vol. 6. Butterworth-Heinemann, 552p.
- [11] Morrison, F. A. 2001 *Understanding Rheology*. Oxford University Press.
- [12] Robison, R. A. V., Huppert, H. E. & Worster, M. G. 2010 Dynamics of viscous grounding lines. *J. Fluid Mech.* DOI: [10.1017/S0022112009993119](https://doi.org/10.1017/S0022112009993119). URL [http://www.journals.cambridge.org/abstract\\_S0022112009993119](http://www.journals.cambridge.org/abstract_S0022112009993119).
- [13] Sayag, R. & Worster, M. G. 2013 Axisymmetric gravity currents of power-law fluids over a rigid horizontal surface. *J. Fluid Mech.* DOI: [10.1017/jfm.2012.545](https://doi.org/10.1017/jfm.2012.545).
- [14] Spannuth, M J, Neufeld, J A, Wettlaufer, J S & Worster, M G 2009 Axisymmetric viscous gravity currents flowing over a porous medium. *J. Fluid Mech.* DOI: [10.1017/S0022112008005223](https://doi.org/10.1017/S0022112008005223). URL [http://www.journals.cambridge.org/abstract\\_S0022112008005223](http://www.journals.cambridge.org/abstract_S0022112008005223).
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