Static and dynamic elastic moduli of organic-rich chalk

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Received May 2017, revision accepted February 2019

ABSTRACT

The elastic moduli and anisotropy of organic-rich rocks are of great importance to geoengineering and geoprospecting of oil and gas reservoirs. In this paper, we probe into the static and dynamic moduli of the Ghareb–Mishash chalk through laboratory measurements and new analytical approaches. We define a new anisotropy parameter, ‘hydrostatic strain ratio’ (Ω), which describes the differential contraction of anisotropic rocks consequent to hydrostatic compression. Ω depends on the C_{11}, C_{12}, C_{13} and C_{33} stiffness constants of a transversely isotropic material, and therefore enables a unique insight into the anisotropic behaviour of TI rocks. Ω proves more sensitive to anisotropy within the weak anisotropy range, when compared with Thomsen’s ε and γ parameters. We use Ω to derive static moduli from triaxial compression tests performed on a single specimen. This is done by novel employment of a hydrostatic-deviatoric combination for transversely isotropic elastic stiffnesses. Dynamic moduli are obtained from acoustic velocities measurements. We find that the bedding-normal velocities are described well by defining kerogen as the load-supporting matrix in a Hashin–Shtrikman model (‘Hashin–Shtrikman (HS) kerogen’). The dynamic moduli of the Ghareb–Mishash chalk in dry conditions are significantly higher than the static moduli. The dynamic/static moduli ratio decreases from ~4 to ~2 with increasing kerogen content. Both the static and dynamic moduli decrease significantly with increasing porosity and kerogen content. The effect of porosity on them is two times stronger than the effect of kerogen.

Key words: Anisotropy, Rock physics, Elastics, Petrophysics.

1 INTRODUCTION

The interest in source rocks has increased during the past decades due to their potential of being self-sourced reservoirs for oil and gas. This is in addition to their important role in the study of petroleum systems and exploration of conventional reservoirs. As a consequence, rock physics studies of source rocks have advanced to address their complex chemical composition and microstructure. Source rocks are typically fine-grained organic-rich sedimentary rocks, mostly shales. Their mineralogy often comprises a mixture of silicates, clays in particular, and carbonates in different proportions. The rock investigated in this study is an organic-rich chalk. Chalks consist of mixtures of skeletal components, dominated by coccoliths and pelagic foraminifera, where the predominant mineral is calcite (Pollastro and Scholle 1986). Upon burial, the porosity of chalks is reduced due to compaction, development of calcite cement and stylolites, so that its stiffness is increased (Fabricius 2003). These chalks may be defined as limestones, owing to the increase in induration (Fabricius 2014). However, the studied organic-rich chalk satisfies both mechanical and sedimentological definitions. The inherent shape of the fossils and the prevalence of fossil fragments in a granular calcite matrix define the texture of chalks (typically mudstone and

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[Correction added on 6 March 2019, after first online publication: Several equations were wrongly quoted; and they have been corrected in this version.]
wackestone). The texture of chalks is quite different from a typical laminated organic-rich shale (e.g. Vernik and Milovac 2011). The organic matter distribution in the matrix of chalks may also differ from shales, and thus might have different mechanical influence. We believe organic-rich chalks combine some of the characteristics of both organic-rich shales and non-organic chalks.

As one would expect, the mechanical properties of organic-rich shales depend strongly on organic matter and clay content (e.g. Vernik and Milovac 2011; Sone and Zoback 2013). Organic-rich shales usually display a laminated microstructure with clay minerals and kerogen particles aligned with the bedding direction. Naturally, the bedding introduces mechanical anisotropy which is often correlated with kerogen and clay contents (Vernik and Nur 1992; Vernik and Landis 1996; Carcione 2000; Dewhurst et al. 2011; Sone and Zoback 2013). Due to this pattern, organic-rich shales are treated as vertically transversely isotropic (VTI) materials where the bedding direction is assumed to be horizontal. Characterization of elastic properties of VTI rocks incorporates five independent elastic stiffness constants (detailed in Section 2.4). Consequently, mechanical or acoustic testing is done in three directions at least, typically 0°, 45° and 90° to the symmetry axis, that is the bedding-normal direction. Anisotropy is quantified using Thomsen’s parameters: ε for vertical versus horizontal P-wave velocity; γ for vertical versus horizontal S-wave velocity; and δ for angular dependence of wave propagation in anisotropic rocks (Thomsen 1986; mathematical definitions are in the Appendix). Thomsen’s parameters were defined to simplify equations that describe phase velocities of P- and S-waves in a VTI media, under the assumption of weak anisotropy (i.e. ε < 0.2).

Theoretical models of shales account for their layered microstructure using Backus or Krief–Gassmann methods (Carcione, Helle and Avseth 2011; Zhao et al. 2016), but can also be modeled using theoretical bounds like Voigt–Reuss and Hashin–Shtrikman models (Hashin and Shtrikman 1963; Mavko, Mukerji and Dvorkin 2009; Sone and Zoback 2013). While shales are mostly composed of clay minerals, which are anisotropic and platy, chalks mainly contain calcite, which is considered isotropic in rock physics modeling. The calcite in chalks occurs in various morphologies, such as flat fragments, equant grains and hollow microfossils. Therefore, chalks might display different anisotropic behaviour than shales. Chalks may exhibit elastic anisotropy (e.g. Talesnick, Hatzor and Tsesarsky 2001; Korsnes et al. 2008) but may also be isotropic (e.g. Røgen et al. 2005; Fabricius et al. 2007; Olsen 2007; Palchik 2013). For example, Niobrara chalk and marl samples with low porosity (~4%), and total organic carbon (TOC) of up to 5%, exhibit elastic isotropy to slight anisotropy (Bridges 2016). Our analysis takes a VTI model into account, with an emphasis on elastic properties in static loading rates and their association with the dynamic elastic properties.

Loading conditions are generally divided into quasi-static (referred to as ‘static’) and dynamic. In static conditions, strain rates are typically $10^{-3} - 10^{-4} \text{s}^{-1}$, and strain amplitudes are at the order of $10^{-2} - 10^{-4}$ (Fjær et al. 2008; Fjær, Stroisz and Holt 2013). In dynamic conditions, strain rates may range between $1 \text{ s}^{-1}$ at ultrasonic frequencies and $10^{-4} \text{s}^{-1}$ at seismic frequencies, and strain amplitudes are at the order of $10^{-7}$. In an ideally homogeneous, linearly elastic material (e.g. metals, glass) the static and dynamic moduli are equal (Simmons and Brace 1965; King 1969; Ledbetter 1993). However, in most rock types static and dynamic moduli are unequal, and the difference between them has been extensively studied in the past (e.g. Zisman 1933; Simmons and Brace 1965; Jizba, Mavko and Nur 1990; Tutuncu and Sharma 1992; Gommessen and Fabricius 2001; Sone and Zoback 2013; Brotons et al. 2016). This discrepancy is due to variation in mechanical properties between the rock components that cause different local stress concentrations, and thus different strain magnitudes (Fjær et al. 2013). The dynamic/static moduli ratio (DSMR) decreases towards unity as stiffness and compactness increase (Brotons et al. 2016). This is mainly attributed to porosity and its various features (e.g. size, shape, distribution, association with cracks) that induce greater impact on rock deformability in static conditions. Cracks in particular increase the DSMR, by affecting the static moduli more than the dynamic moduli (King 1969; Ong et al. 2016; Meléndez-Martínez and Schmitt 2016). The DSMR may approach unity upon closure of discontinuities and tightening of solid–solid contacts (e.g. Cheng and Johnston 1981; Jizba et al. 1990). The issue of stress dependence of the DSMR is therefore complex, as discontinuities may be either formed or closed upon changes in stress conditions. Other possible sources for difference between static and dynamic moduli are mineralogy, specifically clay content (Tutuncu and Sharma 1992) and cement overgrowth (Al-Tahini, Sondergeld and Rai 2004). The DSMR may also be influenced by dispersion in liquid-saturated rocks, but in dry rocks the effect of dispersion is insignificant (Hofmann 2006; Olsen 2007). This suggests that in dry rocks the difference between static and dynamic moduli is controlled by the differences in strain amplitudes (Fjær et al. 2008). In this paper, we examine our results with respect to the above-mentioned mechanisms. In
Elastic moduli of organic-rich chalk

Figure 1 Map of the Shefela basin where organic-rich sequences are thicker than 150 m, delineated by Minster (2009). The thicker sequences are found in the eastern part of the basin, where the Zoharim and Aderet wells are located.

addition, we establish a relationship between the static and dynamic moduli of the Ghareb–Mishash chalk and compare it with relationships for other rock types.

This study focuses on the organic-rich chalk section in the Shefela basin in onshore Israel (Fig. 1). The thickness of the organic-rich sequence in the Shefela basin ranges from 150 m at its margins to 550 m at the depocenter (Gvirtzman, Moshkovitz and Reiss 1985; Minster 2009). The unit is currently at depths of several hundred meters, mostly within the ranges of 250–600 m. It has never experienced significant burial (Burg and Gersman 2016), so that the organic matter in the section is immature, with an average $T_{\text{max}}$ of 412°C (Meilijson et al. 2015). The organic matter mainly consists of a kerogen, classified as Type IIS according to H/C versus O/C path in a Van Krevelen diagram and high sulfur concentration (Spiro 1980). The organic-rich chalk studied here includes the Ghareb Formation and the underlying Mishash Formation of the Mount Scopus Group (Late Cretaceous). There are some occurrences of marl in the Ghareb Formation and chert in the lower Mishash Formation (Meilijson et al. 2014; Burg and Gersman 2016).

Previous petrophysical and mechanical studies of this rock were performed using core material from the Aderet and Zoharim wells in the Shefela basin (Bisnovat et al. 2015; Shitrit et al. 2016). The Ghareb–Mishash chalk was found to have high porosity (20–45%) and high TOC (1–20 wt.%). The rock is considered tight, where permeability is anisotropic and ranges between 0.001 and 1 mD. Mechanical softness is exhibited by low elastic moduli, slow acoustic velocities, and low compressive and tensile strengths. The strength of the rock, both in tension and compression, was found to be independent of the organic matter and strongly dependent on the porosity. In contrast, the dynamic and static elastic parameters were found to be linked with kerogen content. The matrix of the Ghareb–Mishash chalk is considered soft, as indicated by the characteristic parameters obtained from two fluid substitution models: The Biot–Gassmann model with Biot coefficient of $\beta \sim 0.9$, and Marion’s bounding average method model with a normalized stiffness factor of $w \sim 0.2$ (Shitrit et al. 2016).

In this paper, we probe deeper into the elastic behaviour of the organic-rich chalk at dry conditions, focusing on the mechanical effects of porosity, kerogen and microstructure. Elastic anisotropy is examined using a newly defined anisotropy parameter and compared with other organic-rich rock formations. Effective medium and fluid substitution models are used to compare between static and dynamic elastic moduli at dry conditions. We present a novel method for analysis of stress and strain data and derivation of elastic moduli, from a single conventional triaxial compression test using a VTI rock sample.

2 METHODS

2.1 Petrophysical properties

The petrophysical properties were measured to obtain the volumes of the phases that control the mechanical behaviour of the rock, typically the porosity and the kerogen. A Coreval30 poro-permeameter, manufactured by Vinci Technologies, was used to measure porosity and matrix density with nitrogen gas (according to Section 5.3.2.2.1 in American Petroleum Institute 1998). Cylindrical core plugs with a diameter of 1 inch and length of 1 inch, were used for these measurements. Prior to the measurements, the plugs were dried in an oven for 24 h at 105°C. The precision of the Coreval30 system is 0.05 cm$^3$, thus the resultant measurement error is 0.5%. Confining pressure of 400 psi (2.8 MPa) was applied on an elastomer sleeve that surrounds the core plugs. Organic matter content was estimated using TOC measurements performed in a LECO SC632 sulfur-carbon elemental analyser. Note that the terms ‘organic content’ and ‘kerogen content’ throughout this paper refer to volume fractions of these phases. The organic matter
in the Zoharim section is almost entirely made of kerogen, so that organic content is considered equal to the kerogen content, \( f_k \), computed by (Shitrit et al. 2016)

\[
f_k = \frac{\rho_{\text{dry}} \times \text{TOC}}{C_k \rho_k},
\]

where \( \rho_{\text{dry}} \) is the dry bulk density, \( \rho_k \) is the kerogen density and \( C_k \) is the carbon weight fraction in the organic matter (\( \sim 0.7 \)). We previously found that the average kerogen density is 1.43 g/cm\(^3\) (Shitrit et al. 2016). The error on \( f_k \) is estimated to 5.5%, accounting for the accuracy of TOC measurements, and reasonable ranges of kerogen density and \( C_k \) (\( \pm 0.06 \) g/cm\(^3\) and \( \pm 0.05 \), respectively). Based on the porosity and kerogen content, the volume percentage of minerals was calculated by

\[
f_m = 100 - \varphi - f_k.
\]

2.2 Triaxial and hydrostatic compression tests

Compression experiments were performed using a hydraulic, servo-controlled triaxial testing system manufactured by TerraTek Inc. (model FX-S-33090). The stiff load frame was operated using a closed-loop servo control with maximum axial force of 1.4 MN, stiffness of \( 5 \times 10^9 \) N/m, and confining pressure capacity of 70 MPa. The linearity of the load cell, strain transducers, and confining pressure are 0.5%, 1%, and 0.25% full scale, respectively. Axial stress, confining pressure, axial strain and the two orthogonal radial strains were continuously recorded during the tests. Triaxial tests were performed under a constant axial strain rate of \( 1 \times 10^{-5} \) s\(^{-1}\). The confining pressure was controlled by a manual valve. Tests used NX size samples, that is right circular cylinders with nominal diameter of 54 mm and length/diameter ratio of approximately two, and specimen ends flattened to less than \( \pm 0.01 \) mm (International Society for Rock Mechanics 2007). All compression tests were performed with the bedding plane direction normal to the axis of the solid cylinders, thus defining the samples as vertically oriented (Fig. 2). No horizontally oriented samples were available as the core diameter of \( \sim 3 \) inches (\( \sim 75 \) mm) was too short for the experimental setup. After the specimens were drilled and polished, they were kept at room conditions for at least 24 h before the tests. The specimens were not dried in an oven because of their tendency to crack during heating.

Triaxial compression experiments were performed to obtain the static elastic constants and the compressive strength of the rock. In this paper, we focus on the elastic behaviour of the rock and failure processes are not discussed here. Each triaxial experiment started with a hydrostatic (non-deviatoric) compression stage until the desired confining pressure value was reached. Confining pressure values were chosen to be similar to the minimum horizontal stress in the Zoharim section, estimated from hydraulic jacking tests performed by Golder Associates (2011). Continuous recording of the strains throughout the hydrostatic stage allowed us to obtain the static bulk modulus. In the subsequent deviatoric compression stage, the axial stress was elevated while confining pressure was maintained at the final value from the hydrostatic stage. The methodology of elastic moduli derivation is detailed in Section 2.4. We also analysed existing results of uniaxial and triaxial compression experiments performed by Bisnovat et al. (2015) on core samples from the nearby Aderet well (Fig. 1).

2.3 Ultrasonic wave velocities and dynamic moduli

The ultrasonic velocities in VTI rocks are commonly used for calculating the five characteristic stiffness constants (Mavko et al. 2009):
where \( \rho \) is bulk density, \( v_{p0}, v_{s0} \) and \( v_{p90}, v_{s90} \) are the P-wave velocities in the bedding-normal, bedding-diagonal and bedding-parallel directions (respectively), and \( v_{p}, v_{s} \) and \( v_{p90}, v_{s90} \) are the S-wave velocities in the bedding-normal and the fast bedding-parallel directions (respectively). Dynamic moduli of the rock are derived from their ultrasonic P-wave and S-wave velocities, measured on cylindrical core plugs with length of 1 inch and diameter of 1 inch in the Acoustic Velocity System (AVS350) manufactured by Vinci Technologies. The AVS350 system is designed for high-frequency signals using 500 kHz piezoelectric transducers. The waves traverse through pressurized plugs, with individual control on axial, radial and pore pressures. The measurements errors of \( V_p \) and \( V_s \) in dry conditions are 1.9% and 2.3%, respectively, and in brine-saturated conditions 0.7% and 9.1%, respectively.

The ultrasonic velocities data set is divided into two groups: (i) existing data of bedding-normal P-wave and S-wave velocities of brine-saturated core plugs presented in Shitrit et al. (2016); and (ii) measured P-wave and S-wave velocities of dry core plugs in both bedding-parallel and bedding-normal directions. The brine-saturated plugs were measured under axial, radial and pore pressures that were assumed to be the vertical stress, minimum horizontal stress and pore pressure in the field, respectively, as estimated from hydraulic jacking tests (Golder Associates 2011). Pore water had a salinity of 0.6 mgNaCl/gH₂O to simulate the brine in the Zoharim well. Acoustic velocities were measured only on core plugs from the Zoharim well in this study.

The brine-saturated data published in our previous study included the bedding-normal velocities that lead to the vertical oedometer modulus \( M_v \) (i.e. \( C_{13} \)) and the vertical shear modulus \( \mu_v \) (i.e. \( C_{33} \)). Here we combine Marion’s bounding average method (BAM) for fluid substitution with Hashin–Shtrikman (HS) theoretical bounds to calculate the dry rock parameters. The HS upper and lower bounds of bulk and shear moduli are given by Hashin and Shtrikman (1963) and Mavko et al. (2009):

\[
K_{HS}^{+/-} = \left( \frac{1}{\sum_{i=1}^{n} f_i K_i + \frac{1}{2} \mu_{max/min}} \right)^{-1} \frac{2}{3} \mu_{max/min}, \tag{4}
\]

\[
\mu_{HS}^{+/-} = \left( \frac{1}{\sum_{i=1}^{n} f_i \mu_i + \xi_{max/min}} \right)^{-1} \xi_{max/min}, \tag{5}
\]

where ‘+’ and ‘-’ denote upper and lower bounds, \( f_i \) is the volume fraction of each phase and max and min refer to the theoretical moduli of the stiffest and softest phases, respectively. The \( \xi \) of each phase is calculated by Mavko et al. (2009):

\[
\xi = \frac{\mu}{6} \left( \frac{9K + 8\mu}{K + 2\mu} \right). \tag{6}
\]

The upper bound represents the moduli of a homogenous granular package of spheres, where the stiff phase is their shell and softer phases are embedded in them. The average modulus of the mineralogical constituents in this case is denoted by subscript max. The lower bound represents the moduli of a matrix supported by the softest phase. For instance, a fluid-suspended matrix is dominated by the elastic moduli of the pore fluid, denoted by subscript min. Generally, the moduli of any arbitrary geometrical arrangement of rock constituents should plot somewhere between the upper and lower HS bounds. The relative distance of a measured data point between them is a reflection of the geometrical arrangement, hence may be used as a parameter for matrix stiffness quantification (e.g. BAM by Marion 1990). We propose here another approach, based on Hashin–Shtrikman bounds. We hypothesize HS curve that simulates a matrix supported by each of the main phases, assuming the rock is primarily divided into three phases: minerals, organic matter and pores. Just as ‘HS+’ simulates a mineral-supported matrix and ‘HS-‘ simulates a fluid-supported matrix, a similar concept can be used to approximate moduli of a kerogen-supported matrix. Hashin–Shtrikman approximation of such matrix is defined here as ‘HS kerogen’. This was done by introducing the shear modulus of kerogen in equation (4) and the \( \xi \) of kerogen in equation (5):
The elastic moduli of this immature kerogen were assigned based on atomic force microscopy measurements by Emmanuel et al. (2016) on immature type II kerogen. They found the reduced Young’s modulus \( E_r \) to be 7.1 GPa. In order to use these measurements for estimation of the bulk and shear moduli of kerogen, \( E_r \) was converted to \( E \) assuming a reasonable Poisson’s ratio of 0.35. We assumed a Poisson’s ratio of 0.35, following checks within the reasonable range (0.05–0.45; Ahmadov, Vanorio and Mavko 2009), and while comparing to values reported for type II kerogens (e.g. Carcione et al. 2011; Yan and Han 2013). This results in Young’s modulus of 6.4 GPa, which leads to bulk modulus of 7.1 GPa and shear modulus of 2.4 GPa. The organic material in the immature state is approximately 94% kerogen and 6% bitumen (further detailed in Shitrit et al. 2017). Using the elastic moduli of bitumen (Table 1), the bulk and shear moduli of the entire organic phase are estimated to be 6.9 and 2.3 GPa, respectively. Note that the relatively high density of the type IIS kerogen in the Ghareb–Mishash chalk may indicate higher stiffness than calculated. Theoretical elastic moduli used for the calculations are summarized in Table 1. In order to compare the measured \( C_{33} \) and \( C_{55} \) with the models, the oedometer modulus of each HS scenario was calculated by
\[
M_{HS}^{+/k/-} = K_{HS}^{+/k/-} + \frac{4}{3} \mu_{HS}^{+/k/-}.
\]
Once the upper and lower bounds were calculated for the brine-saturated rock, the relative positions of the measured moduli between the upper and lower bounds were estimated using Marion’s BAM model (Marion 1990):
\[
w = \frac{A - A^-}{A^+ - A^-},
\]
where \( w \) is the BAM normalized stiffness factor and \( A \) represents any of the computed elastic moduli. The ‘\( w \)’ of each modulus is a characteristic of the geometrical arrangement of the phases and should not change upon fluid substitution. This way, elastic moduli at any fluid composition can be estimated. Here we calculated the HS bounds at brine-saturated conditions, and then used the measured values of \( C_{33} \) and \( C_{55} \) to calculate the ‘\( w \)’. The dry moduli were calculated based on the HS bounds in dry conditions and using the ‘\( w \)’ from the previous step. The BAM model is used here because we found it suitable for fluid substitution in the Ghareb–Mishash chalk, but it should be noted that it was invented based on a heuristic approach rather than on a physical approach (Olsen et al. 2008b).

The dry rock velocities were measured for cores that had static elastic moduli data but lacked brine-saturated velocities data, in order to have more data for comparison between static and dynamic moduli. Additionally, the measured dry rock velocities provided some information about the acoustic anisotropy of the rock and the applicability of the BAM fluid substitution model. We conducted measurements on four pairs of vertically oriented and horizontally oriented plugs. All dry plugs were measured under axial and confining pressure equal to the vertical stress in the field.

### 2.4 Elastic moduli derivation from triaxial tests

Analysing the elastic behaviour of transversely isotropic rocks can be done by tests on a single core sample using special experimental methods. For example, static moduli can be measured using hollow cylinders testing methodology (Talesnick et al. 2001) or using six strain gauges installed at a specific configuration (Togashi, Kikumoto and Tani 2017), and dynamic moduli can be measured using piezoelectric transducers glued at multiple directions (e.g. Sarout et al. 2007). However, common laboratory apparatuses for static measurements are suited for solid cylinders with strain recorded only in the axial and radial directions. Thus full characterization requires multiple specimens drilled in three directions, usually the bedding-normal, bedding-parallel and bedding-diagonal directions. In some cases, in wells for example, core material

<table>
<thead>
<tr>
<th>Mineral</th>
<th>( %w )</th>
<th>( K ) (GPa)</th>
<th>( \mu ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaolinite</td>
<td>&lt;1</td>
<td>44.2</td>
<td>22.1</td>
</tr>
<tr>
<td>Illite</td>
<td>&lt;1</td>
<td>60.4</td>
<td>25.4</td>
</tr>
<tr>
<td>Smectite</td>
<td>0.5</td>
<td>9.3</td>
<td>6.9</td>
</tr>
<tr>
<td>Calcite</td>
<td>60–80</td>
<td>71</td>
<td>30</td>
</tr>
<tr>
<td>Dolomite</td>
<td>&lt;1</td>
<td>80</td>
<td>48</td>
</tr>
<tr>
<td>Quartz</td>
<td>3–10</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>K-spar</td>
<td>&lt;1</td>
<td>37.5</td>
<td>15</td>
</tr>
<tr>
<td>Plag</td>
<td>&lt;1</td>
<td>75.6</td>
<td>25.6</td>
</tr>
<tr>
<td>Pyrite</td>
<td>&lt;1</td>
<td>147</td>
<td>133</td>
</tr>
<tr>
<td>Apatite</td>
<td>0–20</td>
<td>85</td>
<td>46</td>
</tr>
<tr>
<td>Minerals average</td>
<td></td>
<td>66</td>
<td>32</td>
</tr>
<tr>
<td>Kerogen</td>
<td></td>
<td>7.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Bitumen</td>
<td></td>
<td>3.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Organic matter average</td>
<td></td>
<td>6.9</td>
<td>2.3</td>
</tr>
<tr>
<td>Air</td>
<td></td>
<td>0.0001</td>
<td>0</td>
</tr>
<tr>
<td>Brine</td>
<td></td>
<td>2.32</td>
<td>0</td>
</tr>
</tbody>
</table>

Minerals moduli are taken from Mavko et al. (2009), clay minerals specifically from Wang et al. (2001) and volume percentage of each mineral in the organic-rich chalk from X-ray diffraction analysis (average distribution was provided by Israel Energy Initiatives, Ltd.). Elastic properties of immature kerogen are based on Emmanuel et al. (2016), assuming a Poisson’s ratio of 0.35. The elastic properties of bitumen are taken from Han et al. (2006).
is limited and specimens can be drilled only parallel to the borehole axis. Normally these are vertical wells drilled into horizontal rock beds; hence, the cores are referred to as vertically oriented. There is a considerable amount of uniaxial and triaxial compression tests data obtained on vertically oriented VTI specimens in regular compression testing equipment (Fig. 2). We propose here a method for analysing test results in a manner that maximizes the information available from a single compression test of a vertically oriented VTI specimen.

Transversely isotropic rocks have five independent elastic stiffness constants: $C_{11}$, $C_{12}$, $C_{13}$, $C_{33}$ and $C_{55}$. Alternatively, elasticity can be described using engineering constants, for example the Young’s modulus, Poisson’s ratio and shear modulus in the vertical and horizontal directions: $E_v$, $v_v$, $E_h$, $v_h$ and $\mu_v$ (Lo, Coyner and Toksöz 1986; Higgins et al. 2008; Togashi et al. 2017). A typical deviatoric compression stage, either in uniaxial or triaxial tests, yields two elastic moduli, for instance the Young’s modulus and Poisson’s ratio. Young’s modulus is calculated from the slope of the linear segment of the stress difference - axial strain curve ($\Delta \sigma_r$ vs. $\varepsilon_r$), within the elastic range. Poisson’s ratio is determined as the ratio between radial and axial strains ($-\varepsilon_r/\varepsilon_a$) within the same range (note that compression and contraction are considered positive). Although Young’s modulus and Poisson’s ratio are defined at uniaxial stress conditions, the same methodology is frequently applied on triaxial test results (e.g. Section 7.3 in Fjær et al. 2008). We propose here a solution that incorporates the theoretical role of confining pressure in triaxial tests, considering a vertically oriented VTI sample and assuming linear elasticity.

There are two stages in a standard triaxial test: the hydrostatic compression stage and the subsequent deviatoric compression stage. We consider here the stress-strain relationships of a VTI rock as defined by Hooke’s law (Mavko et al. 2009):

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{66} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4 \\
\varepsilon_5 \\
\varepsilon_6
\end{bmatrix}
$$

where subscripts 1, 2 and 3 denote normal stresses and strains, subscripts 4, 5 and 6 denote shear stresses and strains and $C_{66} = \frac{C_{55} - C_{33}}{2}$. These subscripts are the Voigt-notated directions of stiffness constants in anisotropic elastic materials (Fjær et al. 2008; Mavko et al. 2009) and should not be confused with maximum and minimum principal stresses. In VTI rocks, subscripts 1 and 2 represent the directions within the transverse plane and subscript 3 represents the bedding-normal direction (Fig. 2). In a vertically oriented specimen, direction 3 coincides with the axial direction and directions 1 and 2 with the radial directions. Note that the radial strains in directions 1 and 2 must be equal; otherwise, the material has a lower class of symmetry and transverse isotropy cannot be assumed. Furthermore, in order to perform the analysis, we assume that the axis of the sample is perfectly aligned with the bedding-normal direction and that the faces of the sample are parallel (i.e. perfect cylinder). In this theoretical section, we analyse the special cases of vertically oriented VTI sample in hydrostatic and triaxial stress conditions. Because both cases consider principal stresses, shear stresses and strains are not activated. Therefore, by inverting the matrix in equation (10) introducing the stresses applied on a vertically oriented VTI sample in a typical triaxial test, and focusing on the non-zero terms, the strains are

$$
\begin{bmatrix}
\varepsilon_r \\
\varepsilon_a
\end{bmatrix} = \frac{1}{C_{13} (C_{11} + C_{12}) - 2 C_{13}^2}
\begin{bmatrix}
\frac{C_{13} C_{12} - C_{11}^2}{C_{11} - C_{12}} & \frac{C_{12} C_{13} - C_{11} C_{13}}{C_{11} - C_{12}} & -C_{11} \\
\frac{C_{13} C_{12} - C_{11}^2}{C_{11} - C_{12}} & \frac{C_{12} C_{13} - C_{11} C_{13}}{C_{11} - C_{12}} & -C_{11} \\
-C_{11} & -C_{11} & C_{11} + C_{12}
\end{bmatrix}
\begin{bmatrix}
\sigma_r \\
\sigma_a
\end{bmatrix},
$$

(11)

where the radial strain $\varepsilon_r$ aligns with the confining pressure $\sigma_r$, and the axial strain $\varepsilon_a$ aligns with the axial stress $\sigma_a$, applied by the piston. The resultant strains in triaxial stress conditions are

$$
\varepsilon_a = \frac{\sigma_a (C_{11} + C_{12}) - 2 \sigma_a C_{13}}{C_{13} (C_{11} + C_{12}) - 2 C_{13}^2},
$$

(12)

$$
\varepsilon_r = \frac{\sigma_r C_{13} - \sigma_a C_{13}}{C_{13} (C_{11} + C_{12}) - 2 C_{13}^2}.
$$

(13)

There are four unknown parameters in the matrix: $C_{13}$, $C_{33}$, $C_{11}$ and $C_{12}$. We use the stress-strain relations elaborated here to obtain these parameters. First, we derive Young’s modulus, Poisson’s ratio and bulk modulus, for vertically oriented VTI samples. Defined at uniaxial stress conditions ($\sigma_r = 0$), the vertical Young’s modulus and vertical Poisson’s ratio are

$$
E_v = \frac{\sigma_v}{\varepsilon_v},
$$

(14a)
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Figure 3 Typical stress–strain curves in a deviatoric compression stage of a triaxial experiment. Mathematical expressions are presented for the linear parts of the axial and radial strain curves, adjusted for vertically oriented VTI sample. The example shown originates from 431.5 m depth in the Zoharim well.

\[
\nu_v = -\frac{\varepsilon_r}{\varepsilon_a}. \tag{14b}
\]

Thus

\[
E_v = C_{33} - \frac{2C_{13}^2}{C_{11} + C_{12}}, \tag{15}
\]

\[
\nu_v = \frac{C_{13}}{C_{11} + C_{12}}. \tag{16}
\]

Note that the definition of the vertical Poisson’s ratio, \(\nu_v\), is referred to as \(\nu_{31}\) in many studies (e.g. Sone and Zoback 2013; Yan, Han and Yao 2015). A deviatoric compression stage provides two independent stress–strain relationships (Fig. 3): stress difference versus axial strain (\(\Delta \sigma_a\) versus \(\varepsilon_a\)) and stress difference versus radial strain (\(\Delta \sigma_r\) versus \(\varepsilon_r\)). The \(\Delta \sigma_a\) versus \(\varepsilon_a\) relationship can now be written as

\[
\frac{\Delta \sigma_a}{\varepsilon_a} = \frac{(\sigma_a - \sigma_r)(C_{33}(C_{11} + C_{12}) - 2C_{13}^2)}{\sigma_a(C_{11} + C_{12}) - 2\sigma_r C_{13}}. \tag{17}
\]

Rearranged to a linear equation:

\[
\Delta \sigma_a = \varepsilon_a \left( C_{33} - \frac{2C_{13}^2}{C_{11} + C_{12}} \right) + \sigma_r \left( \frac{2C_{13}}{C_{11} + C_{12}} - 1 \right)
= E_v \varepsilon_a + \sigma_r (2\nu_v - 1). \tag{18}
\]

Similarly, the \(\Delta \sigma_r\) versus \(\varepsilon_r\) curve can be written as

\[
\Delta \sigma_r = \frac{E_r}{\nu_r} \varepsilon_r + \sigma_r \left( \frac{C_{33}}{C_{13}} - 1 \right). \tag{19}
\]

Equations (18) and (19) can be used to describe the linear segment of the stress–strain curves, as demonstrated in Fig. 3. Equations (18) and (19) show that the confining pressure, theoretically, does not influence the slopes of the stress–strain curves. We use only the slopes of these linear relationships for deriving the elastic moduli, as the value of intersection with the vertical axis is sensitive to deviations induced by the experimental procedure. We use equation (19) to derive Poisson’s ratio in order to conform with the basic definition, as shown in equation (16). Equations (12) and (13) indicate that the \(-\varepsilon_r/\varepsilon_a\) ratio in triaxial tests is stress dependent, thus deviates from the basic definition of Poisson’s ratio.

Triaxial tests normally start with a hydrostatic compression stage (\(\sigma_r = \sigma_a = p\)). The bulk modulus is defined as the slope of the linear segment of the confining pressure versus volumetric strain curve (\(p\) vs. \(\varepsilon_v\), Fig. 4):

\[
K = \frac{p}{\varepsilon_v} = \frac{p}{\varepsilon_a + 2\varepsilon_r}. \tag{20}
\]
and by combining equations (12), (13) and (20):

\[ K = \frac{C_{33} (C_{11} + C_{12}) - 2C_{13}^2}{C_{11} + C_{12} - 4C_{13} + 2C_{33}} \]  \hspace{1cm} (21)

Equations (15), (16) and (21) were first presented by King (1964), and their derivations are given here to explain the elaboration of relationships between the measured engineering moduli and Voigt-notated elastic stiffness constants in VTI rocks. Using equations (15), (16) and (21), the parameters \( E_v \), \( \nu_v \) and \( K \) from a single triaxial test can be used to compute \( C_{33} \), \( C_{13} \) and \( C_{11} + C_{12} \):

\[ C_{13} = \frac{2KE_v}{\nu_v - K + 4K (1 - \nu_v)}. \]  \hspace{1cm} (22)

\[ C_{33} = E_v + 2\nu_v C_{13}. \]  \hspace{1cm} (23)

\[ C_{11} + C_{12} = \frac{C_{13}}{\nu_v}. \]  \hspace{1cm} (24)

In a triaxial test, each compression stage provides two independent curves: \( p \) versus \( \varepsilon_v \) and \( \varepsilon_r \) versus \( \varepsilon_a \) from the hydrostatic stage, and \( \Delta\sigma_v \) versus \( \varepsilon_v \) and \( \Delta\sigma_a \) versus \( \varepsilon_a \) from the deviatoric stage. Consequently, we have four equations that can be used to solve the four unknown stiffness constants activated in these compression tests: \( C_{11} \), \( C_{12} \), \( C_{13} \) and \( C_{33} \). Unfortunately, \( C_{11} \) and \( C_{12} \) cannot be separated in this system of equations, and only their sum can be computed when vertically oriented specimens are tested. However, the fourth equation can be used to confirm that both the hydrostatic stage and the deviatoric stage are within the same elastic range of the rock. We do this by means of the ratio between radial strain and axial strain during the hydrostatic stage (Fig. 5), defined here as the ‘hydrostatic strain ratio’, \( \Omega \):

\[ \Omega = \frac{\varepsilon_r}{\varepsilon_a} = \frac{pC_{33} - pC_{13}}{p(C_{11} + C_{12}) - 2pC_{13}} = \frac{C_{33} - C_{13}}{C_{11} + C_{12} - 2C_{13}} \]  \hspace{1cm} (25)

Hence,

\[ C_{11} + C_{12} = \frac{C_{33} - C_{13}}{\Omega} + 2C_{13}. \]  \hspace{1cm} (26)

Equations (24) and (26) must lead to the same result to ensure that the constants are derived correctly. From equation (25), we see that the hydrostatic strain ratio must be positive, and for isotropic rocks should equal one (Fig. 5). In typical VTI rocks, \( \Omega \) should be lower than 1, and it can be
Figure 5 Graphical description of the ‘hydrostatic strain ratio’: schematic radial strain–axial strain curves with a slope of ‘$\Omega$’, that is the hydrostatic strain ratio. The value of $\Omega$ decreases from one (isotropic rock, squared profile turns into a smaller square) with increasing anisotropy of a typical VTI rock (squared profile turns into a flattened rectangle).

used for scaling anisotropy. Equation (25) can be modified to

$$\Omega = \frac{C_{33} + C_{11} - 2C_{13}}{C_{11} + C_{12} - 2C_{13}},$$

so we infer that in a typical VTI source rock $C_{33} + C_{13} \leq C_{11} + C_{12}$.

Although $C_{11}$ and $C_{12}$ could not be isolated, we may define some constraints for their minimum and maximum values. Just as we derive the values of $C_{11} + C_{12}$ and $C_{33}$ from equations (23) and (24), we can also calculate the difference between them:

$$C_{11} + C_{12} - C_{33} = \Delta C_{11-33} + C_{12},$$

where $\Delta C_{11-33}$ equals the difference between $C_{11}$ and $C_{33}$. In a typical VTI material, negative Thomsen’s $s$ values are very uncommon (e.g. Vernik and Liu 1997; Zhao et al. 2017), thus $\Delta C_{11-33} \geq 0$. Based on this logic and recalling that $C_{12}$ must be positive, we can constrain the range of $C_{11}$ and $C_{12}$ using the values of $C_{11} + C_{12}$ and $C_{33}$:

$$C_{12,\text{max}} = C_{11} + C_{12} - C_{33},$$

$$C_{12,\text{min}} = 0,$$

$$C_{11,\text{max}} = C_{11} + C_{12},$$

$$C_{11,\text{min}} = C_{33}.$$

Moreover, since $C_{66} = \frac{C_{11} - C_{12}}{2}$, then

$$C_{66,\text{max}} = \frac{C_{11,\text{max}} - C_{12,\text{min}}}{2},$$

$$C_{66,\text{min}} = \max\left\{\frac{C_{11,\text{min}} - C_{12,\text{max}}}{2}, 0\right\}.$$  

The set of theoretical solutions described in this section is summarized in the procedure shown in Fig. 6. It shows the hydrostatic–deviatoric combination for transversely isotropic elastic stiffnesses (HDC–TIES), employed on results from a single triaxial compression experiment using a vertically oriented sample. Note that the HDC–TIES procedure may be used not only where core material is limited but also where data require repeated analysis or where multiple core samples yield inconsistent results.

2.5 Scanning electron microscopy

Three samples from the Zoharim section were analysed by backscatter scanning electron microscopy (SEM) in order to compare the experimental and analytical observations with microstructural features. Before the scans, the samples were polished using carborundum 1000 abrasive powder to reduce surface roughness to the order of few microns. Afterwards, the surface was cleaned using compressed air and the samples were kept in a desiccator. The samples were scanned at the Ilse-Kats Institute for Nanoscale Science & Technology, Ben-Gurion University of the Negev. All of the samples were scanned in a high-resolution SEM (Verios XHR 460L), and one of them was scanned also in a lower resolution environmental SEM (Quanta 200). The voltage and current of the electron beam were adjusted to obtain good quality images without damaging the organic matter. Note that where the beam is too intense, the organic material heats and stiffens irreversibly. The scans were performed using SEM in backscatter mode (BSEM), so that bright and dark materials in the images are mostly identified as minerals and kerogen, respectively. Phases identification was assisted by an energy dispersive spectroscopy tool that measures the abundance of elements in spotted grains and specified areas.

3 RESULTS

3.1 Static elastic parameters

The results of hydrostatic and triaxial compression experiments are summarized in Table 2. The values derived using
Figure 6 The HDC-TIES method: a flow diagram showing the application of the method, using results from a single triaxial compression experiment, performed on vertically oriented VTI sample.

the hydrostatic-deviatoric combination for transversely isotropic elastic stiffnesses (HDC-TIES) method, \( C_{13}, C_{33} \) and \( C_{11} + C_{12} \), and the bounds of \( C_{11}, C_{12} \) and \( C_{66} \), are given in Table 3. The results include tests on core samples from the Zoharim well and repeated analyses of compression tests performed on samples from the Aderet well. The reported errors are the normalized root mean square (RMS) of the data points with respect to the linear curves, from which the static moduli in Table 2 were derived. In some tests, linear segments were less detectable and a wider range of moduli is reported (e.g. 609.5 m depth in Zoharim well). Consequently, the propagated errors in Table 3 are significantly larger. Note that the error induced by stress and strain measurements is negligibly small and contained in the RMS error analysis. We examine here the dependence of each of the elastic moduli on the soft phases: porosity and kerogen. Since both kerogen
and porosity affect the elastic moduli, we sum them into a single parameter as follows:

\[ \phi_{kf} = \phi + k f_k, \]  

(32)

where \( \phi_{kf} \) is the kerogen-factored porosity and \( k \) is a dimensionless influence factor that scales the effect of kerogen. This sort of summation was defined by Prasad et al. (2011) as ‘porosity-modified kerogen content’, where they examined six different types of organic-rich rocks. We refer to this parameter here as ‘kerogen-factored porosity’, because kerogen effects are scaled to those of porosity and not vice versa. Prasad et al. (2011) suggested an influence factor of \( k = 0.4 \), that is simply the sum of porosity and 40% of the kerogen content. By inserting values of ‘\( R \)’ within the range of \( 0 < k < 1 \) and fitting different types of regressions (linear, logarithmic, exponential and polynomial), we get highest coefficients of determination \( (R^2) \) using an exponential regression and \( k \sim 0.5 \). For this reason, \( k = 0.5 \) is chosen in this study. The kerogen influence factor represents two important features of the kerogen: (i) kerogen-hosted porosity that might be undetected in the porosity measurements and (ii) kerogen is softer than the minerals by an order of magnitude. The results from the Zoharim and Aderet wells seem similar, so we do not distinguish between the two wells in our analysis.

Vertical Young’s modulus and vertical Poisson’s ratio are obtained from triaxial and uniaxial compression tests. Bulk modulus and hydrostatic strain ratio are obtained from triaxial and hydrostatic compression tests. The vertical Young’s modulus shows a strong dependence on both porosity and kerogen content (Fig. 7a,b). It is seen that when the kerogen-factored porosity is used, the scattered data points exhibit better convergence (Fig. 7c). The bulk modulus reveals a similar pattern (Fig. 7d-f). The vertical Poisson’s ratio does not show any clear trend versus porosity or kerogen, but it is seen that at \( \phi < 35\% \) the range is 0.15–0.24 and at a higher porosity it can exceed 0.3 (Fig. 8a,b). The hydrostatic strain ratio, that is the anisotropy, does not exhibit a clear regression on either porosity or kerogen content (Fig. 8c,d). The behaviour of the static vertical Poisson’s ratio

### Table 2 Summary of elastic constants derived from hydrostatic, triaxial and uniaxial tests on cores from Zoharim and Aderet wells

<table>
<thead>
<tr>
<th>Well</th>
<th>Depth (m)</th>
<th>Test</th>
<th>( \phi ) (%)</th>
<th>( f_k ) (%)</th>
<th>( \rho_{dry} ) (g/cm³)</th>
<th>( E_v ) (GPa)</th>
<th>( v_v )</th>
<th>( K ) (GPa)</th>
<th>( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zoharim</td>
<td>332.5</td>
<td>Hydrostatic</td>
<td>42.17</td>
<td>7.7</td>
<td>1.50</td>
<td>1.60 ± 0.15</td>
<td>0.49 ± 0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zoharim</td>
<td>469.2</td>
<td>Hydrostatic</td>
<td>36.88</td>
<td>21.2</td>
<td>1.47</td>
<td>1.37 ± 0.04</td>
<td>0.40 ± 0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zoharim</td>
<td>339.1</td>
<td>Triaxial</td>
<td>38.84</td>
<td>14.7</td>
<td>1.43</td>
<td>1.73 ± 0.01</td>
<td>0.18 ± 0.00</td>
<td>1.16 ± 0.02</td>
<td>0.57 ± 0.05</td>
</tr>
<tr>
<td>Zoharim</td>
<td>360.5</td>
<td>Triaxial</td>
<td>38.00</td>
<td>14.3</td>
<td>1.47</td>
<td>1.66 ± 0.01</td>
<td>0.18 ± 0.00</td>
<td>1.27 ± 0.03</td>
<td>0.45 ± 0.04</td>
</tr>
<tr>
<td>Zoharim</td>
<td>431.5</td>
<td>Triaxial</td>
<td>35.92</td>
<td>24.5</td>
<td>1.38</td>
<td>2.21 ± 0.03</td>
<td>0.20 ± 0.01</td>
<td>1.52 ± 0.15</td>
<td>0.62 ± 0.01</td>
</tr>
<tr>
<td>Zoharim</td>
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<td>Triaxial</td>
<td>35.57</td>
<td>21.1</td>
<td>1.45</td>
<td>2.57 ± 0.01</td>
<td>0.22 ± 0.02</td>
<td>1.63 ± 0.03</td>
<td>0.71 ± 0.00</td>
</tr>
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<td>15.0</td>
<td>1.84</td>
<td>2.34 ± 0.04</td>
<td>0.29 ± 0.01</td>
<td>1.50 ± 0.33</td>
<td>0.40 ± 0.08</td>
</tr>
<tr>
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<td>Triaxial</td>
<td>35.65</td>
<td>11.3</td>
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<td>0.19 ± 0.01</td>
<td>2.11 ± 0.40</td>
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</tr>
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<td>29.54</td>
<td>21.1</td>
<td>1.54</td>
<td>2.39 ± 0.01</td>
<td>0.15 ± 0.00</td>
<td>1.87 ± 0.04</td>
<td>0.47 ± 0.01</td>
</tr>
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<td>Zoharim</td>
<td>588</td>
<td>Triaxial</td>
<td>26.15</td>
<td>8.1</td>
<td>1.92</td>
<td>5.36 ± 0.05</td>
<td>0.17 ± 0.01</td>
<td>3.80 ± 0.11</td>
<td>0.72 ± 0.01</td>
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<td>0.24 ± 0.03</td>
<td>4.40 ± 0.29</td>
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</tr>
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<td>Aderet</td>
<td>274.1</td>
<td>Hydrostatic</td>
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<td>1.46</td>
<td>1.38 ± 0.09</td>
<td>0.57 ± 0.03</td>
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<td>Hydrostatic</td>
<td>38.00</td>
<td>12.7</td>
<td>1.55</td>
<td>3.20 ± 0.32</td>
<td>0.32 ± 0.03</td>
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<td>336.2</td>
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<td>1.69 ± 0.05</td>
<td>0.16 ± 0.01</td>
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<td>0.50 ± 0.01</td>
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<td>Triaxial</td>
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<td>1.78 ± 0.27</td>
<td>0.29 ± 0.04</td>
<td>1.20 ± 0.11</td>
<td>0.53 ± 0.01</td>
</tr>
<tr>
<td>Aderet</td>
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<td>Triaxial</td>
<td>42.78</td>
<td>15.9</td>
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<td>1.46 ± 0.02</td>
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<td>0.64 ± 0.01</td>
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<td>352</td>
<td>Triaxial</td>
<td>36.00</td>
<td>19.6</td>
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<td>2.00 ± 0.20</td>
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<td>8.36 ± 0.42</td>
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Table 3 Values of $C_{13}$, $C_{33}$, and $C_{11} + C_{12}$, and constraints on $C_{11}$, $C_{12}$ and $C_{66}$, as derived using the HDC-TIES method. The errors are propagated from the values in Table 2

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$C_{13}$ (GPa)</th>
<th>$C_{33}$ (GPa)</th>
<th>$C_{11} + C_{12}$ (GPa)</th>
<th>$C_{11}$ (GPa) Minimum</th>
<th>Maximum</th>
<th>$C_{12}$ (GPa) Maximum</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
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<tbody>
<tr>
<td>339.1</td>
<td>0.58 ± 0.03</td>
<td>1.94 ± 0.12</td>
<td>3.18 ± 0.17</td>
<td>1.8</td>
<td>3.4</td>
<td>1.5</td>
<td>0.9</td>
<td>1.7</td>
</tr>
<tr>
<td>360.5</td>
<td>0.67 ± 0.05</td>
<td>1.90 ± 0.14</td>
<td>3.72 ± 0.25</td>
<td>1.8</td>
<td>4.0</td>
<td>2.2</td>
<td>0.9</td>
<td>2.0</td>
</tr>
<tr>
<td>431.5</td>
<td>0.80 ± 0.21</td>
<td>2.53 ± 0.72</td>
<td>4.06 ± 1.11</td>
<td>1.8</td>
<td>5.2</td>
<td>3.4</td>
<td>0.9</td>
<td>2.6</td>
</tr>
<tr>
<td>457.5</td>
<td>0.89 ± 0.20</td>
<td>2.96 ± 0.76</td>
<td>4.09 ± 1.03</td>
<td>2.2</td>
<td>5.1</td>
<td>2.9</td>
<td>1.1</td>
<td>2.6</td>
</tr>
<tr>
<td>517</td>
<td>1.24 ± 0.64</td>
<td>3.07 ± 1.64</td>
<td>4.20 ± 2.17</td>
<td>1.4</td>
<td>6.4</td>
<td>4.9</td>
<td>0.7</td>
<td>3.2</td>
</tr>
<tr>
<td>540</td>
<td>1.22 ± 0.53</td>
<td>4.15 ± 1.85</td>
<td>6.41 ± 2.82</td>
<td>2.3</td>
<td>9.2</td>
<td>6.9</td>
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<td>559.3</td>
<td>0.91 ± 0.06</td>
<td>2.67 ± 0.19</td>
<td>6.03 ± 0.39</td>
<td>2.5</td>
<td>6.4</td>
<td>3.9</td>
<td>1.2</td>
<td>3.2</td>
</tr>
<tr>
<td>588</td>
<td>1.86 ± 0.29</td>
<td>5.99 ± 1.06</td>
<td>11.01 ± 1.84</td>
<td>4.9</td>
<td>12.8</td>
<td>7.9</td>
<td>2.5</td>
<td>6.4</td>
</tr>
<tr>
<td>609.5</td>
<td>2.44 ± 0.80</td>
<td>9.84 ± 3.81</td>
<td>12.40 ± 4.42</td>
<td>6.0</td>
<td>16.8</td>
<td>10.8</td>
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</tr>
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<td>336.2</td>
<td>0.65 ± 0.13</td>
<td>1.89 ± 0.45</td>
<td>4.11 ± 0.85</td>
<td>1.4</td>
<td>5.0</td>
<td>3.5</td>
<td>0.7</td>
<td>2.5</td>
</tr>
<tr>
<td>336.2</td>
<td>0.79 ± 0.38</td>
<td>2.24 ± 1.48</td>
<td>2.74 ± 1.40</td>
<td>0.8</td>
<td>4.1</td>
<td>3.4</td>
<td>0.4</td>
<td>2.1</td>
</tr>
<tr>
<td>346.3</td>
<td>1.13 ± 0.05</td>
<td>2.16 ± 0.12</td>
<td>3.83 ± 0.19</td>
<td>2.0</td>
<td>4.0</td>
<td>2.0</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>352</td>
<td>0.74 ± 0.32</td>
<td>2.27 ± 1.29</td>
<td>4.06 ± 1.90</td>
<td>1.0</td>
<td>6.0</td>
<td>5.0</td>
<td>0.5</td>
<td>3.0</td>
</tr>
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</table>

Figure 7 The effect of soft phases on static vertical Young's modulus and bulk modulus of organic-rich chalk: scatter plots of vertical Young’s modulus (upper panel) and bulk modulus (lower panel), versus porosity (a, d), kerogen volume percentage (b, e) and the kerogen-factored porosity (c, f). An apparent exponential regression of the stiffness constants is best fitted to kerogen-factored porosity with $k = 0.5$.

and the hydrostatic strain ratio is further investigated in the Discussion.

The stiffness constants $C_{13}$, $C_{33}$ and $C_{11} + C_{12}$ can be computed using the HDC-TIES procedure only where linear elastic response is observed in both hydrostatic and deviatoric stages of the triaxial compression experiment. These three stiffness constants depend on both porosity and kerogen and correlate best with the kerogen-factored porosity (Fig. 9). The
Figure 8 The effect of soft phases on static vertical Poisson’s ratio and hydrostatic strain ratio of organic-rich chalk: scatter plots of vertical Poisson’s ratio (upper panel) and hydrostatic strain ratio (lower panel), versus porosity (a, c) and kerogen volume percentage (b, d).

conditions for stability of a transversely isotropic medium (positive strain energy; see Carcione 2014) cannot be fully examined here as not all stiffness constants are known, but it is confirmed that

\[(C_{11} + C_{12}) C_{33} > 2C_{13}^2. \]  

(33)

The minimum and maximum values for \(C_{11}, C_{12}\) and \(C_{66}\) are then calculated according to equations (29)–(31), (Table 3, Fig. 10). Although only the constrained values are given in Fig. 10, it is possible to estimate these parameters using empirical relationships, as indicated in Fig. 6.

We find it interesting to examine the error introduced using two common approximations of bulk modulus rather than the exact expression in equation (21). If isotropy were to be assumed, then the bulk modulus would be calculated by

\[K = \frac{E}{3(1-2v)}. \]  

(34)

The comparison shown in Fig. 11 (‘isotropic medium equation’ series) indicates that the approximation results in under-estimation of the true bulk modulus by about 26%. Bulk modulus of an anisotropic medium is sometimes estimated using the best-isotropic approximation (Cavallini 1999; Carcione 2014):

\[K = \frac{1}{9} \left(C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23})\right) = \frac{1}{9} \left(C_{33} + 4C_{13} + 2(C_{11} + C_{12})\right). \]  

(35)

Note that the latter term in equation (35) is the reduction of the first term in VTI materials. Indeed, we find a good agreement between the best-isotropic approximation and the true bulk modulus of our material (Fig. 11).

3.2 Dynamic elastic parameters

3.2.1 Calculated dry rock properties

Here we use the HS upper and lower bounds, and Marion’s bounding average method (BAM) model as described in Section 2.3 to calculate the dynamic moduli of the dry rock. The HS bounds of the oedometer modulus and the shear modulus are calculated to simulate a brine-saturated rock (Fig. 12). The measured \(C_{33}\) and \(C_{55}\) plot between those bounds and seem to decrease with increasing volume of porosity and...
kerogen. The normalized stiffness factors $w_{C_{33}}$ and $w_{C_{55}}$ are calculated using equation (9), and $C_{33}$ and $C_{55}$ in dry conditions are estimated from the HS bounds in dry conditions (Fig. 13). One of the interesting observations is that the ‘HS kerogen’ model matches the observed data at both brine-saturated and dry conditions (Figs. 12–14). It may suggest that the kerogen particles are positioned in mechanically influential locations in the matrix. The 'HS kerogen' values converge to a continuous curve by using the kerogen-factored porosity with kerogen influence factor of $k = 0.5$. The $w_{C_{33}}$ and $w_{C_{55}}$ values correlate well with each other and range between 0.1 and 0.3 (Fig. 15a). Exceptionally high values of $\geq 0.4$ originate from the deep zone of the Mishash Formation, which is more compacted, cemented and leaner in organic matter. Kerogen seems to affect the normalized stiffness factors (Fig. 15b), although the scatter is considerably large and without any conclusive trend.

### 3.2.2 Measured dry rock properties

The dynamic elastic stiffness constants are calculated based on the measured velocities using equations (3), and their results are summarized in Table 4. Values of $C_{13}$ are unavailable as they require the P-wave velocity at 45° to the bedding direction, which we did not measure. We comment here that full characterization of dynamic elastic moduli of this VTI rock, that is measurements of five independent elastic moduli, is outside the scope of the current study. In Table 4, we add more results by Gordin, Hatzor and Vinegar (2016), measured on core samples from the Zoharim well at the same conditions as in this study.

The measured vertical oedometer and shear moduli $C_{33}$ and $C_{55}$, and horizontal oedometer and shear moduli $C_{11}$ and $C_{66}$, are presented in Fig. 13. The $C_{33}$ and $C_{55}$ plot close to the moduli calculated from the brine-saturated samples using the BAM fluid substitution model. The horizontal oedometer and shear moduli are stiffer than the vertical ones, as expected from typical source rocks where horizontal stiffness is greater than vertical stiffness (Figs. 13–15).

### 3.3 Microstructure observations

SEM micrographs of three samples from the Zoharim well are given to relate the physical properties with microstructural
Figure 10 Constrained elastic stiffness constants: minimum and maximum possible values of $C_{11}$ (a), $C_{12}$ (b) and $C_{66}$ (c) versus the kerogen-factored porosity.

features (Fig. 16). Backscatter mode (BSEM) is used to distinguish between the organic matter and minerals, which appear in dark and bright shades in BSEM, respectively. The selected samples are taken from shallow, middle and deep parts of the Zoharim section, and they differ by porosity, organic content, anisotropy and matrix stiffness. In this section, we present the microstructural features that are most relevant for interpretation of the stiffness of the Ghareb–Mishash chalk.

We previously showed that the porosity in the Zoharim section decreases with depth (Shitrit et al. 2016). The micrographs presented in Fig. 16 reflect this trend, as the shallow, middle and deep samples show features of increasing compaction. In the deep zone, we also identify development of cementation and recrystallization (zoomed area in Fig. 16e), which further contribute to porosity reduction and increased matrix stiffness. Based on the porosity classification used by Loucks et al. (2012), the porosity is mostly associated with inter-particle porosity between the matrix grains. Intra-particle porosity within fossil bodies (Fig. 16a) and organic matter hosted pores also contribute to the total porosity but to a smaller extent. Organic matter hosted pores are hard to detect in our micrographs, but they are known to exist in immature kerogen, based on the findings that type IIS kerogen from the Monterey Formation is inherently porous (Löhr et al. 2015). Moreover, the kerogen influence factor of $k = 0.5$ is partly linked with the effect of isolated un-measurable porosity in the kerogen. The inter-particle porosity is distributed in a mudstone-wackestone textured matrix. Some calcite concentrated zones (Fig. 16b–d) exhibit mudstone texture as in non-organic chalks. Because of the dominance of the inter-particle porosity, we believe the grain contacts in the matrix have the strongest influence on rock stiffness.

The fine-grained matrix mostly includes kerogen and microcrystalline calcite as indicated by energy dispersive
Elastic moduli of organic-rich chalk

Figure 12 Dynamic elastic moduli in brine-saturated conditions of the organic-rich chalk: oedometer modulus (a) and shear modulus (b) plotted versus the kerogen-factored porosity. The upper bound, lower bound and ‘HS kerogen’ curve are the Hashin–Shtrikman curves that represent a matrix supported by minerals, fluid and kerogen, respectively. These values were calculated for each measured core plug individually. The circles represent measured values of $C_{33}$ and $C_{55}$.

spectroscopy. The abundance of kerogen and pores in the fine-grained matrix causes it to appear in darker shades in BSEM (Fig. 16a,c). The difference in shades of kerogen and calcite is most visible in the organic-lean sample (Fig. 16e,f), where the kerogen is dispersed in small accumulations. As assumed in the ‘HS kerogen’ model, the matrix in most of the samples is seen to be composed of kerogen–calcite mixture (most evident in Fig. 16a, c). The matrix may seem homogeneous at a scale of hundreds of microns, but at scale of tens of microns and below it appears heterogeneous. The bedding direction is visible at the scale of tens of microns and above, as inferred from the preferred orientation of elongated skeleton fragments and organic aggregates (Fig. 16a,c). However, many of the elongated particles have various orientations, even vertical. In addition, many of the detrital grains and the pores are more equant rather than flat. Therefore, bedding-aligned fabric is less prominent than in a typical shale (e.g. Vernik and Nur 1992; Sone and Zoback 2013). Anisotropy and stiffness of organic-rich rocks is also frequently associated with microfractures, especially at increased maturity levels (Vernik 1994; Zargari et al. 2013). Microfractures are identified (arrows in Fig. 16a,c, abundant in Fig. 16d). Nevertheless, their origin is arguable as any fracture may have been formed originally in the rock, during core extraction, or by the polishing process of the surface prior to the scans. Unlike most of the intervals in the Ghareb–Mishash sequence, the deepest zone includes none of the above-mentioned anisotropy enhancing features, hence the isotropic nature of this sample. We deduce that the anisotropy of the organic-rich chalk is caused by sub-horizontal preferred orientations of mineral particles and kerogen, and possibly by bedding sub-parallel microfractures as well.

4 DISCUSSION

4.1 The hydrostatic strain ratio as an anisotropy parameter

The hydrostatic strain ratio theoretically should be a good parameter for evaluating the elastic anisotropy because it is defined by the simplest conception of two-dimensional anisotropy: bedding-normal strain over bedding-parallel strain, under hydrostatic compression. To examine its applicability, we combine our results with other organic-rich rocks using acoustic velocities and density measurements at dry conditions, reported by Vernik and Liu (1997). The organic-rich formations included in their data are low porosity samples ($\phi < 10\%$) from the Bakken shale, Bazhenov shale, North-Sea Kimmeridge shale, Woodford shale, Lockatong shale, Niobrara chalk; and high porosity samples ($\phi > 10\%$) from the Monterey Formation and Japan (undeclared geological data). Their data set includes vertical and horizontal P-wave and S-wave velocities, which allow us
to calculate $C_{11}$, $C_{13}$, $C_{15}$, $C_{66}$ and $C_{12}$. Calculation of $C_{13}$ was enabled only where Thomsen’s $\delta$ was reported; hence $C_{13}$ data are fewer (definitions of Thomsen’s anisotropy parameters are in the Appendix). In order to increase the amount of data for comparison, we performed a multiple variable linear regression (as in Fig. 5 in Yan et al. 2016) to obtain a relationship between $C_{13}$ and $C_{11}$, $C_{33}$, $C_{55}$, $C_{66}$, suitable for organic-rich rocks ($R^2 = 0.85$):

$$C_{13} = -0.857 - 0.019C_{11} + 0.663C_{33} - 1.526C_{55} + 0.531C_{66}.$$  

(36)

The $\Omega$ is calculated using the values of $C_{11}$, $C_{12}$, $C_{13}$ and $C_{33}$ (equation (25)). The $C_{13}$ approximation in equation (36) was used only where the measurements by Vernik and Liu did not include Thomsen’s $\delta$. Then, $\Omega$ is compared with Thomsen’s $\varepsilon$ and $\gamma$ to examine the anisotropic behaviour. In order to establish the accuracy of the correlation, the analytical relationship between $\Omega$ and Thomsen’s $\varepsilon$ is elaborated in the Appendix (equation (A8)).

We see that $\Omega$ exhibits a very good correlation with Thomsen’s $\varepsilon$ parameter, based on the relationship in equation (A8) and with $b_2 = 3.58$ ($R^2 = 0.89$; Fig. 17a). This observation can be used to convert between the two anisotropy parameters. The $b_2$ fitting constant (3.58 in Fig. 17a) may be used further to separate between the static $C_{11}$ and $C_{12}$ and

---

**Figure 13** Dynamic elastic moduli in dry conditions of the organic-rich chalk: oedometer modulus (a) and shear modulus (b) plotted versus the kerogen-factored porosity. The upper bound, lower bound and ‘HS kerogen’ curve are the Hashin–Shtrikman curves that represent a matrix supported by minerals, fluid and kerogen, respectively. These values were calculated for each measured core plug individually. The circles represent calculated values of $C_{33}$ and $C_{55}$ based on the BAM model. The measured dry rock dynamic moduli are also presented: $C_{33}$ and $C_{55}$ from bedding-normal wave velocities (diamonds) and $C_{11}$ and $C_{66}$ from bedding-parallel wave velocities (squares).

**Figure 14** Insights on microstructure from dynamic measurements: The values of dynamic moduli predicted using the ‘HS kerogen’ model plotted against the values observed from measurements and fluid substitution. Marked in red circles, are three samples from the organic-lean zone at the bottom of the depth section. This zone is more compacted and cemented, and consequently manifests higher stiffness than the ‘HS kerogen’ predicted values. Note that the stiffer horizontal moduli $C_{11}$ and $C_{66}$ plot to the right of the vertical moduli $C_{13}$ and $C_{55}$.
Elastic moduli of organic-rich chalk

Figure 15 Normalized stiffness factors of the organic-rich chalk: (a) The ‘w’ of the oedometer and shear moduli are comparable in both bedding-normal and bedding-parallel directions. (b) Scatter plots of the normalized stiffness factors \( w_{c11} \), \( w_{c23} \), \( w_{c55} \) and \( w_{c66} \) versus kerogen volume percentage. The ‘w’ of the horizontal moduli \( c_{11} \) and \( c_{66} \) plot higher than those of the vertical moduli \( c_{33} \) and \( c_{55} \).

Table 4 Measured dry rock acoustic velocities

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Porosity (%)</th>
<th>TOC (%)</th>
<th>Dry Density (g/cm(^3))</th>
<th>Confining Pressure (MPa)</th>
<th>( V_P ) 0 (km/s)</th>
<th>( V_S ) 0 (km/s)</th>
<th>( V_P ) 90 (km/s)</th>
<th>( V_S ) 90 (km/s)</th>
<th>( V_SH ) 90 (km/s)</th>
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<tr>
<td>332.5</td>
<td>42.17</td>
<td>5.16</td>
<td>1.50</td>
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<td>2.01</td>
<td>1.28</td>
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<td>1.26</td>
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<td>1.47</td>
</tr>
<tr>
<td>540</td>
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<td>10.6</td>
<td>2.78</td>
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<td>3.31</td>
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<td>2.44</td>
<td>1.46</td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
<td>553(^*)</td>
<td>28.78</td>
<td>11.65</td>
<td>1.67</td>
<td>10.0</td>
<td>2.43</td>
<td>1.32</td>
<td>2.68</td>
<td>1.49</td>
<td>1.60</td>
</tr>
</tbody>
</table>

\(^*\)Data from Gordin et al. (2016).

approximate them, as well as the static \( C_{66} \) (optional step in Fig. 6). Thomsen’s \( \gamma \) parameter shows a similar correlation with \( \Omega \), although a larger scatter is indicated (Fig. 17b) because none of the shear moduli appears in the calculation of \( \Omega \). Figure 17(a,b) reveal that \( \Omega \) decreases sharply in the weak anisotropy range (i.e. \( \epsilon < 0.2 \)), indicating that \( \Omega \) is more sensitive to changes in the magnitude of anisotropy than \( \epsilon \) and \( \gamma \) within this range. Where the anisotropy is higher, \( \Omega \) is less indicative with respect to \( \epsilon \) and \( \gamma \). In Fig. 17(c), we examine the relationship between \( \Omega \) of source rocks with increasing organic content (static values of \( \Omega \) of the Ghereb–Mishash Formations are included). Although the results are scattered, we identify a trend of increased anisotropy with organic enrichment. The scatter is large because anisotropy is not exclusively related to organic content, but also depends on the features of the mineralogical particles and the pores. We also find that the vertical Poisson’s ratio is negatively correlated with elastic anisotropy (Fig. 17d). Interestingly, the decrease in the high-porosity organic-rich formations is sharper than in the low-porosity formations. From Fig. 17, we conclude that the Ghereb–Mishash chalk shares a common anisotropic behaviour with other organic-rich rocks, especially with the high-porosity rocks.

Our analysis points to beneficial features of \( \Omega \) as an anisotropy parameter: (1) it is easily measured in hydrostatic tests; (2) its definition as ratio between bedding-parallel and bedding-normal contraction provides physical intuition; (3) it correlates well with Thomsen’s anisotropy parameters, thus may be used for approximations in the absence of measured data; (4) it quantifies the magnitude of anisotropy by being strongly influenced by \( C_{11} \) and \( C_{33} \) (like \( \epsilon \)), but it also expresses the difference between \( C_{12} \) and \( C_{13} \); and (5) if derived...
Figure 16 BSEM images of core samples from three depth intervals in the Zoharim well. The samples are taken from depths of 339 m (upper panel, a-b), 458 m (middle panel, c-d) and 610 m (lower panel, e-f). Legend: OM, organic material; CA, calcite; HA, halite; PYR, pyrite. Arrows in (a) and (c) point to bedding-parallel microfractures. Arrow in (b) points to a zone that may have been disturbed during the polishing of the surface. Rectangle in (c) delineates a zone of calcite grains, textured as typical chalk mudstone. Zoomed area in (e) reveals textures of calcite recrystallization and cementation.

From a triaxial experiment, it may be used for elastic constants derivation using the hydrostatic-deviatoric combination for transversely isotropic elastic stiffnesses (HDC-TIES) method. Because the hydrostatic strain ratio is calculated using bedding-normal and bedding-parallel strains, it is already available from existing results of hydrostatic tests. Among its advantages, it is important to recall that it assumes linear elasticity and that the strain measurements align with the bedding-normal and bedding-parallel directions. If either of these conditions is not fulfilled, then the derivation of Ω is invalid.
4.2 Hydrostatic strain ratio and Poisson’s ratio

The static elastic horizontal-to-vertical strain ratios, $v_v$, at deviatoric compression and $\Omega$ at hydrostatic compression, are compared in Fig. 18(a). Although there are only 12 observations and the results are scattered, we detect a positive correlation between $\Omega$ and $v_v$. It is inferred that increased anisotropy reduces the vertical Poisson’s ratio. This is also supported by our conclusion from Fig. 17(d). This corresponds to the expectation for a typical vertically transversely isotropic (VTI) material, that increased anisotropy should enhance bedding-normal contraction with respect to bedding-parallel extension (i.e. lower $v_v$). As seen in Fig. 7(a), $v_v$ has a wider range at high porosity. These observations imply that the sensitivity of $v_v$ to anisotropy is enhanced at high porosity. This may explain the improved correlation when vertical Poisson’s ratio is plotted against the product of $\Omega$ and porosity (Fig. 18b). We find this behaviour typical for the Ghareb–Mishash chalk tested here in static conditions, and less representative of other organic-rich rocks that are much less porous and tested in dynamic conditions. The reason for this difference is lithological, as chalk...
matrix exhibits different geometrical arrangement and higher porosity with respect to shales. The dominance of porosity is smaller or absent in clay-rich shales, which are much less porous and more strongly affected by clay and organic content (Sayers 2013).

4.3 Comparison between static and dynamic moduli

The dissimilarity between our static and dynamic data sets limits the comparison between static and dynamic elastic moduli. In our case, elastic constants derived from static measurements are \( C_{13}, C_{33} \) and \( C_{11} + C_{12} \), while from dynamic measurements these are \( C_{13} \) and \( C_{55} \). Derivation of \( C_{13} \) from dynamic measurements is usually prone to significant errors (Yan et al. 2015), while this value is derived more easily and accurately from static measurements using the HDC-TIES method. The shear moduli \( C_{55} \) and \( C_{66} \) are unavailable from the static measurements that we performed. Consequently, the most significant comparison that can be made is between static and dynamic vertical oedometer moduli, \( C_{33} (M_3) \).

As expected, the dynamic \( C_{13} \) are consistently higher than their corresponding static values (Fig. 19a). To allow for comparison with other static-dynamic relations we need to convert \( C_{33} \) to \( E_\nu \). The acoustic measurements of source rocks reported by Vernik and Liu (1997) indicate that \( C_{33} \) is \( \sim 1.11 \) times higher than \( E_\nu \) in dynamic conditions (\( R^2 = 0.99; \) Fig. 19b). Although no direct measurements of dynamic \( E_\nu \) was obtained in this study, we find the correlation in Fig. 19(b) extremely useful in order to approximate it. This way, we compare measured static \( E_\nu \) with approximated dynamic \( E_\nu \) (Fig. 19c). We find that in dry conditions:

\[
C_{13,s} = 0.35 C_{13,d} + 0.1, \quad (37) \\
E_{\nu,s} = 0.35 E_{\nu,d} + 0.02, \quad (38)
\]

where subscripts \( d \) and \( s \) designate dynamic and static elastic moduli, respectively.

Static-dynamic relationships are useful in estimating static elastic moduli for engineering programs, as the dynamic values are easier to obtain (e.g. from well logs or core data). Many relationships have been suggested for these purposes by estimating the static Young’s modulus, \( E_s \), using the dynamic Young’s modulus, \( E_d \) (Mavko et al. 2009; Najibi et al. 2015; Brotons et al. 2016). Soft rocks with static Young’s modulus smaller than 15 GPa exhibit a linear relationship of \( E_s = 0.41 \cdot E_d - 1.06 \) [GPa] (Wang and Nur 2000). Data by Sone and Zoback (2013), obtained on various organic-rich shale formations yield a relationship of \( E_s = 0.97 \cdot E_d - 11.6 \) [GPa] (first loading cycle). Olsen, Christensen and Fabricius (2008) found that in dry North Sea chalk the dynamic and static elastic moduli are nearly equal (first loading cycle, measured with strain gauges). Gommesen and Fabricius (2001) formulated a different trend for North Sea reservoir chalks.
Elastic moduli of organic-rich chalk

Figure 19  Relationships between dynamic and static moduli. (a) Linear correlation between the static and dynamic vertical oedometer modulus of the studied Ghareb–Mishash chalk. Dynamic values include $C_{33,d}$ measured on dry samples and values calculated using HS bounds and BAM model. (b) Compilation of $E_v$ and $C_{33}$ data of organic-rich rocks using the results by Vernik and Liu 1997. This enables the approximation of dynamic $E_v$ from $C_{33}$ measurements. (c) Linear correlation between the static and dynamic vertical Young’s modulus of the Ghareb–Mishash chalk, based on the static measurements and the conversion shown in (b) for dynamic $E_v$. (d) The decrease of the dynamic/static $E_v$ ratio with an increase in the kerogen content. This trend is similar to that of a tight gas sand with an increase in the clay content in Tutuncu and Sharma (1992). Data point marked in red originates from depth of 609.5 m, which is much lower in porosity and kerogen content than the other samples ($\phi = 23.4\%$, $f_k = 2.6\%$) and known to be significantly more compacted and cemented, thus interpreted as anomalous.

(Tor and Hod Formations); at brine-saturated conditions, the static oedometer modulus, $M_s$, is related to dynamic modulus $M_d$: $\ln(M_s) = 1.36 \ln(M_d) - 7.6 \varphi / \varphi_c$, where $\varphi / \varphi_c$ is the ratio between porosity and critical porosity.

The static-dynamic relationship of vertical Young’s modulus of the Ghareb–Mishash chalk appears most similar to the relationship suggested for soft rocks (Wang and Nur 2000; Mavko et al. 2009). It behaves very differently from organic-rich shales and North Sea chalks (Olsen et al. 2008a; Sone and Zoback 2013), in which dynamic and static moduli are much closer to each other. In particular, Niobrara chalk, which has about 25% clay content and lower porosity and organic content, exhibits dynamic/static moduli ratio (DSMR) of 1.18–1.3 (Bridges 2016). These ratios are much lower than the Ghareb–Mishash chalk, which has much higher porosity and total organic carbon (TOC) and very low clay content. This observation highlights the unique behaviour of the Ghareb–Mishash chalk.
The differences between the static and dynamic elastic moduli are generally explained by several mechanisms. Microfractures and low aspect ratio pores cause non-linear deformations during static loading and irreversible strains occur, while in dynamic conditions they have a much smaller effect (Olsen 2007). Moreover, this difference may be caused by viscoelastic and micro-plastic strains in static loading rates, where strain amplitudes affect soft phases more severely (Mashinsky 2003). It was previously reported that clay content decreases the DSMR (Jizba et al. 1990; Tutuncu and Sharma 1992). We find that the DSMR decreases with increasing kerogen content (Fig. 19d). This effect is similar to that of clay content on the DSMR of tight gas sand (Tutuncu and Sharma 1992). We deduce that the DSMR in the Ghareb–Mishash chalk is high primarily due to high porosity, microfractures, soft pore geometry and weak intergranular contacts, as in many other soft sedimentary rocks. Kerogen has an opposite effect by decreasing the DSMR. Yet, the source for difference between static and dynamic moduli may be a good subject for further research, requiring more data of static and dynamic moduli, unloading–reloading cycles, varying clay contents and higher resolution scanning electron microscopy (SEM) images. It would also be interesting to investigate the behaviour of kerogens of different types, maturity levels, lithologies and degrees of mechanical compaction. Furthermore, static-dynamic moduli comparison of horizontally oriented samples of this rock is needed for better understanding of the mechanical behaviour.

5 CONCLUSION

This study focuses on elastic properties of an immature organic-rich chalk, and the mechanical effects imposed by porosity, kerogen and microstructural features. Elastic properties are derived from static and dynamic measurements assuming vertical transverse isotropy (VTI). The research incorporates laboratory measurements, theoretical models and microstructure scanning electron microscopy (SEM) imaging. The analysis employs three novel approaches proposed in this paper: (i) ‘hydrostatic strain ratio’ anisotropy parameter; (ii) ‘HDC-TIES’ method for estimation of VTI elastic moduli; and (iii) the ‘HS kerogen’ model for evaluation of dynamic moduli and acoustic velocities. The main conclusions are as follows:

1. Static elastic parameters are derived reliably using the hydrostatic-deviatoric combination for transversely isotropic elastic stiffnesses (HDC-TIES) procedure, designed here for a single triaxial experiment on a vertically oriented VTI rock sample. Using the experimentally derived values of $E_v$, $v_v$, $K$ and $\Omega$, it is now possible to compute the stiffness constants $C_{33}$, $C_{13}$ and $C_{11} + C_{12}$ and determine ranges of $C_{11}$, $C_{12}$ and $C_{66}$. This procedure is believed to be useful where (i) compression test data require repeated analyses; (ii) multiple specimens from the same core yield inconsistent results; and (iii) core material is limited.

2. We present a new anisotropy parameter: the ‘hydrostatic strain ratio’, denoted by $\Omega$. It describes the anisotropic response of a VTI material during hydrostatic compression and depends on $C_{11}$, $C_{12}$, $C_{13}$ and $C_{55}$. The $\Omega$ parameter is obtained experimentally using both static and dynamic measurements. The $\Omega$ is inversely related to the widely known Thomsen’s $\varepsilon$ and $\gamma$ parameters. The relationship between $\Omega$ and Thomsen’s $\varepsilon$ (i.e. $\Omega = 1/(1 + b_2 \varepsilon)$) is fitted best with $b_2 = 3.58$. The hydrostatic strain ratio is simply defined, easily measured, uniquely informative and highly applicable.

3. As in other source rocks, elastic anisotropy is related to microstructural features such as bedding-parallel alignment of minerals and kerogen particles. The relationships between the anisotropy parameters, and the influence of kerogen content on anisotropy, reveal a similarity to the anisotropic behaviour of other organic-rich rocks.

4. Mechanical anisotropy reduces the vertical Poisson’s ratio ($v_v$). The sensitivity of vertical Poisson’s ratio to anisotropy increases in high porosity source rocks.

5. The ‘HS kerogen’ model predicts well the bedding-normal dynamic moduli. This suggests that the kerogen is effectively dispersed in the matrix, thus forming a load-supporting phase in most of the samples. Backscatter SEM observations confirm that the fine-grained matrix is composed of kerogen–calcite mixture.

6. The static and dynamic moduli depend primarily on the volumetric sum of porosity and 50% of the kerogen content. The use of 50% porosity-like effect of the kerogen also leads to the smooth appearance of the ‘HS kerogen’ curve. Second-order effects are induced by porosity-reducing mechanisms, like compaction and cementation.

7. By combining the Hashin–Shtrikman bounds with Marion’s bounding average method (BAM) fluid substitution model and using the brine-saturated core measurements, the dynamic $C_{13}$ and $C_{55}$ of the dry rock are properly derived.

8. Dynamic moduli are consistently higher than static moduli mostly due to a non-linear dissipative response of porosity, microfractures and intergranular contacts to static loading (higher strain amplitudes). Kerogen introduces a clay-like effect by lowering the DSMR.
ACKNOWLEDGMENTS

This research was funded by Israel Energy Initiatives, Ltd. (IEI) through BGU contract no. 8724481. We thank Dr. Yoav O. Rosenberg, Leonardo Freitas and Edna Danon for professional and technical support. Dr. Yuval Bartov from IEI Ltd. is thanked for providing the cores. Roxana Golan from the Microscopy Unit, Ilse Katz Institute for Nanoscale Science and Technology, Ben-Gurion University of the Negev, is thanked for her support in SEM scanning. Yair Gordin is thanked for being a partner in fruitful discussions. The senior author thanks Israel Ministry of Infrastructures, Energy and Water Resources for a fellowship awarded to graduate students performing research in petroleum geosciences through grant no. 214-01-031. We thank the editor Dr. Best and the associate editor Dr. Chapman for the professional handling of the paper. Finally, we thank Prof. I. Fabricius and another anonymous reviewer for their critical reading of the original manuscript and for their instructive comments.

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REFERENCES


Palchik V. 2013. Is there link between the type of the volumetric strain curve and elastic constants, porosity, stress and strain characteristics? Rock Mechanics and Rock Engineering 46, 315–326.


APPENDIX: THOMSEN’S PARAMETER AND HYDROSTATIC STRAIN RATIO

The widely known Thomsen’s anisotropy parameters $\varepsilon$, $\gamma$ and $\delta$ are (Thomsen 1986)

\[
\varepsilon = \frac{C_{11} - C_{33}}{2C_{33}},
\]

\[
\gamma = \frac{C_{66} - C_{55}}{2C_{55}},
\]

\[
\delta = \frac{(C_{11} + C_{55})^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{55} - C_{33})}.
\]

The hydrostatic strain ratio $\Omega$ (equation (25)) can be expressed with $\varepsilon$:

\[
\Omega = \frac{C_{33} - C_{13}}{C_{33}(1 + 2\varepsilon) + C_{12} - 2C_{13}} = \frac{C_{13} - C_{11}}{C_{11} + C_{12} - 2C_{13} + 2C_{33}\varepsilon}.
\]

This equation can be rearranged for a convenient trend line:

\[
\Omega = \frac{b_1}{1 + b_2\varepsilon},
\]

with:

\[
b_1 = \frac{C_{13} - C_{11}}{C_{11} + C_{12} - 2C_{13}},
\]

\[
b_2 = \frac{2C_{33}}{C_{11} + C_{12} - 2C_{13}}.
\]

The hydrostatic strain ratio theoretically equals 1 when the rock is isotropic (i.e. $\varepsilon = 0$), and $b_1 \rightarrow 1$. Thus, the $\Omega$ versus $\varepsilon$ curve becomes

\[
\Omega = \frac{1}{1 + b_2\varepsilon}.
\]