THE VOUSSOIR BEAM REACTION CURVE

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ABSTRACT: The influence of joint spacing (*s*) on the stability of a Voussoir beam having a fixed span (*B*), total thickness (*d*) and individual bed thickness (*t*) is studied here using DDA. It is found that: A) The required friction angle ($\phi_{req.}$) for stability against shear sliding along the abutments decreases with increasing spacing down to a minimum value beyond which increasing spacing requires increasing joint strength. The corresponding parabolic function is referred to herein as the "Beam Response Function"; B) The axial compressive stress at the beam (σ_n) increases with increasing spacing up to a maximum value beyond which increasing spacing leads to decreasing level of axial thrust and increasing levels of vertical loads

A new concept: the "Beam Reaction Curve" is proposed in which maximum deflection at mid section vs. time is plotted using DDA. Such numerically developed curves can indicate the expected failure modes: shear sliding along the abutments, onset of stable arching after initial shear deformation, or completely stable arching. With the help of such synthetic diagrams monitoring data from real excavations can be analyzed with respect to the ongoing failure mechanisms in the roof, and conclusions can be drawn concerning imminent failure or stabilization.

1 INTRODUCTION

There is no close form solution for the stability of a horizontally bedded roof with vertical joints, a geometric configuration referred to here as a *Laminated Voussoir Beam*. Classic beam theory can be utilized for the calculation of shear and axial stress distribution, as well as amount of deflection, in the analysis of a bedded roof with beds of varying thickness (Obert and Duvall, 1976) and with given friction angle (Goodman, 1989). In these solutions the beam is assumed to be clamped at the ends and therefore the problem is statically determined. When the beam consists of a single bed with vertical joints, the so called *Voussoir* beam is obtained, and the problem becomes statically indeterminate as the beam is free to displace on either the abutments or across mid joints. Evans (1941) developed a design procedure for Voussoir beam geometry, a method which was later extended by Beer and Meek (1982) and is reviewed in detail by Brady and Brown (1993). However in these solutions only a single layer is modeled, and the influence of spacing and friction between the vertical joints are ignored. Little experimental work has been performed on the mechanical strength of a voussoir beam. Passaris *et al*. (1993) studied the crushing strength of the beam, and the mechanism of shear sliding along side walls has been investigated by Ran *et al*. (1991), both of which have used non liner finite element analysis. In both studies the analysis was extended to the case of multiple mid joints and the spacing between joints was considered, but friction along the discontinuities was not modeled.

In order to truly simulate deformation characteristics of a laminated voussoir beam a numerical method must be used. The selected method should allow rigid body movement and deformation to occur simultaneously, and convergence in every time step should be achieved after relatively large block displacement and rotation, without block penetration or tension. Furthermore, the vertical load which is typically assigned explicitly, must be evaluated and updated implicitly in every time step, since it varies with vertical location in the beam, as well as with the progress of beam deformation. Finally, the model must incorporate the influence of joint friction on block displacement and on the arching mechanism.

In this paper the failure of a laminated voussoir beam is back analyzed. All geometrical variables including beam span, beam thickness, joint spacing, and bed thickness are determined from careful field mapping and site investigations. All mechanical parameters are determined from expedient rock mechanics testing. The beam geometry and intact rock properties are used as input parameters in both classic Voussoir Beam Analysis (Beer and Meek, 1982) and in Discontinuous Deformation Analysis (Shi, 1993) and the results are compared. Finally using DDA results some new insights regarding the mechanical behavior of a laminated Voussoir beam are proposed.

2 THE FAILURE OF A LAMINATED VOUSSOIR BEAM - A CASE HISTORY

2.1 *A Brief Description of the Failure*

In the archeological site of Tel Beer Sheva, an ancient city dated back to the Iron stage (1,200 - 700 B.C.) an underground water storage reservoir dated back to approximately 1,000 B.C. was explored. The reservoir was excavated in horizontally bedded chalk with vertical joints clustered in three joint sets. The most abundant joint sets (J1 and J2) are orthogonal with mean spacing of 20 to 25 cm; the mean bed thickness is about 50 cm. The intersection of horizontal layers and vertical joint planes creates a dense network of cubic blocks which form the roof of the excavation. The roof collapsed into the shape of a three dimensional dome, probably during time of construction, and the ancient engineers have erected a massive support pillar in the center of the opening in order to support the remaining roof. The collapsed roof is considered here a failed laminated voussoir beam. Hatzor and Benary (1998) provide details of the failure including maps and cross-sections.

2.2 *Rock Mass Properties*

The studied rock mass consists of a horizontally bedded and vertically jointed chalk, covered by 5m. of a well cemented conglomerate, and by about 3m. of soil in which the archeological remains are found. Individual bed thickness in the chalk is between 30 to 80 cm. with an average thickness of 50 cm. The rock mass RQD values determined from core recovery range between 44 - 100% with typical values between 65 - 80%. The unconfined compressive strength of the chalk is 7 MPa , the Elastic module (E) and Poisson ratio (ν) as measured in unconfined compression are 2GPa and 0.1 respectively. A linear Coulomb-Mohr failure envelope fitted to the peak strength values yield a cohesion of 3.1 MPa and internal friction angle of 32^{o}. The porosity of the chalk is between 27 to 30% and the unit weight is between 18.1 to 20.1 kN/m^3. The Atterberg limits of the interbedded marl indicate relatively low plasticity and low swelling potential. Estimation of input data for rock mass classification methods yields an estimated Q value between 0.4 to 4.0 and an estimated RMR value of 43. These values indicate a fair to poor rock with an expected stand up time of one to several days. The estimated rock mass classification values help explain the historic failure: with the given lithological conditions and considering modern experience we do not expect the rock mass to have been able to sustain the loads which were induced by the attempted excavation, for a significant period of time

2.3 *Estimated Shear Strength of Joints*

Three principal joint sets are defined. The joints are clean and tight with planar surfaces. The roughness of the joint planes is estimated using a profilometer measurements. 10 measured profiles are compared with JRC standards and the mean JRC value is estimated at 9. The residual friction angle of the joints is determined using tilt tests performed on mating saw cut joint planes. The mean residual friction angle is estimated at 35^o. In order to asses the peak friction angle which was available at time of deformation the empirical criterion of Barton is used with the following input parameters: JRC = 9; JCS = 7 MPa; σ_n = 0.25 MPa; $\phi_{residual} = 35^o$. The maximum normal stress active on the joints (σ_n) is a function of beam thickness and is determined here from DDA results (see below). Using the criterion of Barton the dilation angle is expected be about 13^o and therefore the peak friction angle is expected to be about 48^o.

3. VOUSSOIR BEAM ANALYSIS

The assumed stress distribution in a "classic" voussoir beam (Beer and Meek, 1982, Brady and Brown, 1993) is shown in Figure 1, where the beam consists of a single layer . As the number of intermediate joints is irrelevant, the only relevant geometric parameters in the classic analysis are beam span (B) and beam height (d). The geometry of a *laminated* voussoir beam is shown in Figure 2. The principal geometric parameters are beam span (B), overall beam height (d), individual layer thickness (t), and joint spacing (s). This geometry is used in DDA with fixed point location as marked in Figure 2 by the small triangles.

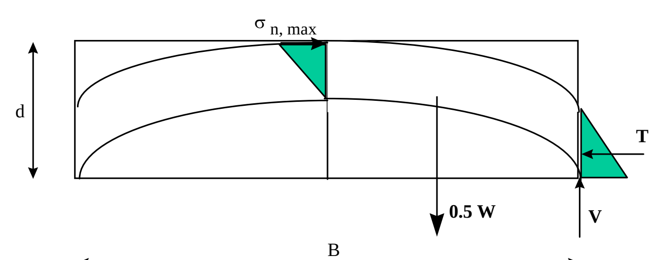

Figure 1a. Assumed stress distribution in classic voussoir beam analysis

The maximum axial stress (σ_n) in the classic voussoir beam analysis is computed for beam spans ranging between 3 to 9 meters and beam thickness between 0.25 to 2.5m. (Figure 3) . It can be seen that in general σ_n increases with increasing beam span and decreases with increasing beam thickness. The calculated results however are only valid for a beam consisting of a single layer. The case of Tel Beer Sheva is shown in the heavy line in Figure 3 for a beam span of 7m. The value of σ_n , obtained for an individual layer thickness of 0.25m, is 2.45 MPa.

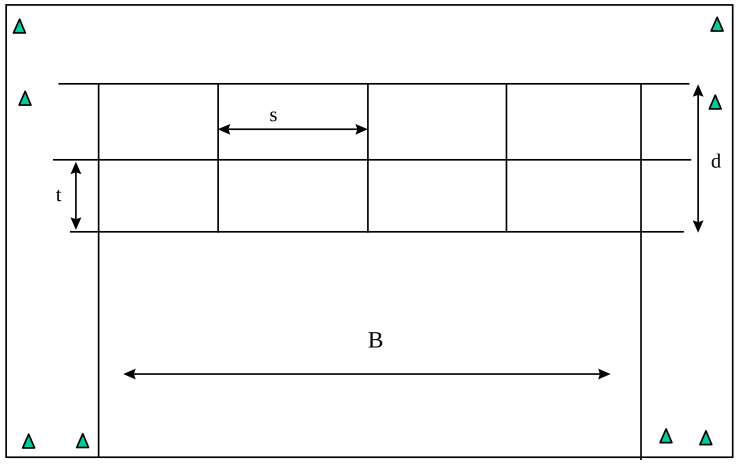

Figure 2. Geometry of a laminated voussoir beam as used in DDA experiments

For a single layer beam with thickness of 2.5m σ_n is 0.244 MPa. At Tel Beer Sheva the average bed thickness is 0.5m. Assuming that each bed transmits the axial thrust independently from the neighboring layers above and below, σ_n within a single layer should be 1.22 MPa. These values are significantly lower than the unconfined compressive strength of the rock which is about 7 MPa, and therefore the beam should be considered safe against failure by local crushing at hinge zones, according to this analysis.

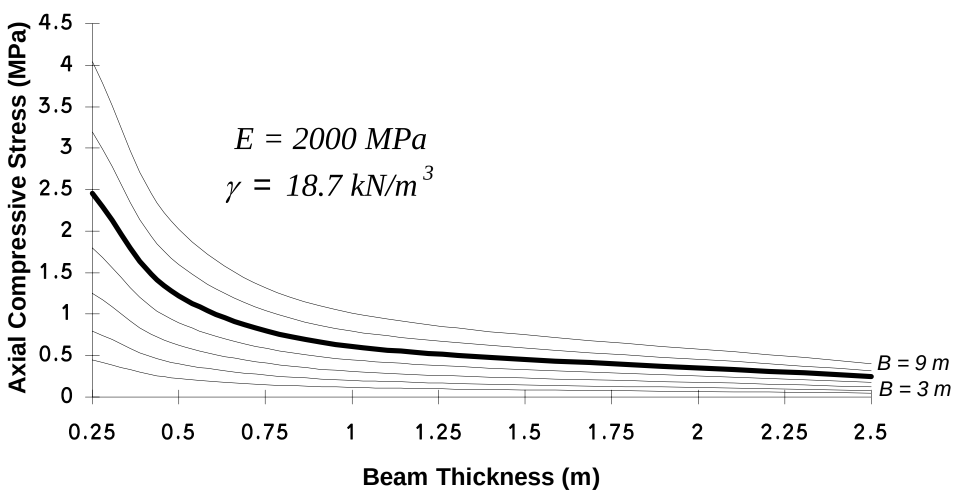

Figure 3. Maximum axial compressive stress (σ_n) as a function of beam thickness according to classic voussoir beam analysis

Brady and Brown (1993) show that for joints and abutments of zero cohesion the factor of safety against failure in shear along the abutments is given by:

$$F.S. = \frac{T \tan\phi}{V} = \frac{0.5\sigma_n n d \tan\phi}{0.5\gamma B d} = \frac{\sigma_n}{\gamma B} n \tan\phi \tag{1}$$

Where: T = resultant horizontal (normal) force; V = resultant vertical (shear) force; ϕ = available friction angle along abutment wall or vertical joint; n = assumed load/depth ratio (compressive zone thickness is given by *n x d*); d = beam thickness; B = beam span; γ = unit weight of rock. The factor of safety against shear failure by sliding along the vertical abutments is calculated in Figure 4 for different values of available friction angle and beam thickness using Eqn. 1. It can be seen that the sensitivity of the factor of safety to beam thickness is quite high and a logarithmic scale is used for better resolution.

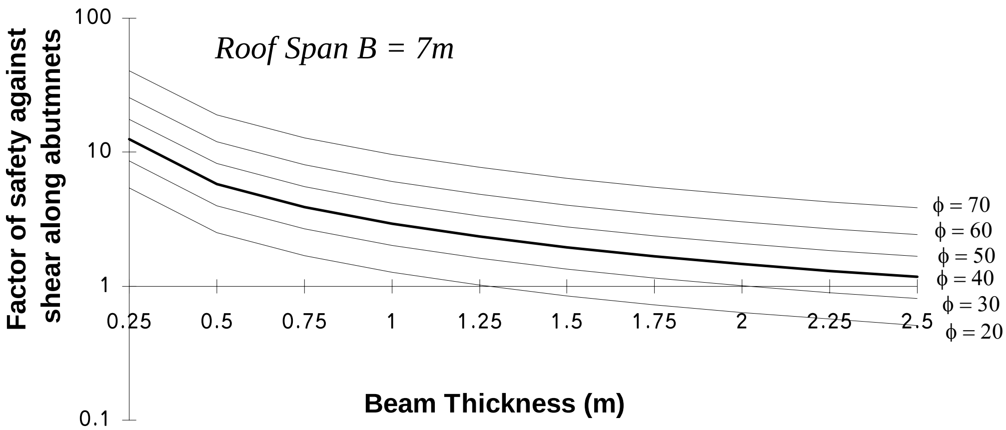

Figure 4. Sensitivity of factor of safety against shear sliding along the abutments to the available friction angle and to the beam thickness according to classic voussoir beam analysis

The available friction angle for Tel Beer Sheva is estimated at 48^o ; a heavy line representing available friction angle of 40^o is shown in Figure 4 above. With a given friction angle of only 40^o the opening should have been safe against shear along the vertical abutments, for any beam thickness between 0.25 to 2.5m. In fact, the required friction angle for stability according to classic voussoir beam analysis is not greater than 36^o, for every beam thickness.

The results obtained using classic voussoir beam analysis can not explain the failure. The local compression which develops at the hinge zones is too low comparing to the available compressive strength; the shear stress which develops along the vertical abutments due to beam weight is lower than the shear strength of the abutments, considering a conservative estimate of friction angle. We must conclude therefore that the approach taken by classic voussoir beam analysis, which ignores the influence of joint spacing and the existence of multiple beds, may prove unconservative, and should not be applied in practice for the analysis of a *laminated* voussoir beam.

4 DISCONTINUOUS DEFORMATION ANALYSIS

4.1 Set up of DDA experiments

The carefully documented geometry of the failed roof is used here in back analysis of the failure. The active span is assumed to be 7m as before, but a distinction is made now between overall beam thickness, and individual layer thickness. The overall beam thickness is represented by the height of the mapped loosened zone, about 2.5m. Individual layer thickness is taken as the average bed thickness, about 50cm. The geometric and mechanical parameters, which are used as variables, are mean joint spacing and joint friction, respectively.

DDA runs are performed for the geometry which is schematically shown in Figure 2, where fixed point location is marked by small triangles. In each analysis a constant mean joint spacing value (*s*) is used, and the value of friction angle along the boundaries is changed until the system shows stability. The stability of the roof is defined by a specified value of maximum deflection at beam mid section, the magnitude of which would not change regardless of the number of time steps. In this research the roof is considered to arrive at stability when a maximum deflection of up to 5.5cm is detected at mid section, after at least 25 time steps. The maximum allowable displacement ratio, namely the maximum allowable displacement per time step divided by half the vertical dimension of the region, is set to 0.01. In the analyzed case study the maximum allowable displacement per time step is 3.5cm.

The joints are considered planar and cohesionless with zero tensile strength, an honest representation of the situation in the field. The friction angle of vertical joints and horizontal bedding planes is assumed equal, merely for simplicity; this is by no means a limitation of the method. The input material parameters are: Mass per unit area = 1900 kg/m^2 ; Weight per unit area = 18.7 kN/m^2; Young's modulus E = 2*10^6 kN/m; and Poisson's ratio ν = 0.1. Seven mean joint spacing values are analyzed: s = 25cm, 50cm, 87.5cm, 116cm, 175cm, 350cm, and 700cm. The roof is modeled for friction angle values between 20^o and 90^o. The maximum deflection at mid section for a given friction angle value is noted in each run. Typically stability and cease of motion is detected after 12 to 16 time steps.

4.2 DDA results

An example of DDA results for a layered beam with s = 25cm. is shown in Figure 5 a, b, c, for available friction angle values of $\phi_{\text{available}}$ = 20^o, 40^o, and 85^o respectively. The beam fails in shear along the abutments when the available friction angle is 20^o (Figure 5a). An available friction angle of 40^o (Figure 5b) is sufficient to induce arching but the roof deflection is excessive, u_{max} = 20cm. The roof remains completely stable only when the available friction angle is increased to 85o (Figure 5c).

The results of all DDA experiment are graphically demonstrated in Figure 6 for a roof of 7m span and 0.5m horizontal layer thickness. Maximum deflections at mid section (u_{max}), arrived at after up to 25 time steps, are plotted against the value of friction angle which was modeled for all discontinuities. The tests are performed for six different vertical joint spacing values (s).

The results of the DDA experiments demonstrate that for the case of Tel Beer Sheva, with mean bed thickness of 0.5m and mean joint spacing of 0.25m, the required friction angle is 80^o, a shear strength which was not available at time of construction and thus the failure.

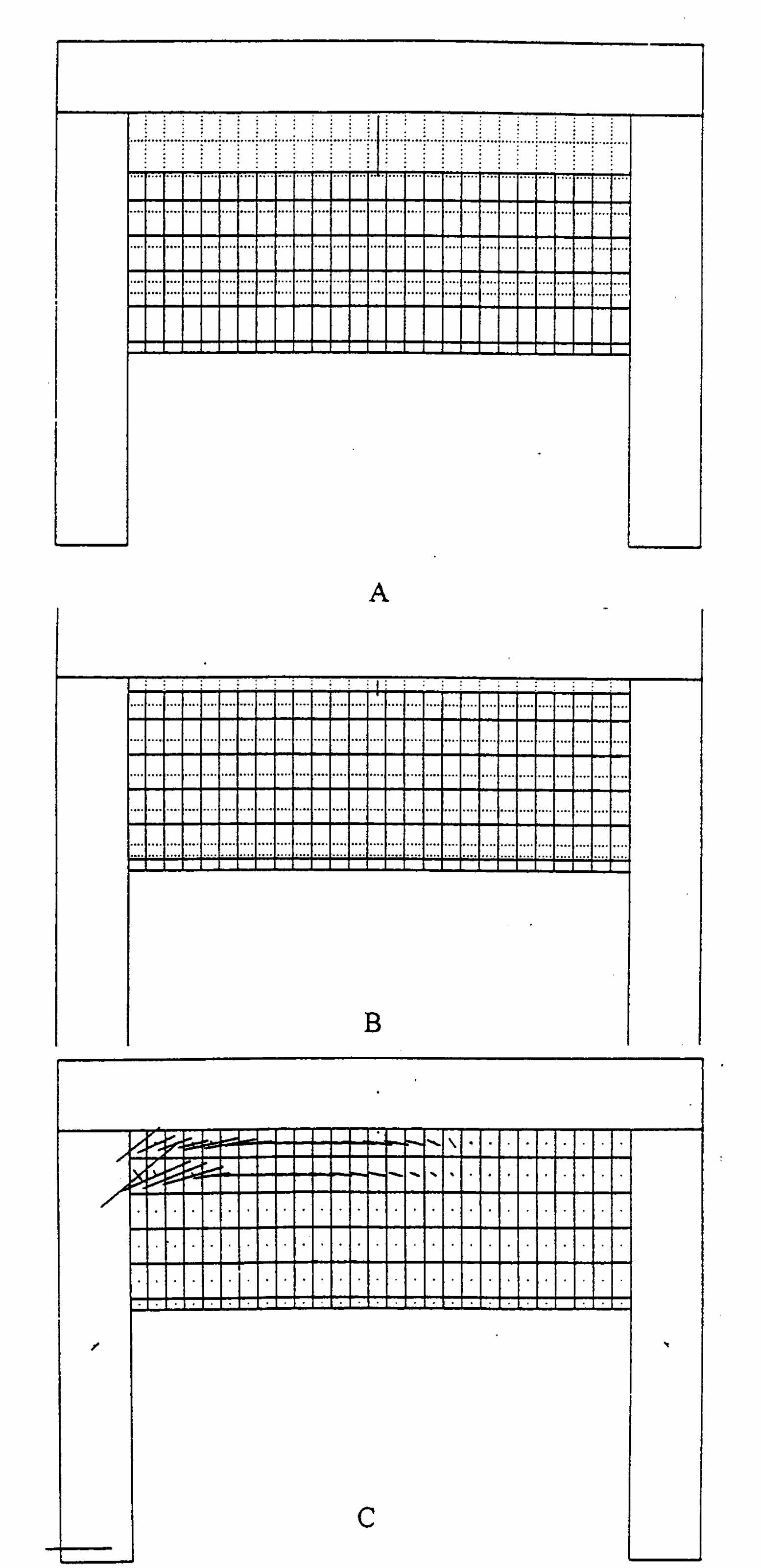

Figure 5. Three DDA experiments for the deformation of a laminated Voussoir beam aftre 25 time steps (see text for details). Beam geometry: B=7m, d=2.5m, t=0.5m, s=0.25m. A) $\phi_{available}$=20^o, u_{max} = 0.83m; B) $\phi_{available}$=40^o, u_{max}=0.2m; C) $\phi_{available}$=85^o, u_{max}=0m.

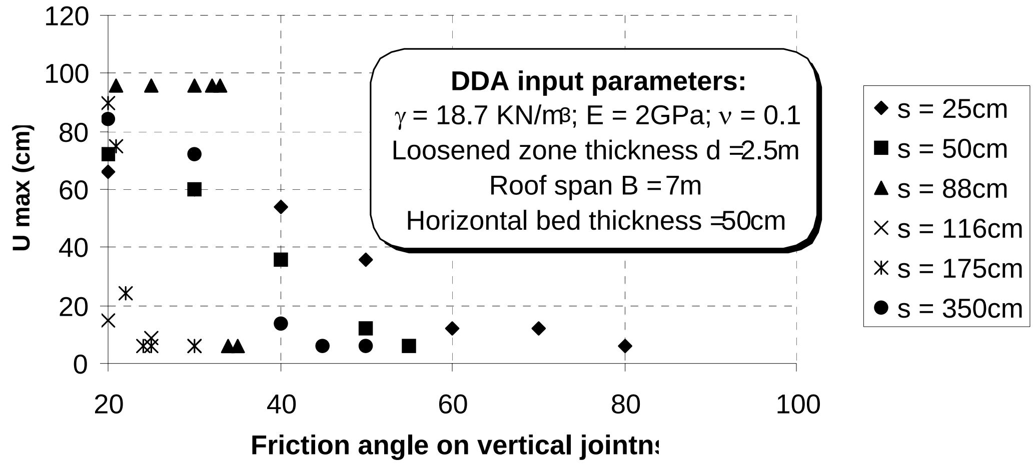

Figure 6. Maximum deflection at beam mid section after up to 25 time steps as predicted by DDA

5 THE VOUSSOIR BEAM RESPONSE FUNCTION

The influence of joint spacing, or block length, on the required friction angle for stability is demonstrated in Figure 7 below. The results indicate that the required friction angle for stability decreases with increasing block length, or joint spacing. However, the empirical function is not monotonously decreasing but presents a minimum, when the number of blocks in an individual layer is 4 (s = 175). When the number of blocks further decreases (block length or joint spacing increases) the required friction angle for stability increases. Ultimately, when each individual layer consists of a single block the required friction angle is 90^o because the abutment walls are vertical with zero cohesion.

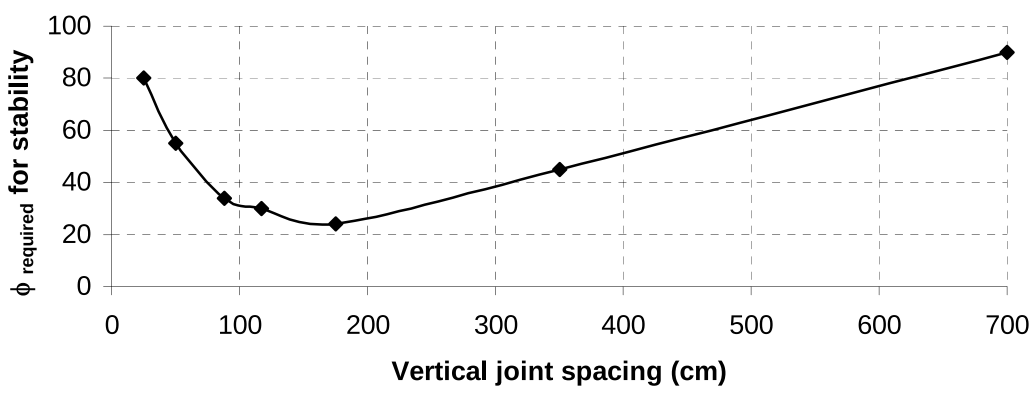

Figure 7. Influence of joint spacing on the stability of a laminated voussoir beam

The empirical function in Figure 7 above is referred to here as the "*Beam Response Function*". The logic behind this parabolic function may be rationalized as follows: with increasing joint spacing the moment arm length in individual blocks increases, and the arcing mechanism by which axial thrust is transmitted through the blocks to the abutments is enhanced. However, above a limiting value of block length, the weight of the overlying blocks becomes more dominant. Consequently, the stabilizing effect of greater axial thrust is weakened by the destabilizing effect of dead load transfer from the weight of overlying blocks. This rational may be tested if we investigate the developed axial thrust at beam mid section (upper hinge point) as a function of block length or joint spacing. Results of DDA calculations (Figure 8 below) confirm the stated rational above.

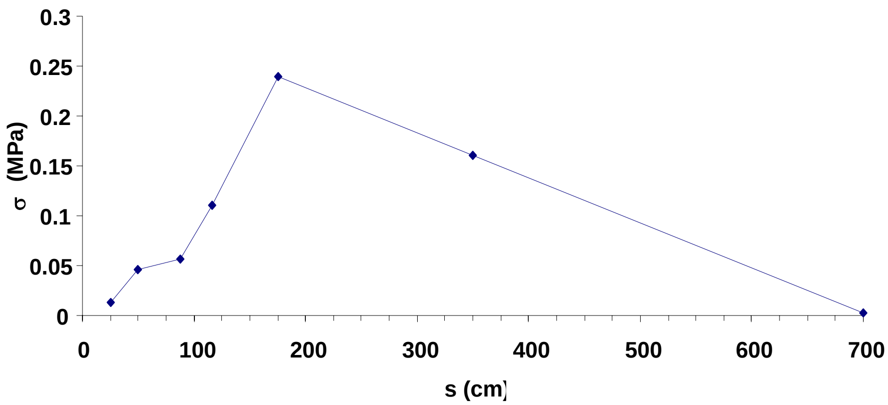

Figure 8. Maximum axial thrust at beam mid section (σ) for a given vertical joint spacing (s) as determined by DDA.

6 THE VOUSSOIR BEAM REACTION CURVE

The dynamic formulation of DDA enables us to gain further insight into the mechanical behaviour of a laminated voussoir beam, in particular its deformation with respect to time. When joint and abutment wall friction is low with respect to the developed vertical shear load continuous sliding deformation along the abutments is expected. With increasing shear resistance onset of arching deformation ensues after initial vertical slip takes place. The beam is completely safe when arching deformation is preceded by minimal vertical slip. The deflection of the beam with time is referred to here as the “*Beam Reaction Curve*” . Three different reaction curves are shown in Figure 9 for three different beam configurations. Such developed curves can indicate the expected failure modes: shear sliding along the abutments, onset of arching after initial shear deformation, or completely stable arching. With the help of such synthetic diagrams monitoring data from real excavations can be analyzed with respect to the ongoing failure mode in the roof, and conclusions can be drawn concerning imminent failure or stabilization.

7. CONCLUSIONS

- DDA is more appropriate and more conservative than classic voussoir beam analysis for the analysis of a *laminated* voussoir beam because it can model joint spacing and joint friction, and it can allow for multiple layers in a beam.
- For a laminated beam with a constant layer thickness the resistance to shear along the abutments improves with increasing joint spacing up to a critical value of block length, beyond which the composite beam becomes less stable.

- Using DDA synthetic Beam Reaction Curves which predict vertical beam deflection with time may be generated. With the help of such synthetic diagrams monitoring data from real excavations can be analyzed with respect to the ongoing failure mechanisms in the roof, and conclusions can be drawn concerning imminent failure or stabilization.

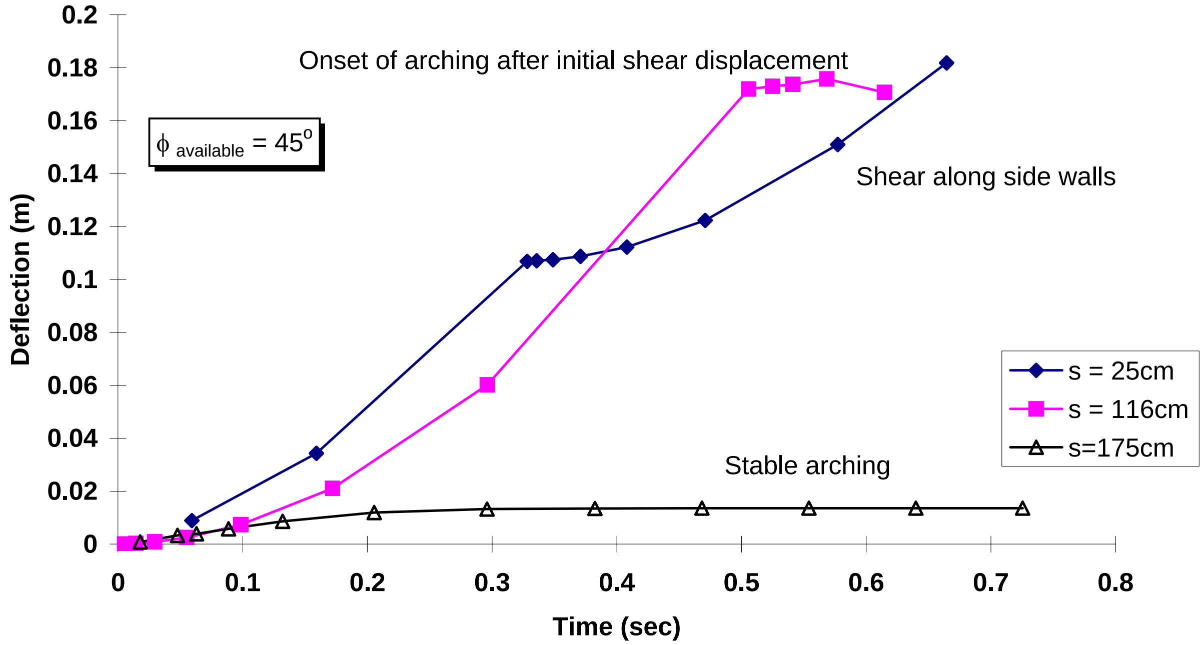

Figure 9. Three different “Beam Reaction Curves” for three different voussoir beam configurations as predicted by DDA.

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