BAYESIAN ESTIMATION OF SPATIAL CONFIGURATION DISTRIBUTION

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE M.Sc. DEGREE

By: Rotem Duani

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BEN GURION UNIVERSITY OF THE NEGEV
FACULTY OF ENGINEERING SCIENCES
DEPARTMENT OF INDUSTRIAL ENGINEERING AND MANAGEMENT

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Abstract

Grasping and manipulating objects in dense unstructured environments is challenging both for humans and for robotic systems. One of the essential components of a successful grasp is the position and orientation (pose) of the wrist, from which the object is grasped. Determining such a goal pose is an important part of reach-to-grasp motion planning. Grasp affordance densities are spatial probability density functions that represent grasp success probability from wrist poses about the object. To represent both position and orientation well, the density function must be based on a mixture model which includes both cyclic and non-cyclic components. We developed a non-parametric Bayesian estimation method suitable for a mixture of such combined density functions composed of Gaussian and Von-Mises-Fisher functions. Non-Parametric Bayesian estimation facilitates joint estimation of both mixture component, number and component parameters and is less prone to being trapped in local minima, or to over-fitting the dataset, than maximum likelihood based estimation methods. The developed method is incorporated in a reach-to-grasp motion planning algorithm and is applied to motion analysis in patients with stroke. For reach-to-grasp motion planning, we integrated the grasp affordance density estimation with bi-direction Randomly exploring Random Tree (RRT) motion planning algorithm. The developed algorithm, grasp affordance-RRT (GA-RRT) facilitates multiple goal configurations with a high grasp success probability. The GA-RRT algorithm was tested in simulation for motion planning towards a mug in five different environments with obstacles. In one of the environments, the algorithm did not find suitable goal configurations, since it was very cluttered and required approach orientations that were not included in the original data-base from which the density was estimated. In the other four environments, 95% of trials led to successful grasps. The grasp affordance estimation method was applied for estimating affordance densities of 15 patients with stroke and 13 healthy, aged-matched controls. Subjects in both groups performed reach-to-grasp movements towards four targets locations. A grasp affordance density was estimated for each group with data from all targets combined. For both groups the estimated densities comprised two components, yet the division of grasp configurations between the components in each density mixture, differed between the groups. For the health group, the configurations of three target locations were allocated to one density mixture component and one target was allocated to a separate component. For the stroke group, subjects were divided between the components based on the effected arm, with which they performed the motion. In addition, the variance of the density components was much higher for the stroke group.

Key words: non-parametric Bayesian estimation, Grasp affordance density, Gaussian Mixture Model, Von Mises-Fisher distribution, Grasping, Motion planning, Stroke.
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Acronyms

ARS  Adaptive rejection sampling
BIC  Bayesian Information Criterion
EM   Expectation Maximization
DOF  Degrees-of-Freedom
ENHANCE  Enhancing brain plasticity for sensorimotor recovery in spastic hemiparesis
FC   Far Centre
FL   Far Left
FR   Far Right
GARRT  Grasp Affordance Rapidly exploring Random Trees
GMM  Gaussian Mixture Model
GRRRT  Grasp Range Rapidly exploring Random Trees
ICL  Integrated Completed Likelihood
IGMM  Infinite Gaussian Mixture Model
KDE  Kernel Density Estimation
NC   Near Center
OP   Overlap Position dataset
OO   Overlap Orientation dataset
OPO  Overlap in Position and Orientation dataset
RRT  Rapidly exploring Random Trees
SE   Special Euclidian
TCP  Tool Center Point
VMF  Von Misses-Fisher
WGR  Workspace Goal Regions
1. Introduction

1.1. Background

Grasping is a fundamental skill needed by both humans and robots for object manipulation, and involves both hand and arm motion. For performing a required grasp, the arm must reach a final wrist configuration in task space and the fingers must reach required contact points on the object. Based on available degrees of freedom, arm and finger joint configuration should also be determined. In the current research, we focus on grasp wrist configurations in task space. Wrist configurations are composed of six parameters, where three parameters represent the position \((X, Y, Z)\) and three parameters represent the orientation \((\sigma, \theta, \phi)\). They can be represented in object-centered coordinates, which facilitates defining grasp configurations based on gripper and object without regard to the environment in which the object is placed.

Grapability maps have been used to store quality grades for object centered, grasp wrist configurations (Eizicovits and Berman 2014). They can be used to generate grasp affordance densities (Detry et al. 2009), which are spatial probability density functions that represent grasp success probability for wrist configurations about the object. Grasp affordance densities are typically mixture models composed of both, Gaussian and cyclic components, used for representing position and orientation respectively (Detry et al. 2009). Previous methods employed for estimating distribution parameters, include Expectation Maximization (EM) (Granville, Fagg, and Southerland 2006) and Kernal Density Estimation (KDE) (Detry et al. 2009). In contrast, the current work investigated the use of non-parametric Bayesian estimation.

There are two main approaches for statistical inference, maximum likelihood (Frequentist) and Bayesian. The Bayesian approach uses Bayes' theorem to combine observational data with prior knowledge, not expressed in observations, (Press 2002). While, the frequentist approach assumes that the sampled data is representative, and a subjective prior may cause bias. One of the main advantages of the Bayesian approach are that data can be used as it comes in. There is no requirement that every contingency be planned for ahead of time, very
useful in machine learning and big Data methods (Orloff and Bloom 2014). In this work we explore the Bayesian approach in the grasp affordance estimation method.

1.2. Goals and Innovation

This research presents a non-parametric Bayesian estimation method which generates grasp affordance densities. The algorithm jointly estimates component parameters, mixture weight, and the number of mixture components. The algorithm developed is suitable for estimation of a mixture model composed of Von Mises-Fisher (VMF) and Gaussian distributions components. It integrates two estimation algorithms, the Infinite Gaussian Mixture Model (Rasmussen 2000) and Infinite VMF Mixture Model (Bangert 2010). The development required adaptation of sampling algorithms due to numeric issues, adaptation of priors and hyper parameters estimators, determination of parameter hierarchy levels and normalization of Gaussian and VMF likelihood probabilities. The grasp affordance density was incorporated in a robotic motion planning algorithm and used for analysis of motion in patients with stroke.

We developed the Grasp Affordance Randomly exploring Random Tree (GA-RRT) algorithm based on the Grasp Region Randomly exploring Random Tree (GR-RRT) algorithm (Reshef, Eizicovits, and Berman 2014). The modifications included, the development and integration of a suitable mixture model random sampling algorithm. The GA-RRT algorithm provides high quality target configurations resulting in greater likelihood of successful grasps.

The analysis of motion of patients with stroke was done as part of the ENHANCE project. The goal of ENHANCE is to establish and clinically validate effective upper limb interventions for recovery of voluntary movement control after stroke. Human grasp configurations of both healthy subjects and patients with stroke were evaluated. Two separate grasp affordance densities were generated, one for each group for motion towards four targets. Analysis of the grasp affordance densities included comparison of the number of clusters, composition of clusters and variance within clusters. By analyzing these measures, we can learn about the number of grasp types and the difference between them. For both groups the estimated densities comprised two components, yet the division of grasp configurations between the components in each density mixture, differed between the groups. For the control group, the
configurations of three target locations were allocated to one density mixture component and one target was allocated to a separate component. For the stroke group, subjects were divided between the components based on the effected arm, with which they performed the motion. In addition, the variance within the components was much higher for the stroke group.

1.3. Research scope and limitations

The current research revolves around the Bayesian distribution estimation method, testing its limits, using it for modifying a path planning method evaluated in simulation, and for the analysis of the motion of patients with stroke. One direction for future research involves further examination of the estimation algorithm by examining optimization of currently, constant, hyper parameters, and providing the option of using previous grasp affordance densities as the prior. For the path planning algorithm, further examination with hardware and comparison to other existing algorithms. For the motion of patients with stroke, examining the post treatment and follow-up data of patients and analyze the progress made throughout the treatment is left to future research, as the ENHANCE project is still in-process and the data is not yet available.

1.4. Thesis outline

This thesis is organized as follows: chapter 2 presents a literature review of the basic concepts and methods, various distribution estimation algorithms are explained, existing, commonly used, motion planning algorithms are presented, and impairment in grasping of patients with stroke is explained. Chapter 3 includes a detailed description of the algorithm developed, and the results of synthetic data used for validation. In chapter 4, an implementation of the algorithm in robotic path planning is presented and analyzed. Finally, in chapter 5, an implementation of the algorithm on post stroke grasp data is presented and analyzed.
2. Literature review

2.1. Overview

This chapter includes the review of different concepts and methods, related to non-parametric Bayesian estimation of grasp affordance densities and the use of such affordance estimation in robotic motion planning and human motion analysis. Theoretical statistical foundations are presented in section 2.1. These include cyclic distributions, mixture models, and three algorithms used for estimating grasp affordance densities which are reviewed and compared. Section 2.2 briefly presents Rapidly exploring Random Trees (RRT) motion planning algorithms. Section 2.3 describes reach-to-grasp characteristics in healthy subjects and in patients with stroke.

2.2. Theoretical foundations

2.2.1. Cyclic distribution functions

There are various practical situations in which observations include orientations, e.g. the orientation from which to grasp an object. In such cases spherical statistics is required for handling the angular data. Cyclic distributions are a tool offered by spherical statistics for expressing the periodic characteristics of angles.

Using quaternion representation is common in cyclic distributions. Quaternions are used to express an orientation of a point in 3D space around an axis. They are composed of four components \((q_0, q_1, q_2, q_3)\), where \(q_0\) represents a scalar and \((q_1, q_2, q_3)\) represent a vector. Quaternions and Euler angles both represent orientation, however using quaternions has several advantages, such as easier conversion to orientation matrices and avoidance of and gimbal lock. In addition, when using quaternions, the order of orientation doesn’t need to be specified like in Euler angles. The functional linkage between Euler angles and quaternions is:

\[
q_0 = \cos\left(\frac{\alpha}{2}\right), \quad q_1 = \sin\left(\frac{\alpha}{2}\right)\cos(\beta_x), \quad q_2 = \sin\left(\frac{\alpha}{2}\right)\cos(\beta_y), \quad q_3 = \sin\left(\frac{\alpha}{2}\right)\cos(\beta_z)
\]

Where \(\alpha\) is orientation angle (the value in radians of the angle of orientation), and \(\cos(\beta_x), \cos(\beta_y)\) and \(\cos(\beta_z)\) are the direction cosines locating the axis of orientation.
2.2.1.1. Von-Mises Fisher (VMF) distribution

The Von Mises - Fisher (VMF) distribution is a commonly used cyclic distribution, as it is the equivalent of the Gaussian distribution in the cyclic world (Figure 2.1). The VMF distribution has two parameters: \( \mu \), the expected value, and \( \kappa \), the concentration of the data.

A \( p \)-dimensional unit random vector \( \vec{X} \) (\( ||\vec{X}|| = 1 \)) with a \( p \)-variate VMF distribution, has a probability density function (pdf):

\[
f(x|\mu, \kappa) = c_p(\kappa) e^{\kappa \mu^T x}
\]

Where \( ||\mu|| = 1 \), \( \kappa \geq 0 \), and \( c_p(\kappa) \) the normalizing constant is given by:

\[
c_p(\kappa) = \frac{\kappa^{p-1}}{(2\pi)^{p/2} I_p(\kappa)}
\]

As \( \kappa \) decreases the pdf becomes more similar to a uniform pdf (Dhillon and Sra 2003).

2.2.2. Mixture models

A mixture model is a probabilistic model used for representing subpopulations within an overall population. Mixture models are used to make statistical inferences about the properties of the sub-populations, given observations on the pooled population, this without sub-population identity information (Everitt 1981). The mixture model assumes the existence of \( M \) densities, where each density is allocated a weight value, and the sum of all weights adds to 1 (Reynolds 2008). When fitting a mixture model to data, the mixture weight and component parameters are determined (Reynolds 2008). The Gaussian Mixture Model (GMM) is a broadly used model in which all \( M \) densities are Gaussian. GMM has been used in many fields, e.g., for modeling vocal-tract related spectral features in a speaker recognition system (Reynolds 2008), human skin colors for security checks (Yang and Ahuja 1998), acoustic units for speech recognition (Torbati, Picone, and Sobel 2013). An example for the probabilistic mixture model follows:
\[
g(x|\theta) = \sum_{i=1}^{k} w_i P_i(x|\theta_i)
\] (3)

Where \( x \) represents the independent identically distributed samples, \( \theta_i \), are the parameters of cluster \( i \)'s distribution and \( w_i \) represents the weight of mixture 'i'. Parameters of mixture models cannot be directly estimated using the classical maximum likelihood estimators, there are several ways to estimate the parameters of a mixture model: Expectation Maximization (EM) (Reynolds 2008), Kernel Distribution Estimation (KDE), and non-parametric Bayesian Estimation.

2.2.2.1. Expectation Maximization (EM)

The EM algorithm is an iterative method for finding maximum likelihood or maximum posteriori estimates of parameters, where the model depends on latent variables. EM operates in two phases. First, E-Step, computes the probability of given samples to belong to the different clusters. Second, set the parameters of the clusters (\( \theta \)) to maximize the log likelihood function (equation 5) (Xu and Jordan 1996):

\[
E(I(\theta)) = E(\log(\prod_{i=1}^{N} P(x_i|\theta))) = E(\sum_{i=1}^{N} \log(p(x_i|\theta)))
\] (5)

Where \( \theta \) indicates the estimated parameters and \( x \) represents a \( d \) dimensional sample. The EM algorithm will attempt to optimize the expression \( E(I(\theta)) \) repeating the two phases described above until reaching convergence or a-priori determined number of repetitions. Using the EM algorithm, to estimate a Gaussian distribution follows (Xu, Jordan, and Hinton 1994). The E step, computes:

\[
h_j^{(k)}(i) = \frac{w_j^{(k)} P(x^{(i)}|\mu_j^{(i)}, \Sigma_j^{(i)})}{\sum_{t=1}^{M} w_t^{(k)} P(x^{(i)}|\mu_t^{(i)}, \Sigma_t^{(i)})}
\] (6)

The M step, finds a new estimate for parameters \( \theta = \{w_j, \mu_j, \Sigma_j\} \) using equations 7-9.

\[
w_j^{(k+1)} = \frac{\sum_{i=1}^{N} h_j^{(k)}(i)}{N}
\] (7)
\[ \mu_j^{(k+1)} = \frac{\sum_{i=1}^{D} h_j^{(k)}(i)x^{(i)}}{\sum_{i=1}^{D} h_j^{(k)}(i)} \]

\[ \Sigma_j^{(k+1)} = \frac{\sum_{i=1}^{D} h_j^{(k)}(i)[x^{(i)} - \mu_j^{(k)}][x^{(i)} - \mu_j^{(k)}]^T}{\sum_{i=1}^{D} h_j^{(k)}(i)} \] 

(8)  

(9)

Where \( w_j \) indicates the weight of component \( 'j' \), the mean vectors are represented by \( \mu_j \), and the covariance matrices \( \Sigma_j \). In the EM algorithm, the number of components is set a-priori. To find the most suitable model, given the number of components is not known, the algorithm is repeated several times with a different number of clusters. The best model is determined using statistical criterion indicator. Two common statistical criterions are the Bayesian information criterion (BIC) and the Integrated Completed Likelihood (ICL).

The Bayesian Information Criterion (BIC) was developed as an approximation to the log marginal likelihood of a model, and therefore, the difference between two BIC estimates may be a good approximation to the natural log of the Bayes factor (Posada and Buckley 2004). Choosing the model with the smallest BIC is equivalent to selecting the model with the maximum posterior probability. The BIC is formally defined as

\[ BIC = k \ln(n) - 2\ln(\hat{L}) \]

(10)

Where \( \hat{L} \) is the maximized value of the likelihood function of the model M i.e. \( \hat{L} = p(x|\hat{\theta}, M) \) where \( \hat{\theta} \) are the parameter values that maximize the likelihood function, \( x \) is the observed data, \( n \) is the sample size and \( k \) is the number of free parameters to be estimated. The BIC criterion suffers from three main limitations: the approximation is only valid for sample size \( n \) much larger than the number of parameters, \( k \) in the model, the BIC cannot handle complex collections of models, as in the variable selection problem in high-dimension and it often over estimates the number of clusters (Posada and Buckley 2004).

Like the BIC, Integrated Completed Likelihood (ICL) prefers models which explain the training data and punish complexity. The ICL criterion, gives an answer to the tendency of BIC to overestimate the number of clusters. This is done by replacing the maximum a
posteriori probability estimator of a sample $x_i$ to belong to a cluster $k$ with the missing cluster indicators $z_{ik}$ (Itti, Koch, and Niebur 2010):

$$z_{ik} = \begin{cases} 1 - \text{if } \arg \max(\hat{\theta}) = k \\ 0 - \text{otherwise} \end{cases}$$

(10)

2.2.2.2. Kernel Density Estimation

Kernel Density Estimation (KDE) estimates non-parametric models. A non-parametric model, does not assume a fixed number of parameters. As the number of samples grows the number of parameters grows. In parametric models, the goal is to model the data in the best way, given a known number of clusters. For example, in Figure 2.2A, a parametric model is presented, the model attempts to fit one cluster to all observations in the best way possible. In non-Parametric models (Figure 2.2B), each sample can be represented by a cluster. Some samples will merge (after smoothing) to represent one cluster. Theoretically, enabling the model to have a cluster per sample (Detry et al. 2009).

![Figure 2.2: Parametric Vs Non-Parametric Visualization](image)

2.2.2.3. Bayesian estimation

Bayesian estimation is based on Bayes’ rule:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

(11)
Where H and D are events and P(D)$\neq 0$. An additional interpretation is available, where H is a hypothesis and D is data which may give evidence for or against H. The prior $P(H)$ is the probability that H is true before the data is considered. The posterior $P(H|D)$ is the probability that H is true after the data is considered. The likelihood $P(D|H)$ is the evidence about H provided by the data D. $P(D)$ is the total probability of the data considering all possible hypotheses (Orloff and Bloom 2014). In most experiments, the prior probabilities on hypotheses are not known. Thus, priors can be determined using trial and error or maximum likelihood methods.

Each observation is composed of some pattern (in distribution estimation the parameters of the distribution is the pattern and their values are the hypothesis (H)) plus an independent noise. Assuming the observations are independent and identically distributed (IID) is required, however in this case it is clear the observations are not IID, they are conditionally independent (Gelman et al. 2014). Conditional independence means that given the parameter information the data is independent, identically distributed and exchangeable (changing the order in which the data was entered does not change the joint distribution). Conditional independence is sufficient according to de Finety (de Finetti 1995)

The prior $P(H)$ is previous information we have or assume about the model. In case there is no prior knowledge, it is common to either guess or use the likelihood. Conjugate priors are commonly used when estimating distributions as using them generates analytical equations (Murphy 2007). For example, the weight parameters of a mixture model are distributed multinomial as they are positive numbers which add to one therefore their conjugate prior is the Dirichlet distribution where $\alpha$ and $G_0$ represent the priors in the Dirichlet distribution (Ferguson 2014).
2.2.2.4. Comparison of model estimation methods

The EM estimation requires prior information regarding both the type of distribution as well as the number of components examined. When using the EM algorithm, we are prone to over-fitting as an optimization to the data is performed. Additionally, initial values influence the results and we may encounter local minima. Granville and Fagg (Granville, Fagg, and Southerland 2006), generated grasp affordance densities using EM estimation along with model selection criteria for mixture-model parameter estimation, trying to identify representative components.

KDE, on the other hand, is an algorithm which initially represents each sample with a cluster, causing high dimensionality. Thus, attempting to compute an analytical equation to model the distribution is very complicated, time consuming and often, impossible (Shalizi 2016). Detry, (Detry et al. 2009) applied kernel-density estimation methods for vision based grasp learning successfully.

When using the non-parametric Bayesian estimation, the number of clusters is among the estimated parameters and need not be set a-priori. Additionally, the use of conjugate priors derives, easily computed, analytical equations (describing the distribution). Furthermore, the parameters are drawn at each iteration, thus marginalizing over all possibilities.
2.3. A Non-Parametric Bayesian estimation method for GMMs

A non-parametric Bayesian estimation method for infinite GMMs was suggested by Rasmussen (2000). It has been successfully implemented for examining neuron signals.

The algorithm works as follows (Figure 2.3), first, priors are generated using maximum likelihood estimators. During algorithm initiation, the priors are calculated given that all observations belong to a single cluster. Second, the parameters and hyper parameters of clusters are sampled, using the conditional posterior distributions and the Gibbs sampler. The Gibbs sampler is a technique for generating random variables from a marginal distribution indirectly, without having to calculate the density (Casella and George 1992).

Once the parameters and hyper parameters of each cluster are sampled, a new cluster is sampled for each observation. This is also done using the Gibbs sampler, where the conditional posterior distributions depend on the parameters of the cluster the observation was last allocated to. The clusters sampled for each observation now pose an alternative for the current cluster the observation is allocated.

The probability for each observation to originate from an existing cluster or a new cluster is estimated using the Dirichlet Process Posterior:

Figure 2.3: Non-parametric Bayesian estimation- flow chart
\[
\theta_{n+1} | \theta_1, \ldots, \theta_n \sim \frac{1}{n + \alpha} \sum_{j=1}^{n} \delta_{\theta_j} (\theta_{n+1}) + \frac{\alpha}{n + \alpha} \mathcal{G}_0 (\theta_{n+1})
\]

(12)

\[
p(x_{n+1} | x_1, \ldots, x_n) = \sum_{k=1}^{K_n} \frac{n_k}{n + \alpha} p(x_{n+1} | \theta_k^*) + \frac{\alpha}{n + \alpha} \int p(x_{n+1} | \theta) \mathcal{G}_0 (\theta) d\theta
\]

(13)

Where \( \theta \) represents the parameters of a specific cluster, \( n \) is the number of samples and \( x \) represents the observations. The estimation in equation 12 will be used and inserted into equation 13. We can now sample using equation 13, the probability that the data point has originated from an already existing cluster (first expression in equation 13), and the probability that it is a new cluster (the second expression in the equation). Once a new cluster is proposed the probability of each data point to belong to the new cluster (likelihood) is calculated. The new cluster is now proposed to all data observations, the decision to belong to the new cluster does not depend solely on the likelihood, the likelihood is used as input to the multinomial distribution. At each iteration, for each data point a multinomial allocation is drawn and the allocation to cluster is determined (\( C_i \) indicates the cluster allocation for observation \( i \)). To conclude, at different iterations a data point may be affiliated with one cluster and at the following iteration it may belong to another.

Each iteration generates a valid estimation of the distribution model. However, to reach convergence and stable allocation of observations, many iterations need to be executed. To reduce complexity and computation time, at each iteration only one new cluster can be proposed to all observations (Mandel 2005).

2.4. Robotic reach-to-grasp motion

To successfully grasp and manipulate an object, a robot must bring its end-effector to a pose (position and orientation) from which a high-quality grasp can be formed. When the object to be grasped is known, grasps (end-effector configurations) can be synthesized a-priori. When the object location with respect to the manipulator is additionally known, manipulator grasp poses and configurations can additionally be computed a-priori. A collision-free path to a grasp configuration can be computed a-priori only in a known, static environment.
The path planning problem is the robot’s attempt to plan the arrival to its target position while avoiding obstacles along the way. Given a starting position, a target position and obstacles a path planning algorithm attempts to find a continues path while avoiding collisions. The common procedures for motion planning are executed in iterations, in each iteration, an attempt is made to advance towards the target position. If the attempt is successful, then it will be made otherwise another move will be examined.

There are many algorithms for motion planning available. Many motion planning algorithms resembles the problem to sampling and searching t possibilities trees. Such as the Rapid Exploring Random Tree (RRT) and its variants (Lavalle 2006).

RRT is used frequently due to its efficiency and ability to deal with constraints. RRT belongs to the Rapidly Exploring Dense Trees (RDT) family and differs from other sample and search algorithms in its ability to gradually improve its space coverage. The tree will eventually, densely cover the space (Lavalle 2006).

RRT algorithm attempts to find a free-collision path from the origin ($q_0$) to the goal ($q_{goal}$) using a sequence of random samples (where $a_i$ is the $i^{th}$ sample). In each iteration $a_i$ is connected to the graph (at point $q_n$) via the shortest path. If $q_n$ is a vertex, then it connects an edge between the two ($a_i$ and $q_n$). Otherwise if $q_n$ is an edge, $a_i$’s connection to the graph will generate two new vertices (the second one being the point on the edge where $a_i$ was connected) (Figure 2.4).

![Figure 2.4: Adding a new node to an existing graph (Lavalle 2006)](image)

When the path between $a_i$ and $q_n$ is not feasible due to the presence of an obstacle, an edge will be made from $q_n$ to $q_s$ ($q_s$ being the last point possible before hitting the obstacle). The closeness of $q_s$ to the obstacle depends on both the algorithm chosen to check for collision as well as the algorithm chosen to find the nearest point (exact or approximate) (Figure 2.5).
Single tree search uses the algorithm above to expand the tree from $q_0$. In each iteration, the algorithm will either select $a_i$ or $q_{goal}$ and check whether it is possible to connect the RRT to the $q_{goal}$. The selection between $a_i$ and $q_{goal}$ can be implemented in different ways.

The balanced, bidirectional search method usually outperforms the single tree search. The search produces two RRT trees. The first tree ($T_a$) begins at the origin $q_0$ and the second tree ($T_b$) begins at $q_{goal}$. After several iterations $T_a$ and $T_b$ are swapped, hence the allocation ($T_b, T_a$) is not permanent.

The Grasp Region Rapid exploring Random Trees’ (GR-RRT) (Reshef, Eizicovits, and Berman 2014) purpose is to incorporate quality of the target configurations into the RRT motion planning algorithm. Thus, allowing more intelligent sampling. The algorithm includes two phases, the offline grasp region determination (GR) and the online planning (Figure 2.6).

The offline GR determination primarily produces graspibility maps for the required objects. Eventually the configurations are divided into two groups the successful configurations and the not successful configurations this is done using an empirical threshold. The GR is the area encompassing all successful configuration. GR-RRT uses the EM algorithm with BIC criteria to estimate the distributions (Schwarz 1978). The parameters estimated are then delivered to the planner to allow him to sample accordingly.

The online planning phase can now use the offline information (grasp affordance densities) and plan an intelligent path. To insure only successful configurations are selected whenever an out of GR bounds configuration is selected, it is discarded, and new sampling occurs.
This algorithm, though slightly more time consuming, produces significantly better results both in path selection and in target configuration selection (Reshef, Eizicovits, and Berman 2014).

2.5. Reach-to-grasp motion of patients with stroke

A stroke can cause temporary or permanent disabilities, depending on how long the brain lacks blood flow and which part was affected. One of the common complications are paralysis or loss of muscle movement. A post stroke patient may become paralyzed on one side of his body, or lose control of certain muscles, such as those on one side of the face or one arm. (Thrasher et al. 2008). Most post stroke patients experience impairment of arm movement, (Lin 2008, Carr and Shepherd 1998) as a result of the impairment, patients often avoid using the affected arm and compensate the impairment using the other arm or trunk (Michaelsen Dannenbaum, and Levin 2005). The reach to grasp act is often impaired in patients with stroke.

Reaching a specific endpoint arm position can be repeated in different ways. A subject may execute varying movement speeds or one constant speed to different distances (Liebermann et al. 2010). During unimpaired execution of motion, there is a variance in path selection i.e. speed and route, however, after re-executing a similar motion numerous times, movements may follow specific, least effort paths. Patients with stroke demonstrate temporal inefficiency in preplanning and executing movements and rely heavily on feedback control of reaching and grasping (Lin 2008). Liebermann (Liebermann et al. 2010) compared the number of sub movements when reaching to grasp a target in healthy and post stroke
subjects, and concluded that post stroke patients showed more sub movements. Lieberman also noticed jerkier movements for patients with stroke.

The orientation adopted by a grasping hand is known to depend on the shape and orientation of the object to be grasped (Desmurget, Prablanc, and Prablanc 1997). In addition, recent studies have demonstrated that it also depends on the spatial characteristics of the task such as the location of the object and initial hand position. It is suggested that not only object location and hand initial position affect variation in hand orientation, but also the movement direction (Bennis and Levin 2003).

Patients with stroke often have weaknesses in distal muscles used to stabilize the wrist, decreased grip strength and lack of fine finger control. These impairments may lead to the development of alternative grasping strategies such as anchoring the fingers on the object to achieve a passive grasp (Roby-Brami et al. 1997).
3. Non-parametric Bayesian estimation of grasp affordance

3.1. Overview

In this chapter, the non-parametric Bayesian estimation of grasp affordance algorithm developed is presented in section 3.2. Various tests conducted to examine the limitations and abilities of the algorithm are presented in section 3.3.

3.2. Non-parametric Bayesian estimation algorithm description

The algorithm developed integrates Rasmussen's estimation method, Infinite Gaussian Mixture Model (IGMM) (Rasmussen 2000) and Bangart's VMF estimation method (Bangert 2010). The new combined estimation algorithm (Figure 3.1) is suitable for finding parameters of a combined infinite mixture model, where several degrees of freedom (DOF) are Gaussian and several are VMF. Such a combined mixture model is suitable for representing grasp configurations, in which the three position DOF are modelled using Gaussian distribution components and the three orientation DOF are modelled using VMF distribution components. In Figure 3.1, The bold box indicates an addition to the algorithm and the broken line boxes indicates a stage modification.

![Figure 3.1: Non-parametric Bayesian estimation algorithm – flow chart](image-url)
The algorithm starts with an initiation stage, an assumption that all data points are allocated to one cluster is made (Rasmussen 2000). The parameters are initiated using maximum likelihood estimators given the original data. The original VMF estimation method set the hyper parameters to constants, we attempted to modify the estimation of the hyper parameters. However due to numeric problems we only changed the hyper parameters (m,t) to be initiated using maximum likelihood estimators, and left the hyper parameters (a,b) constant.

For the Von Misses-Fisher distribution, kappa, the concentration parameter is estimated using:  \[ \hat{\kappa} = \frac{d - \frac{3}{r}}{1 - \frac{3}{r^2}} \]  (Banerjee 2005) Where d is the number of dimensions and \( r = \sum_i x_i \).

The VMF mean parameter \( (u_{cy}) \) is initiated using \( \hat{\mu} = \frac{r}{\sqrt{r^2}} \) where \( r = \sum_i x_i \). The hyper parameters for the mean parameter (m,t) are initiated using maximum likelihood, where m (expected direction parameter) is equal to \( \hat{\mu} \) and t (concentration parameter) is equal to the initiation of the parameter kappa. For the Gaussian parameters the initiation of the location was done following Rasmussen's work (Rasmussen 2000).

The conjugate prior distributions are presented in table 1. The parameters of a specific cluster \( i (\mu_i, \sigma_i, \kappa_i, \mu_{cy_i}) \) are estimated using the data observations allocated to the i'th cluster. Differently, the hyper parameters, are estimated using all data observations regardless of the allocation to clusters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution chosen for parameters</th>
<th>Distribution chosen for estimation of hyper - parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite GMM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu )-Mean parameter</td>
<td>Normal ( (\lambda, r^{-1}) ) Where ( \lambda ) is the mean parameter and ( r ) is precision parameter</td>
<td>( p(\lambda) \sim \text{Normal} (\mu_y, \sigma_y^2) ) where ( \mu_y ) is the mean of all observations and ( \sigma_y^2 ) is the variance of all observations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( p(\lambda) \sim \text{Gamma} (1, \sigma_y^{-2}) ) where ( \sigma_y^{-2} ) is the inverse variance of all observations</td>
</tr>
</tbody>
</table>
The posterior conditional distributions (equations 14-19) follow Bangart and Rasmussen's work with adaptations (Bangert 2010, Rasmussen 2000). Similarly to the initiation stage, in Bangart's algorithm (Bangert 2010) the hyper parameters of VMF distribution $(m, t, a, b)$ remain constant throughout all estimation iterations. Thus, the same modification explained in initiation was performed for the hyper parameter estimation.
Translation parameters and hyper parameters (Rasmussen 2000)

\[
p(\mu_j|c, y, s_j, \lambda, r) \sim \text{Normal} \left( \frac{y_j n_j s_j + \lambda r}{n_j s_j + r}, \frac{1}{n_j s_j + r} \right)
\]

\[
p(\lambda|\mu_1, ..., \mu_k, r) \sim \text{Normal} \left( \frac{\mu y \sigma^2 + r \sum_k \mu_j}{\sigma^2 + kr}, \frac{1}{\sigma^2 + kr} \right)
\]

\[
p(r|\mu_1, ..., \mu_k, \lambda) \sim \text{Gamma}(k + 1, \left[ \frac{1}{k+1} (\sigma_y^2 + \sum_{j=1}^k (\mu_j - \lambda)^2) \right]^{-1})
\]

\[
p(s_j|c, y, u_j, \beta, w) \sim \text{Gamma}(\beta + n_j, \left[ \frac{1}{\beta + n_j} (w\beta + \sum_{i:c_i=j} (y_i - u_j)^2) \right]^{-1})
\]

\[
p(w|s_1, ..., s_k, \beta) \sim \text{Gamma}(k\beta + 1, \left[ \frac{1}{k\beta + 1} (\sigma_y^2 + \beta \sum_{j=1}^k s_j) \right]^{-1})
\]

\[
p(\beta|s_1, ..., s_k, w) \propto I \left( \frac{\beta - k}{2} \right) \exp \left( \frac{1}{2\beta} \left( \frac{\beta - k - 2}{2} \right) \prod_j s_j w^2 \right)
\]

Where \(y_i\) represents the three position parameters of data sample 'i', \(n_j\) is the number of observations allocated to cluster 'j' and 'k' indicates the number of clusters, the rest of the parameters are presented in Table 1. The latter density is not of standard form, but it can be shown that \(p(\beta|s_1, ..., s_k, w)\) is log-concave, so we may generate independent samples from the distribution for \(\log(\beta)\) using Adaptive Rejection Sampling (ARS) technique.

Orientation parameters (Bangert 2010):

\[
p(T_j|a, b, \{x_{i\in k}\}, \mu_j) \propto \left\{ \frac{T_j}{4\pi \sinh(T_j)} \right\}^{a+n_j} \exp(T_j \ast \left( b + \sum_{i\in k} \mu_c y_c T_k x_i \right))
\]

\[
p(\mu_j|\{x_{i\in k}\}, T_j, m_o, t_0) \propto \left| \frac{m_o t_0 + T_k \sum_{i\in k} x_i |}{4\pi \sinh((m_o t_0 + T_j) \sum_{i\in k} x_i)} \right| \exp(T \left( \frac{m_o t_0 + T_j \sum_{i\in k} x_i}{|m_o t_0 + T_j \sum_{i\in k} x_i|} \right)^T x)
\]

Where \(x_i\) is the orientation parameters of data sample 'i', the rest of the parameters are presented in Table 1. The kappa parameter's posterior conditional distribution is a complex function (equation 20), requiring a suitable sampling method such as slice sampling algorithm (Neal 2003). Still, when encountering a cluster with many observation samples, numeric problems occur. Therefore, when the algorithm encounters a cluster with many observations, the algorithm randomly selects a portion of observations from the cluster.
Mixing parameters:

Components with more than one data sample allocated

Gaussian: \[ p(c_i = j | c_{-i}, u_j, s_j, a) \propto p(c_i = j | c_{-i}, a) p(y_i | c_{-i}, u_j, s_j) \] (22)

VMF: \[ p(c_i = j | c_{-i}, u_j, s_j, a) \propto p(c_i = j | c_{-i}, a) p(x_i | c_{-i}, u_j, \kappa_j) \] (23)

All other components combined:

Gaussian \[ p(c_i \neq c_{i'} \text{ for all } i \neq i' | c_{-i}, a) \int p(y_i | u_j, s_j) p(u_j, s_j | \lambda, r, \beta, w) du_j ds_j \] (24)

VMF: \[ p(c_i \neq c_{i'} \text{ for all } i \neq i' | c_{-i}, a) \int p(x_i | u_j, \kappa_j) p(u_j, \kappa_j | a, b, t, m) du_j d\kappa_j \] (25)

where the subscript \(^{-i}\) indicates all indexes except \(i\), \(x_i\) is the orientation parameters of data sample \(i\), \(y_i\) is the position parameters of data sample \(i\). The rest of the parameters are presented in Table 1. The estimation of the parameters for the translation and orientation (equations 24-25) were done independently. However, determining the observation allocation to a cluster and whether a new cluster arouse, considers both translation and orientation parameters. To combine the Gaussian and VMF probabilities we normalized the probabilities and gave weights to both translation and orientation probabilities. This stage is illustrated in the bold box in Figure 3.1. Once a new cluster is proposed, reallocation of the data occurs, and a new iteration begins. Only clusters which represent more than one sample remain in the following iteration.

3.3. Validation experiment

To validate the algorithm an examination of each, position and orientation, based estimation were tested alone and combined. Data sets with different levels of overlap between clusters were examined to understand the abilities and limitations of the algorithm.

3.2.1. Data

Four different sets of synthetic data were generated, the first set of synthetic data generated was from distinctly different clusters both in orientation and in position. The first data set
was used for validation and three types of estimations were carried, estimation by position only, estimation by orientation only and estimation integrating both position and orientation. The second data set, tested the algorithm’s performance when an overlap in position existed and a distinct separation in orientation (OP). The third data set tested the algorithm given an overlap in orientation and a distinct separation in position (OO). Lastly, the fourth data set tested the algorithm given an overlap in both position and orientation (OPO).

500 samples were generated from each data set (data set parameters presented in appendix 1) using the random mixture model sampling algorithm specified in section 4.2.1 and then used as input to the non-parametric Bayesian algorithm specified in section 3.2.

3.3.2. Experimental protocol

15000 iterations were performed using the grasp affordance density algorithm, hyper parameters of the VMF concentration parameter, kappa, are constants, and set to a=5, b=4.7 the values were determined following Bangart (Bangert 2010). And the hyper parameter of the Dirichlet distribution is drawn from an inverse gamma distribution G∼(1,1). The likelihoods (VMF and Gaussian) determining the allocation of a sample to cluster were given equal weights (0.5,0.5). Sampling kappa using the slice sampling algorithm (VMF concentration parameter) was done based on one hundred randomly selected samples with a burning in value of 5. The final iteration of the algorithm is used as the selected model where clusters which weigh less than 5% are discarded. To compare the goodness of fit visual graphs are generated and examined.

3.3.3. Results

3.3.3.1. Validation Set: Distinct separation of both orientation and position

For estimation by position only, the algorithm converged after about 3500 samples. The affordance densities established, includes three clusters (Figure 3.2-3.3, table 2). The position expected value is 0.57 ± 0.01 cm away from the ground truth. The weights of the cluster are (0.33,0.338,0.332) compared with (0.33,0.33,0.33) and the variance is also very similar (Appendix 1). Though the allocation of data did not consider the orientation components the estimated expected value of the orientation is very close to the ground truth
and is only $0.65 \pm 0.14$ degrees apart. The concentration of the orientation data is lower than the concentration of the ground truth (Appendix 1).

**Figure 3.2: Position only - mixture model**

**Figure 3.3: Position only - distribution by component**

The ellipsoid represents the cluster which the algorithm found fit where the center of the ellipsoid represents the expected value and the circumference is one standard deviation away from the center.

| TABLE 2: Position only - parameters per cluster |

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.33</td>
<td>0.338</td>
<td>0.332</td>
</tr>
<tr>
<td><strong>Position</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>[100.03, 99.85, 100.08]</td>
<td>[0.95, 0.98, 1.07]</td>
<td>[199.9, 199.84, 199.94]</td>
</tr>
<tr>
<td>S</td>
<td>1.2, -0.02, 0.04</td>
<td>0.95, 0.01, 0.10</td>
<td>1.05, 0.04, 0.07</td>
</tr>
<tr>
<td>Kappa</td>
<td>107.88</td>
<td>100.75</td>
<td>94.42</td>
</tr>
<tr>
<td>$\mu_{\text{cyc}}$</td>
<td>[0.1, -0.01, 0]</td>
<td>[0, -0.01, 0.71]</td>
<td>[0.01, 0.72, 0.7]</td>
</tr>
</tbody>
</table>
For estimation by orientation only, the algorithm converged after about 3000 samples. The affordance densities established, includes three clusters (Figure 3.4-3.5, table 3). The orientation expected value is $0.81 \pm 0.41$ degrees away from the ground truth, the weights of the cluster are (0.33,0.338,0.332) compared with (0.33,0.33,0.33) and the concentration of the orientation data is lower than the concentration of the ground truth (Appendix 1). Though the allocation of data did not consider the position components the position expected value is very close to the ground truth and is only $0.21 \pm 0.13$ cm apart and the variance is also very similar (Appendix 1).

![Convergence](image1.png)

**Figure 3.4:** Orientation only- mixture model and convergence

The black lines represent the direction (expected value) of the cluster

![Distribution by component](image2.png)

**Figure 3.5:** Orientation only - distribution by component

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.338</td>
<td>0.33</td>
<td>0.332</td>
</tr>
<tr>
<td>Position</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>[0.88,0.87,1.11]</td>
<td>[99.96,99.98,100.08]</td>
<td>[200.04,199.99,200.07]</td>
</tr>
<tr>
<td>$S$</td>
<td>1.06,0.19,0.02</td>
<td>1.08,0.09,0.03</td>
<td>0.89,0.02,0.16</td>
</tr>
<tr>
<td></td>
<td>0.19,0.93,-0.03</td>
<td>0.09,0.84,0.09</td>
<td>0.02,1.01,0</td>
</tr>
</tbody>
</table>

**TABLE 3:** Orientation only-parameters per component
Estimation given both position and orientation integrated, the algorithm converged after about 200 iterations. The affordance densities established, includes three clusters (Figure 3.6-3.7, table 4). The position expected value is $0.27 \pm 0.09$ cm away from the ground truth, the weights of the cluster are $(0.33, 0.338, 0.332)$ compared with $(0.33, 0.33, 0.33)$ and the variance is also very similar (Appendix 1). The orientation expected value is very close to the ground truth and is only $0.89 \pm 0.24$ degrees apart. The concentration of the orientation data is lower than the concentration of the ground truth (Appendix 1).

<table>
<thead>
<tr>
<th>Kappa</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>118.39</td>
<td>0.02,0.03,1.47</td>
</tr>
<tr>
<td>74.09</td>
<td>0.03,0.09,1.3</td>
</tr>
<tr>
<td>81.11</td>
<td>0.16,0.17</td>
</tr>
</tbody>
</table>

$\mu_{cyc}$

| $[-0.01,0,0.7,0.71]$ | $[0,1,0.01,0.01]$ | $[-0.01,0.71,0.7,0.02]$ |

Figure 3.6: integrated algorithm- Convergence and mixture model

Quaternions and positions are colored according to the cluster they are allocated to. The black lines in figure C represent the direction (expected value) of the cluster.

Figure 3.7: Integrated algorithm- distribution by component

The ellipsoid represents the cluster which the algorithm found fit where the center of the ellipsoid represents the expected value and the circumference is one standard deviation away from the center.
<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.338</td>
<td>0.332</td>
<td>0.33</td>
</tr>
<tr>
<td>Position</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu)</td>
<td>[0.94,0.95,1.12]</td>
<td>[200.07,199.89,200.2]</td>
<td>[100.21,100,100]</td>
</tr>
<tr>
<td>(S)</td>
<td>1.17,0.13,0.07</td>
<td>1.17,0.16,0.19</td>
<td>1.4,0.12,0.05</td>
</tr>
<tr>
<td></td>
<td>0.13,1.08,0.09</td>
<td>0.16,1.09,0.23</td>
<td>0.12,0.92,-0.01</td>
</tr>
<tr>
<td></td>
<td>0.07,0.09,1.21</td>
<td>0.19,0.23,1.32</td>
<td>0.05,-0.01,1.03</td>
</tr>
<tr>
<td>Orientation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kappa</td>
<td>98.75</td>
<td>108.3</td>
<td>101.56</td>
</tr>
<tr>
<td>(\mu_{cye})</td>
<td>[0.005,-0.013,0.718,0.696]</td>
<td>[-0.006,0.716,0.698,-0.007]</td>
<td>[-0.001,1,-0.01,0.007]</td>
</tr>
</tbody>
</table>

**TABLE 4**: Integrated algorithm - parameters per component

3.3.3.2. Overlap Position dataset (OP): Distinct separation of orientation and overlap in position

The samples generated are demonstrated in Figure 3.8, the model parameters used to generate the samples are presented in Appendix 1.

![Figure 3.8: Position overlap - samples generated](image)

Estimation given both position and orientation integrated, the algorithm converged after about 300 iterations. The affordance densities established, includes three clusters (Figure 3.9-3.10, table 5). The position expected value is 0.66 ± 0.23 cm away from the ground truth, the weights of the cluster are (0.322,0.39,0.288) compared with (0.33,0.33,0.33) and the variance is also very similar (Appendix 1). The orientation estimated expected value is very close to the ground truth and is only 3.24 ± 1.9 degrees apart. The concentration of the orientation data is lower than the concentration of the ground truth (Appendix 1).
Figure 3.9: Translation overlap - convergence and mixture model -
Quaternions and positions are colored according to the cluster they are allocated to. The black lines in figure C represent the direction (expected value) of the cluster.

Figure 3.10: Translation overlap - distribution by component
The ellipsoid represents the cluster which the algorithm found fit where the center of the ellipsoid represents. The expected value and the circumference is one standard deviation away from the center.
### 3.3.3.3. Overlap Orientation dataset (OO): Distinct separation of position and overlap in orientation

The samples generated are demonstrated in Figure 3.11 the model parameters used to generate the samples are presented in Appendix 1.

![Samples generated - Positions](image1)

![Samples generated - Orientations](image2)

**Figure 3.11 Orientation overlap- samples generated**

The algorithm converged after 300 iterations. The affordance densities established, includes three clusters (Figure 3.12-3.13, table 6). The position expected value is $0.19 \pm 0.04$ cm away from the ground truth, the weights of the cluster are $(0.348,0.364,0.288)$ compared with $(0.33,0.33,0.33)$ and the variance is also very similar (Appendix 1). The orientation estimated expected value is very close to the ground truth and is only $2.62 \pm 1.88$ degrees apart. The concentration of the orientation data is lower than the concentration of the ground truth (Appendix 1).

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.322</td>
<td>0.39</td>
<td>0.288</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$[2.67,3.25,3.11]$</td>
<td>$[6.24,5.85,6.03]$</td>
<td>$[-0.16,-0.42,-0.3]$</td>
</tr>
<tr>
<td>$S$</td>
<td>3.73,0.1,0.12</td>
<td>2.78,-0.01,0.39</td>
<td>2.95,-0.32,0.06</td>
</tr>
<tr>
<td></td>
<td>0.1,3.11,0.13</td>
<td>-0.01,2.57,0.12</td>
<td>-0.32,2.97,0.39</td>
</tr>
<tr>
<td></td>
<td>0.17,0.13,2.99</td>
<td>0.39,0.12,2.6</td>
<td>0.06,0.39,4.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orientation</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kappa</td>
<td>45.2</td>
<td>42.16</td>
<td>49.74</td>
</tr>
<tr>
<td>$\mu_{cy}!c$</td>
<td>$[0.737,0.474,-0.235,0.4190]$</td>
<td>$[0.86,0.361,0.34,-0.12]$</td>
<td>$[0.469,0.195,-0.845,-0.169]$</td>
</tr>
</tbody>
</table>
Figure 3.12: Orientation overlap – convergence and mixture model
quaternions and positions are colored according to the cluster they are allocated to. The black lines in figure C represent the direction (expected value) of the cluster.

Figure 3.13 Orientation overlap - distribution by component
The ellipsoid represents the cluster which the algorithm found fit where the center of the ellipsoid represents. The expected value and the circumference is one standard deviation away from the center.
TABLE 6: Orientation overlap- parameters per component

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.348</td>
<td>0.364</td>
<td>0.288</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>μ</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[ 10.06,10.09,10.07]</td>
<td>[ 20.05,19.9,20.06]</td>
<td>[ 1.02,1.04,1.08]</td>
</tr>
<tr>
<td>S</td>
<td>1.03,0.25,0.15</td>
<td>1.27,0.08,0.12</td>
<td>1.53,0.48,0.57</td>
</tr>
<tr>
<td></td>
<td>0.25,1.16,0.21</td>
<td>0.08,0.95,0.21</td>
<td>0.48,1.29,0.56</td>
</tr>
<tr>
<td></td>
<td>0.15,0.21,1.32</td>
<td>0.12,0.21,1.21</td>
<td>0.57,0.56,1.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orientation</th>
<th>Kappa</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>31.25</td>
<td>30.09</td>
<td>28.43</td>
</tr>
<tr>
<td>μ_ycy</td>
<td>[ -0.036,-0.999,0.007,0.017]</td>
<td>[ 0.838,0.342,-0.383,0.185]</td>
<td>[0.687,-0.726,-0.024,0.012]</td>
</tr>
</tbody>
</table>

3.3.3.4. Overlap in Position and Orientation dataset (OTR)

The samples generated are demonstrated in Figure 3.14 the model parameters used to generate the samples are presented in Appendix 1.

![Samples generated - Positions](image1)
![Samples generated - Orientations](image2)

Figure 3.14: Total overlap - samples generated

The algorithm converged after about 300 iterations (Figure 3.15A) however the convergence was not distinct, and the algorithm’s number of clusters varied between 2-3 clusters. The affordance densities established, includes three clusters (Figure 3.15-3.16, table 7). The position expected value is 2.62 ± 2.36 cm away from the ground truth, the weights of the cluster are (0.41,0.13,0.46) compared with (0.33,0.33,0.33) and the variance is also very similar (Appendix 1). The orientation estimated expected value (of clusters 1,3) is very close to the ground truth and is only 2.08 ± 0.97 degrees apart and for cluster two, 78 degrees apart. The concentration of the orientation data is lower than the concentration of the ground truth (Appendix 1)
Figure 3.15: Total overlap-convergence and mixture model-
Quaternions and positions are colored according to the cluster they are allocated to. The black lines in figure C represent the direction (expected value) of the cluster. The ellipsoids in figure B represents the cluster which the algorithm found fit, where the center of the ellipsoid represents The expected value and the circumference is one standard deviation away from the center.

Figure 3.16: Total overlap- distribution by component
The ellipsoid represents the cluster which the algorithm found fit where the center of the ellipsoid represents the expected value and the circumference is one standard deviation away from the center.
3.3.4. Discussion

The estimated models resemble the ground truth but with higher dispersion in the orientation components (kappa). A relatively small overlap between clusters in orientation parameters, significantly impacts the algorithm’s ability to distinguish between clusters and estimate its true values. A plausible explanation is that the variance in position does not correspond to the variance in orientation. The space characteristics are different for these two distributions, a Gaussian distribution parameter is defined between \([-\infty, \infty]\) and a VMF parameter distribution is defined between \([-180, 180]\).

As for the convergence, it occurs very early i.e. before iteration 1000 in all cases. Thus, even though Rasmussen (Rasmussen 2000) suggested to use 30000 iterations we decided to use 15000 iterations. Furthermore, to avoid a large amount of small, unstable clusters, clusters with a weight of 5% and less are discarded. In table 8 the parameters chosen to initiate the algorithm in the following applications, robotic reach to grasp motion planning and analysis of reach to grasp motion of patients with stroke, are presented.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of iterations</td>
<td>15000</td>
</tr>
<tr>
<td>a- Hyper parameter VMF</td>
<td>5</td>
</tr>
<tr>
<td>b- Hyper parameter VMF</td>
<td>4.7</td>
</tr>
<tr>
<td>a- Hyper parameter Dirichlet</td>
<td>$G^{-1} \sim (1,1)$</td>
</tr>
<tr>
<td>VMF likelihood weight</td>
<td>0.5</td>
</tr>
<tr>
<td>GMM likelihood weight</td>
<td>0.5</td>
</tr>
<tr>
<td>Slice sampling burning in value</td>
<td>5</td>
</tr>
<tr>
<td>Number of randomly selected observations</td>
<td>100</td>
</tr>
<tr>
<td>for slice sampling</td>
<td></td>
</tr>
</tbody>
</table>
4. Robotic reach-to-grasp motion planning

4.1. Overview

The grasps affordance densities used in the GR-RRT algorithm, were modeled by a six-dimensional Gaussian mixture model, and were found separately for each region using EM (Reshef, Eizicovits, and Berman 2014). The division of data between the regions, along with the use of a non-cyclic distribution for estimation of orientation led to models with a high number of components. This complicated the sampling stage during path planning and led to poor generalization. We replaced these models with densities functions modeled using Gaussian functions for positions and VMF functions for orientation, and applied non-parametric Bayesian estimation for model parameter estimation as illustrated in chapter 3. This new method is termed grasp affordance-RRT (GA-RRT). The rest of this chapter is organized as follows: section 4.2 describes the GA-RRT algorithm including the mixture model random sampling algorithm developed. Section 4.3 describes an experiment for testing the GA-RRT algorithm. Results are presented in section 4.4 and a discussion is presented in section 4.5.

4.2. GA-RRT algorithm

Like the GR-RRT algorithm the GA-RRT algorithm has two phases, a-priori offline estimation phase (Figure 3.1) and a run-time planning phase (Figure 4.1). The grasp affordance density is derived using non-parametric Bayesian estimation based on graspability maps in the offline estimation phase. During the run-time planning phase, a collision-free path is found using the bi-directional RRT algorithm, where goal configurations are sampled from the grasp affordance density.

Figure 4.1: Run time planning phase flow chart
The sampling algorithm has two stages (pseudo code below). First, a cluster is sampled with selection probability based on its weight. Second, a configuration is sampled from the selected cluster, where the location and orientation are sampled separately. The location is sampled from the GMM using mvrnd, the built-in function for sampling GMMs in Matlab™. The orientation is sampled from the VMF mixture model using the random Mises-Fiser sampling algorithm (Jung 2009). To ensure only high quality grasp configurations are used, samples which are far from the mean for both orientation and position are discarded and sampling is re-iterated. For the position, samples within 3σ range are kept. Orientation is represented by a quaternion \((q_0, q_1, q_2, q_3)\). The distance from the mean is evaluated separately for the direction vector and for rotation angle \(q_0\). Only samples where both are within \(\frac{1}{3+\kappa}\) range are kept.

---

**Algorithm random configuration sampling**

\[
[\mu_i, \sigma_i, \kappa_i, \mu_{cyc_i}] = \text{getClusterDist}()
\]

The function \(\text{getClusterDist}()\), draws a cluster ID, randomly according to cluster weight and sets the parameter values according to the parameters of the selected cluster

Do

\[
[xyz] = \text{mvnrnd}(\mu_i, \sigma_i)
\]

While \((\mu_i - 3\sigma_i > xyz \text{ or } \mu_i + 3\sigma_i < xyz)\)

Do

\[
[\text{quaternion}] = \text{vmfrnd}(\kappa_i, \mu_{cyc_i})
\]

\[
\theta = \arccos\left(\frac{\overline{\mu_{cyc_i}} \cdot \text{quaternion}[1:3]}{\left|\overline{\mu_{cyc_i}}\right| \cdot \left|\text{quaternion}[1:3]\right|}\right)
\]

While \(\frac{-1}{3+\kappa} > \theta \text{ or } \frac{1}{3+\kappa} < \theta \text{ or } \frac{-1}{3+\kappa} > \text{quaternion}[0] \text{ or } \frac{1}{3+\kappa} < \text{quaternion}[0]\)

Return \(\text{sample} = [xyz, \text{quaternion}]\)
4.3. Experiment

4.3.1. Object and Environment

A mug with multiple grasping regions and a jaw gripper were modeled in simulation (Figure 4.2-4.3). The simulated environment was modeled based on the Telerobotics lab, the Industrial Engineering Dept. at the Ben-Gurion University (Figure 4.4). It comprised a six DOF manipulator (UP6, MOTOMAN, Japan) with a two-jaw gripper (HGPL-25-60-A, FESTO, Germany), a table placed within the robot’s reach, and wooden blocks that served as obstacles. Five compositions of the blocks and the object location were created for each object (Figure 4.5). (Eizicovits and Berman 2014). A graspibility map containing 1500 grasps was generated for the gripper and mug by a robotic expert, (Figure 4.6). The grasps included three general types of grasps, one grasping the mug handle, and two grasps types grasping the mug body. The estimation algorithm and simulation were executed with MATLAB (Version 2016A, Mathworks, USA) using an Intel® i7-5500U, 2.4GHz CPU, with 8GB 2.4GHz RAM running Windows 10.

![Figure 4.2: Mug’s dimensions](image1)

![Figure 4.3: Grippers dimensions](image2)
4.3.2. Experimental protocol

Distribution estimation parameters were initiated according to table 8. The probability of choosing to add a configuration over expanding the RRT tree, $P_{\text{sample}}$, was set to 0.15. Planning was executed 20 times per environment composition (compositions one to five). The maximum number of iterations was set to 20,000. Each resulting path was smoothed using a path smoothing method based on vertices removal (Reshef and Berman 2013). The configuration used for analysis is the Tool Centre Point (TCP).

4.3.3. Analysis

Algorithm performance was evaluated in terms of computation time, path quality, and grasp success. Path quality was quantified by the final path length in the configuration space, using.
two normalized distance measures: Euclidean distance, $ND_e$ (equation 26) and City-block distance, $ND_{cb}$ (equation 27).

$$ND_e = \frac{\sum_{i=2}^{V} \left( \sum_{j=1}^{J} (x_{i,j} - x_{i-1,j})^2 \right) \sqrt{\sum_{j=1}^{J} (x_{i,j} - x_{V,j})^2}}{-1}$$ (26)

Where $x_{i,j}$ (in deg) is the position of the $i^{th}$ vertex in the $j^{th}$ dimension of the configuration space, $V$ is the total number of vertices in the path, and $J$ is the total number of manipulator joints (six in our experiment).

$$ND_{cb} = \frac{\sum_{i=2}^{V} \sum_{j=1}^{J} |x_{i,j} - x_{i-1,j}|}{\sum_{j=1}^{J} |x_{i,j} - x_{V,j}|} - 1$$ (27)

Both measures are normalized to start at zero, for the shortest possible path. The Euclidean distance, $ND_e$, reflects the distance with respect to the minimal movement length and thus is related to shortest mission execution time. While the City-block distance, $ND_{cb}$, reflects distance with respect to minimal movement of each joint, and thus is related to a minimal motor effort.

A considerable number of paths were along the line-of-sight to the target pose (both $ND_e$ and $ND_{cb}$ are 0), thus the analysis of path quality was divided into two categories. Grasps where the Euclidean distance was equal to the line-of-sight and others.

Grasp success was evaluated by projecting the sampled pose back onto the graspability map. The grasp was determined as successful (‘1’) in case of a grasp of quality grade of 0.7 or above, and as unsuccessful (‘0’) otherwise.

4.4. Results

The algorithm converged very fast after about 100 samples. The affordance densities established, includes three clusters (Figure 4.7-4.8, table 9). The estimated model represents the data comprehensively and demonstrates the three grasp types which were expected.
Figure 4.7: Mug convergence and mixture model
Data points and quaternions are colored according to the cluster they are allocated to. The center of the ellipsoid represents the mean and the circumference is one standard deviation away from the center.

Figure 4.8: Mug distribution by component

<table>
<thead>
<tr>
<th>TABLE 9: Mug-Parameters per component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster</td>
</tr>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>Position</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
For environment 5, the algorithm did not find a target point on the mug. In the remainder of the results we will refer to the environments which found at least one target point either successful or not (environments 1,2,3,4).

Average planning time was 4.97±4.6 seconds (Table 10). Path quality: out of the generated paths, 50% were along the line-of-sight to the target pose. In environment composition one (no obstacles) and three (an obstacle in front of the object) all paths were along the line-of-sight. The average values for both distance measures, $ND_e$ and $ND_{cb}$, were 0.665 and 0.666 respectively (Table 10), which means that for both distance measures the average path required 66%-67% more effort to execute than the lower bound. For grasp success, the average grasp success rate was 95% (Table 10). Examples of grasps per environment are shown in Figures 4.9-4.12.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Euclidean distance, $ND_e$</th>
<th>City-block distance, $ND_{cb}$</th>
<th>Time (seconds)</th>
<th>grasp quality grade</th>
<th>Grasp success*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2.38</td>
<td>0.720</td>
<td>95</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1.6</td>
<td>0.723</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>0.619</td>
<td>0.650</td>
<td>7.21</td>
<td>0.732</td>
<td>95</td>
</tr>
<tr>
<td>4</td>
<td>0.711</td>
<td>0.682</td>
<td>8.66</td>
<td>0.809</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not line-of-sight path average</td>
<td>0.665</td>
<td>0.666</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Grasp success calculated assuming quality cutoff for successful grasps.
Figure 4.9: Environment 1 - representative grasps - the grasp grades are: A =0.77, B=0.88, C=0.75

Figure 4.10: Environment 2 - representative grasps - the grasp grades are: A =0.76, B=0.61, C=0.75

Figure 4.11 Environment 3 - representative grasps - the grasp grades are: A =0.81, B=0.79, C=0.80

Figure 4.12: Environment 4 - representative grasps - the grasp grades are: A =0.77, B=0.76, C=0.75
4.5. Discussion

The estimated model converged fast (after less than 100 iterations), and represents the data comprehensively. Furthermore, the integration introduced by the sampling algorithm performed as expected, and produced configurations that grasped the mug successfully. For one environments, no feasible target poses were found by the algorithm, as all grasps were discarded, due to obstacles, in the path planning stage. This one environment, was very cluttered and required approach orientations that were not included in the original database from which the density was estimated. Having a richer graspiability map could help the algorithm in such environments. While in some cases manually defining grasp pose regions may be feasible (Berenson et al. 2009), it is not necessarily representative of the complete distribution of poses that afford high quality grasps. For the environments which a grasp was obtained, the GA-RRT demonstrates a high grasp success rate (95%).

The current work presented an alternative Bayesian based estimation path planning algorithm. Future work should test the GA-RRT algorithm with a richer graspiability map based on additional demonstrations.
5. Analysis of reach-to-grasp motion of patients with stroke

5.1. Overview

The study was part of the ENHANCE project (Enhancing brain plasticity for sensorimotor recovery in spastic hemiparesis). The ENHANCE project aims to test the effectiveness of a personalized rehabilitation training program on recovery of voluntary motion of patients with stroke. The training program is based on combination of, motion adaptation to patient capabilities, brain stimulation, and virtual reality. Patient progress is monitored based on clinical and kinematic measurements. The kinematic measurements are based on reach-to-grasp movements performed towards four targets. The current work analyzes wrist configurations at the end of the reach-to-grasp motion. Motion data of patients with stroke was collected prior to the rehabilitation training and after the training (in post training and follow-up sessions). The current work is based only on data recorded before the training, as the post and follow-up trials are sealed until the end of the project. Motion of healthy, control subjects were recorded to from a baseline for comparison. The rest of this chapter is organized as follows: section 5.2 describes the experiment. Results are presented in section 5.3 and discussed in section 5.4.

5.2. Method

5.2.1. Subjects

Participants included 15 subjects with stroke at the subacute stage, 0 to 6 months post stroke of which 6 are left handed (9 males, age 57.4±11 years). And 13 healthy age-matched controls, all right handed (9 males, age 60.46±8.68 years), with no other neurological, sensorimotor, or orthopedic impairments. Subjects with stroke were included in the experiment if they have a first ever stroke in the middle cerebral artery area territory, aged 25-75 years, in the sub-acute stage of the stroke (three weeks to six months post stroke), have arm paresis, able to perform voluntary elbow flexion extension movement of at least 30 degrees, have elbow spasticity and are able to provide informed consent. Subjects were excluded due to clinical issues such as orthopedic problem or pain, major cognitive deficits, history of psychiatric disorders or under medicine treatment. Demographic data of the subjects is presented in Appendix 2.
5.2.2. Environment

Subjects sat on a chair with feet supported and their hand resting alongside body (elbow extended to 180 degrees). The chair had a back support that did not restrict trunk movements. Four targets (standard hollow cones about 10 mm radius × 30 mm height) were placed on the table in front of the subject (Figure 5.1). Two target locations were in the mid-sagittal plane, one at 2/3rd of arm’s length (Target 1 – Near Center target (NC)) and one at arm’s length (Target 2 – Far Center target(FC)). Target 3 - Far Left (FL) and Target 4 - Far Right target (FR) targets were placed at arm’s length, about 20-30 cm to the left, depending on and within reaching distance, respectively (Figure 5.2).

Movements were recorded with a wireless electromagnetic tracking system G4™ Polhemus (Figure 5.3). The reported root mean square static accuracy of this system is 0.08 inches for position and 0.50 degrees for orientation when used within 1 meter of the source. Each sensor has 6 degrees-of-freedom and is tracked at 120Hz. Five sensors (denoted M1-M5) were used to track the position of the upper limb, shoulder girdle, and trunk in real-time. Sensors were placed on the metacarpi-phalangeal (MCP) joint of the index finger (M1), on the dorsal surface of the forearm (1/3 of the length of the forearm proximal to the head of the ulna), on the lateral surface of the upper arm at about the middle of the upper-arm (M3), on the mid-point of the superior-lateral border of the acromion (M4), and on the mid-sternum (M5). All experiment recordings were saved as MicrosoftExcel™ files using Matlab™.
5.2.3. Experimental Protocol

Each procedure started with calibration of the sensors for the markers locations on the subject’s body, which included seven movement types: elbow flexion-extension, elbow supination-pronation, wrist flexion-extension, wrist abduction-adduction, shoulder flexion-extension, shoulder pronation-supination, and shoulder abduction-adduction. A static calibration was performed for the 4 targets positions and for the chair. Subjects were instructed to perform a reach to grasp to the targets, based on a visual signaling. They were requested to rest between the sets and allowed to rest when needed between trials. Two sets of 40 trials (10 trials per target, fixed random order) were recorded for a total of 80 movements per subject. The order of the sets was counter-balanced between subjects.

5.2.4. Analysis

Movements were determined as erroneous in several cases: there was a recording failure, the experimenter noted during task execution that the subject did not wait after grasping the cone, the target was misplaced, the experimenter determined that the subject did not perform the task well, or the error was identified during segmentation.

Movement trajectories were filtered using a Butterworth filter with 6 Hz cutoff frequency. The filtered profiles were used for determination of motion onset and offset. Tangential velocity was computed by differentiating position samples. Motion onset and offset were defined as the times at which the wrist (forward arm sensor) tangential velocity exceeded and remained above, or decreased and remained below 10% peak wrist tangential velocity. Hand closure was defined as the time at which the hand angular velocity decreased below 1% the hand peak angular velocity. The threshold was iteratively increased by 1% in case a hand closure was not identified. The segmentation was performed semi-autonomously. An automatic procedure was developed for initial segmentation and the segmentation results were all manually screened (Figure 5.4).
The wrist configurations, were based on the recording of the distal arm sensor (sensor 2). For each subject, the configurations for all targets were translated to a unified, target-centered coordinate frame origin, based on target location, recorded during the calibration phase.

Two data sets were formed, one for each group (stroke and healthy). A grasp affordance was estimated for each group based on the method detailed in chapter 3. The parameters were defined according to table 8.

5.3. Results

Out of 996 reach-to-grasp movements of healthy subjects, 941 were used (94.5%) and 55 erroneous movements were discarded. The number of movements per target are NC=237, FC=239, FL=234 and FR=231. Out of 863 reach-to-grasp movements of patients with stroke, 829 were used (96.1%) and 34 erroneous movements were discarded. The number of movements per target are NC=204, FC=210, FL=211 and FR=204.

5.3.1. Grasp affordance density of healthy subjects

The algorithm converged very fast after about 100 samples. The affordance densities established, includes two clusters (Figure 5.5-5.7, table 11). The two cluster positions are very close with a Euclidian distance of 2.6 inches. The orientation distinctly differs between the clusters, the mean vectors are 43.3 degrees apart. The composition of clusters by target is presented in Figure 5.7.
Quaternions and positions are colored according to the cluster they are allocated to. The red colored data points above form a cluster weighing less than 5%.

The center of the ellipsoid represents the expected value and the circumference is one standard deviation away from the center.

Figure 5.7: Healthy subjects- Component composition by target
5.3.2. Grasp affordance density of patients with stroke

The algorithm converged after about 2000 iterations (Figure 5.8A). The convergence graph displays only clusters with a weight larger than 5%. The affordance densities established, includes two clusters (Figure 5.8-5.10, table 12). The two cluster positions are with a Euclidian distance of 5.1 inches. The orientation distinctly differs between the clusters, the mean vectors are 156.5 degrees apart. The composition of clusters by target is presented in Figure 5.10. Each subjects' dominant cluster and affected side are presented in table 13.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.215</td>
<td>0.782</td>
</tr>
<tr>
<td>Position</td>
<td></td>
<td></td>
</tr>
<tr>
<td>µ</td>
<td>[-2.86,1.18,-4.74]</td>
<td>[-0.82,2.71,-5.01]</td>
</tr>
<tr>
<td>S</td>
<td>0.33,0.18,-0.29</td>
<td>1.05,-0.06,-0.32</td>
</tr>
<tr>
<td></td>
<td>0.18,0.26,0.07</td>
<td>-0.06,0.43,0.42</td>
</tr>
<tr>
<td></td>
<td>-0.29,0.07,2.03</td>
<td>-0.32,0.42,2.33</td>
</tr>
<tr>
<td>Orientation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kappa</td>
<td>99.74</td>
<td>57.52</td>
</tr>
<tr>
<td>µ_cyc</td>
<td>[0.881,0.164,0.266,0.3550]</td>
<td>[0.89,-0.166,0.114,0.409]</td>
</tr>
</tbody>
</table>

58

**Figure 5.8:** Patients with stroke - convergence and mixture model

Quaternions and positions are colored according to the cluster they are allocated to. The red colored data points above form a cluster weighing less than 5%.
TABLE 12: Patients with stroke - mixture model parameters

<table>
<thead>
<tr>
<th>Cluster</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.53</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Position

<table>
<thead>
<tr>
<th></th>
<th>Cluster 1</th>
<th>Cluster 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>[-0.66, 2.44, -3.79]</td>
<td>[0.18, 0.91, -0.04]</td>
</tr>
<tr>
<td>( S )</td>
<td>4.65, -0.9, -0.79</td>
<td>4.23, 0.43, 2.86</td>
</tr>
<tr>
<td></td>
<td>-0.9, 2.94, 2.18</td>
<td>0.43, 6.85, 8.11</td>
</tr>
<tr>
<td></td>
<td>-0.79, 2.18, 5.58</td>
<td>2.86, 8.11, 20.83</td>
</tr>
</tbody>
</table>

Orientation

<p>| Kappa | 21.22 | 9 |</p>
<table>
<thead>
<tr>
<th>Subject</th>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Dominant Cluster</th>
<th>Affected Side (R-right, L-left)</th>
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<td>1</td>
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</tr>
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<td>3</td>
<td>57</td>
<td>2</td>
<td>L</td>
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</tr>
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<td>L</td>
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<td>1</td>
<td>R</td>
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<tr>
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<td>44</td>
<td>2</td>
<td>L</td>
</tr>
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<td>1</td>
<td>79</td>
<td>2</td>
<td>L</td>
</tr>
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<tr>
<td>15</td>
<td>47</td>
<td>4</td>
<td>1</td>
<td>R</td>
</tr>
</tbody>
</table>

| $h_{cyC}$ | [0.876,-0.17,0.047,0.45] | [0.964,0.045,0.071,-0.252] |

**TABLE 17:** The dominant cluster of patients with stroke

5.4. Discussion

The affordance models of both the healthy subjects and patients with stroke are both composed of two clusters. However, the division of configurations between clusters is very different in the two models.

For the healthy group, the position Euclidian distance between the clusters of 2.6 inches, and orientation expected value difference of 43.3 degrees demonstrate a division between the
clusters, hence, different grasp types. Grasp wrist configuration to three targets (FL, NC and FC) were allocated to the same cluster of the affordance density. The subjects used different configurations to the fourth target, (FR), and these were allocated to the second cluster of the affordance density. The division of clusters by target indicates there are different motion profiles executed towards different targets, this could be due to the different trajectory profiles. The variance in the data, both for orientation and position, is relatively small (concluded based on naked-eye observation (Figure 5.5) because calculating the exact variance of the data will not incorporate division to clusters and may be misleading). The small variance expected is reflected in the estimated model, cluster one: \( \sigma_x^2 = 0.3, \sigma_y^2 = 0.3, \sigma_z^2 = 2.03, \text{kappa}=99.7 \) cluster two: \( \sigma_x^2 = 1, \sigma_y^2 = 0.4, \sigma_z^2 = 2.3, \text{kappa}=57.5 \). Making it relatively easy for the algorithm to distinguish between the different grasps.

The affordance density of the stroke group was also composed of two clusters, (position Euclidian distance of 5.1 inches and orientation expected value difference of 156.5 degrees indicate a distinct division between the clusters. However, unlike in the healthy affordance density, the clusters are not divided by targets. Both clusters were composed of all targets' samples (FL, NC, FC and FR). Instead subjects were divided between clusters based on the hand with which they performed the movement (their effected arm). The variance in the data, both for orientation and position, is relatively high (naked-eye observation, Figure 5.8). The high variance expected is reflected in the estimated model, cluster one: \( \sigma_x^2 = 4.7, \sigma_y^2 = 2.9, \sigma_z^2 = 5.6, \text{kappa}=21.2 \), cluster two: \( \sigma_x^2 = 4.2, \sigma_y^2 = 6.9, \sigma_z^2 = 20.8, \text{kappa}=9 \). It seems the stroke group had the same grasp type regardless of the target’s location.
6. Bibliography


Gelman, Andrew, John Carlin, Hal Stern, David Dunson, and Aki Vehtari. 2014. *Bayesian Data Analysis (Vol. 3).*


Shalizi, Cosma Rohilla. 2016. “From an Elementary Point of View.”


Appendix 1: Synthetic data sets

The synthetic data sets were generated using the random mixture model sampling algorithm developed (4.2.1). The synthetic validation set - distinct separation in both orientation and position was generated using table A.1. The synthetic overlap in position dataset (OP) was generated using table A.2. The synthetic overlap in orientation dataset (OO) was generated using table A.3 and synthetic overlap in position and orientation dataset (OPO) was generated using table A.4.

TABLE A.1: Total separation - original model

<table>
<thead>
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<th>Cluster</th>
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</tr>
</thead>
<tbody>
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<td>Weight</td>
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<td>µ</td>
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<td>[100,100,100]</td>
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<td>0,1,0,1,1</td>
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<tr>
<td>Orientation</td>
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<td>200</td>
<td>200</td>
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<td>[0,1,0,0]</td>
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</table>

TABLE A.2: Position overlap - original model

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</thead>
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TABLE A.3: Orientation overlap- original model

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<tr>
<td>Kappa</td>
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<td>( \mu_{\text{cyc}} )</td>
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TABLE A.4: Total overlap- original model

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**Appendix 2: Demographic data**

Demographic data of subjects with stroke and healthy control subjects are presented in table A.5 and table A.6 respectively.

**TABLE A.5: Subject with stroke- demographic data**

<table>
<thead>
<tr>
<th></th>
<th>Age (Years)</th>
<th>Country</th>
<th>Gender</th>
<th>Effected Side (R-Right, L-Left)</th>
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<tr>
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TABLE A.6: Healthy subjects- demographic data

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Mean 60.4  
SD 8.5
תקציר

אחת התפירות של אובייקטים בסביבה סבוכה ולא מובנית היא המשימה מאתגרת עבור בני אדם ומערכות רובוטיות. אחד המרכיבים הכרחיים לאחיזה מוצלחת הוא המיקום והאוריינטציה (קונפיגורציה) של מפרק כף היד, שם האובייקט נאחז. קביעת קונפיגורציה לאחיזה היא חלק אלמנטרי מתהליך תכנון התנועה של הושטה אל עבר אחיזה (reach-to-grasp).

צפיפות התפלגות אחיזות מוצלחות (grasp affordance densities) הן פונקציות התפלגות מרחביות המציגות הסתברויות לאחיזה מוצלחת של מפרק כף היד מסביב לאובייקט. על מנת לייצג את המיקום והאוריינטציה_forumבziaית, פונקציית התפלגות מצויה לטווחים שונים_NET מחשבים עם משולש ריבועי ומעורב (mixture model). שיערוך של סדר הרכיבים ופונקציות ההתפלגות במודל מעורב (Gaussian) מאפשר שיערוך של מספר הרכיבים וערכי הפרמטרים במודל מעורב. בניגוד לשיערוך המבוסס על נראות מקסימאלית, השיערוך הבייסיני, שלא כמו שיערוך של נראות מקסימאלית, פחות מועד לתופעות של התאמה יריתרית. המודל который בשר缑ו השיערכה הואアルゴリズム של אלגוריתם תכנון תנועה אל עבר אחיזה רובוטי, שילב את השיערוך בין החופשות עם אלגוריתם תכנון התנועה (bidirectional Random Trees (RRT). המודל który בשר缑ו השיערכה הואアルゴリズム של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. המודל который בשר缑ו השיערכה הואアルゴリזם של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. }

שיערוך בייסיאני אפרמטרי, לא פרמטרי, מתאימה לשיעורים של מספר הריבועים המרוכבים של המודלים ומעורבים. פונקציית התפילה של פונקציות המיקום והתקנים של komponentים במודל מעורב (mixture model) מאפשר שיעוך של מספר הרכיבים וערכי הפרמטרים במודל מעורב. בניגוד לשיעוך של מודל נראות מקסימאלית, השיעוך הבייסיני, meno מועד לתופעות של התאמה יריתרית. המודל который בשר缑ו השיערכה הוא אלגוריתם של 어떤 אלגוריתם תכנון תנועה אל עבר אחיזה רובוטי, שילב את השיעוך בין החופשות עם אלגוריתם תכנון התנועה (bidirectional Random Trees (RRT). המודל который בשר缑ו השיערכה הואアルゴリズム של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. המודל который בשר缑ו השיערכה הואアルゴリズム של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. }

שיעוך בייסיאני, לא פרמטרי, מתאימה לשיעורים של מספר הריבועים המרוכבים של המודלים ומעורבים. פונקציית התפילה של פונקציות המיקום והתקנים של komponentים במודל מעורב (mixture model) מאפשר שיעוך של מספר הרכיבים וערכי הפרמטרים במודל מעורב. בניגוד לשיעוך של מודל נראות מקסימאלית, השיעוך הבייסיני, meno מועד לתופעות של התאמה יריתרית. המודל который בשר缑ו השיערכה הוא אלגוריתם של 어떤 אלגוריתם תכנון תנועה אל עבר אחיזה רובוטי, שילב את השיעוך בין החופשות עם אלגוריתם תכנון התנועה (bidirectional Random Trees (RRT). המודל который בשר缑ו השיערכה הואアルゴリズム של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. המודל который בשר缑ו השיערכה הואアルゴリズム של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. }

שישונים ב частности, שיעוך של מספר הריבועים המרוכבים של המודלים ומעורבים. פונקציית התפילה של פונקציות המיקום והתקנים של komponentים במודל מעורב (mixture model) מאפשר שיעוך של מספר הרכיבים וערכי הפרמטרים במודל מעורב. בניגוד לשיעוך של מודל נראות מקסימאלית, השיעוך הבייסיני, meno מועד לתופעות של התאמה יריתרית. המודל который בשר缑ו השיערכה הוא אלגוריתם של 어떤 אלגוריתם תכנון תנועה אל עבר אחיזה רובוטי, שילב את השיעוך בין החופשות עם אלגוריתם תכנון התנועה (bidirectional Random Trees (RRT). המודל который בשר缑ו השיערכה הואアルゴリズם של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. המודל который בשר缑ו השיערכה הואアルゴリズם של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. }

מילים ממחата: שיעוך בייסיאני, לא פרמטרי, פונקציות התפילה של פונקציות המיקום והתקנים של komponentים במודל מעורב (mixture model) מאפשר שיעוך של מספר הרכיבים וערכי הפרמטרים במודל מעורב. בניגוד לשיעוך של מודל נראות מקסימאלית, השיעוך הבייסיני, meno מועד לתופעות של התאמה יריתרית. המודל который בשר缑ו השיערכה הוא אלגוריתם של 어떤 אלגוריתם תכנון תנועה אל עבר אחיזה רובוטי, שילב את השיעוך בין החופשות עם אלגוריתם תכנון התנועה (bidirectional Random Trees (RRT). המודל который בשר缑ו השיערכה הואアルゴリズם של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. המודל который בשר缑ו השיערכה הואアルゴリズם של GA-RRT, מספק מספר רב של קונפיגורציות מטרה עם סיכוי גבוה לאחיזה מוצלחת. }

ממס פшир, אהוב, הטונת, שבר.
שערוך בייסיאני של פילוג קונפיגורציות מרחביות

生态圈 המדכאות מחודש למדכאות מחודש

חתימת המחבר:....................................................................
תאריך:.................

אישור המנחה:.....................................................................
תאריך:............

אישור יו"ר ועדת תואר שני מחלקתית:.....................................
תאריך:.................

אוקטובר 2017

חתירת תשב"וח
שערוך בייסיאני של פילוג Конфігураций Материалов

חיבור זה מהווה חלק מהדרישות לקבלת תואר מגיסטר בהנדסה

מאת: רותם דואני

אוקטובר 2017
תשרי תשע"ז

מאמר: רותם דואני

אוגוסט 2017
תש"ע