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Abstract

We study all-pay auctions under incomplete information in which the designer can impose taxes or subsidies, and his expected payoff is the contestants' expected total effort minus the cost of subsidies, or, alternatively, plus the tax payment. When contestants have linear effort cost functions, we show that taxing the winner's payoff is profitable for the contest designer, and particularly more profitable than the same model with no taxation or the same model with contestants' effort taxation. When the contestants' effort cost functions are convex and the taxation rate is relatively low, we show that the designer should tax the winner's payoff while subsidizing all of the other contestants' effort costs. As a result, contest organizers should think about combining taxation and subsidies in their contests because they complement rather than substitute each other.

JEL CLASSIFICATION: C72, D44, H25

KEYWORDS: All-pay auctions, incomplete information, taxation, subsidies

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1 Introduction

Taxation is an important policy of governments in modern economies, but it is also a well-known policy that is used in many other economic environments (see, for example, Sav, 2004, Zuniga Vicente et al. 2014, and Bisceglia, 2020). A contest is one example of such an environment. In general, regardless of the amount, prizes and awards in contests are subject to ordinary income taxation by the U.S. federal government. This means that whatever percentage you are taxed for your regular income, that same rate will apply to the prize money you received. For example, if an agents earns $43,000 per year, his federal tax rate is 22%. If he wins $1,000 in a contest, his total income rises to $44,000, and his tax rate remains 22%. However, a large prize could push his income into a higher tax bracket. There are some exclusions, such as prize money awarded to a U.S. athlete by the United States Olympic Committee (USOC), as well as the fair market value of any gold, silver, or bronze medal received as a result of competing in the Olympic or Paralympic Games. However, Olympic athletes must pay taxes if they earn $1 million or more per year. Likewise, a scholarship or a grant awarded for academic studies or research in the academy is not taxable income, but employers (i.e., universities) may charge a tax. In Israel, for example, a university charges each researcher a tax rate of 16 percent of the winning amount for each research grant received from the Israel Science Foundation (ISF) or any other science foundation. In this paper, we attempt to provide some answers on the profitability of contest designers using economic taxation policies.

In the area of contest taxation, not much research has been done. The majority of the research was conducted in a complete information environment (see, for example, Glazer and Konrad 1999, Konrad 2000, Fu et al. 2012, Mealem and Nitzan 2014, Carpenter et al. 2016, and Thomas and Wang 2017). To the best of our knowledge, our previous work (Minchuk and Sela 2023) is the only work that studied contests (all-pay auctions) with incomplete information, non-linear effort costs, and two types of subsidies/taxes, either on the cost of effort or on the size of the prize.[1] The expected payoff of the designer in their model is the total effort of the contestants minus the cost of subsidy or, alternatively, plus the tax payment, and they demonstrate that when the effort cost function is convex, the expected payoff for the designer in all-pay auctions with

[1] Fu et al. (2012) call this form of subsidy on the cost of effort an "efficiency-enhancing subsidy."

both types of taxation is greater than in the same contest without any taxation.[2] When the effort cost functions are concave, the expected payoff for the designer in all-pay auctions with both types of subsidies is higher than in the same contest without any subsidies. In other words, when the effort cost functions are convex, taxation is efficient; when they are concave, subsidy is efficient. In this paper, we extend and improve these findings of Minchuk and Sela (2023) by demonstrating that different types of taxation, which are some combinations of taxation and subsidy, are the best ways to apply taxes and subsidies in contests held in an environment with incomplete information.

There is a significant distinction between designing in environments with complete and incomplete information when a designer wants to impose taxes in addition to subsidies. The reason for this is that, whereas in a complete information environment, the designer can apply a different subsidy/tax for each contestant based on his type (see, for example, Konrad 2000, and Nitzan and Mealem 2014), in an incomplete information environment, the contestants' types are ex-ante identical where each contestant knows his type (which is private information), and thus the designer who does not know the contestants' types must apply a uniform policy of taxation for all the contestants without the ability to discriminate among them. In our model, there is incomplete information on the contestants' types in the first stage, which are revealed in the second stage, so the designer must apply a uniform taxation policy in the first stage but has the option of imposing different taxes on the winner in relation to all other contestants. This type of policy taxation is what we propose in our all-pay contests with incomplete information.

We study all-pay contests (auctions) with incomplete information with n contestants who have either linear or non-linear effort cost functions.[3] The designer who does not know the contestants' values informs them the tax rate in the first stage, and the contestants choose their efforts; the contestant with the highest effort wins, and all contestants pay the cost of their efforts. In the second stage, the contestants' values are revealed and the designer receives a tax payment from the winner's profit, which is equal to the winner's winning value minus the cost of his effort. The designer's expected payoff is the contestants' expected total

[2] Runkel (2006) and Ritz (2008) also show that a policy that uniformly raises the effort costs of the contestants can result in an increase in total effort.

[3] Among others, Amman and Leininger (1996), Krishna and Morgan (1997), Moldovanu and Sela (2006), Kirkegaard (2012), and Liu and Lu (2017) have studied all-pay contests (auctions) with incomplete information.

effort plus the tax on the winner's payoff. In our model because that the contestants' types are revealed in the second stage, the higher the tax rate, the higher the expected payoff for the designer, and we assume that the tax rate is fixed and is not an endogenous parameter, so the designer accepts it as given. We will refer to this contest as the winner's payoff taxation model.

The winner's payoff taxation has ambiguous effects on the expected payoff of the designer. On the one hand, the designer receives a tax payment by imposing a tax on the winner's payoff, but on the other hand, by imposing a tax, he reduces the contestants' equilibrium efforts. We show that, when effort cost functions are linear, regardless of tax rate, the expected payoff for the designer in the winner's payoff taxation model is greater than in the same model without taxation. We also demonstrate that this result holds true even when the effort cost functions are convex and the tax rate is low enough. To demonstrate the efficiency of the winner's payoff taxation, we compare it to the model studied by Minchuk and Sela (2023), in which the designer imposes a tax on the cost of efforts of the contestants, and we show that if the tax rate is the same in both models, the designer's expected payoff in our winner's payoff taxation model is greater than that in the model of effort cost taxation.

In the next step, we give the designer the option of taxing the winner's profit and subsidizing the other contestants' effort costs. This competition is referred to as the winner's payoff taxation and subsidies model. Subsidies, on the one hand, increase the equilibrium efforts of the contestants, but on the other hand, the designer must pay this subsidy payment. We show that when effort cost functions are linear, the expected payoff for the designer is the same in both models of winner's payoff taxation with and without subsidies. Furthermore, even if we allow the contest designer to set different tax and subsidy rates, the expected payoff for the contest designer is the same for both models with and without subsidies. In other words, the additional subsidies to the winner's payoff taxation model have no effect on the designer's expected payoff. On the other hand, when the effort cost functions are convex, we show that for sufficiently low tax rates, the winner's payoff taxation and subsidies model yields a higher expected payoff for the designer than without subsidies. As a result, when the effort cost functions are convex, the designer should tax the winner's profit while subsidizing all other contestants' effort costs.

Finally, we assume that, in addition to the tax payment, the designer benefits from the contestants' highest effort rather than their total effort. In that case, the winner's effort is significant, whereas the efforts of the other contestants are not. As a result, it appears that the subsidies for all contestants except the winner are ineffective; however, we show that when the contestants' effort costs are convex, the model of the winner's payoff taxation with subsidies is the best option for the designer. The reason for this is that subsidies for the contestants' costs of effort increase their efforts, forcing the winner to increase his effort as well, so subsidies are required in addition to taxation of the winner's profit. As a result, we show that optimal collaboration between taxation and subsidies is very efficient in all-pay contests with incomplete information, and we believe it is efficient in other types of contests with both complete and incomplete information.

As previously stated, there are other works dealing with taxation in competitions, but they consider environments with complete information. Glazer and Konrad (1999) consider a model of taxation for firms that engage in rent-seeking contests. They investigate two types of taxation: effort taxation and profit taxation. They do not analyze the effect of these taxes on the designer's payoff in the same way that we do, but they do find situations in which taxes have no effect on the contestants' efforts. Konrad (2000) investigates a trade all-pay contest between two firms in which each country can impose taxes or subsidies on an exporting firm. He discovers that governments' strategic policy is to subsidize one firm while taxing the other. Later Mealem and Nitzan (2014) investigate the optimal taxation policy in Konrad's model of all-pay contest and show that, given a revenue-maximizing contest designer and a balanced-budget constraint, the optimal taxation scheme is to tax one of the contestants while subsidizing the other, and that the contestants' total efforts are greater than those obtained under almost any pure-strategy equilibrium in the Tullock contest. In comparison to our model, the contest designer's constraint of a balanced budget limits the design of the optimal tax policy in their model. Fu et al. (2012) study research contests in which two firms compete on the quality of their products. They demonstrate that combining prizes and subsidies is optimal for the designer who wants to maximize the quality of the winning product. As previously stated, the most relevant paper is Minchuk and Sela (2023), who investigate the use of tax and subsidy in an all-pay auction with incomplete information and show that one is useful when contestants' costs of effort are concave and the other when these costs of effort are convex. This work departs from our previous work by demonstrating the use of carrots (subsidies) and sticks (taxations) in conjunction, rather than just carrots (subsidies) or just sticks (taxes). In other words, combining tax and subsidy is far more beneficial to the designer who wishes to maximize the contestants' total effort than using either tax or subsidy alone. There is another significant distinction between our model and that of Minchk and Sela (2023). Minchuk and Sela (2023) study a one-stage contest with incomplete information, whereas this paper looks at a two-stage model in which the first stage is incomplete and the private information is revealed in the second. The reason for this is that the designer taxed the winner, who must report his actual profit. However, this assumption complicates our model; without it, if the winner can manipulate his true profit, our model would be simpler and comparable to a one-stage contest.

In our paper, the designer uses taxation and subsidies to increase his expected payoff, which is based on the total effort of the contestants plus the taxation payment minus the subsidy payment. Similar methods of reimbursing some of the contestants' effort costs have been shown in the contest literature to be beneficial for the contest designer, who wants to maximize the contestants' total effort minus the cost of the reimbursement (see, among others, Cohen and Sela 2005, Matros and Armanious 2009, Matros 2012, Minchuk 2018, and Minchuk and Sela 2020). Obviously, the designer has different ways, other than taxation and subsidies, to enhance the contestants' efforts. For example he can decide how to distribute the entire prize sum among the contestants by allocating number of prizes and punishments (see, among others, Moldovanu and Sela, 2001, Moldovanu et al. 2012, Olszewski and Siegel 2016, Sela 2020, Liu et al. 2018, and Liu and Lu 2023), he can also limit the number of contestants by setting a minimum effort level (see, among others, Taylor 1995, Fullerton and McAfee 1999, Fu et al. 2015, and Kirkegaard 2022), or, alternatively, he can impose a maximum effort level (see, among others, Che and Gale 1998, Gavious et al. 2003, Megidish and Sela 2014, and Olszewski and Siegel 2019).

The rest of the paper is organized as follows. In Section 2, we examine the winner's payoff taxation model, and in Section 3, we examine the winner's payoff taxation model with subsidies. We examine additional objectives of the designer in these models in Section 4. Section 5 concludes. The appendix contains the proofs.

2 The winner's payoff taxation model

Consider $n \geq 2$ contestants competing for a single prize in an all-pay contest (auction). Contestant i's winning value is v_i, $i = 1, .., n$, and is private information. The contestants' values are drawn independently of each other from the interval $[0, 1]$ according to the distribution function F which is common knowledge. We assume that F is continuously differentiable and that $f(x) = F'(x) > 0$ for all $0 \leq x \leq 1$. The contest is divided into two stages, which are as follows: In the first stage, the designer, who is unaware of the contestants' winning values, informs them of the (exogenous) tax rate $0 < (1-\beta) < 1$ on the winner's payoff, and then the contestants simultaneously choose their efforts, where the contestant with the highest effort wins and all the contestants pay the cost of their efforts, where an effort of x has a cost of $\gamma(x), \gamma' > 0, \gamma(0) = 0$ in monetary units. In other words, γ transfers x units of effort to $\gamma(x)$ monetary units. We denote $g = \gamma^{-1}$. In the second stage, contestants' types are revealed, and the designer receives a tax payment of $(1-\beta)(v-\gamma(x))$ from the winner's payoff, who has a winning value of v and an effort cost of $\gamma(x)$. The designer's expected payoff is the contestants' expected total effort plus the tax on the winner's payoff. This model will be referred to as the winner's payoff taxation model. In this model, player i with a winning value of v_i has an expected payoff,

$$U(v_i) = \beta \left(v_i - \gamma(x(v_i))\right) G(v_i) - (1 - G(v_i))\gamma(x(v_i)), \tag{1}$$

where $G(v_i) = F^{n-1}(v_i)$ is the probability that the value v_i is the highest among all the n contestants, and the tax rate $1 - \beta$ satisfies $0 < 1 - \beta \leq 1$. The first term in (1) describes the profit in the case of winning multiplied by the probability of winning, while the second describes the cost of losing multiplied by the probability of losing. The designer's expected payoff in effort units is

$$R_{tax} = TE + E_{\max}(g\left((1 - \beta)v - \gamma(x(v))\right)), \tag{2}$$

where TE is the contestants' expected total effort, and $E_{\max}(g((1-\beta)\left(v - \gamma(x(v))\right)))$ is the designer's expected profit from the winner's payoff taxation in effort units. The expected payoff increases with the tax rate $(1-\beta)$, but this parameter is assumed to be exogenous and the designer accepts it as given.

Assume that there is a symmetric monotonically increasing equilibrium effort function $x(v_i)$. This assumption will be confirmed later. Then, the maximization problem of contestant $i, i = 1, .., n$, is

$$\max_s \beta \left(v_i - \gamma(x(s))\right) G(s) - (1 - G(s))\gamma(x(s)).$$

The first order condition (FOC) of the maximization problem of contestant i's expected payoff is

$$\beta G'(s^*) v_i - ((1 - (1 - \beta)G(v_i))\gamma(x(v_i)))' = 0.$$

In equilibrium $s^* = v_i$, thus, we obtain

$$(1 - (1 - \beta)G(v_i))\gamma(x(v_i)) = \beta \int_0^{v_i} sG'(s)ds + k.$$

Since $\gamma(x(0)) = 0$, we have

$$\gamma(x(v_i)) = \frac{\beta}{(1 - (1 - \beta)G(v_i))} \int_0^{v_i} G'(s)sds.$$

Rearranging, yields the equilibrium effort of contestant i, $i = 1, 2, ..., n$ as follows:

$$\begin{aligned} x_i(v_i) &= g\left(\frac{\beta}{(1 - (1 - \beta)G(v_i))} \int_0^{v_i} sG'(s)sds\right) \\ &= g\left(\frac{\beta}{(1 - (1 - \beta)G(v_i))} \left[v_i G(v_i) - \int_0^{v_i} G(s)ds\right]\right). \end{aligned} \tag{3}$$

Deriving (3) gives

$$\begin{aligned} \frac{\partial x_i(v_i)}{\partial v_i} &= g'\left(\frac{\beta}{(1 - (1 - \beta)G(v_i))} \left[v_i G(v_i) - \int_0^{v_i} G(s)ds\right]\right) \\ &\times \left[\frac{\beta(1 - \beta)G'(v_i)}{(1 - (1 - \beta)G(v_i))^2} \left[v_i G(v_i) - \int_0^{v_i} G(s)ds\right] + \frac{\beta v_i G'(v_i)}{(1 - (1 - \beta)G(v_i))}\right]. \end{aligned}$$

It can be easily verified that $\frac{\partial x_i(v_i)}{\partial v_i} > 0$, and therefore we can confirm our assumption that the equilibrium effort is monotonically increasing.

Inserting (3) into (1) gives us the expected payoff of contestant i with a winning value of v_i as follows:

$$\begin{aligned} U(v_i) &= \beta v_i G(v_i) - (1 - (1 - \beta)G(v_i))\gamma(x(v_i)) \\ &= \beta v_i G(v_i) - \beta \left[v_i G(v_i) - \int_0^{v_i} G(s)ds\right] = \beta \int_0^{v_i} G(s)ds. \end{aligned} \tag{4}$$

In the standard all-pay auction with linear cost functions and without any taxation (see Krishna 2010) the expected payoff of contestant i with a winning value of v_i is $\int_0^{v_i} G(s)ds$. Therefore, the ratio of a contestant's expected payoffs in our all-pay contest model with winner's payoff taxation and in the standard all-pay auction without any taxation is equal to $\beta < 1$ when the tax rate is $(1-\beta)$. Because the winner must pay a portion of his payoff in the all-pay contest with winner's payoff taxation, the winner's equilibrium effort appears to be lower in this scenario than in the standard all-pay contest with no taxation. However, as demonstrated by the following result, the effect of taxation on the contestants' equilibrium effort is not completely straightforward.

Proposition 1 *The equilibrium efforts in the winner's payoff taxation model are less than or equal to those in the same model without taxation. The taxation, however, has no effect on the equilibrium efforts of the contestants with the highest winning values.*

The reasoning behind this result is that when a contestant increases his effort, there are two possible outcomes. On the one hand, he will have to pay a lower tax to the designer, but his effort will cost him more if he loses. Because the contestants with the highest winning values win almost always, these two opposing effects balance each other out so that the contestants with the highest winning values have no reason to change their efforts as a result of taxation.

We now analyze the designer's expected payoff. Inserting (3) in (2) gives the designer's expected payoff in the winner's payoff taxation model as follows:

$$R_{tax} = n\int_0^1 g\left(\frac{\beta}{(1-(1-\beta)G(v))}\int_0^v sG'(s)ds\right)f(v)dv + E_{\max}(g\left(\left(1-\beta\right)\left(v-\left(x(v)\right)\right)\right)). \tag{5}$$

By comparing the designer's expected payoff in models with and without taxation, we demonstrate the benefit of the winner's taxation for the contest designer.

Proposition 2 *When the effort cost functions are linear, regardless of tax rate, the expected payoff of the designer in the winner's payoff taxation model is greater than in the same model without taxation.*

The result in Proposition 2 shows that a designer can increase his expected payoff in the all-pay contest with linear cost functions by taxing the winner's payoff. This result indicates that the Revenue Equivalence Theorem (RET) (see Myerson 1981 and Riley and Samuelson 1981) does not hold in the current setting and the reason is that, in our model, the winner's payment to the contest designer is based on his payoff and effort rather than just his effort.

In order to demonstrate the advantage of the winner's payoff taxation we will show that it is also more profitable than the model of cost taxation as studied by Minchuk and Sela (2023). In the model of cost taxation where the tax rate is $(\delta - 1)$, $\delta > 1$, the payoff function of contestant $i, i = 1, .., n$, is

$$U(v_i) = v_i G(v_i) - \delta\gamma(x(v_i)).$$

Then, the designer's expected payoff is

$$\begin{aligned} R_{ct} &= n\int_0^1 g\left(\frac{1}{\delta}\left(vG(v) - \int_0^v G(s)ds\right)\right) f(v)dv + E\left(g(n(\delta-1)\gamma(x))\right) \qquad (6) \\ &= n\int_0^1 g\left(\frac{1}{\delta}\left(vG(v) - \int_0^v G(s)ds\right)\right) f(v)dv + \int_0^1 g\left(\frac{n(\delta-1)}{\delta}\left(vG(v) - \int_0^v G(s)ds\right)\right) f(v)dv. \end{aligned}$$

In that case of cost taxation, the RET holds. Thus, by Proposition 2, we can conclude that

Corollary 1 *When the effort cost functions are linear and the tax rate is the same in the winner's payoff taxation model and the cost taxation model, the designer's expected payoff in the winner's payoff taxation model is greater than that in the cost taxation model.*

The RET is no longer valid when the effort cost functions are non-linear, and in particular, the outcome of Corollary 1 may not hold. The following result, however, demonstrates that even when the effort cost functions are non-linear, the winner's payoff taxation model still outperforms the standard model with cost taxation when the tax rates are relatively low.

Proposition 3 *When the effort cost functions are convex, and the tax rate is sufficiently low and the same in both the winner's payoff taxation model and the cost taxation model, the expected payoff in the winner's payoff taxation model is greater than in the cost taxation model.*

The intuition behind this result is that since $g = \gamma^{-1}$ is concave, the derivative of the concave function g' is a decreasing function. Thus, when the cost function is changed from linear to convex, the marginal decrease in contestants' efforts is less than the marginal increase in contestants' payoff as a result of taxation for high-type contestants (contestants with high winning values) and the opposite for low-type contestants (contestants with low winning values). Therefore, when the cost function is convex, it is profitable for the designer to impose a tax on the winner's payoff.

According to Minchuk and Sela (2023), when the contestants have convex effort cost functions, the designer's expected payoff in the cost taxation model is higher than in the same model without any taxation. Thus, by Proposition 3, we can conclude that

Corollary 2 *When the effort cost functions are convex and the tax rate is sufficiently low, the designer's expected payoff in the winner's payoff taxation model is greater than in the same model without any taxation.*

3 The winner's payoff taxation and subsidies model

Consider $n \geq 2$ contestants competing in the same all-pay contest model described in the previous section, but now, in the second stage, the designer receives a tax payment of $(1-\beta)(v_j - \gamma(x_j)) - (1-\beta)\sum_{\substack{i=1 \\ i\neq j}}^{n} \gamma(x_j)$ where $(1-\beta)(v_j - \gamma(x_j))$ is the winner's tax payment received by the designer and $(1-\beta)\sum_{\substack{i=1 \\ i\neq j}}^{n} \gamma(x_j)$ is the designer's subsidies paid to all contestants who did not win. In that case, both the tax and subsidy rates are $0 < (1-\beta) < 1$, and this model will be referred to as the winner's payoff taxation and subsidies model. In this model, the payoff function of contestant $i.i = 1, .., n$ is given by

$$U(v_i) = \beta\left(v_i G(v_i) - \gamma(x(v_i))\right). \tag{7}$$

The equilibrium effort of the contestants will not be dependent on β, namely, they are the same as in the standard all-pay contest without any taxation/subsidies, and is given by

$$x(v_i) = g\left(v_i G(v_i) - \int_0^{v_i} G(s)ds\right).$$

The designer's expected payoff is then

$$R_{tax}^{all} = TE + E(g\left((1-\beta)\left[(v_1 - \gamma(x(v_1))) + \sum_{i=2}^{n}(-\gamma(x(v)))\right]\right), \tag{8}$$

where TE is the contestants' expected total effort and $E\left(g\left((1-\beta)\left[(v_1 - \gamma(x(v_1))) + \sum_{i=2}^{n}(-\gamma(x(v)))\right]\right)\right)$ is the designer's expected payoff from the winner's taxation (the winner is arbitrarily chosen as contestant 1) minus the subsidies to all other contestants.

A comparison of the designer's expected payoffs with and without subsidies is not obvious because, in the winner's payoff taxation model, the designer's payoff from the contestants' equilibrium efforts decreases in comparison to the standard model without any taxation, but the designer receives an additional payoff from the winner's payoff tax. In the winner's payoff taxation and subsidies model, the designer's payoff from the contestants' equilibrium efforts is the same as in the standard model without any taxation, but the designer must pay some subsidies to the contestants who did not win. Indeed, we show that

Proposition 4 *When the effort cost functions are linear and both models of winner's payoff taxation with and without subsidies have the same tax rate, the designer's expected payoff is the same in both models.*

In the winner's payoff taxation and subsidies model, the tax rate for taxation and subsidies is not necessarily the same. In that case, the tax rate of the winner's payoff is $1-\beta$, and the subsidy rate of all other contestants' costs is $1-\delta$. Then, the payoff function of contestant $i, i = 1, ..., n$ is given by

$$U(v_i) = \beta\left(v_i - \gamma(x(v_i))\right)G(v_i) - \delta(1 - G(v_i))\gamma(x(v_i)). \tag{9}$$

The first order condition (FOC) of the maximization problem of contestant i's expected payoff is

$$\beta G'(v_i)v_i - ((\delta - (\delta - \beta)G(v_i))\gamma(x(v_i)))' = 0.$$

Using the same arguments as in the equilibrium analysis in the previous cases, we obtain the contestants' equilibrium effort as follows:

$$\begin{aligned} x_i^{all\_dif}(v_i) &= g\left(\frac{\beta}{(\delta - (\delta - \beta)G(v_i))}\int_0^{v_i} sG'(s)ds\right) \qquad (10) \\ &= g\left(\frac{\beta}{(\delta - (\delta - \beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right). \end{aligned}$$

The following result shows that when the effort cost functions are linear, the option of setting different taxation and subsidy rates provides no advantage to the contest designer.

Proposition 5 *When the effort cost functions are linear, regardless of the tax and subsidy rates, the designer's expected payoff in the winner's payoff taxation models with and without subsidies is the same.*

Following Proposition 5, we return to our initial assumption that taxation and subsidies have the same rates. Then, when the cost functions are convex, the designer's expected payoff is given by

$$
\begin{aligned}
R^{all}_{tax} \quad &= \quad n\int_0^1 g\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv \qquad (11)\\
&+\int_0^1 g\left((1-\beta)\left(v-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)dG_{n,n}\\
&+\sum_{i=1}^{n-1}\int_0^1 g\left((1-\beta)\left(-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)dG_{n-i,n}.
\end{aligned}
$$

where $G_{k,n}(x) = \sum_{j=k}^{n}\binom{n}{j}F(x)^j[1-F(x)]^{n-j},\ k = 1,2,...,n,$ denotes the distribution of the k-th order statistic out of n independent variables independently distributed according to F. In contrast to the outcome of Proposition 4, when effort cost functions are non-linear, the subsidies for contestants who did not win have a positive effect on the designer's expected payoff.

Proposition 6 *When the effort cost functions are convex, the designer's expected payoff in the winner's payoff taxation model is less than in the winner's payoff taxation and subsidies model for the same sufficiently low tax rate.*

The intuitive explanation for the last result is that, because $g = \gamma^{-1}$ is concave, the derivative of the concave cost function g' is a decreasing function, and g' is a meaningful parameter that affects the designer's expected payoff. This implies that subsidies for high-type contestants are relatively small, while subsidies for low-type contestants are relatively large, and thus total subsidies are smaller than in the case of linear effort costs, making subsidies profitable in the case of convex effort costs.

By Proposition 6 and Corollary 2 we obtain that

Corollary 3 *When the effort cost functions are convex and the tax rate is low enough, the designer's expected payoff in the winner's payoff taxation and subsidies model is greater than in the same model without any taxation or subsidies.*

4 Extensions

In the previous section, we assumed that the designer's expected payoff is based on the tax payment plus the total effort of the contestants. In some cases, the designer is more concerned with the contestants' highest effort than their total effort. Because the contestants' equilibrium efforts do not depend on the designer's goal, they are the same as in the previous sections, and thus, in the winner's payoff taxation model, the designer's expected payoff is

$$\begin{aligned} R_{tax}^{\max} &= E_{\max}(g\left(x(v)\right)) + E_{\max}(g\left(\left(1-\beta\right)\left(v-\gamma(x(v))\right)\right)) \qquad (12) \\ &= \int_0^1 g\left(\beta\frac{vG(v)-\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v) + \int_0^1 g\left((1-\beta)\frac{v(1-G(v))+\beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v), \end{aligned}$$

where $E_{\max}(g\left(x(v)\right))$ is the contestants' expected highest effort and $E_{\max}(g((1-\beta)\left(v-\gamma(x(v))\right)))$ is the expected taxation of the winner's payoff. The following result demonstrates that when the designer is concerned with the contestants' highest effort, the winner's payoff taxation is profitable.

Proposition 7 *When the effort cost functions are either convex or linear, and the designer benefits from the highest effort, his expected payoff in the winner's payoff taxation model is greater than in the same model without any taxation.*

It is worth noting that Proposition 7 holds true for any tax rate, though we assume throughout this paper that when effort costs functions are non-linear the tax rate is relatively low and, more importantly, bounded. This assumption is consistent with our model because, as in the previous section, the designer's expected payoff increases with the tax rate, and without a cap on its value, contestants will have no incentive to participate in the contest. According to Proposition 7 when the designer benefits from the contestants' highest effort, taxation of the winner's payoff is profitable for him. Now, we want to compare the winner's payoff taxation model with and without subsidies. When there are subsidies for all the contestants who did not win, the designer's expected payoff is

$$\begin{aligned} R_{tax}^{all\_\max} &= \int_0^1 g\left(vG(v)-\int_0^v G(s)ds\right)dG_{n,n} \qquad (13) \\ &+ \int_0^1 g\left((1-\beta)\left(v-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)dG_{n,n} \\ &+ \sum_{i=1}^{n-1}\int_0^1 g\left((1-\beta)\left(-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)dG_{n-i,n}. \end{aligned}$$

Below we show, just as when the designer benefits from the total effort of the contestants, that taxation of the winner's payoff in addition to subsidies for the remaining contestants is more profitable for the designer than the same model without taxation and subsidies.

Proposition 8 *When the effort cost functions are linear, if the designer benefits from the highest effort, then his expected payoff in the winner's payoff taxation model with taxation and subsidies is greater than in the same model without any taxation and subsidies.*

Proposition 8 shows that when the designer benefits from the highest effort, it is profitable for the designer to tax the winner while subsidizing losers. One of the reasons for this is that the designer taxes the highest effort, which, in the case of linear cost functions, can extract the majority of the total effort in the competition.[4] Thus, in that case, because the effort remains the same as in the absence of taxation and subsidies, the gain from the winner taxation exceeds the total cost of subsidies.

The following result shows that combining taxes and subsidies is even more profitable when contestants have convex cost functions.

Proposition 9 *When the effort cost functions are convex and the tax rate is sufficiently low, if the designer benefits from the highest effort, then his expected payoff in the winner's payoff taxation model with subsidies is greater than in the same model without any subsidies.*

The outcome of Proposition 9 is counterintuitive because, if the designer benefits from the highest effort, why should he subsidize the effort costs of all the other contestants where he does not benefit from their efforts? The explanation is that by subsidizing all of the other contestants' effort costs, they increase their effort, and as a result, the winner also increases his effort, which benefits the contest designer.

5 Conclusion

Taxation and subsidies are well-known tools for increasing the payoff of designers in various organizations. Each appears to be useful in different environments, The main conclusion of this work is that it demonstrates how taxation and subsidies can be usefully combined in contests with incomplete information. In contests with complete information, it is quite clear that this collaboration of taxation and subsidy is beneficial

[4] The highest effort in the linear case may reach 50% of the total efforts in all pay auctions with incomplete information (see Gavious and Minchuk, 2014).

because, by imposing taxes on the strong contestants while providing subsidies to the weak contestants, the contest becomes more balanced, and the total effort of the contestants increases. In contrast, in contests with incomplete information, the types of contestants are private information, making tax discrimination for high-type contestants and subsidy discrimination for low-type contestants more complicated. As a result, we distinguish the winner from all the other contestants and then levy a tax on the winner's payoff while providing subsidies to all the other contestants. As a result, an improvement in our model of the winner's taxation and subsidies for all the other contestants could be accomplished in two stages: the first is to divide the contestants into different classes based on their efforts, and then each of these classes will be either taxed or subsidized, with tax and subsidy rates potentially differing for each of the contestants based on their classified classes.

6 Appendix

6.1 Proof of Proposition 1

By (3), the equilibrium effort is

$$x_i(v_i) = g\left(\frac{\beta}{(1-(1-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right).$$

If we derive the equilibrium effort with respect to β we obtain

$$\begin{aligned}\frac{dx_i}{d\beta} &= g\prime\left(\frac{\beta}{(1-(1-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right) \\ &\quad \times\frac{1-G(v_i)}{(1-(1-\beta)G(v_i))^2}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right] \\ &\geq 0.\end{aligned}$$

Thus, the equilibrium effort of every contestant decreases in the tax rate $(1-\beta)$. However, when v_i approaches 1, we obtain $\frac{dx_i(v_i)}{d\beta} = 0$, namely, the contestants with the highest winning values do not change their equilibrium efforts as a result of the taxation.

6.2 Proof of Proposition 2

By (5), we have

$$R_{tax} = n\int_0^1 \left(\beta\frac{vG(v)-\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) f(v)dv + \int_0^1 \left((1-\beta)\frac{v(1-G(v))+\beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) dF^n(v),$$

where the LHS of the integrand represents the total effort of the contestants and the RHS of the integrand represents the tax payment. If we rewrite this expression, the expected payoff for the designer is

$$R_{tax} = n\int_0^1 \left(vG(v)-\beta\int_0^v G(s)ds\right) f(v)dv. \tag{14}$$

According to Krishna (2010), the designer's expected payoff in an all-pay contest with no taxes and linear effort cost functions is

$$R = n\int_0^1 \left(vG(v)-\int_0^v G(s)ds\right) f(v)dv.$$

The difference between the designer's expected payoffs in these two cases is

$$R_{tax} - R = n\int_0^1 \left((1-\beta)\int_0^v G(s)ds\right) f(v)dv > 0.$$

Thus, when the effort cost functions are linear, the designer's expected payoff in the all-pay contest with any tax rate $(1-\beta)$ is greater than in the same contest without any taxation.

6.3 Proof of Proposition 3

By (5), the designer's expected payoff in the winner's payoff taxation model is

$$R_{tax} = n\int_0^1 g\left(\beta\frac{vG(v)-\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) f(v)dv + \int_0^1 g\left((1-\beta)\frac{v(1-G(v))+\beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) dF^n(v).$$

Differentiating this expression with respect to the taxation parameter δ yields

$$\begin{aligned}\frac{\partial R_{tax}}{\partial \beta} &= n\int_0^1 g'\left(\beta\frac{vG(v)-\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)\left(\frac{1-G(v)}{(1-(1-\beta)G(v))^2}\beta\left(vG(v)-\int_0^v G(s)ds\right)\right) f(v)dv \\ &\quad +n\int_0^1 g'\left((1-\beta)\frac{v(1-G(v))+\beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) \\ &\quad \times\frac{-v(1-G(v))+\left[(1-2\beta)-(1-3\beta)(1-\beta)G(v)\right]\int_0^v G(s)ds}{(1-(1-\beta)G(v))^2} f(v)G(v)dv.\end{aligned}$$

Notice that in case of $\beta = 1$ we get the designer's expected payoff in the standard model without any taxation (for more details see Minchuk and Sela , 2020; Minchuk and Sela, 2023). Thus, when β approaches 1, we obtain that the designer's expected payoff is

$$\begin{aligned} \lim_{\beta \to 1} \frac{\partial R_{tax}}{\partial \beta} &= n\int_0^1 g'\left(\left(vG(v) - \int_0^v G(s)ds\right)\right)\left(vG(v) - \int_0^v G(s)ds\right)(1 - G(v))\, f(v)dv \\ &\quad - n\int_0^1 g'(0)\left(v(1 - G(v)) + \int_0^v G(s)ds\right) G(v)f(v)dv. \end{aligned} \tag{15}$$

Differentiating the designer's payoff in the cost taxation model given by (6) with respect to the taxation parameter δ yields

$$\begin{aligned} \frac{\partial R_{ct}}{\partial \delta} &= -n\int_0^1 g'\left(\frac{1}{\delta}\left(vG(v) - \int_0^v G(s)ds\right)\right)\left(vG(v) - \int_0^v G(s)ds\right)\frac{1}{\delta^2} f(v)dv \\ &\quad + n\int_0^1 g'\left(\frac{n(\delta - 1)}{\delta}\left(vG(v) - \int_0^v G(s)ds\right)\right)\left(vG(v) - \int_0^v G(s)ds\right)\frac{1}{\delta^2} f(v)dv. \end{aligned}$$

Then, taking the limit when δ approaches 1 gives us

$$\begin{aligned} \lim_{\delta \to 1}\left(-\frac{\partial R_{ct}}{\partial \delta}\right) &= n\int_0^1 g'\left(\left(vG(v) - \int_0^v G(s)ds\right)\right)\left(vG(v) - \int_0^v G(s)ds\right) f(v)dv \\ &\quad - n\int_0^1 g'(0)\left(vG(v) - \int_0^v G(s)ds\right) f(v)dv. \end{aligned} \tag{16}$$

We compare $\lim_{\beta \to 1} \frac{\partial R_{tax}}{\partial \beta}$ with $\lim_{\delta \to 1}(-\frac{\partial R_{ct}}{\partial \delta})$. The reason is that when we increase the value of β we decrease the tax rate $(1 - \beta)$ in the winner's payoff taxation model, but when we increase the value of δ we increase the tax rate $(\delta - 1)$ in the cost taxation model. Thus, the parameters β and δ have opposite effects on the tax rates in both models. The difference between (16) and (15) is

$$\begin{aligned} &\lim_{\beta \to 1} \frac{\partial R_{tax}}{\partial \beta} - \lim_{\delta \to 1}\left(-\frac{\partial R_{ct}}{\partial \delta}\right) \\ &= n\int_0^1 g'\left(\left(vG(v) - \int_0^v G(s)ds\right)\right)\left(vG(v) - \int_0^v G(s)ds\right) G(v)f(v)dv \\ &\quad - n\int_0^1 g'(0)\left(\left(vG(v) + \int_0^v G(s)ds\right) G(v) - \int_0^v G(s)ds\right) f(v)dv. \end{aligned}$$

If γ is convex and strictly increasing, its inverse function $g=\gamma^{-1}$ is concave and then g' is decreasing such that $g'\left(vG(v)-\int_0^v G(s)ds\right)\leq g'(0)$. Thus,

$$\begin{aligned}
\lim_{\beta\to 1}\frac{\partial R_{tax}}{\partial\beta}-\lim_{\delta\to 1}(-\frac{\partial R_{ct}}{\partial\delta}) &\leq n\int_0^1 g'(0)\left(vG(v)-\int_0^v G(s)ds\right)G(v)f(v)dv \\
&\quad -n\int_0^1 g'(0)\left(\left(vG(v)+\int_0^v G(s)ds\right)G(v)-\int_0^v G(s)ds\right)f(v)dv \\
&= n\int_0^1 g'(0)\left(1-2G(v)\right)\left(\int_0^v G(s)ds\right)f(v)dv.
\end{aligned}$$

Notice that $\int_0^v G(s)ds\leq G(v)$ and $G(v)=F^{n-1}(v)$. Thus, we have

$$\begin{aligned}
\lim_{\beta\to 1}\frac{\partial R_{tax}}{\partial\beta}-\lim_{\delta\to 1}(-\frac{\partial R_{ct}}{\partial\delta}) &\leq n\int_0^1 g'(0)\left(1-2G(v)\right)\left(\int_0^v G(s)ds\right)f(v)dv \\
&\leq ng'(0)\left[\int_0^1\left(1-2G(v)\right)G(v)f(v)dv\right] \\
&= g'(0)\left[\int_0^1 dF^n(v)-\frac{2n}{2n-1}\int_0^1 dF^{2n-1}(v)\right]=-\frac{g'(0)}{2n-1}<0.
\end{aligned}$$

Thus, we can conclude that $\lim_{\beta\to 1}\frac{\partial R_{tax}}{\partial\beta}-\lim_{\delta\to 1}(-\frac{\partial R_{ct}}{\partial\delta})\leq 0$. Since according to (5) and (6), $(R_{tax}-R_{ct})\mid_{\beta=\delta=1}=0$, we obtain the result that when the tax rates are sufficiently low, then the designer expected payoff in the winner's payoff taxation R_{tax} is greater than in the the same model with cost taxation R_{ct}.

6.4 Proof of Proposition 4

By (8), when the effort cost functions are linear, the designer's expected payoff in the winner's payoff taxation and subsidies model is

$$\begin{aligned}
R_{tax}^{all} &= n\int_0^1\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv \\
&\quad +(1-\beta)\int_0^1\left(v-\left[vG(v)-\int_0^v G(s)ds\right]\right)dG_{n,n} \\
&\quad +(1-\beta)\sum_{i=1}^{n-1}\int_0^1\left(-\left[vG(v)-\int_0^v G(s)ds\right]\right)dG_{n-i,n},
\end{aligned} \tag{17}$$

where $G_{k,n}(x)=\sum_{j=k}^{n}\binom{n}{j}F(x)^j[1-F(x)]^{n-j}$ denotes the distribution of the k-th order statistic out of n independent variables independently distributed according to F. Note that $G_{n,n}$ is the highest order statistic distribution. Rearranging (17) where $G_{n,n} = F^n(v), G(v) = F^{n-1}(v)$ and $\sum_{i=0}^{n-1} dG_{n-i,n} = nf(v)$ gives us

$$\begin{aligned} R_{tax}^{all} &= n\int_0^1 \left(vG(v) - \int_0^v G(s)ds\right) f(v)dv + (1-\beta)n\int_0^1 vG(v)f(v)dv \\ &+(1-\beta)n\int_0^1 \left(-\left[vG(v) - \int_0^v G(s)ds\right]\right) f(v)dv. \end{aligned}$$

This yields

$$R_{tax}^{all} = n\int_0^1 \left(vG(v) - \beta\int_0^v G(s)ds\right) f(v)dv. \tag{18}$$

This last expression of the designer's expected payoff in the winner's payoff taxation and subsidies model is exactly the same as the designer's expected payoff in the winner's payoff taxation without subsidies given by (14).

6.5 Proof of Proposition 5

The designer's expected payoff in the winner's payoff taxation and subsidies model with a tax rate of $0 < (1-\beta) < 1$ and a subsidy rate of $0 < (1-\delta) < 1$ is

$$\begin{aligned} R_{tax}^{all\_dif} &= n\int_0^1 \left(\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right) f(v)dv \\ &+(1-\beta)\int_0^1 \left(v - \left[\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right]\right) dG_{n,n} \\ &+(1-\delta)\sum_{i=1}^{n-1}\int_0^1 \left(-\left[\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right]\right) dG_{n-i,n}. \end{aligned} \tag{19}$$

Rearranging (19) when $G_{n,n} = F^n(v), G(v) = F^{n-1}(v)$ and $\sum_{i=0}^{n-1} dG_{n-i,n} = nf(v)$ gives us

$$\begin{aligned} R_{tax}^{all\_dif} &= n\int_0^1 \left(\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right) f(v)dv \\ &+(1-\beta)\int_0^1 \left(v - \left[\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right]\right) nG(v)f(v)dv \\ &-(1-\delta)n\int_0^1 \left(\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right) f(v)dv \\ &+(1-\delta)\int_0^1 \left(\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i) - \int_0^{v_i} G(s)ds\right]\right) nG(v)f(v)dv, \end{aligned}$$

or, alternatively,

$$R_{tax}^{all\_dif} = n\int_0^1 \left(\frac{\beta}{(\delta-(\delta-\beta)G(v_i))}\left[v_iG(v_i)-\int_0^{v_i}G(s)ds\right]\right)((\delta-(\delta-\beta)G(v_i)))\,f(v)dv \tag{20}$$

$$+(1-\beta)\int_0^1 vnG(v)f(v)dv$$

$$= n\int_0^1 \left(\beta\left[v_iG(v_i)-\int_0^{v_i}G(s)ds\right]\right)f(v)dv+(1-\beta)\int_0^1 vnG(v)f(v)dv$$

$$= n\int_0^1 \left(vG(v)-\beta\int_0^{v}G(s)ds\right)f(v)dv.$$

We can see that the designer's expected payoff when the taxation and subsidy rates are different, as given by (20), is exactly the same as when the taxation and subsidy rates are the same, as given by (18).

6.6 Proof of Proposition 6

Rearranging the designer's expected payoff in the winner's payoff taxation and subsidies model given by (11) yields

$$R_{tax}^{all} = n\int_0^1 g\left(vG(v)-\int_0^{v}G(s)ds\right)f(v)dv$$

$$+\int_0^1 g\left((1-\beta)\left(v-\left[vG(v)-\int_0^{v}G(s)ds\right]\right)\right)dG_{n,n}$$

$$+\sum_{i=0}^{n-1}\int_0^1 g\left((1-\beta)\left(-\left[vG(v)-\int_0^{v}G(s)ds\right]\right)\right)dG_{n-i,n}$$

$$-\int_0^1 g\left((1-\beta)\left(-\left[vG(v)-\int_0^{v}G(s)ds\right]\right)\right)dG_{n,n}.$$

Since $\sum_{i=0}^{n-1} dG_n^{n-i} = nf(v)$, we get

$$\begin{aligned}
R_{tax}^{all} &= n\int_0^1 g\left(vG(v) - \int_0^v G(s)ds\right) f(v)dv && (21)\\
&+\int_0^1 g\left((1-\beta)\left(v - \left[vG(v) - \int_0^v G(s)ds\right]\right)\right) dG_{n,n}\\
&+n\int_0^1 g\left((1-\beta)\left(-\left[vG(v) - \int_0^v G(s)ds\right]\right)\right) f(v)dv\\
&-\int_0^1 g\left((1-\beta)\left(-\left[vG(v) - \int_0^v G(s)ds\right]\right)\right) dG_{n.n}.
\end{aligned}$$

Differentiating (21) with respect to the taxation parameter β gives us

$$\begin{aligned}
\frac{\partial R_{tax}^{all}}{\partial \beta} &= -\int_0^1 g\prime\left((1-\beta)\left(v - \left[vG(v) - \int_0^v G(s)ds\right]\right)\right)\left(v - \left[vG(v) - \int_0^v G(s)ds\right]\right) dG_{n,n}\\
&+n\int_0^1 g\prime\left((1-\beta)\left(-\left[vG(v) - \int_0^v G(s)ds\right]\right)\right)\left[vG(v) - \int_0^v G(s)ds\right] f(v)dv\\
&-\int_0^1 g\prime\left((1-\beta)\left(-\left[vG(v) - \int_0^v G(s)ds\right]\right)\right)\left[vG(v) - \int_0^v G(s)ds\right] dG_{n,n}.
\end{aligned}$$

Taking the limit when β approaches 1 yields

$$\begin{aligned}
\lim_{\beta\to 1} \frac{\partial R_{tax}^{all}}{\partial \beta} &= -n\int_0^1 g'(0)\left(v - \left[vG(v) - \int_0^v G(s)ds\right]\right) G(v)f(v)dv && (22)\\
&\quad +n\int_0^1 g'(0)\left(vG(v) - \int_0^v G(s)ds\right)(1-G(v))f(v)dv\\
&= -n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right) f(v)dv.
\end{aligned}$$

The difference between (15) and (22) is

$$\begin{aligned}
\lim_{\beta\to 1}\left(\frac{\partial R_{tax}}{\partial \beta} - \frac{\partial R_{tax}^{all}}{\partial \beta}\right) &= n\int_0^1 g'\left(\left(vG(v) - \int_0^v G(s)ds\right)\right)\left(vG(v) - \int_0^v G(s)ds\right)(1-G(v))\, f(v)dv\\
&-n\int_0^1 g'(0)\left(v\,(1-G(v)) + \int_0^v G(s)ds\right) G(v)f(v)dv\\
&+n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right) f(v)dv.
\end{aligned}$$

If γ is convex and strictly increasing, its inverse function $g = \gamma^{-1}$ is concave, g' is decreasing and therefore $g'\left(vG(v) - \int_0^v G(s)ds\right) \leq g'(0)$. Thus,

$$
\begin{aligned}
\lim_{\beta \to 1}\left(\frac{\partial R_{tax}}{\partial \beta} - \frac{\partial R_{tax}^{all}}{\partial \beta}\right) &\leq n\int_0^1 g'(0)\left(vG(v) - \int_0^v G(s)ds\right)(1-G(v))\,f(v)dv \\
&\quad -n\int_0^1 g'(0)\left(v(1-G(v)) + \int_0^v G(s)ds\right)G(v)f(v)dv \\
&\quad +n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right)f(v)dv \\
&= 0
\end{aligned}
$$

Since by Proposition 4, $\left(R_{tax} - R_{tax}^{all}\right)|_{\beta=1} = 0$, then the inequality $\lim_{\beta\to 1}\left(\frac{\partial R_{tax}}{\partial \beta} - \frac{\partial R_{tax}^{all}}{\partial \beta}\right) \leq 0$ implies that when the effort cost functions are convex, the expected payoff in the winner's payoff taxation model is smaller than in the same model with subsidies for the same tax rate that is sufficiently low.

6.7 Proof of Proposition 7

By (12), the designer expected payoff in the winner's payoff taxation model is

$$
R_{tax}^{\max} = \int_0^1 g\left(\beta\frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v) + \int_0^1 g\left((1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v). \tag{23}
$$

We will divide the proof into two cases: one with linear cost functions and one with convex cost functions.

1) Linear cost functions: If the cost function is linear, (23) can be rewritten as follows:

$$
\begin{aligned}
R_{tax}^{\max} &= \int_0^1 \left(\beta\frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v) + \int_0^1 \left((1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v) \qquad (24) \\
&\geq \int_0^1 \left(\beta\frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right)dF^n(v) + \int_0^1 \left((1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}G(v)\right)dF^n(v) \\
&= \int_0^1 \left(vG(v) - \beta\int_0^v G(s)ds\right)dF^n(v).
\end{aligned}
$$

According to Minchuk and Sela (2020) and Minchuk and Sela (2023), the designer's expected payoff which is the expected highest effort in the all-pay auction without any taxation for any cost function is

$$R^{\max} = \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dF^n(v). \tag{25}$$

Thus, by (24) and (25), we obtain that $R^{\max}_{tax} - R^{\max} > 0$.

2) Convex cost functions: Define

$$\begin{aligned} A &= \beta \frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))} \\ B &= (1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}. \end{aligned}$$

Notice that $A, B \geq 0$. Since γ is convex and $g = \gamma^{-1}$ is concave, we get

$$\begin{aligned} g(A) + g(B) &= g(A\frac{A+B}{A+B}) + g(B\frac{A+B}{A+B}) \\ &\geq \frac{A}{A+B}g(A+B) + \frac{B}{A+B}g(A+B) = g(A+B). \end{aligned}$$

Thus, we have

$$\begin{aligned} R^{\max}_{tax} &= \int_0^1 g\left(\beta\frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) + g\left((1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) dF^n(v) \qquad (26) \\ &> \int_0^1 g\left(\beta\frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))} + (1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}\right) dF^n(v) \\ &\geq \int_0^1 g\left(\beta\frac{vG(v) - \int_0^v G(s)ds}{(1-(1-\beta)G(v))} + (1-\beta)\frac{v(1-G(v)) + \beta\int_0^v G(s)ds}{(1-(1-\beta)G(v))}G(v)\right) dF^n(v) \\ &= \int_0^1 g\left(vG(v) - \beta\int_0^v G(s)ds\right) dF^n(v). \end{aligned}$$

Then, according to (23) and (26) we obtain that

$$R^{\max}_{tax} > \int_0^1 g\left(vG(v) - \beta\int_0^v G(s)ds\right) dF^n(v). \tag{27}$$

On the other hand, the designer's expected payoff in the all-pay auction without any taxation is

$$R^{\max} = \int_0^1 g\left(vG(v) - \int_0^v G(s)ds\right) dF^n(v).$$

Because that $\beta < 1$, we obtain that $R_{tax}^{\max} - R^{\max} > 0$, that is, the designer's expected payoff in the winner's payoff taxation is higher than in the same model without any taxation.

6.8 Proof of Proposition 8

By (13), when the cost functions are linear, the designer's expected payoff in the winner's payoff taxation model is

$$\begin{aligned}
R_{tax}^{all_\max} &= \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dG_{n,n} \\
&\quad + (1-\beta) \int_0^1 \left(v - \left[vG(v) - \int_0^v G(s)ds \right] \right) dG_{n,n} \\
&\quad - (1-\beta) \sum_{i=1}^{n-1} \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dG_{n-i,n}. \\
&= \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dG_{n,n} \\
&\quad + (1-\beta) \int_0^1 v dG_{n,n} \\
&\quad - (1-\beta) \sum_{i=0}^{n-1} \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dG_{n-i,n}.
\end{aligned}$$

Because that $\sum_{i=0}^{n-1} dG_{n-i,n} = nf(v), G_{n,n} = F^n(v)$ and $\int_0^1 v dG_{n,n} = n \int_0^1 vF^{n-1}(v) f(v) dv$, we get

$$\begin{aligned}
R_{tax}^{all_\max} &= \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dF^n(v) \\
&\quad + (1-\beta) n \int_0^1 \left(\int_0^v G(s)ds \right) f(v) dv \\
&> \int_0^1 \left(vG(v) - \int_0^v G(s)ds \right) dF^n(v) = R^{\max}.
\end{aligned} \tag{28}$$

where $R^{\max}$ represents the designer's expected payoff in the model without taxation and subsidies. Thus, when the cost functions are linear, taxation and additional subsidies for all contestants benefit the designer.

6.9 Proof of Proposition 9

Differentiating the designer's expected payoff in the winner's payoff taxation and subsidies model given by (13) with respect to the taxation parameter β yields

$$\begin{aligned}
\frac{\partial R_{tax}^{all_\max}}{\partial \beta} &= -\int_0^1 g'\left((1-\beta)\left(v-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)\left(v-\left[vG(v)-\int_0^v G(s)ds\right]\right)dG_{n,n} \\
&\quad +n\int_0^1 g'\left((1-\beta)\left(-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)\left[vG(v)-\int_0^v G(s)ds\right]f(v)dv \\
&\quad -\int_0^1 g'\left((1-\beta)\left(-\left[vG(v)-\int_0^v G(s)ds\right]\right)\right)\left[vG(v)-\int_0^v G(s)ds\right]dG_{n,n}.
\end{aligned}$$

The limit when β approaches 1 is

$$\lim_{\beta\to 1}\frac{\partial R_{tax}^{all\_\max}}{\partial \beta} = -n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right)f(v)dv. \tag{29}$$

Taking the difference between the models of the winner's taxation models without subsidies (12) and with subsidies (29) give us

$$\begin{aligned}
\lim_{\beta\to 1}\left(\frac{\partial R_{tax}^{\max}}{\partial \beta}-\frac{\partial R_{tax}^{all_\max}}{\partial \beta}\right) &= n\int_0^1 g'\left(\left(vG(v)-\int_0^v G(s)ds\right)\right)\left(vG(v)-\int_0^v G(s)ds\right)(1-G(v))\,G(v)f(v)dv \\
&\quad -n\int_0^1 g'(0)\left(v\,(1-G(v))+\int_0^v G(s)ds\right)G(v)f(v)dv \\
&\quad +n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right)f(v)dv.
\end{aligned}$$

If γ is convex and strictly increasing, its inverse function $g=\gamma^{-1}$ is concave, g' is decreasing and therefore $g'\left(vG(v)-\int_0^v G(s)ds\right)\leq g'(0)$. Thus,

$$\begin{aligned}
\lim_{\beta\to 1}\left(\frac{\partial R_{tax}^{\max}}{\partial \beta}-\frac{\partial R_{tax}^{all_\max}}{\partial \beta}\right) &\leq n\int_0^1 g'(0)\left(vG(v)-\int_0^v G(s)ds\right)(1-G(v))\,G(v)f(v)dv \\
&\quad -n\int_0^1 g'(0)\left(v\,(1-G(v))+\int_0^v G(s)ds\right)G(v)f(v)dv \\
&\quad +n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right)f(v)dv \\
&= -n\int_0^1 g'(0)\left(vG(v)-\int_0^v G(s)ds\right)(1-G(v))^2 f(v)dv.
\end{aligned}$$

Therefore, for a designer who cares about the highest effort, additional subsidies for all the contestants is profitable.

$$\begin{aligned}
\lim_{\beta\to 1}\left(\frac{\partial R_{tax}^{\max}}{\partial\beta}-\frac{\partial R_{tax}^{all_\max}}{\partial\beta}\right) &\le n\int_0^1 g'(0)\left(vG(v)-\int_0^v G(s)ds\right)(1-G(v))\,G(v)f(v)dv \\
&\quad -n\int_0^1 g'(0)\left(v\,(1-G(v))+\int_0^v G(s)ds\right)G(v)f(v)dv \\
&\quad +n\int_0^1 g'(0)\left(\int_0^v G(s)ds\right)f(v)dv \\
&= -n\int_0^1 g'(0)\left(vG(v)-\int_0^v G(s)ds\right)(1-G(v))^2 f(v)dv.
\end{aligned}$$

Therefore, for a designer who cares about the highest effort, additional subsidies for all the contestants is profitable.

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