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Abstract

We say that an alternative is socially acceptable if the number of individuals who rank it among their most preferred half of the alternatives is at least as large as the number of individuals who rank it among the least preferred half. A Condorcet winner may not be socially acceptable. However, if preferences are single-peaked or satisfy the single-crossing property, any Condorcet winner is socially acceptable.

Keywords: Condorcet winner, single-peaked preferences, single-crossing.

JEL Classification Numbers: D71, D72.

1 Introduction

An alternative is a Condorcet winner if there is no other alternative that is preferred to it by more than half of the individuals. More generally, for $q \geq 1/2$, an alternative is a q-Condorcet winner if there is no alternative that is preferred to it by more than a proportion q of the individuals. On the other hand, an alternative is socially acceptable if it is ranked among the top half of the alternatives by at least half of the individuals. Failing to be socially acceptable can be regarded as a drawback since a majority of voters may be uncomfortable

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with such an alternative. It turns out that unless q is high enough, a q-Condorcet winner may not be socially acceptable. In this paper we show, however, that if preferences are single-peaked or if they satisfy the single-crossing property, any Condorcet winner is socially acceptable.

2 Definitions

Let $A = \{a_1, \ldots, a_K\}$ be a set of K alternatives and let $N = \{1, \ldots, n\}$ be a set of individuals. Also, let $\mathcal{P}$ be the subset of complete, transitive and antisymmetric binary relations on A. We will refer to the elements of $\mathcal{P}$ as preference relations or simply as preferences. A *preference profile* is a mapping $\pi = (\succ_1, \ldots, \succ_n)$ of preference relations on A to the individuals in N. For any subset $C \subseteq \mathcal{P}$ of preference relations, $\mu_\pi(C) = |\{i \in N : \succ_i \in C\}|$ is the number of individuals whose preference relations are in C. Also, $\pi(N) = \{\succ \in \mathcal{P} : \exists i \in N \text{ s.t. } \succ_i = \succ\}$ is the set of preferences that are present in the profile π. For any two alternatives $a, a' \in A$, $C(a \to a') = \{\succ \in \mathcal{P} : a \succ a'\}$ denotes the set of preference relations according to which a is preferred to a'.

Definition 1 Let π be a preference profile, and let $a \in A$ be an alternative. We say that a is a *Condorcet winner* for π if for every alternative $a' \in A$ the number of individuals who prefer a to a' is at least as large as the number of individuals who prefer a' to a.

For any preference relation $\succ$ and for any alternative $a \in A$, the rank of a in $\succ$, denoted by $\text{rank}_\succ(a)$, is 1 + the number of alternatives that are strictly preferred to a according to $\succ$. Formally, $\text{rank}_\succ(a) = K - |\{a' \in A \setminus \{a\} : a \succ a'\}|$. Alternatives whose ranks in $\succ$ are less than $(K+1)/2$ are said to be placed *above the line* by $\succ$ and those whose ranks are greater than $(K+1)/2$ are said to be placed *below the line* by $\succ$. For instance, if $K = 5$ and a voter's preference relation is given by $a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5$, he places alternatives a_1 and a_2 above the line and alternatives a_4 and a_5 below the line.

We now define the concept of social acceptability.

Definition 2 Let π be a preference profile. We say that alternative $a \in A$ is *socially acceptable with respect to* π if the number of individuals whose preferences place a above the line is at least as large as the number of individuals whose preferences place it below the line.

Mahajne and Volij [6] showed that every preference profile has a socially acceptable alternative. The next example shows that a Condorcet winner may not be socially acceptable.

Example 1 Assume $A = \{a, b, c, d\}$ and consider the preference profile $(abcd, acbd, cdab, cbad, bdac)$. It can be seen that whereas alternative a is a Condorcet winner, it is not socially acceptable. The only socially acceptable alternatives are b and c.

The concept of a Condorcet winner can be strengthened by requiring that the alternative be preferred to any other alternative by at least a given proportion of the voters.

Definition 3 Let $\pi = (\succ_1, \ldots, \succ_n)$ be a preference profile, and let $a \in A$ be an alternative. For $q \in (1/2, 1)$ we say that a is a q-*Condorcet winner* for π if for every alternative $a' \in A$ the number of individuals who prefer a to a' is greater or equal to a fraction q of the number of individuals. Namely, if $\mu_\pi(C(a \to a')) \geq qn$ for all $a' \in A \setminus \{a\}$.

The concepts of q-Condorcet winner and q-majority equilibrium have been studied in Greenberg [5], Sari [10], Baharad and Nitzan[1], and Courtin et al.[3]. The following result shows that for large enough q, any q-Condorcet winner is socially acceptable.

Proposition 1 Let $q \geq (3K - 4)/(4K - 4)$ and let π be a preference profile for which alternative a is a q-Condorcet winner. Then a is socially acceptable.

Proof : Let a be a q-Condorcet winner for π and let

$$W_\pi(a) = \sum_{a' \in A \setminus \{a\}} \mu_\pi(C(a \to a')).$$

Since $\mu_\pi(C(a \to a')) \geq qn \geq \frac{3K-4}{4K-4}n$ for all $a' \in A \setminus \{a\}$ we have that

$$W_\pi(a) \geq n\frac{3K-4}{4}. \tag{1}$$

Assume by contradiction that a is not socially acceptable. Then, there are proportions α and β, with $\alpha < \beta$ such that a proportion α of individuals places a below the line and a proportion β places a above the line. Let π' be the preference profile that is obtained from π by sending alternative a to the top of each preference relation that places it above the line and by sending a to the $(K+1)/2+1$-th rank (just below the line) of each preference relation that places it below the line. By construction, a is not socially acceptable for π' and

$$\begin{aligned} W_\pi(a) &\leq W_{\pi'}(a) \\ &= \alpha n(K-1) + \beta n(\frac{K-1}{2} - 1) + (1-\alpha-\beta)n\frac{K-1}{2} \\ &= \alpha n(K-1) + (1-\alpha)n\frac{K-1}{2} - \beta n \\ &< \alpha n(K-1) + (1-\alpha)n\frac{K-1}{2} - \alpha n \\ &= \alpha n(\frac{K-3}{2}) + n\frac{K-1}{2} \\ &< n\frac{3K-5}{4} \\ &< n\frac{3K-4}{4} \end{aligned}$$

where we have used the fact that $1/2 < \alpha < \beta$. This inequality contradicts inequality 1. □

The bound $(3K-4)/(4K-4)$ cannot be improved. To see this, let $K=4$ so that the bound equals $2/3$. Let $q < 2/3$. We will construct a preference profile for which alternative a is a q-Condorcet winner but is not socially acceptable. Let m be a positive integer such that $q \leq (4m+1)/(6m+3)$ and let π be a preference profile with m individuals having preference $(abcd)$, m individuals with preference $(adbc)$, m individuals with preference $(acdb)$, $m+1$ individuals with preference $(bcad)$, $m+1$ individuals with preference $(dbac)$, and $m+1$ individuals with preference $(cdab)$. The number of individuals is $n = 6m+3$. The number of individuals who prefer a to b is $4m+1$. The same number of individuals prefer a to c and a to d. Therefore, a is a q-Condorcet winner. However, a is not socially acceptable.

3 Single-peaked preferences

In this section we restrict attention to single-peaked preferences and show that in this case, any Condorcet winner is socially acceptable.[1]

Definition 4 Let A be a set of K alternatives and let $\leq$ be a linear order on A. We say that the preference relation $\succ$ is single-peaked with respect to $\leq$ if there is an alternative $p \in A$ such that [2]

$$(a < b \leq p \text{ or } p \leq b < a) \Rightarrow b \succ a.$$

Theorem 1 Let $\leq$ be the linear order on A and assume without loss of generality that $a_1 < \cdots < a_K$. Let π be a preference profile of single-peaked preferences with respect to $\leq$, and let $a \in A$ be a Condorcet winner with respect to π. Then a is socially acceptable for π.

Proof : Case 1: $a \neq a_{(K+1)/2}$.

Case 1.1: $a = a_k$ for some $k \in \{1, \ldots, \lceil \frac{K-1}{2} \rceil\}$. Let $b = a_{\lceil \frac{K-1}{2} \rceil + k}$ and let $\succ$ be a preference relation in the profile. It cannot be the case that both a and b are placed above the line by $\succ$. For, in that case, since preferences are single-peaked with respect to $\leq$, a_ℓ would be above the line for all $\ell = k, \ldots, \lceil \frac{K-1}{2} \rceil + k$. But this means that more than $K/2$ alternatives would be above the line, which is impossible. On the other hand, it cannot be the case that both a and b are placed below the line by $\succ$. For, in that case, since preferences are single-peaked with respect to $\leq$, the number of alternatives placed above the line by $\succ$ would be at most $(\lceil \frac{K-1}{2} \rceil + k) - 1 - k) = \lceil \frac{K-3}{2} \rceil$ which is less than $\frac{K-1}{2}$, which is impossible. Finally, if a is on the line, then b is above the line. For if it was below the line, the number of alternatives above the line would be at most $(\lceil \frac{K-1}{2} \rceil + k) - 1 - k) = \lceil \frac{K-3}{2} \rceil$ which is impossible. As a result, $\succ$ places a above the line if and only if $a \succ b$. Consequently, the number of voters

[1]Single-peaked preferences were introduced by Black [2].

[2]For any two alternatives, $a, b \in A$, $a < b$ means $a \leq b$ and not $b \leq a$.

who place a above the line equals the number of voters in the profile who prefer a to b. Since a is a Condorcet winner, this number is at least $n/2$ and therefore it is at least as large as the number of voters that place a below the line. In other words, a is socially acceptable with respect to π.

Case 1.2: $a = a_k$ for some $k \in \{\lfloor \frac{K+1}{2} \rfloor + 1, \ldots, K\}$. Let $b = a_{k - \lceil \frac{K-1}{2} \rceil}$ and let $\succ$ be a preference relation in the profile. It cannot be the case that both a and b are placed above the line by $\succ$. For, in that case, since preferences are single-peaked with respect to $\leq$, a_ℓ would be above the line for all $\ell = k - \lceil \frac{K-1}{2} \rceil, \ldots, k$. But this means that more than $K/2$ alternatives would be above the line, which is impossible. On the other hand, it cannot be the case that both a and b are placed below the line by $\succ$. For, in that case, since preferences are single-peaked with respect to $\leq$, the number of alternatives placed above the line by $\succ$ would be at most $(k-1) - (k - \lceil \frac{K-1}{2} \rceil) = \lceil \frac{K-3}{2} \rceil$ which is less than $\frac{K-1}{2}$, which is impossible. Finally, if a is on the line, then b is above the line. For if it was below the line, the number of alternatives above the line would be at most $(k-1) - (k - \lceil \frac{K-1}{2} \rceil) = \lceil \frac{K-3}{2} \rceil$ which is less than $\frac{K-1}{2}$, which is impossible. As a result, $\succ$ places a above the line if and only if $a \succ b$. Consequently, the number of voters who place a above the line equals the number of voters in the profile who prefer a to b. Since a is a Condorcet winner, this number is at least $n/2$ and therefore it is at least as large as the number of voters that place a below the line. In other words, a is socially acceptable with respect to π.

Case 2: $a = a_{(K+1)/2}$. In this case, alternative a cannot be below the line for any of the voters. For suppose that $\text{rank}_\succ(a_{\frac{K+1}{2}}) > \frac{K+1}{2}$ for some preference relation $\succ$ that is single-peaked with respect to $\leq$. Then we must have that $\text{rank}_\succ(a_k) > (K+1)/2$ either for all $k \leq (K+1)/2$ or for all $k \geq (K+1)/2$. In either case we would have that more than half of the alternatives have a rank higher than $K/2$, which is impossible. As a result, a is socially acceptable with respect to π. □

4 Single-Crossing Preferences

We now restrict attention to the class of preferences that satisfy the single-crossing property. This class has been introduced by Roberts [7] and has been shown to admit a majority voting equilibrium. See for instance Rothstein [8, 9], Gans and Smart [4], as well as Saporiti and Tohmé [11]. Roughly speaking, a set of preferences satisfies the single-crossing property if both the preferences and the alternatives can be ordered from "left" to "right" so that if a rightist preference prefers an alternative that is to the left of another alternative, then so do all preferences that are to the left of this preference.

Definition 5 Let $\leq$ be a linear order on A. Let $C \subseteq \mathcal{P}$ be a nonempty subset of preferences and let $\sqsubseteq$ be a linear order on C. We say that the preference relations in C satisfy the *single-crossing property with respect to* $(\leq, \sqsubseteq)$ if for all pairs of alternatives $a, b \in A$ and for all pairs of preferences $\succ, \succ' \in C$, we have [3]

$$\left.\begin{array}{c} a < b \\ \succ \sqsubset \succ' \end{array}\right\} \Rightarrow (b \succ a \Rightarrow b \succ' a).$$

We also say that the profile $\pi = \{\succ_1, \ldots, \succ_n\}$ *satisfies the single-crossing property* if there is a linear order $\leq$ on A and a linear order $\sqsubseteq$ on the set $\pi(N)$ of preferences in the profile, such that the preferences in $\pi(N)$ satisfy the single crossing property with respect to $(\leq, \sqsubseteq)$.

Example 2 Let the set of alternatives be $A = \{a, b, c\}$ with the linear order given by $a < b < c$. Consider the subset $C \subseteq \mathcal{P}$ that contains the following four preference relations:

$$\begin{array}{rclcrcl} \succ_1 & = & a, b, c & \quad & \succ_2 & = & a, c, b \\ \succ_3 & = & c, a, b & \quad & \succ_4 & = & c, b, a \end{array}$$

with the linear order given by $\succ_1 \sqsubset \succ_2 \sqsubset \succ_3 \sqsubset \succ_4$. It can be checked that the preferences in C satisfy the single-crossing property with respect to $(\leq, \sqsubseteq)$. Indeed, note that the preferences

[3] For any two preferences, $\succ, \succ' \in \mathcal{P}$, $\succ \sqsubset \succ'$ b means $\succ \sqsubseteq \succ'$ and not $\succ' \sqsubseteq \succ$.

in C that rank alternative c over alternative a are $\succ_3$ and $\succ_4$. Similarly, the preferences in C that rank alternative c over alternative b are $\succ_2$, $\succ_3$ and $\succ_4$. Finally, the only preference in C that ranks alternative b over alternative a is $\succ_4$. The reader can check that the preferences in C are not single-peaked with respect to any linear order on A.

The following claim states a useful property of single-crossing preferences. Namely, if two individuals agree on the ranking of two alternatives, so do all individuals who are ideologically "between" them.

Claim 1 Let $\pi = \{\succ_1, \ldots, \succ_n\}$ be a profile of preferences that are single-crossing with respect to $(\leq, \sqsubseteq)$ for some linear orders $\leq$ on A and $\sqsubseteq$ on $\pi(N)$. Let $i, j, k \in N$ with $\succ_i \sqsubset \succ_j \sqsubset \succ_k$. Then for any two alternatives $a, b \in A$,

$$\text{if both } a \succ_i b \text{ and } a \succ_k b, \text{ then also } a \succ_j b.$$

Proof : Assume that $b \succ_j a$. If $a < b$ then by single-crossing we must have that $b \succ_k a$. If $b < a$ by single-crossing we must have that $b \succ_i a$. □

When a set of preferences is ordered by $\sqsubseteq$, one can define its median. Formally,

Definition 6 Let $\pi = \{\succ_1, \ldots, \succ_n\}$ be a profile of preferences and let $\sqsubseteq$ be a linear order on $\pi(N)$. We say that $\succ_m \in \pi(N)$ is a median preference relation of π if

$$\mu_\pi(\{\succ \in \pi(N) : \succ \sqsubseteq \succ_m\}) \geq n/2 \quad \text{and} \quad \mu_\pi(\{\succ \in \pi(N) : \succ_m \sqsubseteq \succ\}) \geq n/2.$$

In other words, $\succ_m$ is a median preference of π if it belongs to $\pi(N)$, and at least half of the individuals have preferences that are at least as to the "right" as $\succ_m$ and at least half of the individuals have preferences that are at least as to the "left" as $\succ_m$. It is clear that any preference profile that satisfies the single-crossing property has a median preference.

We are now ready to state our second result.

Theorem 2 Let π be a preference profile that satisfies the single-crossing property and let $a \in A$ be a Condorcet winner with respect to π. Then a is socially acceptable for π.

Proof : Let $(\leq, \sqsubseteq)$ be the linear orders on A and $\pi(N)$, respectively, with respect to which π is single-crossing.

Lemma 1 Let $\succ_m \in \pi(N)$ be a median preference of π. Its top alternative is socially acceptable.

Proof : Let a be the top alternative of the median preference $\succ_m$ and let i be an individual who ranks a at least as low as any other individual. That is, $\text{rank}_{\succ_i}(a) \geq \text{rank}_{\succ_j}(a)$ for all $j \in N$. If $\text{rank}_{\succ_i}(a) \leq (K+1)/2$, a is socially acceptable since no individual places it below the line. So assume that $r = \text{rank}_{\succ_i}(a) > (K+1)/2$. There are $r-1 > (K-1)/2 \geq K/2$ alternatives $b_1, \ldots, b_{r-1}$ such that $b_k \succ_i a$ for $k = 1, \ldots, r-1$. Assume that $\succ_i \sqsubset \succ_m$. The case where $\succ_m \sqsubset \succ_i$ is similar and is left to the reader. Since preferences are single-crossing, and $a \succ_m b_k$ for $k = 1, \ldots, r-1$, by Claim 1 we must have that $a \succ_j b_k$ for $k = 1, \ldots, r-1$, for all $j \in N$ such that $\succ_m \sqsubseteq \succ_j$. Consequently, since such individuals constitute at least half of the voters and since $r - 1 > K/2$, a is above the line by at least half of the individuals. Namely, it is socially acceptable. □

Lemma 2 Let $\succ_m \in \pi(N)$ be a median preference. Its top alternative is a Condorcet winner.

Proof : Let a be preference $\succ_m$'s top alternative and let b be another alternative. If $b < a$ then, by the single-crossing property, $a \succ_i b$ for all i such that $\succ_m \sqsubseteq \succ_i$. And if $a < b$ then $a \succ_i b$ for all i such that $\succ_i \sqsubseteq \succ_m$. In either case a is preferred to b by at least half the individuals. □

Let a be a Condorcet winner. If a is the top alternative for some median preference, by Lemma 1 it is socially acceptable. Therefore assume that a is not the top preference for any median preference. Since by Lemma 2 the top alternative of any median preference is a Condorcet winner, we conclude that there are multiple Condorcet winners. This also implies that there are exactly two different median preferences in π. Denote them by $\succ_{m_1}$ and $\succ_{m_2}$ and assume without loss of generality that $\succ_{m_1}\sqsubset\succ_{m_2}$. Notice that there is no preference relation $\succ_i\in \pi(N)$ such that $\succ_{m_1}\sqsubset\succ_i\sqsubset\succ_{m_2}$.

Let $\succ_m$ be the preference relation that is obtained from $\succ_{m_1}$ by sending a to the top. That is, $a \succ_m b$ for all $b \in A \setminus \{a\}$ and $b \succ_m b' \Leftrightarrow b \succ_{m_1} b'$ for all $b, b' \in A \setminus \{a\}$. Also let π' be the preference profile that is obtained from π by adding a single voter with preference $\succ_m$, and extend the order $\sqsubseteq$ to $\pi'(N)$ by setting $\succ_{m_1}\sqsubset\succ_m\sqsubset\succ_{m_2}$. It can be checked that profile π' satisfies the single-crossing property with respect to $\leq$ and to the extended order $\sqsubseteq$. To see this, it is enough to check comparisons only involving a since, restricted to $A\setminus\{a\}$, profiles π and π' contain the same preferences. Let $b \in A \setminus \{a\}$ and let $\succ_i\in \pi(N)$. We know that $a \succ_m b$. Assume first that $b < a$ and $\succ_m\sqsubset\succ_i$. We need to show that $a \succ_i b$. If $a \succ_{m_1} b$, then by the single-crossing property of π, $a \succ_i b$. If, on the other hand, $b \succ_{m_1} a$, we must have that $a \succ_{m_2} b$, because otherwise, if $b \succ_{m_2} a$, by the single-crossing property of π we would have that $b \succ_j a$ for all $\succ_j\in \pi(N)$ such that $\succ_j\sqsubset\succ_{m_2}$. Since $\succ_{m_2}$ is the "right" median preference, this means that more than half of the individuals would prefer b to a, which contradicts the fact that a is a Condorcet winner. Summarizing, we have that $b < a$ and $a \succ_{m_2} b$. Then, by the single-crossing property of π, $a \succ_j b$ for all $\succ_j\in \pi(N)$ such that $\succ_{m_2}\sqsubseteq\succ_j$. In particular $a \succ_i b$. Similarly, assume now that $a < b$. Since $a \succ_m b$, we need to show that $a \succ_i b$ for all $\succ_i$ such that $\succ_i\sqsubset\succ_m$. If $a \succ_{m_2} b$, then by the single-crossing property of π, $a \succ_i b$ for all $\succ_i$ such that $\succ_i\sqsubset\succ_m$. If, on the other hand, $b \succ_{m_2} a$, we must have that $a \succ_{m_1} b$, because otherwise, if $b \succ_{m_1} a$, by the single-crossing property of π we would have that $b \succ_j a$ for all $\succ_j\in \pi(N)$ such that $\succ_{m_1}\sqsubset\succ_j$. Since $\succ_{m_1}$ is the "left" median preference, this means that more than half of the individuals would prefer b to a, which contradicts the fact that a is a Condorcet winner. Summarizing, we have that $a < b$ and $a \succ_{m_1} b$. Then, by the single-crossing property of $\pi(N)$, $a \succ_i b$ for all $\succ_i\in \pi(N)$ such that $\succ_i\sqsubseteq\succ_{m_1}$. We conclude that π' satisfies the single-crossing property.

We can now prove that a is socially acceptable for π. The proof is similar to that of Lemma 1. Indeed, by construction $\succ_m$ is a median preference of π' and a is its top alternative. Let i be an individual who ranks a at least as low as any other individual. That is, $\text{rank}_{\succ_i}(a) \geq \text{rank}_{\succ_j}(a)$ for all $j \in N$. If $\text{rank}_{\succ_i}(a) \leq (K+1)/2$, a is socially acceptable since it is not placed below the line by any individual. Therefore, assume that $r = \text{rank}_{\succ_i}(a) > (K+1)/2$. Then, there are $r-1 > (K-1)/2 \geq K/2$ alternatives $b_1, \ldots, b_{r-1}$ such that $b_k \succ_i a$ for $k = 1, \ldots, r-1$. Assume that $\succ_i \sqsubset \succ_m$. The case where $\succ_m \sqsubset \succ_i$ is similar and is left to the reader. Since preferences are single-crossing for π', and $a \succ_m b_k$ for $k = 1, \ldots, r-1$, by Claim 1 we must have that $a \succ_j b_k$ for $k = 1, \ldots, r-1$, for all $j \in N$ such that $\succ_m \sqsubseteq \succ_j$. One such individual is m_2. Therefore, $a \succ_j b_k$ for $k = 1, \ldots, r-1$, for all $j \in N$ such that $\succ_{m_2} \sqsubseteq \succ_j$. Since $r - 1 \geq (K+1)/2$, all these individuals place a above the line. Since such individuals constitute at least half of the voters in π, a is socially acceptable.

$\square$

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