All-Pay Matching Contests

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Discussion Paper No. 23-13

August 2023

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January 10, 2023

Abstract

We study two-sided matching contests with two sets, each of which includes two heterogeneous players with commonly known types. The players in each set compete in all-pay contests where they simultaneously send their costly efforts and then are assortatively matched. A player has a value function that depends on his type as well as his matched one. This model always has a corner equilibrium in which the players do not exert efforts and are randomly matched. However, we characterize the interior equilibrium and show that although players exert costly (wasted) efforts, this equilibrium might be either welfare superior or inferior to the corner equilibrium. We analyze the cross-effects of the players' types on their expected payoffs and also their effect on the players' expected total effort. We then show,that each of the players' types might have either a positive or a negative marginal effect on their expected total effort.

Keywords: Two-sided matching, All-pay contests.

JEL classification: D44, J31, D72, D82

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1 Introduction

In one-sided contests, which are also referred to as standard contests, agents exert efforts, and then commonly known fixed prizes are awarded to the agents who win according to the contest rules. Each agent knows exactly what his prize will be if he comes in first, second, or any other place. Such one-sided contests are applied, for example, in rent-seeking models, lobbying in organizations, R&D races, political contests, promotions in labor markets, trade wars, and military and biological wars of attrition. In contrast, in two-sided matching contests, agents from two sides exert efforts but the prizes for players are not commonly known and depend on the results from the other set.[1] These types of contests are applied, for example, in the academic arena where one side can consist of universities that invest in hiring top researchers and teachers, and the other side of international student candidates who aspire to be accepted to higher education institutions. In other areas, we can mention accounting or law students on one side, and firms on the other, and models, actors, and artists on one side, and talent agencies on the other.

There is an extensive literature on one-sided contests which include Tullock contests (see Tullock 1980, Skaperdas 1996, Clark and Riis 1998,, Baye and Hoppe 2003, and Gallice 2017), all-pay contests (see Baye et al. 1993, Che and Gale 1998, Moldovanu and Sela 2001, 2006, Siegel 2009, and Odegaard and Anderson 2014), rank-order tournaments (see Lazear and Rosen 1981, Rosen 1986, and Akerlof and Holden 2012). However, the literature on two-sided matching contests in which the agents compete according to the rules of the standard contests is more limited and is mainly seen when players compete in rank-order tournaments (see Bhaskar and Hopkins 2016), in all-pay contests under incomplete information (see Hoppe et al. 2009, Hoppe et al. 2011, and Dizdar et al. 2019), and in Tullock contests (see Cohen et al. 2020). A crucial issue in all these models is the uncertain matching between the two sides, the reasons being that in the rank-order tournament there is noise in each player's output, in the all-pay contest (auction) under incomplete

[1] Peters (2007) showed that equilibrium efforts in a very large finite two-sided matching model can be quite different from the equilibrium efforts in the continuum model.

information the information of each player is private, and in the Tullock contest the contest success function is stochastic.

We study a two-sided matching contest in which the players compete in the all-pay contest under complete information. There are two sets of players, each of which includes two heterogeneous players with commonly known types. They simultaneously exert their efforts, and then they are assortatively matched, namely, the winners as well as the losers from both sets are matched with each other. A player has a value which is a function of his own type and the type of his match, and a player who is matched has a payoff of his value minus the cost of his effort. What makes our model interesting is that the players use mixed strategies which are the reason for the uncertain matching between the two sides.

Since the players' equilibrium strategies and their expected payoffs depend on the players' types from both sets, the cross-effects of these types on the players' expected payoffs as well as on their expected total effort is not straightforward. Our analysis is feasible since the players' types are commonly known and is derived by that of the standard all-pay contest under complete information (see Hillman and Samet 1987, Hillman and Riley 1989, and Baye et al. 1996). Under incomplete information such an analysis is not applicable if the players compete in the all-pay contest (see Hoppe et al. 2009).

We first study the assortative all-pay matching contest with an additive value function according to which the value of each pair of players who are matched is the sum of their types. We show that in this case since the value function is not strictly super-modular, there is only a corner (trivial) equilibrium in which all the players in both sets do not exert efforts and the players are randomly matched. In contrast to this case, in other forms of the values function, the corner equilibrium is not unique and there are also interior equilibria in which players exert positive efforts, and at least some of the players prefer the interior equilibrium. Then, if some payers exert efforts, all the players have an incentive to exert efforts in order to get the best match.

We proceed by analyzing the assortative all-pay matching contest with a multiplicative value function according to which the value of each pair of players who are matched is the product of their types. We show that in contrast to the player with the lower type in his set, the expected payoff of the player with the higher type does not necessarily increase in all the other players' types in the other set. This result is not straightforward given that the larger are the types of the players in the other set, the larger are the players' values of winning. Its intuition is that when a player has a high probability to be matched with the higher type in the other set, if the lower type in the other set increases, this player's probability to be matched with the higher type decreases and as such his expected payoff decreases as well.

Afterwards, we examine the marginal effects of the players' types on their expected total effort. In the one-sided all-pay contest with two players under complete information, the higher type has a negative marginal effect on the players' expected total effort, since when the higher type increases, the balance of the contest (the difference between the players' types) decreases, and this yields a decrease in the total effort. Likewise, the lower type has a positive marginal effect on the players' expected total effort, since increasing the lower type increases the balance of the contest. In our two-sided all-pay matching contest, each type, either the higher or the lower one, might have a positive or a negative marginal effect on the players' expected total effort. The reason is that when we change each of the players' types, the balance of the contest changes, but, the values of winning of the players in the other set change as well. These two parallel changes have opposite effects on the expected total effort, and depending on the players' types, each player might be either stronger or weaker than the other, such that any change in a player's type might either increase or decrease the players' expected total effort. Thus in the two-sided all-pay matching contest, a player's type has a much more complex marginal effect on the results, such as the players' expected payoffs and their expected total effort, than in the standard one-sided all-pay contest.

Since in our model there is always a corner equilibrium according to which all the players do not exert efforts and then they are randomly matched, the question arises is therefore if an interior equilibrium could be welfare superior to the random matching given that the players have costly (wasted) efforts in the assortative matching contest. In that case, the net welfare is equal to players' expected total value minus the expected total effort. We show that depending on the players' types, the assortative matching contest might be either welfare superior or inferior to the random matching. In particular, the assortative matching contest is welfare inferior to the random matching when the variance between the players' types is sufficiently small. On the other hand, if the low-type players in both sets have sufficiently small types then the assortative matching contest is welfare superior to the random matching.

In our assortative matching contest there are always players who prefer the assortative matching over the random one. We show that while the low-type players prefer the random matching over the assortative matching, but the high-type players in both sets prefer the assortative matching if the variance of the players' types in each set is sufficiently large. The comparison between the assortative matching and the random matching gives an incentive for the study of assortative matching contests although there is always a corner equilibrium in which the players are randomly matched without exerting any effort.

The rest of the paper is organized as follows: in Section 2 we analyze the equilibrium for the general assortative matching contest. In Sections 3 and 4 we analyze the equilibrium with multiplicative and additive value functions. In Section 5 we compare between the assortative matching and the random matching. Section 6 concludes. Some of the proofs appear in the Appendix.

2 The assortative all-pay matching contest

We consider two sets, A and B, each of which includes two players. The players' types in set A are $a_i \in R_+$, $i = 1, 2$, where $a_1 \geq a_2$, and the players' types in set B are $b_j \in R_+$, $j = 1, 2$ where $b_1 \geq b_2$. All of these types are commonly known. The contest proceeds as follows: Each player, a_i, $i = 1, 2$, in set A exerts an effort x_i, and each player, b_j, $j = 1, 2$, in set B exerts an effort y_j.

Efforts are submitted simultaneously. Then, the players with the highest efforts from both sets are matched with each other, and the players with the lowest efforts from both sets are also matched with each other. If player a_i from set A is matched with player b_j from set B after exerting efforts of x_i and y_j, correspondingly, the utility of player a_i is $f(a_i, b_j) - x_i$, and the utility of player b_j is $g(a_i, b_j) - y_j$, where $f, g : R_+^2 \to R^1$ are the value functions. These value functions are assumed to be non-decreasing in the types of both players who are matched. We also assume that the value functions are super-modular that satisfy the following conditions:$(f_{a_1}(a_1, b_1) - f_{a_1}(a_1, b_2))$

$$1.\ [f(a_1, b_1) - f(a_1, b_2)] > [f(a_2, b_1) - f(a_2, b_2)] \text{ for all } a_1 > a_2 \text{ and } b_1 > b_2, \tag{1}$$

and

$$2.\ [g(a_1, b_1) - f(a_2, b_1)] > [g(a_1, b_2) - g(a_2, b_2)] \text{ for all } a_1 > a_2 \text{ and } b_1 > b_2. \tag{2}$$

These conditions are necessary for the existence of an interior equilibrium. We say that this assortative all-pay matching contest has an equilibrium if every player chooses an effort that maximizes his expected utility given the efforts of the other players in both sets.

3 The equilibrium analysis

The equilibrium analysis of the assortative all-pay matching contest is derived by that of the standard all-pay contest (see Hillman and Riley 1989, and Baye et al. 1996). We denote by $p_i^a, i = 1, 2$, player a_i's probability of winning in set A, and by $p_j^b, j = 1, 2$, player b_j's probability of winning in set B. Then, in set A, player a_i's expected values of winning w_i^a and losing $l_i^a, i = 1, 2$, are

$$\begin{aligned} w_i^a &= f(a_i, b_1)p_1^b + f(a_i, b_2)p_2^b \\ l_i^a &= f(a_i, b_1)p_2^b + f(a_i, b_2)p_1^b. \end{aligned}$$

In the following we show that Condition 2 implies that $p_1^b > p_2^b$ and then $w_i^a > l_i^a, i = 1, 2$. Comparing the differences between the values of winning and losing for both players in set A yields

$$\begin{aligned} & (w_1^a - l_1^a) - (w_2^a - l_2^a) \\ = \; & (p_1^b - p_2^b)[(f(a_1, b_1) - f(a_1, b_2)) - (f(a_2, b_1) - f(a_2, b_2))]. \end{aligned}$$

Thus, by Conditions 2 and 1, $w_1^a - l_1^a > w_2^a - l_2^a$. In that case, by Baye et al. (1996), player a_1 in set A chooses an effort from the interval $[0, w_2^a - l_2^a]$ according to the cumulative distribution function $F_1^a(x)$ which is given by

$$w_2^a F_1^a(x) + l_2^a(1 - F_1^a(x)) - x = l_2^a, \tag{3}$$

where l_2^a is the expected payoff of player a_2. Player a_2 in set A chooses an effort from the same interval $[0, w_2^a - l_2^a]$ according to the cumulative distribution function $F_2^a(x)$ which is given by

$$w_1^a F_2^a(x) + l_1^a(1 - F_2^a(x)) - x = w_1^a - w_2^a + l_2^a, \tag{4}$$

where $w_1^a - w_2^a + l_2^a$ is the expected payoff of player a_1. Player a_1's probability of winning in set A is then

$$p_1^a = 1 - \frac{w_2^a - l_2^a}{2(w_1^a - l_1^a)} = 1 - \frac{f(a_2, b_1) - f(a_2, b_2)}{2(f(a_1, b_1) - f(a_1, b_2))}. \tag{5}$$

Here we can see that Condition 1 implies that $p_1^a > 0.5$ which is a necessary condition for the existence of a mixed-strategy equilibrium in our model, since if $p_1^a = p_2^a = 0.5$, the players in set B do not have an incentive to exert positive efforts.

Similarly, in set B, player b_i's expected value of winning w_i^b and losing $l_i^b, i = 1, 2$, are

$$\begin{aligned} w_i^b &= g(a_1, b_i)p_1^a + g(a_2, b_i)p_2^a \\ l_i^b &= g(a_1, b_i)p_2^a + g(a_2, b_i)p_1^a. \end{aligned}$$

Comparing the differences between the values of winning and losing for both players in set B yields

$$\begin{aligned} & (w_1^b - l_1^b) - (w_2^b - l_2^b) \\ = \; & (p_1^a - p_2^a)[(g(a_1, b_1) - g(a_2, b_1)) - (g(a_1, b_2) - g(a_2, b_2))]. \end{aligned}$$

We showed that Condition 1 implies that $p_1^a > p_2^a$. Thus, by Conditions 1 and 2, $w_1^b - l_1^b > w_2^b - l_2^b$. In that case, according to Baye et al. (1996), player b_1 in set B chooses an effort from the interval $[0, w_2^b - l_2^b]$ according to

the cumulative distribution function $F_1^b(x)$ which is given by

$$w_2^b F_1^b(x) + l_2^b(1 - F_1^b(x)) - x = l_2^b, \tag{6}$$

where l_2^b is the expected payoff of player b_2. Player b_2 in set B chooses an effort from the interval $[0, w_2^b - l_2^b]$ according to the cumulative distribution function $F_2^b(x)$ which is given by

$$w_1^b F_2^b(x) + l_1^b(1 - F_2^b(x)) - x = w_1^b - w_2^b + l_2^b, \tag{7}$$

where $w_1^b - w_2^b + l_2^b$ is the expected payoff of player b_1. Player b_1's probability of winning in set B is then

$$p_1^b = 1 - \frac{w_2^b - l_2^b}{2(w_1^b - l_1^b)} = 1 - \frac{g(a_1, b_2) - g(a_2, b_2)}{2(g(a_1, b_1) - g(a_2, b_1))}. \tag{8}$$

Here, condition (2) implies that $p_1^b > 0.5$ which is a necessary condition for the existence of mixed strategy equilibrium in our model. Thus, under conditions (1) and (2), by the above analysis, in the assortative all-pay matching contest with two sets, $A = \{a_1, a_2\}$ and $B = \{b_1, b_2\}$, there is a mixed strategy equilibrium in which the players' equilibrium efforts in set A are distributed according to the cumulative distribution functions

$$F_1^a(x) = \frac{x}{w_2^a - l_2^a} = \frac{x}{(f(a_2, b_1) - f(a_2, b_2))(2p_1^b - 1)}. \tag{9}$$

$$F_2^a(x) = \frac{x + (w_1^a - w_2^a) - (l_1^a - l_2^a)}{w_1^a - l_1^a} = \frac{x + (f(a_1, b_1) - f(a_1, b_2) - (f(a_2, b_1) - f(a_2, b_2)))(2p_1^b - 1)}{(f(a_1, b_1) - f(a_1, b_2))(2p_1^b - 1)}, \tag{10}$$

and the players' equilibrium efforts in set B are distributed according to the cumulative distribution functions

$$F_1^b(x) = \frac{x}{w_2^b - l_2^b} = \frac{x}{(g(a_1, b_2) - g(a_2, b_2))(2p_1^a - 1)}. \tag{11}$$

$$F_2^b(x) = \frac{x + (w_1^b - w_2^b) - (l_1^b - l_2^b)}{w_1^b - l_1^b} = \frac{x + (g(a_1, b_1) - g(a_2, b_1) - (g(a_1, b_2) - g(a_2, b_2)))(2pa_1 - 1)}{(g(a_1, b_1) - g(a_2, b_1))(2p_1^a - 1)}, \tag{12}$$

where p_1^a and p_1^b are given by (5) and (8), respectively.[2]

The players' expected total effort in set A is

$$\begin{aligned} TE_A &= \frac{w_2^a - l_2^a}{2}(1 + \frac{w_2^a - l_2^a}{w_1^a - l_1^a}) \\ &= (f(a_2, b_1) - f(a_2, b_2))\frac{p_1^b - p_2^b}{2}(1 + \frac{f(a_2, b_1) - f(a_2, b_2)}{f(a_1, b_1) - f(a_1, b_2)}), \end{aligned} \tag{13}$$

and the players' expected total effort in set B is

$$\begin{aligned} TE_B &= \frac{w_2^b - l_2^b}{2}(1 + \frac{w_2^b - l_2^b}{w_1^b - l_1^b}) \\ &= (g(a_1, b_2) - g(a_2, b_2))\frac{p_1^a - p_2^a}{2}(1 + \frac{g(a_1, b_2) - g(a_2, b_2)}{g(a_1, b_1) - g(a_2, b_1)}). \end{aligned} \tag{14}$$

Example 1 *Assume that the players have the same multiplicative value function $f(a_i, b_j) = g(a_i, b_j) = a_i b_j$, $i = 1, 2$, $j = 1, 2$.[3] In that case, Conditions 1 and 2 are satisfied since for all $a_1 > a_2$ and $b_1 > b_2$,*

$$[f(a_1, b_1) - f(a_1, b_2)] = a_1(b_1 - b_2) > a_2(b_1 - b_2) = [f(a_2, b_1) - f(a_2, b_2)],$$

and

$$[g(a_1, b_1) - f(a_2, b_1)] = b_1(a_1 - a_2) > b_2(a_1 - a_2) = [g(a_1, b_2) - g(a_2, b_2)].$$

Then, by (9) and (10), in set A, player a_1's cumulative distribution function is

$$F_1^a(x) = \frac{b_1 x}{a_2(b_1 - b_2)^2},$$

[2] By Baye et al. (1996) this equilibrium is unique if $\frac{d}{da}(f(a, b_1) - f(a, b_2)) > 0$ for all $b_1 \geq b_2$, and $\frac{d}{db}(g(a_1, b) - g(a_2, b)) > 0$ for all $a_1 \geq a_2$.

[3] Our results in this section can be immediately extended to value functions of the form $f(a, b) = \delta(a)\rho(b)$, where δ and ρ are strictly increasing and differentiable functions.

and player a_2's cumulative distribution function is

$$F_2^a(x) = \frac{b_1 x + (a_1 - a_2)(b_1 - b_2)^2}{a_1(b_1 - b_2)^2}.$$

Similarly, by (11) and (12), in set B, player b_1's cumulative distribution function is

$$F_1^b(x) = \frac{a_1 x}{b_2(a_1 - a_2)^2},$$

and player a_2's cumulative distribution function is

$$F_2^b(x) = \frac{a_1 x + (b_1 - b_2)(a_1 - a_2)^2}{b_1(a_1 - a_2)^2}.$$

The expected payoffs of the players in set A are

$$\begin{aligned} u_1^a &= a_1 \frac{2b_1^2 - b_1 b_2 + b_2^2}{2b_1} - a_2 \frac{(b_1 - b_2)^2}{b_1} \\ u_2^a &= \frac{a_2 b_2}{2}(3 - \frac{b_2}{b_1}), \end{aligned} \tag{15}$$

and the expected payoffs of the players in set B are

$$\begin{aligned} u_1^b &= b_1(\frac{2a_1^2 - a_1 a_2 + a_2^2}{2a_1}) - b_2 \frac{(a_1 - a_2)^2}{a_1} \\ u_2^b &= \frac{b_2 a_2}{2}(3 - \frac{a_2}{a_1}), \end{aligned} \tag{16}$$

where by (5) and (8), the probabilities of players a_1 and b_1 to win are

$$\begin{aligned} p_1^a &= 1 - \frac{a_2}{2a_1} \\ p_1^b &= 1 - \frac{b_2}{2b_1} \end{aligned} \tag{17}$$

We now want to empasize the role of Conditions 1 and 2 concerning the existence of equilibrium in our model. Suppose that these conditions are not satisfied and instead there exist

$$\begin{aligned} [f(a_1, b_1) - f(a_1, b_2)] &= [f(a_2, b_1) - f(a_2, b_2)] \\ [g(a_1, b_1) - f(a_2, b_1)] &= [g(a_1, b_2) - g(a_2, b_2)]. \end{aligned} \tag{18}$$

For example, if the players have the same additive value function $f(a_i, b_j) = g(a_i, b_j) = a_i + b_j$, $i = 1, 2$, $j = 1, 2$, then

$$
\begin{aligned}
[f(a_1, b_1) - f(a_1, b_2)] &= (b_1 - b_2) = [f(a_2, b_1) - f(a_2, b_2)], \\
[g(a_1, b_1) - f(a_2, b_1)] &= (a_1 - a_2) = [g(a_1, b_2) - g(a_2, b_2)].
\end{aligned}
$$

When (18) is satisfied, then by (5) and (8) there exists

$$
\begin{aligned}
p_1^a &= 1 - \frac{f(a_2, b_1) - f(a_2, b_2)}{2(f(a_1, b_1) - f(a_1, b_2))} = 0.5 \\
p_1^a &= 1 - \frac{g(a_1, b_2) - g(a_2, b_2)}{2(g(a_1, b_1) - g(a_2, b_1))} = 0.5
\end{aligned}
$$

In that case, the players in both sets do not have an incentive to exert positive efforts and we have

Proposition 1 *In the assortative all-pay matching contest with two sets, $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$, and value functions f and g that satisfy (18), there is only a corner equilibrium in which in both sets every player exerts an effort of zero.*

4 Players' probabilities of winning and their expected payoffs

Consider that Conditions 1 and 2 are satisfied. We want to examine the players' probabilities of winning and their expected payoffs in our model. Denote by $f_{a_1}(a_i, b_j)$, $f_{a_2}(a_i, b_j)$, and $g_{b_1}(a_i, b_j), g_{b_2}(a_i, b_j)$, $i = 1, 2$, $j = 1, 2$, the partial derivatives of the value functions f and g respectively. Then we have

Proposition 2 *In the assortative all-pay matching contest with two sets $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$ and value functions f and g that satisfy conditions (1) and (2), the winning probability of the player with the higher type in his set (a_1,b_1) increases in his own type, and decreases in the other player's type in his set. The winning probabilities of players depends on the players' types in the other sets as follows:*

1) $\frac{dp_1^a}{db_1} \geq 0,\ or\ \frac{dp_2^a}{db_1} \leq 0$ *iff*

$$f_{b_1}(a_1,b_1)(f(a_2,b_1)-f(a_2,b_2))-f_{b_1}(a_2,b_1)(f(a_1,b_1)-f(a_1,b_2)) \geq 0. \quad (19)$$

2) $\frac{dp_1^a}{db_2} \geq 0,\ or\ \frac{dp_2^a}{db_2} \leq 0$ *iff*

$$f_{b_2}(a_2,b_2)(f(a_1,b_1)-f(a_1,b_2))-f_{b_2}(a_1,b_2)(f(a_2,b_1)-f(a_2,b_2)) \geq 0. \quad (20)$$

3) $\frac{dp_1^b}{da_1} \geq 0,\ or\ \frac{dp_2^b}{da_1} \leq 0$ *iff*

$$g_{a_1}(a_1,b_1)(g(a_1,b_2)-g(a_2,b_2))-g_{a_1}(a_1,b_2)(g(a_1,b_1)-g(a_2,b_1)) \geq 0. \quad (21)$$

4) $\frac{dp_1^b}{da_2} \geq 0,\ or\ \frac{dp_2^b}{da_2} \leq 0$ *iff*

$$g_{a_2}(a_2,b_2)(g(a_1,b_1)-g(a_2,b_1))-g_{a_2}(a_2,b_1)(g(a_1,b_2)-g(a_2,b_2)) \geq 0. \quad (22)$$

Proof. *See Appendix.* ■

If the conditions (19),(20), (21), and (22) equal to zero, then the players' winning probabilities do not depend on the players' types in the other sets. For example, if the value functions are multiplicative $f(a_i,b_j) = g(a_i,b_j) = a_i b_j$, $i = 1,2$, $j = 1,2$, then these conditions equal to zero. Note that if $a_i, i = 1,2$ are close enough and $b_j, j = 1,2$ are close enough, then conditions (19),(20), (21), and (22) are equal to zero for any value functions f and g.

In our all-pay matching contest, since a player's value of winning is based on the players' types in the other set, the marginal effects of the players' types on each of the player's expected payoff are not straightforward. The following result show that the effect of the the players ' types in their set on their expected payoffs is not different than in the standard all-pay contest.

Proposition 3 *In the assortative all-pay matching contest with two sets $A = \{a_1,a_2\}$, $B = \{b_1,b_2\}$ and value functions f and g that satisfy conditions (1) and (2), if the players' winning probabilities do not depend on the players' types in the other sets, the expected payoff of the player in every set increases in his own type and decreases in the type of the other player in his set.*

Proof. *See Appendix.* ■

Proposition 3 does not say anything about the effects of types from one set on the agents' expected payoffs in the other sets. These cross effect are very complex. Consider the player with the higher type in his set whose expected payoff might either increase or decrease in the lower type of the player in the other set. It might decrease, for example, when player a_1 has a significantly higher type than the other player in his set a_2, and the lower type in the other set b_2 is relatively low. Then, if b_2 increases, the chance of the player with the lower type in the other set b_2 to win increases and since the winning probability of the player with the higher type a_1 is high, his payoff loss from matching with the player with the lower type b_2 instead of b_1 is relatively high, and therefore the expected payoff of the player with type a_1 decreases in the the value of b_2. Hence, we can say that increasing all the players' types in one set of the assortative matching all-pay contest does not necessarily enhance the expected payoff of the player with higher type in the other set. Furthermore, it turns out that increasing all the players'types in one set does not necessarily enhance even the expected payoff of the player with the lower type.

5 Players' Total effort

Now, we examine the marginal effect of the players' types on their expected total effort. By (13) and (14), the players' expected total effort is

$$\begin{aligned} TE &= TE_A + TE_B \qquad (23) \\ &= (f(a_2,b_1) - f(a_2,b_2))\frac{p_1^b - p_2^b}{2}(1 + \frac{f(a_2,b_1) - f(a_2,b_2)}{f(a_1,b_1) - f(a_1,b_2)}) \\ &\quad + (g(a_1,b_2) - g(a_2,b_2))\frac{p_1^a - p_2^a}{2}(1 + \frac{g(a_1,b_2) - g(a_2,b_2)}{g(a_1,b_1) - g(a_2,b_1)}) \end{aligned}$$

In the standard all-pay contest with two players, the expected total effort always decreases in the higher type (value of winning) but increases in the lower one. In our assortative matching all-pay contest, we need sufficient conditions in order to ensure similar results.

Proposition 4 *In the assortative all-pay matching contest with two sets* $A = \{a_1, a_2\}$*,* $B = \{b_1, b_2\}$*, and winning value functions that satisfy conditions (1) and (2), then*

1) If

$$g_{a_1}(a_1, b_1)(g(a_1, b_2) - g(a_2, b_2)) - g_{a_1}(a_1, b_2)(g(a_1, b_1) - g(a_2, b_1)) \leq 0$$

the total effort in set A *decreases in the higher type of the players in that set*

2) If

$$g_{a_2}(a_2, b_2)(g(a_1, b_1) - g(a_2, b_1)) - g_{a_2}(a_2, b_1)(g(a_1, b_2) - g(a_2, b_2)) \geq 0$$

the total effort in set A *increases in the lower type of the player in that set.*

Similarly, the effects of the types in set B *on the total effort in set* B*.*

Proof. See Appendix. ■

While by proposition 4 the marginal effect of each player's type on the expected total effort in his set is clear, the marginal effect of each player's type on the expected total effort in the other set is not clear. Furthermore, the marginal effects of the players' types on the expected total effort in both sets together are more complicated as we can see in the following result.

Proposition 5 *In the assortative all-pay matching contest with two sets* $A = \{a_1, a_2\}$*,* $B = \{b_1, b_2\}$*, and winning value functions that satisfy conditions (1) and (2), each of the players' types might have either a positive or negative marginal effect on the players' expected total effort.*

Proof. See Appendix. ■

The reason being the above result is that when the value of the higher type increases, the contest is less balanced, which has a negative marginal effect on the players' expected total effort. On the other hand, when the value of the higher type increases, all the players' values of winning in the other set increase, which has a positive marginal effect on the players' expected total effort. Thus, when the difference of the players' types in a set is relatively small such that the contest is balanced, increasing the value of the higher type upsets this balance which has a negative marginal effect on the expected total effort. However, when the difference of the players' types in a set is relatively large such that the contest is already unbalanced, increasing the higher type has a positive marginal effect on the players' expected total effort.

Likewise, by Proposition 5, the lower type in a set might also have a positive or a negative marginal effect on the players' expected total effort. The reason is that when the difference of the players' types in a set is relatively small, increasing the value of the lower type increases the balance of the contest which has a positive marginal effect on the players' expected total effort. On the other hand, when the difference of the players' types in a set is relatively high and the contest is already unbalanced, by increasing the lower type, the difference of these types is reduced which has a negative marginal effect on the expected efforts in the other set. Thus, increasing the lower type might have also a negative marginal effect on the players' expected total effort.

6 Assortative matching vs. random matching

Now we compare the equilibrium outcomes of assortative matching based on costly efforts to those of random matching without efforts at all. While the total matching output (the players' expected total value) generated through an assortative matching is larger than the one obtained through a random matching, an assortative matching involves the cost of efforts. The net total welfare in the assortative matching contest is

$$\begin{aligned} W_{all} &= (f(a_1,b_1)+f(a_2,b_2))(p_1^a p_1^b + p_2^a p_2^b) \\ &\quad +(f(a_1 b_2)+f(a_2,b_1)(p_1^a p_2^b + p_2^a p_1^b) \\ &\quad -TE_A - TE_B \end{aligned} \tag{24}$$

where p_1^a and p_1^b are given by (5) and (8) respectively, and TE_A, TE_B are given by (13) and (14) respectively.

On the other hand, the total welfare in the random matching contest with multiplicative value functions is

$$W_r = \frac{f(a_1, b_1) + f(a_1, b_2) + f(a_2, b_1) + f(a_2, b_2)}{2} \tag{25}$$

Then, we have

Proposition 6 *The assortative all-pay matching contestwith two sets $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$, and winning value functions, f and g that satisfy conditions (1) and (2), depending on the players' types, might be either welfare superior or inferior to the random matching.*

Proof. See Appendix. ■

The intuition behind Propoposition 6 is as follows. Suppose that the lower types in each set are sufficiently low, then the goal of the designer who maximizes welfare is to match the players with the higher types in both stages and he can do it better by the assortative all-pay matching contest which is in that case welfare superior to the random matching. On the other hand, when two types in one set are quite close to each other, the final matching is not so important and then it is better for the designer who maimizes the welfare not to west efforts such that in that a case the assortative matching contest is welfare inferior to the random matching. The Proof of Proposition 6 actually illustartes these situations for multiplicative value functions.

7 Concluding remarks

In the one-sided all-pay contest in which players compete against each other to win one of the fixed prizes on the other side, the players prefer that their types (values of winning) be large and their opponents' types be small. The designer of such a contest who wishes to maximize the players' total effort, prefers large values of the players' types and also that the difference between them will be small. In this paper, we demonstrated that in a two-sided assortative matching all-pay contest with a multiplicative value function, the players' preferences about their opponents types as well as the designer's preference about these types different from those in the one-sided all-pay contest: While each player's type has a positive effect on the expected payoff of the player with the lower type in the other set, the player with the lower type in his set might have either a positive or a negative effect on the expected payoff of the player with the higher type in the other set. Furthermore, each player's type might have either a positive or a negative marginal effect on the players' expected total effort. As such, in the two-sided all-pay matching contest, it could be difficult to anticipate the effects of any change of the players' types on the results. Last, although there is always the random matching in our model in which all the players do not exert costly efforts, we show that our assortative all-pay matching with costly efforts might be welfare superior to the random matching.

8 Appendix

8.1 Proof of Proposition 2

1) By (5), player a_1's probability of winning in set A is

$$p_1^a = 1 - \frac{f(a_2, b_1) - f(a_2, b_2)}{2(f(a_1, b_1) - f(a_1, b_2))}.$$

Then,

$$\frac{dp_1^a}{da_1} = \frac{2(f_{a_1}(a_1, b_1) - f_{a_1}(a_1, b_2))(f(a_2, b_1) - f(a_2, b_2))}{(2(f(a_1, b_1) - f(a_1, b_2)))^2}.$$

By Condition (1) $(f_{a_1}(a_1, b_1) - f_{a_1}(a_1, b_2)) > 0$, thus, we obtain that $\frac{dp_1^a}{da_1} > 0$ or, alternatively, $\frac{dp_2^a}{da_1} < 0$.

2) Similarly,

$$\frac{dp_1^a}{da_2} = \frac{-(f_{a_2}(a_2, b_1) - f_{a_2}(a_2, b_2))(f(a_1, b_1) - f(a_1, b_2))}{(2(f(a_1, b_1) - f(a_1, b_2)))^2}.$$

By Condition (1) $(f_{a_2}(a_2, b_1) - f_{a_2}(a_2, b_2)) > 0$, thus, we obtain that $\frac{dp_1^a}{da_2} < 0$ or, alternatively, $\frac{dp_2^a}{da_2} > 0$.

3) By (5) we have

$$\frac{dp_1^a}{db_1} = \frac{-2f_{b_1}(a_2, b_1)(f(a_1, b_1) - f(a_1, b_2)) + 2f_{b_1}(a_1 b_1)(f(a_2, b_1) - f(a_2, b_2))}{(2(f(a_1, b_1) - f(a_1, b_2)))^2}.$$

Thus, $\frac{dp_1^a}{db_1} \geq 0$, or $\frac{dp_2^a}{db_1} \leq 0$ iff $f_{b_1}(a_1 b_1)(f(a_2, b_1) - f(a_2, b_2)) - f_{b_1}(a_2, b_1)(f(a_1, b_1) - f(a_1, b_2)) \geq 0$.

4) Similarly,

$$\frac{dp_1^a}{db_2} = -\frac{-2f_{b_2}(a_2, b_2)(f(a_1, b_1) - f(a_1, b_2)) + 2f_{b_2}(a_1 b_2)(f(a_2, b_1) - f(a_2, b_2))}{(2(f(a_1, b_1) - f(a_1, b_2)))^2}$$

Thus, $\frac{dp_1^a}{db_2} \geq 0$, or $\frac{dp_2^a}{db_2} \leq 0$ iff $f_{b_2}(a_2, b_2)(f(a_1, b_1) - f(a_1, b_2)) - f_{b_2}(a_1 b_2)(f(a_2, b_1) - f(a_2, b_2)) \geq 0$.

Likewise, we obtain the results for the players' prbabilities in set B. $Q.E.D.$

8.2 Proof of Proposition 3

By (4) player a_1's expected payoff is

$$u_1^a = w_1^a - w_2^a + l_2^a = p_1^b(f(a_1, b_1) - f(a_2, b_1) + f(a_2, b_2)) + p_2^b(f(a_1, b_2) - f(a_2, b_2) + f(a_2, b_1))$$

Thus,

$$\begin{aligned} \frac{du_1^a}{da_1} &= f_{a_1}(a_1, b_1)p_1^b + \frac{dp_1^b}{da_1}(f(a_1, b_1) - f(a_2, b_1) + f(a_2, b_2)) \\ &\quad + f_{a_1}(a_2, b_2)p_2^b + \frac{dp_2^b}{da_1}(f(a_1, b_2) - f(a_2, b_2) + f(a_2, b_1)) \end{aligned}$$

Since $\frac{dp_1^b}{da_1} = -\frac{dp_2^b}{da_1}$ we obtain that

$$\begin{aligned} \frac{du_1^a}{da_1} &= f_{a_1}(a_1, b_1)p_1^b + f_{a_1}(a_2, b_2)p_2^b \\ &\quad + \frac{dp_1^b}{da_1}(f(a_1, b_1) - f(a_1, b_2) - 2(f(a_2, b_1) - f(a_2, b_2))) \end{aligned}$$

If $\frac{dp_1^b}{da_1} = 0$, we obtain that $\frac{du_1^a}{da_1} > 0$.

2) By (4) we have

$$
\begin{aligned}
\frac{du_1^a}{da_2} &= f_{a_2}(a_2,b_1) - f(a_2,b_2))(p_2^b - p_1^b) \\
&\quad \frac{dp_1^b}{da_2}(f(a_1,b_1) - f(a_2,b_1) + f(a_2,b_2)) \\
&\quad \frac{dp_2^b}{da_2}(f(a_1,b_2) - f(a_2,b_2) + f(a_2,b_1)) \\
&= f_{a_2}(a_2,b_1) - f_{a_2}(a_2,b_2))(p_2^b - p_1^b) \\
&\quad + \frac{dp_1^b}{da_2}(f(a_1,b_1) - f(a_1,b_2) - 2f(a_2,b_1) + 2f(a_2,b_2))
\end{aligned}
$$

Since $(p_2^b - p_1^b) < 0$, and by Condition (1) we obtain that $f_{a_2}(a_2,b_1) - f_{a_2}(a_2,b_2))(p_2^b - p_1^b) < 0$. If, $\frac{dp_1^b}{da_2} = 0$ we have $\frac{du_1^a}{da_2} < 0$.

3) Likewise,

$$
\begin{aligned}
\frac{du_1^a}{db_1} &= -\frac{d}{db_1}(f(a_1,b_1) - 2f(a_2,b_1))p_2^b \\
&\quad -(f(a_1,b_1) - f(a_2,b_1) - (f(a_1,b_2) - f(a_2,b_2)) + f(a_2,b_2)\frac{dp_2^b}{db_1} \\
&\quad + \frac{d}{db_1}(f(a_1,b_1) - f(a_2,b_1))
\end{aligned}
$$

By (7), player a_2's expected payoffis

$$
u_2^a = l_2^a = f(a_2,b_1)p_2^b + f(a_2,b_2)p_1^b
$$

Thus,

$$
\begin{aligned}
\frac{du_2^a}{da_1} &= f(a_2,b_1)\frac{dp_2^b}{da_1} + f(a_2,b_2)\frac{dp_1^b}{da_1} \\
&= (f(a_2,b_1) - f(a_2,b_2))\frac{dp_2^b}{da_1}
\end{aligned}
$$

If $\frac{dp_2^b}{da_1} = 0$ we obtain that $\frac{du_2^a}{da_1} = 0$.

4) Last,

$$\frac{du_2^a}{da_2} = (f_{a_2}(a_2, b_1) - f_{a_2}(a_2, b_2))p_2^b + (f(a_2, b_1) - f(a_2, b_2))\frac{dp_2^b}{da_2}$$

Since by Condition 1 $(f_{a_2}(a_2, b_1) - f_{a_2}(a_2, b_2)) > 0$, if $\frac{dp_2^b}{da_2} = 0$ then $\frac{du_2^a}{da_2} > 0$.

Likewise, we obtain the results on the players' expected payoffs in set B. $Q.E.D.$

8.3 proof of Proposition 4

1) By (13), the players' expected total effort in set A is

$$TE_A = (f(a_2, b_1) - f(a_2, b_2))\frac{p_1^b - p_2^b}{2}(1 + \frac{f(a_2, b_1) - f(a_2, b_2)}{f(a_1, b_1) - f(a_1, b_2)})$$

Then

$$\begin{aligned}\frac{dTE_A}{da_1} &= \frac{d}{da_1}(\frac{2p_1^b - 1}{2})(f(a_2, b_1) - f(a_2, b_2))(1 + \frac{f(a_2, b_1) - f(a_2, b_2)}{f(a_1, b_1) - f(a_1, b_2)}) \\ &\quad + (f(a_2, b_1) - f(a_2, b_2))(\frac{p_1^b - p_2^b}{2})\frac{-(f_{a_1}(a_1, b_1) - f_{a_1}(a_1, b_2))(f(a_2, b_1) - f(a_2, b_2))}{(f(a_1, b_1) - f(a_1, b_2))^2}\end{aligned}$$

By (21) we have $\frac{dp_1^b}{da_1} \leq 0$ iff

$$g_{a_1}(a_1, b_1)(g(a_1, b_2) - g(a_2, b_2)) - g_{a_1}(a_1, b_2)(g(a_1, b_1) - g(a_2, b_1)) \leq 0$$

and by Condition 1 we have

$$f_{a_1}(a_1, b_1) - f_{a_1}(a_1, b_2) > 0$$

Thus, if condition (21) satisfies, then we obtain that $\frac{dTE_A}{da_1} < 0$.

2) By (13),

$$\begin{aligned}\frac{dTE_A}{da_2} &= \frac{d}{da_2}(\frac{1 - 2p_2^b}{2})(f(a_2, b_1) - f(a_2, b_2))(1 + \frac{f(a_2, b_1) - f(a_2, b_2)}{f(a_1, b_1) - f(a_1, b_2)}) \\ &\quad + (f(a_2, b_1) - f(a_2, b_2))(\frac{p_1^b - p_2^b}{2})\frac{(f_{a_2}(a_2, b_1) - f_{a_2}(a_2, b_2))f(a_1, b_1) - f(a_1, b_2)}{f(a_1, b_1) - f(a_1, b_2)}\end{aligned}$$

By (22), $\frac{dp_2^b}{da_2} \leq 0$ iff

$$g_{a_2}(a_2, b_2)(g(a_1, b_1) - g(a_2, b_1)) - g_{a_2}(a_2, b_1)(g(a_1, b_2) - g(a_2, b_2)) \geq 0$$

and by Condition 1 we have

$$f_{a_2}(a_1, b_1) - f_{a_2}(a_1, b_2) > 0$$

Thus, if condition (22) satisfies, we obtain that $\frac{dTE_A}{da_2} > 0$.

Q.E.D.

8.4 Proof of Proposition 5

We prove this proposition by assuming the multilicative value function $f(a, b) = g(a, b) = ab$. By (13) and (14) the expected total effort is

$$TE = \frac{a_2(b_1 - b_2)^2}{2b_1}(\frac{a_1 + a_2}{a_1}) + \frac{b_2(a_1 - a_2)^2}{2a_1}(\frac{b_1 + b_2}{b_1}).$$

By (23), the marginal effect of player a_1's type on the players' expected total effort is

$$\begin{aligned} \frac{dTE}{da_1} &= \frac{d}{da_1}(\frac{a_2(b_1 - b_2)^2}{2b_1}(\frac{a_1 + a_2}{a_1}) + \frac{b_2(a_1 - a_2)^2}{2a_1}(\frac{b_1 + b_2}{b_1})) \\ &= \frac{1}{2a_1^2 b_1}\left(a_1^2 b_1 b_2 + a_1^2 b_2^2 - a_2^2 b_1^2 + a_2^2 b_1 b_2 - 2a_2^2 b_2^2\right). \end{aligned}$$

We can see that $\frac{dTE}{da_1} \geq 0$ iff $z = \left(a_1^2 b_1 b_2 + a_1^2 b_2^2 - a_2^2 b_1^2 + a_2^2 b_1 b_2 - 2a_2^2 b_2^2\right) \geq 0$. When a_2 approaches zero we obtain that,

$$\lim_{a_2 \to 0} z = a_1^2 b_1 b_2 + a_1^2 b_2^2.$$

Thus, the marginal effect of player a_1's type on the players' expected total effort is positive. On the other hand, when a_2 approaches a_1 we have

$$\lim_{a_2 \to a_1} z = -a_1^2(b_1 - b_2)^2.$$

Thus, the marginal effect of player a_1's type on the players' expected total effort is negative.

Similarly, the marginal effect of player a_2's type on the players' expected total effort is

$$\begin{aligned}\frac{dTE}{da_2} &= \frac{d}{da_2}(\frac{a_2(b_1-b_2)^2}{2b_1}(\frac{a_1+a_2}{a_1})+\frac{b_2(a_1-a_2)^2}{2a_1}(\frac{b_1+b_2}{b_1})) \\ &= \frac{1}{2a_1b_1}\left(a_1b_1^2-a_1b_2^2+2a_2b_1^2+4a_2b_2^2-4a_1b_1b_2-2a_2b_1b_2\right).\end{aligned}$$

We can see that $\frac{dTE}{da_1}\geq 0$ iff $z=\left(a_1b_1^2-a_1b_2^2+2a_2b_1^2+4a_2b_2^2-4a_1b_1b_2-2a_2b_1b_2\right)\geq 0$. When a_2 approaches zero we obtain that

$$\lim_{a_2\to 0} z = \frac{1}{b_1}((b_1-b_2)^2-2b_1b_2,$$

where the last term might be either positive or negative. That is, the marginal effect of player a_2's type on the players' expected total effort is either positive or negative. On the other hand, when a_2 approaches a_1 we have

$$\lim_{a_2\to a_1} z = 3a_1(b_1-b_2)^2.$$

Thus, the marginal effect of player a_2's type on the players' expected total effort is positive. Hence, we can conclude that each of the players' types might have either a positive or a negative marginal effect on the players' expected total effort. *Q.E.D.*

8.5 Proof of Proposition 6

In order to prove this proposition that the assortative all-pay matching contest, depending on the players' types, might be either welfare superior or inferior to the random matching, it is sufficient to illustrate this by assuming a specific value functions such as multiplicative functions, namely, $f(a,b)=g(a,b)=ab$. Then, by (24) we have

$$\begin{aligned}W_{all} &= (a_1b_1+a_2b_2)((1-\frac{a_2}{2a_1})(1-\frac{b_2}{2b_1})+(\frac{a_2}{2a_1})(\frac{b_2}{2b_1})) \\ &\quad +(a_1b_2+a_2b_1)((1-\frac{a_2}{2a_1})(\frac{b_2}{2b_1})+(\frac{a_2}{2a_1})(1-\frac{b_2}{2b_1})) \\ &\quad -\frac{a_2(b_1-b_2)^2}{2b_1}(\frac{a_1+a_2}{a_1})-\frac{b_2(a_1-a_2)^2}{2a_1}(\frac{b_1+b_2}{b_1}).\end{aligned}$$

and by (25) we have

$$W_r = \frac{a_1b_1 + a_1b_2 + a_2b_1 + a_2b_2}{2}.$$

Comparing the total net welfare in both cases yields

$$W_{all} - W_r = -\frac{1}{2a_1b_1}\begin{pmatrix} -a_1^2b_1^2 + 3a_1^2b_1b_2 + 3a_1a_2b_1^2 \\ -8a_1a_2b_1b_2 + a_1a_2b_2^2 + a_2^2b_1b_2 + a_2^2b_2^2 \end{pmatrix}$$

It can be easily verified that

$$\lim_{a_2,b_2\to 0} W_{all} - W_r = \frac{a_1b_1}{2} > 0$$

and, on the other hand,

$$\lim_{a_2\to a_1} W_{all} - W_r = -\frac{a_1}{b_1}(b_1 - b_2)^2 < 0$$

Q.E.D.

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