COMMON-VALUE GROUP CONTESTS WITH ASYMMETRIC INFORMATION

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Abstract

We study contests between two groups where all the players have a common value of winning. In each group one of the players has an information advantage over the other. This player is referred to as the dominant player. We show that a group contest is equivalent to a contest between the dominant players, and, as such, the expected total effort of both groups is always the same, while their probabilities of winning as well their expected total payoff are not.

JEL CLASSIFICATION: C72, D44, D82

KEYWORDS: Group contests, asymmetric information..

1 Introduction

In many cases, contests are held between groups where the players in each group have a common interest. Competitions between groups have been extensively modeled by Tullock contests under complete information (see, among others, Tullock 1980, Katz et al. 1990, Mitzan 1991, Riaz et al. 1995, Nti 1998, Esteban and Ray 2001, and Ryvkin 2011).

We consider a Common-value contest between two groups under asymmetric information in which the value of winning is the same for all the players in the same state of nature, but the information about which state of nature is realized can be different. The information of a player about the value of winning is

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described by a partition of the space of states of nature which is assumed to be finite (see, among others, Einy et al. 2015, 2017).[1] The advantage of our framework is that we are able to determine which player's realized information is more favorable for each state. Accordingly, we assume that in each group one of the players has an information advantage over the other, namely, the partition of this player is finer than the partition of the other. In each group, the player who has an information advantage over all the other players is referred to as the dominant player. We also assume that one of the dominant players has an information advantage over the dominant player from the other group.

The work most related to our setting is Baik (2008) who studied a Tullock contest between groups with linear cost functions and demonstrated that only the player with the highest valuation in each group exerts a positive effort in equilibrium. As such, a group contest is actually reduced to a contest between the strongest players in these groups (the players with the highest values of winning) while all the other players completely free-ride. We similarly demonstrate that with asymmetric information a group contest is actually reduced to a contest between their dominant players. We apply the results of Aiche et al. (2019) about Tullock contests between players with asymmetric information, and conclude that in a group contest in which the dominant player in group A has an information advantage over the dominant payer in group B, the expected total effort in both groups is the same and the probability of group A to win is smaller than that of group B, but the expected total payoff of the the players in group A is larger than in group B.

2 The model

Consider two groups of players, group $A = \{1, 2, ..., m\}$ of $m \geq 2$ players and group $B = \{1, 2, ..., n\}$ of $n \geq 2$ players. The groups compete in a Tullock contest in which the players in both groups simultaneously exert efforts, and group A's probability of winning is

$$p_A = \frac{\sum_{i=1}^{m} x_i}{\sum_{i=1}^{m} x_i + \sum_{i=1}^{n} y_i}$$

where $(x_1, ..., x_m)$ are the efforts of the players in group A, and $(y_1, ..., y_n)$ are the efforts of the players in group B. Group B's probability of winning is $P_B = 1 - P_A$. All the players bear the costs of their efforts.

[1] Jackson (1993) and Vohra (1999) showed that this partition representation is equivalent to the more common Harsanyi-type formulation of Bayesian games.

The value of winning is common for all the players in each group, and this common value is a function $v : \Omega \to \mathbb{R}_+$ where Ω is a finite set of states of nature, and p is the probability distribution over Ω (w.l.o.g. $p(\omega) > 0$ for every $\omega \in \Omega$). If the state of nature $\omega_j \in \Omega$, $j = 1, 2, ..., k$ is realized, then the value of winning is $v_j = v(\omega_j)$ for all the players. The private information of player $i, i = 1, 2, ..., n$ is described by a partition Π_i of Ω.

We denote this group contest by G. It begins with the move of nature which selects one from $k \geq 2$ states of nature ω from Ω according to the probability distribution p. The players do not observe the state of nature ω that actually occurs, but each player i is informed of the element $\pi_i(\omega)$ from his partition Π_i which contains ω (the players will typically have different information partitions). Then player $i \in A$ chooses an effort $x_i \in \mathbb{R}_+$, and player $i \in B$ chooses an effort $y_i \in \mathbb{R}_+$. The utility (payoff) of player $i \in A$ is given by the function $u_i : \Omega \times \mathbb{R}^m_+ \times \mathbb{R}^n_+ \to \mathbb{R}$ as follows:

$$u_i(\omega, x, y) = v(\omega) \frac{\sum_{i=1}^m x_i}{\sum_{i=1}^m x_i + \sum_{i=1}^n y_i} - x_i \quad .$$

where $x = (x_1, ..., x_m)$ and $y = (y_1, ..., y_n)$. Similarly the utility of player $i \in B$ is given. Thus, we can say that a common-value group contest with asymmetric information is described by the collection $G = (A, B, (\Omega, p), \{u_i\}_{i \in A}, \{\Pi_i\}_{i \in A}, \{u_i\}_{i \in B}, \{\Pi_i\}_{i \in B})$.

A pure strategy of player i is a function $x_i : \Omega \to \mathbb{R}_+$ which is measurable w.r.t. Π_i such that x_i (y_i) is constant on every element of Π_i. A Bayesian equilibrium (in pure strategies) of the contest G is a m-tuple $x^* = (x^*_1, ..., x^*_m)$ and a n-tuple $y^* = (y^*_1, ..., y^*_n)$ of pure strategies (efforts) such that for every player $i \in A$ and every pure strategy x_i of i there exists

$$E_i(x^*, y^*) = E\left[u_i\left(\cdot, x^*\left(\cdot\right), y^*\left(\cdot\right)\right)\right] \geq E\left[u_i\left(\cdot, x^*_{-i}\left(\cdot\right), x_i\left(\cdot\right), y^*\left(\cdot\right)\right)\right]$$

We say that player $i \in K, K = A, B$ has an information advantage over player $j \in N$ if partition Π_i is finer than partition Π_j. Thus, if i has an information advantage over j, then $\pi_i(\omega) \subset \pi_j(\omega)$ for every $\omega \in \Omega$, i.e., player i knows the realized state of nature with at least the same precision as player j. We assume that in each group one of the players has an information advantage over the other or they have the same information. A player with the information advantage over all the other players in his group is referred to as a dominant player. Without loss of generality we assume that there is one dominant player in each group.[2]

[2]If there are more than one dominant player in a group, randomly, one of them will be referred as the dominant player.

We also assume that the dominant player in group A has an information advantage over the dominant player in group B.

Proposition 1 *The expected total effort in either group A or B is obtained by the solution of the first order conditions of the dominant payer's maximization problem in that group.*

Proof. Let player 1 be the dominant player in group A. Thus, there exists $\pi_1(\omega) \subseteq \pi_i(\omega)$ for all $\omega \in \Omega$ and for all $i \in A$. Without loss of generality, we assume that the partition of player 1 is $\Pi_1 = \{\{\pi_{11}\}, ..., \{\pi_{1k}\}\}$, which is the finest partition. We also assume that the total effort of group B in the state of nature ω_j, $j = 1, 2, ..., k$ is β_j. Then, for all $j = 1, 2, ..., k$ player 1's maximization problem is

$$\max_{x_{1j}} v_j \frac{\sum_{i=1}^{m} x_{ij}}{\sum_{i=1}^{m} x_{ij} + \beta_j} - x_{1j}$$

where x_{ij} is the effort of player i from group A in the state of nature ω_j. The F.O.C. of player 1's maximization problem are

$$v_j \frac{\beta_j}{(\sum_{i=1}^{m} x_{ij} + \beta_j)^2} \leq 1 \quad j = 1, ..., k \tag{1}$$

For every player s, $s = 2, ..., m$ in group A, there is a partition of the states of nature $\Pi_s = \{\{\{\pi_{s1}\}, ..., \{\pi_{s,l_s}\}\}$ that includes $l_s < k$ elements. If $\omega \in \pi_{st}, t \leq l_s$, player s solves the following maximization problem

$$\max_{x_{st}} \sum_{j \in \pi_{st}} \frac{p_j}{\sum_{i \in \pi_{st}} p_i} v_j \frac{\sum_{i=1}^{m} x_{ij}}{\sum_{i=1}^{m} x_{ij} + \beta_j} - x_{st}$$

The F.O.C. of player s' maximization problem is

$$\sum_{j \in \pi_{st}} \frac{p_j}{\sum_{i \in \pi_{st}} p_i} v_j \frac{\beta_j}{(\sum_{i=1}^{m} x_{ij} + \beta_j)^2} \leq 1 \tag{2}$$

Note that (1) implies (2). Thus, in every state of nature, given the F.O.C. of player 1's maximization problem, the F.O.C. of player s' maximization problem does not add any additional information about the players' equilibrium strategies. Thus, the solution of player 1's maximization problem yields the solution of group A's maximization problem. ∎

By Proposition 1 we have

Proposition 2 *In a group contest, the expected total effort of both groups is the same as the expected total effort of their dominant players when they compete against each other.*

Proof. Assume that player 1 is the dominant player in group A, player 2 is the dominant player in group B, and player 1 has an information advantage over player 2, namely, $\pi_1(\omega) \subseteq \pi_2(\omega)$ for all $\omega \in \Omega$. Without loss of generality, we assume that player 1 knows exactly the state of nature.

According to Proposition 1, the solutions of the dominant players' maximization problems provide the expected total efforts in both groups. Then, for all $j = 1, 2, ..., k$, player 1's maximization problem is

$$\max_{x_{1j}} v_j \frac{\sum_{i=1}^{m} x_{ij}}{\sum_{i=1}^{m} x_{ij} + \sum_{i=1}^{n} y_{ij}} - x_{1j}$$

where x_{ij} is the effort of player i from group A in the state of nature ω_j, and y_{ij} is the effort of player i from group B in the state of nature ω_j. The F.O.C. of player 1's maximization problem are

$$v_j \frac{\sum_{i=1}^{n} y_{ij}}{(\sum_{i=1}^{m} x_{ij} + \sum_{i=1}^{n} y_{ij})^2} \leq 1 \quad j = 1, ..., k \tag{3}$$

Suppose that the partition of player 2 from group B is $\Pi_2 = \{\{\pi_{21}\}, ..., \{\pi_{2q}\}\}$ where $q \leq k$. If $\omega \in \pi_{2t}, t \leq q$, then player 2 solves the following maximization problem

$$\max_{x_{2t}} \sum_{j \in \pi_{2t}} \frac{p_j}{\sum_{i \in \pi_{2t}} p_i} v_j \frac{\sum_{i=1}^{n} y_{ij}}{\sum_{i=1}^{m} x_{ij} + \sum_{i=1}^{n} y_{ij}} - x_{2t}$$

The F.O.C. of player 2's maximization problem is

$$\sum_{j \in \pi_{2t}} \frac{p_j}{\sum_{i \in \pi_{2t}} p_i} v_j \frac{\sum_{i=1}^{n} y_{ij}}{(\sum_{i=1}^{m} x_{ij} + \sum_{i=1}^{n} y_{ij})^2} \leq 1 \tag{4}$$

Denote the total effort of group A in the state of nature ω_j by X_j, and the total effort of group B in the state of nature ω_j by Y_j. Then, by (3) and (4) we obtain

$$\begin{aligned} v_j \frac{Y_j}{(X_j + Y_j)^2} &\leq 1 \quad j = 1, ..., k \\ \sum_{j \in \pi_{2t}} \frac{p_j}{\sum_{i \in \pi_{2t}} p_i} v_j \frac{X_j}{(X_j + Y_j)^2} &\leq 1 \,, \quad t = 1, ..., q \end{aligned} \tag{5}$$

The solution of equations (5) is identical to the solution of a direct contest between players 1 and 2 when they have the same information as they have in their groups. ■

By Proposition 2 and the results obtained by Aiche et al. (2019) about common-value Tullock contests between players with asymmetric information we have

Proposition 3 *In a group contest in which the dominant player in group A has an information advantage over the dominant player in group B, the expected total effort in both groups is the same, the probability of group A to win is smaller than that of group B, but the expected total payoff of the the players in group A is larger than in group B.*

In contrast to group Tullock contests with complete information, in our setting with asymmetric information, in equilibrium not only the two strongest players (the dominant players) are the active ones, but similarly to group contests with complete information, the group with the strongest player (the player with the information advantage over all the other players in the contest) is the group with the higher expected total payoff.

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