Wage Growth Decomposition: Mobility, Learning, and Loss of Human Capital in Unemployment Preliminary

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26 November 2015, 11:15-12:30, bld. 72, room 465

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Abstract

This paper studies sources of cumulative wage growth and its sensitivity to unemployment history for two education groups in the US. I assume that ex ante homogenous workers stochastically live through several stages of career, giving rise to ex post within-age and between-age heterogeneity. Mobility rates and the ability to acquire and retain skills differ across career stages, and firms post stage-specific offers taking these differences into account. In the calibrated model the equilibrium offers range is very wide initially, it contracts in mid-career, and shifts down later on. With a wide job ladder mobility can account for 57 percent of the total wage growth over the first career decade and it can become a negative factor afterwards because workers move to search within a lower range of offers. I find that workers lose up to 6.6 percent of their lifetime wage increase due both to depreciation of skills in unemployment and foregone skills accumulation. Finally, lifetime earnings of college graduates are found to be especially sensitive to unemployment history.

1   Introduction

In their review of post-schooling wage growth in the United States, Rubinstein and Weiss (2007) stipulate that two major mechanisms shape increasing and concave log-wage profiles: human capital accumulation and mobility.[1] Human capital accumulation implies that workers become more productive as they gain experience. Mobility captures the idea that matches do not last forever and

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[1]Rubinstein and Weiss (2007) also mention a third potential mechanism, namely, gradual learning about match quality, and the fact that good workers are promoted once their quality is revealed to an employer, and are therefore less likely to leave the firm. However Rubinstein and Weiss focus on the first two channels, as does the literature that my model relates to.

usually workers find themselves in a new job, either as a result of their old match being terminated, or as a result of receiving an attractive outside offer and agreeing to move. In recent years, a whole literature has emerged trying to assess the relative importance of these two major mechanisms for wage growth.[2] This exercise involves the decomposition of the observed wage profile into two unobserved components - human capital and frictional component that changes whenever a worker changes jobs. Using both reduced-form and structural approaches, most of the literature finds that human capital accumulation explains the lion's share of wage growth over first 10 years of career. I demonstrate that this will not be the case once the distributions from which the workers sample offers are endogenized in a wage-posting equilibrium framework as in Burdett, Mortensen (1998).

I revisit the question of relative importance of human capital accumulation and mobility in wage growth, and I study the impact of unemployment history on wage growth and earnings of high-school and college graduates in the U.S. The link between these questions arises naturally because I assume loss of skills in unemployment as an additional factor of wage change. The motivation is twofold. First, there is a vast body of empirical research documenting the substantial and persistent negative impact of unemployment episodes on subsequent wages (reviewed in Section 2). If one of the reasons for these losses is depreciation of skills, then we should account for it when decomposing wage growth into the effects of human capital and mobility. Second, for the unemployed, skills depreciation is an additional incentive to seek employment, which lowers their reservation wage and is relevant for the analysis of mobility and its role in wage growth. I get that the calibrated negative returns to each additional instant of unemployment are zero for most young workers, while many older workers lose skills in unemployment at about quarter the rate that they acquired them on the job. This substantial depreciation of human capital drives down the reservation wages of the senior unemployed workers. On average, by the end of 40 years of a career workers lose from 3 to 6 percent of their wage growth due to unemployment history, depending on education level, and these losses are driven both by the direct human capital depreciation and by the foregone accumulation of skills. Finally, in a simulation I demonstrate that though college graduates enjoy higher lifetime earnings than high-school graduates, their earnings are more sensitive to their unemployment histories. Careers of college graduates with above-average unemployment will yield a much lower lifetime value than an average career. This is explained by the fact that college graduates usually accumulate long unemployment histories in those periods in career when it is most destructive for their human capital.

The second central assumption of the model deals with the life-cycle nature of a career. Empirically, younger workers tend to be much more mobile than

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[2] Adda et al. (2013), Altonji et al. (2013), Bagger et al. (2013), Bowlus and Liu (2013), Buchinsky et al. (2010), Carrillo-Tudela(2012), Schonberg (2010), Yamaguchi (2010)

older workers.[3] They also have different returns to human capital than older workers.[4] Rubinstein and Weiss (2007) provide evidence that suggests that these between-age life-cycle differences can be related to changes in the within-age heterogeneity of workers' outcomes. They report that the share of wage gainers goes down with age, the share of wage losers goes up, while the size of the gain of gainers and the loss of losers remain virtually unchanged. A baseline wage ladder model can not explain these facts (the share of gainers will go down, but it looks extremely hard to justify an increasing share of losers), even when augmented by some human capital accumulation function with decreasing returns (even if predicts that workers lose human capital when old, does not account for the dynamic co-existence of gainers and losers within age). I account for these life-cycle properties by assuming that over the course of a career workers move across a finite number of stages, starting from being "young," and finishing being "old." Human capital and mobility parameters are taken as a given technology in each stage, implying that as the workers age, their opportunities change (generally, they deteriorate), and these changes are imminent and irreversible. The timing of these changes for a particular worker is a matter of chance in the model: some workers are lucky to stay "young" for a very long time (i.e., they acquire skills quickly, keep them in unemployment, and they find jobs easily), whereas others quickly move on to later stages, becoming "old" (i.e., they find themselves unable to learn much on-the-job, to keep their skills in unemployment, or to search efficiently after only several years of a career). Within-age heterogeneity changes with age, as workers in later stages of career become more and more prevalent, and they will be more likely to be wage losers. These gradual transitions over career stages explain observed changes in the curvature of wage-experience profile and observed life-cycle dynamics of mobility rates. As opposed to deterministic ageing, the model is stationary and tractable.

Third, for all stages of a career, I derive the endogenous distribution of wage offers as an equilibrium in a wage-posting game like in Burdett, Mortensen (1998). The critical assumption that I make here is that each stage of career is a separate labor market and the the workers who have become "old" can no longer search in the labor market for the "young". Under this segmentation assumption the distribution of offers in each career stage reflects the arrival rates of offers and the rates of human capital accumulation and loss specific for this stage, as well as the expected horizon of the workers.

Bowlus and Liu (2013) highlight the importance of the interaction between search and human capital accumulation over the life-cycle. They show that when the reservation cutoff of the unemployed is endogenous, it dramatically enhances the role of mobility because workers at the beginning of a career will be ready to accept very low offers in order to get employed and efficiently invest

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[3] Menzio, Telyukova, and Visschers (2012) note that "there are dramatic differences in the extent of labor reallocation for workers of different ages."

[4] reflected, for example, in Mincer wage function (Mincer (1974))

in their human capital on-the-job. In my model, the interaction between search and human capital is reflected in the entire endogenous distribution of wage offers, which changes over the life-cycle.

I demonstrate that the distributions of offers differ considerably across stages. At the beginning of a career, the lowest offer is very low: young workers face a long expected horizon and are eager to move to employment and to enjoy the gains from rapid early learning throughout their lifetime. The highest offer is high: intensive on-the-job search increases between-firm competition for workers. With a broad range of offers there is much room for improvement through upward mobility. My estimate is that mobility accounts for 57 percent of total wage growth of high-school graduates in the first decade of career - twice as high as in most comparable studies.[5] In mid-career, the expected horizon shortens, human capital accumulation slows down, loss of skills is still moderate, and on-the-job search becomes less efficient while the offers still arrive frequently to the unemployed. All these factors make the unemployed increase the minimal acceptable offer and the equilibrium support contracts. Late in a career, productivity loss in unemployment becomes very severe, and offers arrive at a low rate to the unemployed, stimulating the workers to be less picky and causing the minimal acceptable offer to go down. As a result, the entire distribution of offers shifts down relative to mid-career. As more and more workers reach their last career stage, late-life mobility turns out to be a *negative* factor in wage growth, on average.[6]

The contribution of my work is in highlighting the importance of interactions between search and human capital processes when analyzing the components of cumulative wage growth. In a cross-section, the decomposition of observed wage into the unobserved worker and firm effects is to some extent arbitrary, in the sense that once one distribution (for example, workers' productivities) is assumed, the other can be derived as a residual. Different decompositions will, however, have different implications for the role of mobility versus the role of human capital in wage growth over the life-cycle. By allowing the distribution of offers (which are firm-specific wage components in the model) to be determined in equilibrium one puts more discipline on wage decomposition exercise and is able to take into account endogenous links between changing human capital and workers' mobility opportunities and planning horizon on the one hand, and available offers on the other. As the results above show, this exercise yields novel predictions about the role of mobility, both in qualitative and quantitative terms. From the theoretical point of view, my work contributes to the literature by offering a novel approach to a career, which combines life-cycle with stationarity and tractability.

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[5] Still, lower than the estimate in Bowlus and Liu (2013).

[6] Notably, this novel effect arises only for college graduates, since according to the calibration, unemployment is especially damaging, and offers arrival rate is especially low for these workers when they are old.

The paper is organized as follows. In Section 2, I review the background literature on wage growth decomposition and the impact of unemployment on subsequent earnings. In Section 3, I present a formal model of a career where a worker's life-cycle involves transitions across a number of stages, which are each characterized by a different search and learning environment. I characterize a wage-posting equilibrium for each stage, regarding it as a separate submarket. In Section 4, I choose the preferred number of environments that a worker is exposed to over the course of a career, and calibrate their parameters in terms of human capital and transition rates. In Section 5, I discuss how equilibrium distributions of offers differ across career stages. In Section 6 I simulate the model by letting the workers live out their careers, move across stages, and sample offers from changing distributions. I analyze the resulting average wage path, and its three additive components, namely, the (positive) impact of actual experience accumulation, the (negative) impact of unemployment history, and the impact of mobility. Section 7 concludes.

2   Background Literature

This research paper is related to two literatures. The first one is a (very recent) literature on sources of cumulative wage growth. Studies in this field try to decompose observed wage profiles into the effects of unobserved human capital and job shopping.[7] The second is the literature on the impact of unemployment on careers, which focuses on the differences between career outcomes of workers who lost or kept their jobs. I briefly review these two literatures below. Finally, since one of the calibration outcomes is the rates of human capital accumulation over age, in the subsection 2.3 I also refer to the literature on learning abilities and productivity over the life-cycle.

2.1   Wage growth decomposition

Reduced-form studies that try to disentangle returns to mobility from returns to experience do so by estimating a set of regressions. The main equation is usually the Mincer wage function, which describes decreasing returns to experience and sometimes tenure, while controlling for education and demographics. Schonberg (2007) estimates this equation and measures the impact of mobility using the differences between wage growth for job-stayers, job-switchers, and those who move to unemployment.[8] Buchinsky et al. (2010) highlight that experience and seniority are themselves endogenous results of workers' decisions, and they explicitly include the equations that describe a decision to move to

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[7]Additional components are sometimes included, such as learning about match quality or effects of tenure

[8]Using NLSY data.

employment or job-to-job. Adda et al. (2013) estimate a wage equation including experience and tenure effects, allowing all the transition rates to differ by business cycle state, experience, and level of skills.[9] Altonji et al. (2013) estimate a rich model of earnings dynamics,[10] which includes the characterization of wage rates, work hours, employment, and job changes over the life-cycle. Except for Schonberg (2007), who exploits the wage differential between stayers and movers, the impact of search in the papers mentioned above is measured by first estimating the parameters of the unobserved offers distribution, usually assumed to be log-normal, and then simulating the model generating offers from this distribution.

The reduced-form analysis above concludes that human capital is by far the dominant driver of earnings growth, both at the beginning of a career (except, perhaps, the very first year or two in the market) and over a longer horizon. A common result is that for high-school graduates - the most frequently analyzed group of workers - human capital accumulation accounts for approximately three-quarters of wage growth over the first decade of a career.

Structural models that combine mobility and human capital accumulation are few. One of the pioneering studies combining these two mechanisms is Burdett, Coles, and Carrillo-Tudela (2011). In this paper workers accumulate experience at a given rate when employed and face an exogenous arrival rate of offers on-the-job. The authors show that this rich wage process results in the convenient decomposition of wage into the two additive components of human capital and mobility. The focus of Burdett, Coles, and Carrillo-Tudela (2011) is on cross-sectional wage dispersion.[11] In my work I use a framework that is technically related to their paper, and apply it to the analysis of individual wage profiles.

The existing papers that adopt a structural approach to individual wage dynamics differ in the way they model the two mechanisms of human capital accumulation and mobility. Yamagouchi (2010) and Bagger et al. (2013) combine human capital accumulation and on-the-job search in the framework of multilateral bargaining, as developed in Postel-Vinay and Robin (2002). Menzio et al. (2012) adopt a directed search framework with finite life.[12] Yamagouchi (2010) and Menzio et al. (2012) find that the role of mobility is minor and is concentrated at the very beginning of a career. Bagger et al. (2013) arrive at the opposite conclusion using the data from Denmark rather than the U.S.

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[9]Using German administrative panel data.

[10]Using PSID data.

[11]Carrillo-Tudela (2012) extends the work of Burdett et al. (2011) by allowing for firm productivity heterogeneity, and by calibrating the model to match the average wage profiles of young British workers. Still, the focus is on wage variance decomposition. The same focus is maintained in the paper by Tjaden and Wellschmied (2014), who add to it the depreciation of human capital in unemployment.

[12]Menzio et al.'s main focus is on explaining the decline in mobility over the life-cycle. They claim that mobility falls endogenously with age, because over time workers tend to search for a job in a submarket where pay is high, but they need to wait longer to get an offer.

Bowlus and Liu (2013) demonstrate that given the same US data as in previous studies, and accounting for the endogenous interactions between search behavior and investment in human capital produces novel results in terms of wage growth decomposition. Bowlus and Liu (2013) show that the presence of human capital accumulation has a dramatic impact on reservation rates, driving them down at the beginning of a career, when the returns to human capital accumulation are high. With initially low reservation rates, workers benefit a lot from search, and mobility explains a hefty 75 percent of wage growth of high-school graduates in the first decade of their career.

My paper is conceptually related to Bowlus and Liu (2013), in the sense that it focuses on the interactions between search and human capital processes. However, my work differs from that of Bowlus and Liu (2013) in several respects. First, not only the reservation wage of the unemployed but also the entire distribution of offers, which changes over the course of a career, is endogenous in my analysis. Second, I include the process of productivity loss in unemployment, which has an impact both on the reservation cutoff of the unemployed and on the entire distribution of offers at various stages of a career. Finally, the entire structure of a career is differerent in my model, which is a steady-state one, and has a stochastic, rather than a deterministic, life-cycle.

2.2 Unemployment and subsequent career outcomes

In a parallel strand of literature on individual wage outcomes, researchers have tried to estimate the effects of layoffs on wage profiles in econometric reduced-form studies. Addison and Portugal (1989) find, using the U.S. Displaced Workers Survey, that a 10 percent increase in unemployment duration lowers accepted wages by about 1 percentage point. Jacobson et al. (1993), using U.S. administrative data, record losses of 25 percent of pre-displacement wage even five years after displacement. Gregory and Jukes (2001) estimate the effects of unemployment on the subsequent earnings of British men and find that wage penalty after a six-month unemployment spell is 13 percent for the young and almost twice as high for the old. Davis and vonWachter (2011) explore how earnings losses in the years following separation differ depending on the timing of the separation within the business cycle. They estimate that workers in the U.S. can lose from 10 to 24 per-cent of lifetime earnings as a result of displacement. Jarosch (2015) shows that in Germany, even 20 years after separation, the wage of a displaced worker is 10 percent lower than the wage of a stayer. Other recent studies documenting substantial and persistent losses of earnings following displacement are Jung and Kuhn (2013) and Saporta (2013).

Despite ample evidence on the persistent negative impact of unemployment episodes on wages, both structural and reduced-form models of individual wage dynamics focusing on sources of wage growth have not included any mechanism

that would generate such an effect.[13] In my work the negative impact of unemployment is reflected in the loss of human capital, and it is thus a process that runs counter to human capital accumulation on the job. The role of this human capital depreciation is two-fold. First, it is a direct component of wage profile decomposition. Here the model implies that this direct negative component is moderate on average, but combined with foregone earnings it results in a non-negligible loss of cumulative wage growth even despite the fact that the average unemployment history accumulated by the end of a typical career in the U.S. is not long. Second, the depreciation of human capital is an incentive for workers to lower their reservation rate when the loss is substantial. This is the main force that drives down the reservation cutoffs of the unemployed workers in their last career stage, shifting down the entire distribution of offers later in the life-cycle.

2.3 Age, learning ability, and individual productivity

The calibration indicates that human capital accumulation rates are declining with age, and for a significant share of workers in both education groups late-in-career productivity falls in employment (negative human capital accumulation). This outcome is related to the research on the effects of aging on learning abilities, and on productivity. Within the first, psychological literature, there is a consensus that some abilities do not decline with age,[14] but the robust result is that some crucial learning-related abilities decline over the life-cycle, such as encoding new memories of episodes or facts, working memory and processing speed (Hedden (2004), Salthouse (2004)). In more recent research Janascek et al. (2012) study the ability to unconsciously recognize regularities and patterns, and find that this learning ability declines over age. Finally, Craik and Bialystok (2006), referring to the development of cognitive ability, note that "change can occur at any time [and] development depends on interactions among genetic, environmental and social factors." Indeed, the stochastic nature of ageing in the model conforms to this fact. The second relevant branch of literature deals with the impact of age on individual productivity. The general finding is that job performance often does decrease at older ages, but not for all tasks and less so in occupations where age-resistent abilities (such as verbal skills) are important (see Skirrbekk, 2003, 2008) for a survey of this literature). In a recent study, Gobel and Zwick (2009) use linked employer-employee German data, and find

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[13]The only exception is Altonji et al. (2013), who include the deterioration of general human capital in their wage equation. They find that the average impact of this wage component is small, and that cumulative losses of human capital after 30 years of career are negligible on average.

Fujita (2012) builds a search model whereby he explains the downward trend in the separation rate in the US over the past 30 years by the increased probability of loss of skills in unemployment.

[14]For example, autobiographical memory, semantic knowledge, emotional processing, and vocabulary skills.

that establishment productivity increases with the share of employees until the age of 50 - 55 years and decreases slightly afterwards.

This concludes the review of the related literature. In the next section I present the theoretical model.

3   Theoretical Model

3.1   General setup

A carrer in the model has two basic properties. First, it is based on the idea that workers of different ages are heterogeneous in their ability to acquire and retain skills, and in their chances to move across states[15]. The second property of a career has to do with within-age heterogeneity of workers: some workers exhibit high learning ability and search efficiency till old age, while others quickly slow down in their learning and become much less mobile in the labor market. To capture these two ideas I assume that over the life-cycle the workers move across several stages of career, and their human capital and mobility opportunities change accordingly. These changes are taken as given, I treat them as arising from exogenous reasons, such as biological aging, life-cycle changes in family circumstanes, etc. Furthermore, transitions between submarkets are random events that the workers have no control over. Therefore, two initially identical workers who simultaneously started their careers may be in different stages at a point in time, even though they have the same potential experience. At this point in time these two workers accumulate human capital at different rates, and get job offers at different rates. Though unobserved directly, the gradual movement of workers across career stages will explain the average life-cycle log-wage profile, as well as the life-cycle dynamics of transition rates in the labor market.

The workers in the model know the parameters of their current stage, and know how these parameters will change in the future, though they do not know exactly when this change will occur (transitions are random). Given that information, the workers decide, at each stage, what the lowest offer that they will accept is, given the distribution of available offers.

The firms operate a constant returns to scale technology and post offers for each type of workers (each firm hires all types of workers, and has separate wage offers for each type). I assume that each worker can only perform a job that was advertised for the workers of his type (i.e., stage), so that there is no problem of

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[15] Menzio et al., 2015, note that there are "marked differences in the degree of labor reallocation between ages"

incentive compatibility, and from a worker's point of view being in each stage of career means participating in a separate labor market. The firms know how mobile workers are in each career stage and what their human capital rates are, and post offers for each type as in Burdett, Mortensen (1998) and Burdett et al. (2011). In equilibrium the lowest offer that they post will equal the lowest offer that is acceptable for each stage, and the highest offer will give the firm the same expected profit. As a result, different distributions of offers arise endogenously, depending on search and human capital parameters of each career stage, and the expected horizon of the workers.

My approach is different from the deterministic ageing concept. Under deterministic aging the model would not be steady-state, and I would not be able to solve for equilibrium distributions of offers at different career stages. Second, under deterministic aging all the workers of the same potential experience have the same rates of human capital accumulation and depreciation, and search with the same efficiency. This implausible property is usually overcome in the exisiting literature by explicitly assuming within-age heterogeneity (see, for example, Bagger, Fontaine, Postel-Vinay, and Robin (2013), or Bowlus and Liu (2013)). In my model this witin-age heterogeneity arises naturally as part of the setting and it changes across ages.

Note, finally, that though there is no *deterministic link* between potential experience and the stages of a career, workers with higher potential experience are more likely to have already moved on to later stages, so that there is a *positive correlation* between potential experience and the stage.

I formalize the general setting below.

3.2   The workers

Assume that time is continuous and the economy is in a steady state. The life of a worker is divided into N stages $\{S_1, ..., S_N\}$. A unit measure of workers participates at each point in time in stage $S_k, k \in \{1, ..., N\}$, so that the total measure of workers in the economy is N. An individual worker starts his career in the first stage, S_1. There is a measure ϕ of workers starting their career at each instant. A worker leaves S_1 at Poisson rate ϕ, and moves to S_2, where he remains until the transition shock ϕ happens again, whereupon he moves to S_3, and so on. When a worker reaches the last stage, S_N, he stays there until the transition shock ϕ happens for the last time in his career and he exits the economy for good. These assumptions guarantee that the measure of workers in each stage remains constant all the time.

A career starts for each worker from the first stage S_1, which he enters as an unemployed worker with a unit productivity, $y_0 = 1$, the same for all workers. Once the career begins, the productivity starts to evolve. Productivity is

general human capital, which grows when a worker is employed and accumulates experience, and declines when he is unemployed and loses his skills. The assumption is that when a worker is employed, his productivity grows automatically due to learning-by-doing, whie the opposite occurs in unemployment. When a person takes a break from accumulating experience, he loses part of his skills, or, alternatively, he finds it more difficult to keep up with the advancing techological frontier, his productivity goes down, and unlearning by not doing occurs.[16] This damage increases with the duration of unemployment. The rates of human capital accumulation and depreciation are ρ and η, respectively, so that x periods of employment increase productivity by a factor of $\exp(\rho \cdot x)$, and q periods off the job decrease productivity by a factor of $\exp(-\eta \cdot q)$. Human capital technology (the rates ρ and η) is exogenous and stage-specific. This is different from the model of Ben Porath (1967) where workers in each period decide on the amount of time they want to invest in human capital accumulation. In my model the workers take it as given that as they move to later stages, they will be exposed to different (exogenous) returns to employment and unemployment. Note that in a given stage the order of employment and unemployment spells does not matter for productivity, only the cumulative durations of these two states are what matters.

Human capital is not lost in job-to-job transitions, and it is carried across stages - a worker starts a new stage in a career with the same productivity with which he finished his previous stage. Each worker's productivity summarizes his entire labor market history, including periods of employment and non-employment, over all stages that he has lived through up to this point in time. For tractability I assume that each transition across stages involves unemployment. When a transition shock hits, an employed worker gets separated from his current job and becomes an unemployed worker in the next stage, and an unemployed worker remains unemployed but starts searching in the next stage.

Search technology is stage-specific and for each stage it is defined by Poisson arrival rates of offers for unemployed and employed workers, λ_0 and λ_1, respectively. The offers are piece rates $\theta \in [0, 1]$, stipulating a share of the flow productivity y, that the worker receives at each instant, so that his wage is $\theta \cdot y$. Piece rate offers originate from the cumulative distribution $F(\theta)$ - a stage-specific distribution found in a wage-posting equilibrium as in Burdett and Mortensen (1998) and Burdett et al. (2011) (derived in detail below). A worker who is in stage s can only sample offer from the distribution $F^s(\theta)$,

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[16]The term "unlearning by not doing" was used by Coles and Masters (2000). They show that when there is a loss of skills during unemployment, the firm that opens a vacancy and hires a worker does not take into account the externality that they create for other firms, by increasing the quality of their potential workforce. In this setting a number of Pareto-rankable equilibria arise, where everyone would be better off if firms posted many vacancies, and workers did not spend much time in unemployment and did not lose much human capital. The worst case is an equilibrium with few vacancies, and longer unemployment spells and worse average workforce quality.

he can not perform the jobs that are advertised for the workers in other stages. This rules out the problem of incentive compatibility for the workers and implies that each stage is a separate labor submarket. Jobs are destroyed exogenously at a stage-specific Poisson rate δ. A worker's instantaneous discount rate is r. Income in unemployment is a fixed share b of a worker's human capital.

The value function of an unemployed worker in stage s who has productivity y, $W^{U,S}(y)$, is:

$$
\begin{aligned}
rW^{U,s}(y) &= b \cdot y + \frac{\partial W^{U,s}(y)}{\partial t} + \\
&\quad + \lambda_0^s \int_{\underline{\theta}^s}^{\overline{\theta}^s} \max\left[W^{E,s}(y,\theta') - W^{U,s}(y), 0\right] dF^s(\theta') + \\
&\quad + \phi \cdot \left(W^{U,s+1}(y) - W^{U,s}(y)\right)
\end{aligned}
\tag{1}
$$

The first element in the flow value of unemployment is the flow income in unemployment, $b \cdot y$. Note that the parameter b that determines the income of the unemployed worker is not stage-specific, i.e., unemployed workers in all submarkets get the same share of their productivity as their income in unemployment. The second element, $\frac{\partial W^{U,s}(y)}{\partial t}$, is the negative change in the value over time, due to depreciation of human capital that reduces current productivity. Note that the value of unemployment $W^{U,s}(y)$ will be decreasing over time as long as the rate of human capital depreciation is not equal to zero. The third element on the right-hand side of equation (1) above is the search option of the unemployed in stage s. At rate λ_0^s unemployed workers face an offer θ' sampled from the stage-specific distribution $F^s(\cdot)$, which they either accept and become employed with the value $W^{E,s}(y,\theta')$, or decline and remain unemployed with their current value $W^{U,s}(y)$. Finally, an unemployed worker in stage s might experience a transition shock ϕ, whereupon he moves to the next stage and his value is the value of being unemployed in stage $s+1$ with the same human capital [17] y.

The value function of an employed worker in stage s depends both on the worker's productivity y and the piece rate that a firm pays, θ:

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[17] In the last stage, i.e., when $s = N$, the value in the event of a transition shock becomes zero, and the last element in the value function is simply $-\phi W^{U,s}(y)$.

$$
\begin{aligned}
rW^{E,s}(y,\theta) \;=\; & \theta \cdot y + \frac{\partial W^{E,s}(y,\theta)}{\partial t} + \\
& + \lambda_1^s \int_{\underline{\theta}^s}^{\overline{\theta}^s} \max\left[W^{E,s}(y,\theta') - W^{E,s}(y,\theta), 0 \right] dF^s(\theta') + \\
& + \delta^s \cdot \left(W^{U,s}(y) - W^{E,s}(y,\theta) \right) + \\
& + \phi \cdot \left(W^{U,s+1}(y) - W^{E,s}(y,\theta) \right)
\end{aligned}
\tag{2}
$$

The first element on the right-hand side of (2) is the flow wage of the worker, which is a share θ of his productivity. The second dynamic element, $\frac{\partial W^{E,s}(y,\theta)}{\partial t}$, is a positive increment to the value due to learning-by-doing. Since each instant of employment adds to the productivity of the worker, and the value of employment depends on productivity, the value grows over time. The third element is the on-the-job search option. An employed worker receives an outside offer θ' that comes from the stage-specific distribution $F^s(\theta')$, and he compares the value of remaining with the current firm, $W^{E,s}(y,\theta)$, and the value of moving to the poaching firm, $W^{E,s}(y,\theta')$. Note that since there are no firm-specific skills, the entire stock of human capital y is moved to the new job in case a worker decides to accept the offer. Exogenous separation shock δ^s might destroy the match. Finally, an employed worker can experience a transition shock ϕ, in which case he loses his value and becomes an unemployed worker in the next submarket, $s + 1$.

Note that both value functions are stage-specific (hence superscript s), and that they account for the effect of a shortening horizon - through the future component $W^{U,s+1}(y)$. This component is the expected value of being unemployed in the next submarket, and, by the chain rule, it includes the values of all subsequent stages as well. In the last stage, N, the value of the future component is zero, since transition event ϕ entails a permanent exit from the market. In the next-to-last stage, $N - 1$, the future component is the value of being unemployed in stage N; in the previous submarket $N - 2$ the future component is the value of unemployment in stage $N-1$, which includes also the value of unemployment in stage N, and so on. It can be seen that the future component $W^{U,s+1}(y)$ declines from stage to stage: it is high in the beginning since it subsumes many future stages, while in the last submarket it is simply zero.

3.3   Optimal strategies

The optimal strategies of the workers are simple. All unemployed workers in stage s will accept any offer above the reservation cutoff $\theta^{R,s}$. The cutoff arises because the value of unemployment is independent of the piece rate, whereas

the value of employment is an increasing function of it. Notably, this cutoff is common for all workers who are in the same stage, since flow incomes, and dynamic components in the value functions are proportionate to productivity, and individual human capital has no impact on the *relative* attractiveness of employment over unemployment.[18]

The cutoff in each stage does depend, however, on the key stage-specific mobility and human capital parameters $\lambda_0, \lambda_1, \delta, \rho, \eta$, as well as on the expected horizon of the workers.The reservation cutoff will be lower in stage where the relative value of unemployment is lower. For example, a low arrival rate of offers λ_0, or a high loss of skills rate η, make the unemployed workers more eager to move to employment, thereby lowering $\theta^{R,s}$. If a stage is characterized by a higher arrival rate of offers on-the-job λ_1, a lower match destruction rate δ, and a higher human capital accumulation rate ρ, then the unemployed workers in this stage will also accept lower offers.

The effect of the shortening horizon is present in the model despite the fact that the length of a career is stochastic, because a permanent exit from the market can only happen in the last stage of a career. Workers are forward-looking and take into account that the human capital accumulated in the first stage will serve them throughout their entire career, whereas productivity losses will be a permanent drag on value over their lifetime. It is especially relevant at the beginning of a career, when the expected horizon is the longest. This enhances the relative attractiveness of employment and drives down the reservation cutoff, especially in the first stage, with the effect becoming weaker in subsequent stages.[19]

For the employed workers the optimal strategy is trivial, namely, accept any offer that is above their current piece rate. This is because human capital is perfectly transferable across firms, and all firms are identical.

Optimal strategies of the workers are important because they are taken as given by the firms, which decide which offers to post. In particular, as will be shown below, the minimal posted offer is the reservation cutoff of the unemployed, while high frequency of job-to-job transitions urges firms to set higher piece rates in order to better retain workers.

The problem of the firm is described in the next subsection.

---

[18]See Appendix A for the complete derivation.

[19]Mathematically, it can be seen from the value functions, that the same additional future component, $\phi W^{U,s+1}$, which is present in both states, will increase the value of employment more than it increases the value of unemployment (due to the dynamic components of value functions). Therefore, this discrepancy will be higher in the first stages of a career.

3.4 Firms

There is measure one of identical firms, operating under the same constant returns to scale technology. Each firm posts N offers - one for each type of workers. Within each submarket, the firm hires everyone to whom the respective offer is attractive enough; see Burdett and Mortensen (1998). The expected profit from posting an offer θ in a submarket s is[20]

$$
\begin{aligned}
\pi^s(\theta) &= \overline{y}_0^s \cdot (1-\theta) \cdot \lambda_0^s U^s \cdot \left[\int_{x=0}^{\infty} \int_{q=0}^{\infty} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^2 P^{U,s}(x,q)}{\partial x \partial q} \int_{\tau=0}^{\infty} e^{-r\tau} e^{-q^s(\theta) \cdot \tau} e^{\rho^s \tau} d\tau \right] + \\
&\quad + \overline{y}_0^s \cdot (1-\theta) \cdot \lambda_1^s (1-U^s) \cdot \left[\int_{x=0}^{\infty} \int_{q=0}^{\infty} \int_{\underline{\theta}^s}^{\theta} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^3 P^{E,s}(x,q,\theta)}{\partial x \partial q \partial \theta} \int_{\tau=0}^{\infty} e^{-r\tau} e^{-q^s(\theta)\tau} \cdot e^{\rho^s \tau} d\theta \right]
\end{aligned}
$$
(3)

The first component of the profit equation is the expected profit from hiring a worker from the pool of the unemployed. The measure of unemployed workers in each stage, U^s, can easily be found from inflow-outflow steady-state conditions and it equals $\frac{\phi+\delta^s}{\phi+\delta^s+\lambda_0^s}$. The term $e^{\rho^s \cdot x} e^{-\eta^s \cdot q}$ is the initial productivity of the worker at the time when he is hired by the firm. The unemployed workers accumulate stochastic labor market histories in stage s, and their cumulative employment (x) and unemployment (q) spells are distributed in steady state according to $P^{U,s}(x,q)$. Therefore, the integral $\int_{x=0}^{\infty} \int_{q=0}^{\infty} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^2 P^{U,s}(x,q)}{\partial x \partial q}$ summarizes the average productivity of a worker hired from unemployment in stage s. In a firm, as long as the match survives, the productivity of a worker will grow at rate ρ^s. The match will last at least τ with probability $e^{-q^s(\theta) \cdot \tau}$, where $q^s(\theta) = \phi + \delta^s + \lambda_1^s \cdot (1-F^s(\theta))$ is the total separation rate, which includes transition shock ϕ, exogenous separation δ^s, and job-to-job transition $\lambda_1^s \cdot (1-F^s(\theta))$. Therefore, the integral $\int_{\tau=0}^{\infty} e^{-r\tau} e^{-q^s(\theta) \cdot \tau} e^{\rho^s \tau} d\tau$ summarizes the expected productivity gain of a worker in this match.

The second component of the profit equation is the expected profit from poaching an employed worker. Here it is important that only those workers who are currently employed at piece rates below θ will be attracted by the offer θ. In this case the joint distribution of employment and unemployment spells, and piece rates, denoted by $P^{E,s}(x,q,\theta)$, has to be taken into account. The term $\int_{x=0}^{\infty} \int_{q=0}^{\infty} \int_{\underline{\theta}^s}^{\theta} e^{\rho^s \cdot x} e^{-\eta^s \cdot q} \frac{\partial^3 P^{E,s}(x,q,\theta)}{\partial x \partial q \partial \theta}$ thus summarizes the average productivity of a worker poached from a firm that paid him a piece rate below θ.

Finally, $\overline{y}_0^s$ is the average productivity with which the workers start stage s. It has no impact on the distribution of offers in stage s (see Appendix C).

---

[20] This expected profit does not take into account stage-specific costs of posting an offer. In order to close the model I assume that these stage-specific costs are such that the firms are indifferent between posting offers for any type, and, WLOG, they post in all the submarkets.

3.5 Steady-state equilibrium

Following Burdett and Mortensen (1998), I impose a constant profit condition in order to derive the equilibrium distribution of offers. The idea is that a firm posting a very high offer gets low profit per worker, but is very successful in attracting and retaining workers, and the measure of its employees at each instant will be high. Similarly, a firm that posts a very low offer will enjoy high profit per worker but will have a smaller measure of employees, since the workers will frequently accept outside offers and leave. A constant profit condition requires that both these extremes, as well as any offer between them, yield the same expected profit. Given that other firms post according to this distribution, any individual firm is indifferent between all offers. In this way an entire non-degenerate distribution of offers arises across initially identical firms. Burdett et al. (2011) apply this type of equilibrium to their model with on-the-job search and human capital accumulation, and study cross-sectional wage dispersion. Carrillo-Tudela (2012) addresses the same question but adds firm productivity differentials to the model of Burdett et al. (2011). To study life-cycle wage profiles, I apply this concept in my model with on-the-job search, human capital accumulation, human capital depreciation, and segmented markets.

For each submarket $s, s \in \{1, ..., N\}$ a steady-state equilibrium is a tuple: $\left\{ \theta^{R,s}, F^s(\cdot), U^s, P^{U,s}(x,q), P^{E,s}(x,q,\theta) \right\}$ such that

(i) $\theta^{R,s}$ is the optimal reservation piece rate of any unemployed worker in stage s.

(ii) $F^s(\cdot)$ satisfies the constant profit condition:

$$
\begin{aligned}
\pi^s(\theta) &= \overline{\pi}^s > 0 \text{ for all } \theta \text{ where } dF^s(\theta) > 0 \\
\pi^s(\theta) &\leq \overline{\pi}^s \qquad \text{ for all } \theta \text{ where } dF^s(\theta) = 0
\end{aligned}
\tag{4}
$$

(iii) $U^s, P^{U,s}(x,q)$ and $P^{E,s}(x,q,\theta)$ are consistent with steady-state turnover.

The following useful result from Burdett et al. (2011) applies here as well:

Lemma 1. In the equilibrium defined above, for all $s, s \in \{1, ..., N\}$: (i) $F^s(\cdot)$ contains no mass points, (ii) $F^s(\cdot)$ has a connected support, and (iii) $\underline{\theta}^s = \theta^{R,s}$. Condition (iii) means that in each submarket (stage) the lowest offer in the distribution equals the stage-specific reservation cutoff of the unemployed. The proof is relegated to the Appendix B.

The characterization of the equilibrium distribution of offers, including its upper and lower bounds, can be found in Burdett, Coles, and Carrillo-Tudela (2011). It is applicable here as well, with two major complications. First, since in my model both actual experience, and unemployment history matter for

productivity, the solution involves finding the steady-state joint distributions of histories, and piece rates $P^{U,s}(x,q)$ and $P^{E,s}(x,q,\theta)$, for each submarket. Second, since the workers are forward-looking, their optimal reservation cutoff in each stage (submarket) depends on the expected value of the future. Therefore, the cutoff and the entire distribution of offers has to be found first for the last stage, and then, going backwards, for all preceding stages. Appendix C contains a full description of the solution.

At this point, it is clear that the equilibrium distributions of offers at different stages of a career, the resulting life-cycle profile of the piece rate (the mobility component of a wage), and the life-cycle profile of productivity gains and losses (the human capital component of a wage) all depend on the stage-specific parameters. In the next section I calibrate the model for two education groups in the United States.

4 Calibration

The period in the model is a quarter. I assume that the number of stages in a life-cycle is three - "young", "middle" and "old". I pick the transition parameter $\phi = 1/36$ so that the composition of the population would change symmetrically over a career, from "young" being most prevalent in the beginning, to "old" being most prevalent towards 40 years of potential experience - the end of the career window that I analyze. This transition rate implies that on average each career stage will last 9 years for each individual worker (with a standard deviation of 18 years). Figure 1 below illustrates how shares of worker types change over a life-cycle.
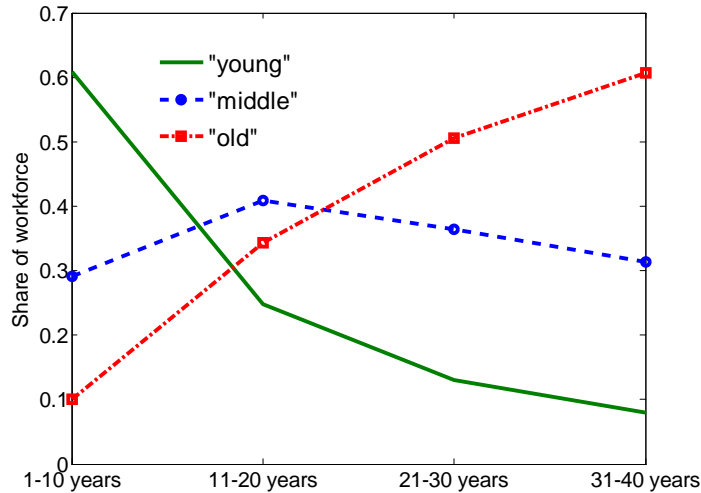
Figure 1: Labor force composition by type

Figure 1 is a graphic illustration of the fact that career stages and potential experience are not deterministically linked, but positively correlated through workforce composition, with workers who have already reached the last stage of their career becoming more and more prevalent as potential experience increases.

There are five stage-specific parameters to calibrate, for each stage $s \in \{1,2,3\} : (\lambda_0^s, \lambda_1^s, \delta^s, \rho^s, \eta^s)$. The parameters can be divided into two categories: mobility rates $\lambda_0^s, \lambda_1^s, \delta^s$, and human capital accumulation and depreciation rates ρ^s, η^s. I use the data on mobility by age in order to infer information about $\lambda_0^s, \lambda_1^s, \delta^s$, and I calibrate human capital parameters ρ^s, η^s in such a way that the average log-wage profile in the simulation matches the one from the data. In the next subsection I present the details of the calibration exercise. Non-stage-specific parameters are set at $b = 0.4$, and $r = 0.0099$ per quarter.

4.1   Mobility parameters $\lambda_0^s, \lambda_1^s, \delta^s$

Given the model, an age-specific transition rate observed in the data is a combination of the transition rates of three underlying types - "young", "middle", and "old" - with the weights set according to dynamic workforce composition, as in Figure 1 above. I use this correspondence in order to calibrate the stage-specific rates $\lambda_0^s, \lambda_1^s, \delta^s$. Note that in equilibrium δ^s- events correspond to the flow from employment to unemployment, λ_0^s - events correspond to the flow from unemployment into employment. These two (weighted) rates can be directly compared with age-specific transition probabilities in the data. Job-to-job flows, however, arise in the model in the combined event of getting an offer and accepting it (the probability depends on the current wage). Therefore, the actual job-to-job transition rate should be compared with a weighted aggregation of transition rates at all possible wage levels in the model. The detailed solution is relegated to the Appendix, and it follows the lines of Nagypal (2008), Hornstein et al. (2011), and Ortego-Marti (2012).

The dashed lines in Figure 2 below represent actual transition rates for college graduates and high-school graduates over 40 years of a career (calculated by Menzio et al. (2015) using the 1996 wave of SIPP panel. I thank Ludo Visschers for providing me with these data). The solid lines lines are the transition rates implied by the calibration.
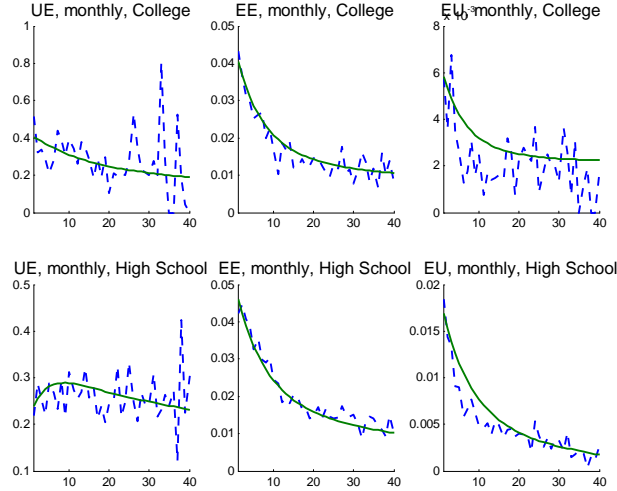
Figure 2: Mobility over the life-cycle: actual and fitted transition rates

Mobility generally declines with potential experience, as Figure 2 illustrates. This dynamics, is driven by the changes in the type composition of workforce - more mobile "young" workers become extinct later in career. Calibrated Poisson rates $\lambda_0^s, \lambda_1^s, \delta^s$ differ substantially across stages:

<div align="center">

Table 1

	δ^s		λ_0^s		λ_1^s	
	HSG	**CG**	**HSG**	**CG**	**HSG**	**CG**
"*young*"	0.062	0.024	0.778	1.586	0.574	0.580
"*middle*"	0.008	0.003	1.656	1.181	0.084	0.039
"*old*"	0.0003	0.008	0.462	0.334	0.044	0.073

</div>

Three points are noteworthy regarding calibrated mobility parameters. First, once workers stop being "young", on-the-job search becomes much less efficient. Second, for both education levels the chance to leave unemployment drops sharply upon becoming a worker of an "old" type. Finally, for college graduates job-finding rate declines monotonically over the life-cycle, whereas for high-school graduates it is highest in the middle stage. The latter fact is driven by a slight hump in the empirical job-finding rate profile of high-school graduates (see Figure 2).

The next step is to calibrate human capital accumulation and depreciation parameters ρ^s, η^s.

4.2 Human capital parameters ρ^s, η^s

Given the transition rates, I now calibrate human capital parameters in order to match the life-cycle log-wage profiles of high-school graduates and college graduates in the United States. In what follows I describe the construction of average log-wage profiles, and the matching criterion.

4.2.1 Data on life-cycle career profiles

I take repeated cross-sections from CPS March Supplements for the years 1996 - 2006. I limit the subsample to white male workers, who are employed full-time, and have positive potential experience (potential experience = age - years of education - 6). The observations with a missing value of real hourly wage are dropped. I limit the sample further by dropping observations for which hourly wage is below the federal minimum, and limit the potential experience to no more than 40 years. There are 59,162 observations for college graduates (completed 16 years of education), and 86,177 observations for high-school graduates (completed 12 years of education). These samples combine people who entered the labor market as early as 1956 and as late as 2005. In order to take account of the cohort effects that can bias wage experience profiles due to an increase in the returns to higher education that occurred in the second half of the twentieth century, I run the following simple regression for both education levels:

$$\ln w_{i,C,X,t} = \sum_{C=1956}^{2005} \beta_C \cdot D_{C,i} + \sum_{X=1}^{40} \beta_X \cdot D_{X,i,t} + \varepsilon_{i,t} \tag{5}$$

The regression decomposes the log-wage of individual i, who belongs to cohort $D_{C,i}$ and has potential experience X at time t, into the effect of his cohort $D_{C,i}$, the effect of his experience at time t $D_{X,i,t}$, and the individual i.i.d. error term $\varepsilon_{i,t} \sim N(0, \sigma^2)$. Dummy variable $D_{C,i}$ equals 1 if individual i belongs to cohort C, defined according to the year of labor market entry. Dummy variable $D_{X,i,t}$ equals 1 if individual i has potential experience X in year t. The profile of coefficients β_X over potential experience levels is the life-cycle log-wage profile net of cohort effects. Notably, as Figure 5 illustrates, the removal of cohort effects has little impact on the wage profile of less educated workers, who have only a high-school degree. By contrast, the profile of those with a college diploma is corrected upwards for high experience levels, reflecting the fact that most experienced workers are those who belong to the earliest cohorts, i.e., those who entered the labor market before the period of growth of the returns to higher education. These workers are, therefore, "disadvantaged" relative to less experienced college graduates from more recent cohorts. The regression with cohort effects corrects for this bias.
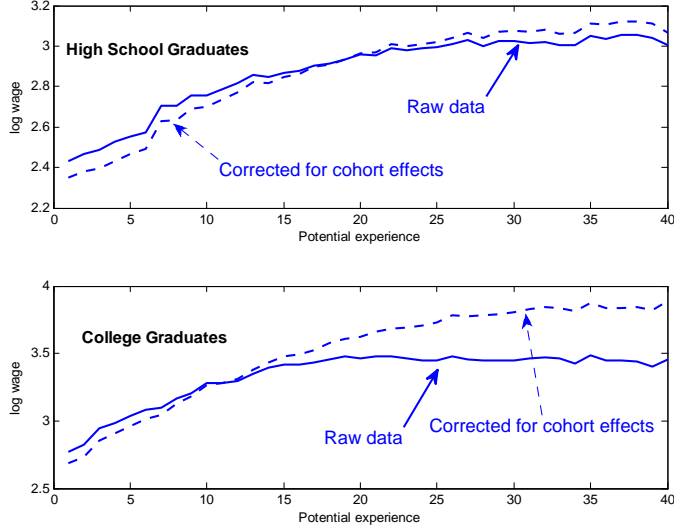
Figure 3: Wage profiles and cohort effects

For each education group, I search for combinations of ρ^s, η^s for all $s = 1, 2, 3, 4$ that would minimize the distance between the simulated average log-wage profile and the actual log-wage profiles net of cohort effects constructed above:

$$\min_{\rho^s, \eta^s} MSE = \frac{1}{40} \sum_{x=1}^{40} \left(\ln w_x^{data} - \ln w_x^{simulation} \right)^2, \qquad (6)$$

where $\ln w_x^{data}$ is the β_X coefficient from the regression (5) above, and $\ln w_x^{simulation}$ is the average log-wage profile obtained from simulating careers for an artificial sample of 10,000 workers over 40 years of potential experience. Table 2 summarizes the resulting calibrated human capital parameters:

Table 2. Calibrated stage-specific human capital parameters

	ρ^s		η^s	
	HSG	**CG**	**HSG**	**CG**
"$young$"	0.008	0.017	0.000	0.000
"$middle$"	0.008	0.012	0.002	0.004
"old"	-0.006	-0.015	0.015	0.02

Several points stand out based on Table 2.

First, regarding negative returns to an additional period of unemployment, η, few benchmarks exist in the literature. Ortego-Marti (2012) uses PSID and runs

a regression relating current log wage to accumulated unemployment history, and reports an estimate that is equivalent to $\eta = 0.036$ in my model. Saporta-Eksten (2013) calibrates the loss-of-skills rate to be 0.0125 per quarter.[21] These estimates are irrespective of experience. They approximate to my calibrated values for the last career stage.

Second, the calibration shows that returns to experience become negative in the last career stage - the productivity declines even if a person is working. This decline is stronger for college graduates than for high-school graduates. The reason behind it might be that college graduates are employed in occupations where the role of technology is relatively high and as the technological frontier advances older workers find it especially difficult to keep up with it. It is important to keep in mind that since each potential experience level combines workers of all the three types, declining productivity of the "old" does not mean that all workers late in their career will be losing skills even in employment. What it does mean, is that over the course of a career there is an increasing share of workers whose productivity declines even in employment, reaching 60 percent of the workforce with 31 to 40 years of potential experience.

Third, in the first stage workers do not lose skills when unemployed, but from the second stage onwards, the losses are non-zero and increase from stage to stage. The rate of unlearning by not doing is never higher than the rate of learning. This pattern of negative returns to unemployment increasing with age is consistent with the evidence for the U.S. in Davis and Von Wachter (2011). They find that the losses of lifetime earnings upon displacement are twice as high for workers aged 51-60 as for 21-30-year-olds. Gregory and Jukes (2001) report a similar pattern using data from the U.K.

Summing up, like mobility rates, human capital technology differs substantially over the life-cycle. The negative impact of unemployment on wages is always driven more by the foregone human capital accumulation on-the-job, than by skills depreciation in unemployment, because ρ is always higher than η. This is especially true for young workers, most of whom do not lose skills in unemployment at all while learning quickly on-the-job. The last stage is characterized by human capital losses in employment, and by even larger depreciation in unemployment. Finally, human capital processes are overall more intensive for college graduates, than for high-school graduates.

Given all the parameters I obtain the following normalized[22] profiles of average log wage for the two education levels in the simulation of 100,000 workers over 40 years of career .

_____

[21] 5% annually.
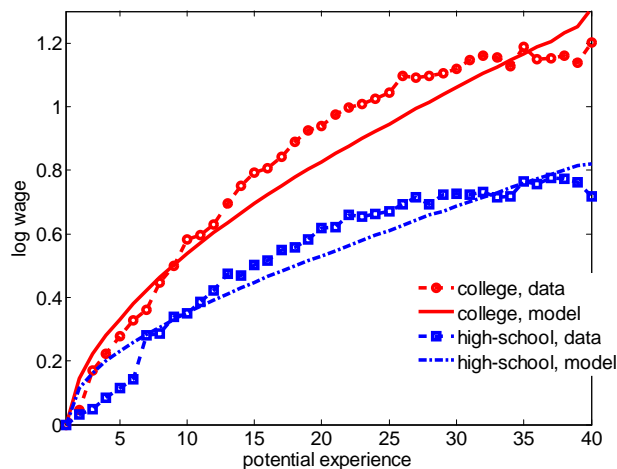
[22] The profiles are normalized to start from 0.

Figure 4: Cumulative wage growth: data and calibrated model

The model well captures the empirical life-cycle cumulative wage growth. The mean square errors are $MSE(HSG) = 0.004$ (s.d. 0.0002), and $MSE(CG) = 0.006$ (s.d. 0.0007) for high-school and college graduates, respectively.

In the next Section 5, I discuss the implied endogenous equilibrium distributions of offers. In Section 6, I decompose the log wage profiles above into the effects of job search, human capital accumulation, and human capital depreciation in unemployment.

5 Stage-specific equilibrium distributions of offers, $F^s(\theta)$

The stage-specific parameters of human capital and mobility determine the endogenous equilibrium distributions of offers in each stage. Appendix C contains all the technical details of the equilibrium solution, outlined in Section 3.3. Figures 5a and 5b below present the bounds of the equilibrium $F(\theta)$ for each of the three stages for the two education levels.
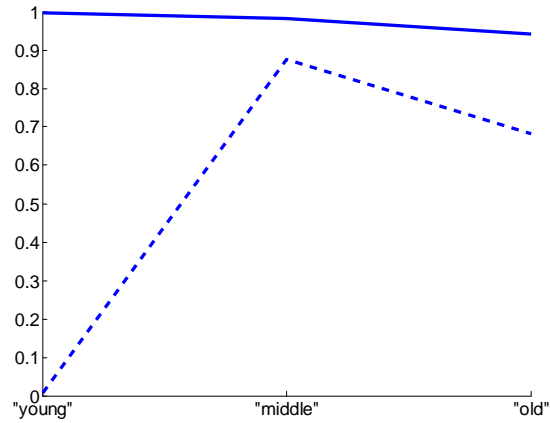
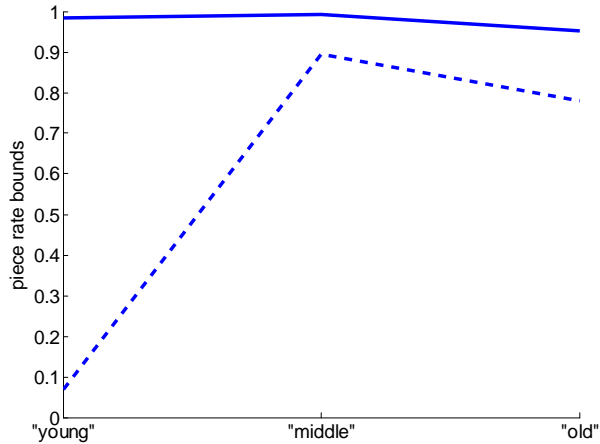**Figure 5a: College graduates: bounds of the equilibrium offers distribution**

Figure 5b: High-school graduates, bounds of the equilibrium offers distribution

There are marked differences between the distributions from which the workers in different stages sample wage offers, as Figure 5 illustrates. This is true for both education groups.

At the beginning of a career, human capital accumulation rate ρ is especially high and the expected lifespan is long, which makes employment very attractive since human capital is general and each additional unit accumulated will serve a worker throughout his career, increasing earnings. Offer arrive frequently on the

job, as reflected in high λ_1. Taken together, these factors drive down the reservation rate of the unemployed workers in the first submarket and, correspondingly, the lower bound in the equilibrium distribution $F^1(\theta)$. The fact that workers of "young" type are verymobile on-the-job increases the competition among the firms, increasing the highest equilibrium offer. As a result, the range of offers for the workers in their first stage of career is very wide. In mid-career, the expected horizon shortens, human capital accumulation slows down while skills losses in unemployment are still moderate, on-the-job search becomes less efficient (lower λ_1), while job-finding rate is not much lower than for the "young" (high-school graduates it is even higher). All this makes "middle-aged" unemployed workers value unemployment relatively more and they increase their reservation cutoff. Consequently, the range of offers contracts. Finally, those who reach the "old" stage of their life-cycle face far worse conditions in terms of the job-finding rate in unemployment λ_0, and, especially, human capital depreciation η. All this makes them lower their cutoffs again in the last stage, despite the fact that employment itself is associated with some loss of productivity - the conditions in unemployment are even worse. Therefore, relative to the "middle" stage of career, workers in the last stage sample offers from a distribution that is shifted down.

Comparing distributions of offers in the model to the data is tricky. First, piece rate offers can not be observed in the data. Second, the real distribution of offers is also unobserved - only accepted offers show up in the surveys. Given these constraints, I proxy the distribution of wages by the distribution of wages of low-tenured workers. The data are real hourly wages of white male full-time workers taken from the CPS Outgoing Rotation Groups for 1999 - 2006, see Appendix for details. I restrict the sample to employed workers who were unemployed in the previous month. For some potential experience levels there are only few observations of such newly-hired workers in the CPS data, and I bunch potential experience into eight 5-year intervals in order to increase the size of each bin. I compare these workers' wages with the wages of newly-hired workers in the simulation (the product of offered piece rate and unemployed worker's human capital). I use two statistics: the 5th percentile of the distribution, and the coefficient of variation. I chose the 5th percentile because as the figures above suggest much of the differences in offers distributions across stages happen in the lower part of the distribution. Comparing the coefficient of variation allows to abstract from the scale of wages in the data and in the model.
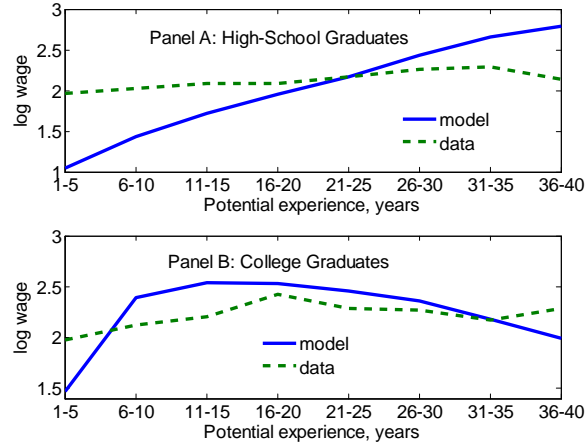
Figure: 5th percentile of log wage offers: model and data
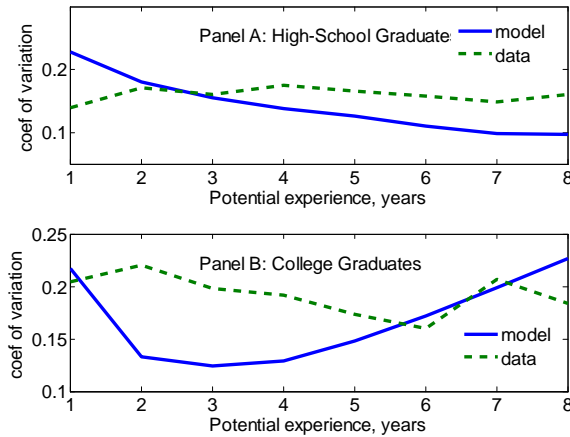

Figure: Coefficient of variation of log-wage offers

The figures above indicate that the observed wage offers distributions for college graduates change differently over career that those of high-school graduates. There is a slight hump-shape in the 5th percentile, and a U-shape in the coefficient of variation. The model reflects these non-monotonic patterns. For high-school graduates both the 5th percentile and the coefficient of variation stay almost constant over career, while the model predicts that they change monotonically. The results of the comparison should be treated with caution because there is very much noise in the data - even with bunching by 5-year intervals the number of just-hired workers in the CPS outgoing rotation groups is very small - on average around 200 points in each interval for high-school graduates, and around 60 points for college graduates (for comparison, in the simulated sample it is over 37,000 for high-school graduates and over25,000 for

college graduates).

In the next subsection, I simulate the model, letting the workers move across stages and sample from the equilibrium distributions, in order to decompose the average actual wage profile into the effects of job search, human capital growth, and human capital loss in unemployment.

6 Simulation and wage growth decomposition

I simulate an artificial sample of 100,000 workers, who start from the first stage, S_1, with a corresponding distribution of offers, and stochastically move on to later stages. I follow each worker's unemployment history, including, at each level of potential experience, current stage, employment status, actual productivity, productivity losses accumulated due to unemployment episodes up the present, and current piece rate in case of employment. Since the wage in the model is a combination of piece rate and actual productivity, which in turn is the product of total human capital accumulated, and total human capital lost, I can easily decompose the actual log wage of an employed worker in the simulation into the additive effects of mobility ($\ln \theta$), human capital accumulated, and human capital lost due to unemployment. In the following three sub-sections I analyze each of the components in detail.
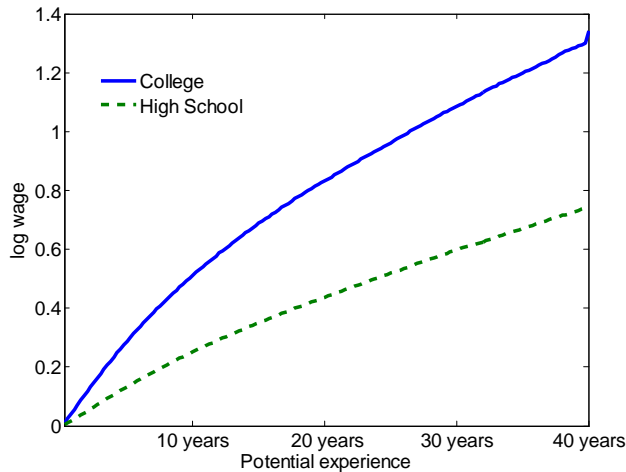
6.1 Human capital accumulation

Figure 6a: human capital accumulation

Figure 6a presents the productivity profile resulting from actual experience accumulation. One can see how the gap between college graduates and high-school graduates opens up over time. The reason is twofold. First, (positive)

returns to experience are always higher for college graduates than for high-school graduates. Therefore, a given level of actual experience will give more to college graduates in terms of human capital. Second, matches tend to last longer, and unemployment spells tend to be shorter for college graduates (according to calibrated λ_0 and δ) and, therefore, given potential experience, college graduates will have accumulated more actual experience than less educated workers. For college graduates, productivity growth due to actual experience accumulation accounts for 62 percent of wage increase over the first 10 years (44 percent for high-school graduates), and for 85 percent over 40 years (72 percent for high-school graduates). Human capital plays an important role in total wage increase. However, as will become clear shortly, its estimated role is still much more modest, than most previous studies have concluded.
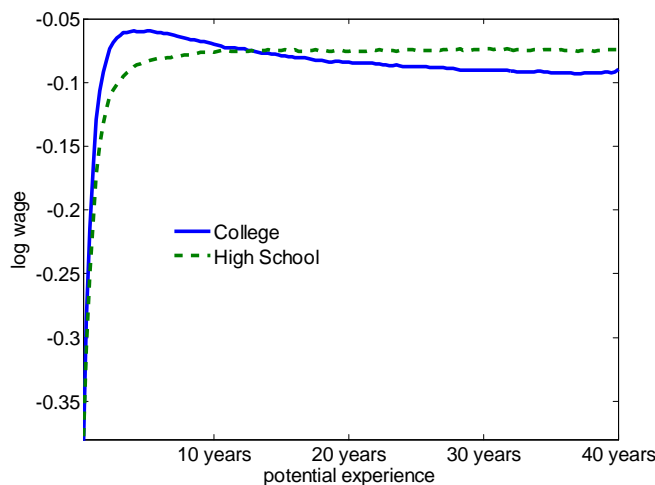
6.2 Mobility

Figure 6b: Mobility component of wages

Figure 6b presents log piece rate profiles for college and high-school graduates, i.e., the mobility component of log wages. Initially this component increases concavely, reflecting diminishing returns to search as workers succeed in climbing up the wide ladder of offers being still in their first career stage. For college graduates the average piece rate starts to *decrease* after initial rise, and drops by 0.03 log points till the end of career. Such a prediction can never arise in a framework where all workers sample job offers from one and the same distribution. Indeed, a common finding in the literature is that the average search component stays flat after an initial rise, because the returns to moving up the given wage ladder are quickly exhausted. Here, within each submarket, a similar dynamics applies; however, in the last stage, compared to mid-career, the entire

distribution of offers endogenously shifts down, especially for college graduates who lose a lot in terms of job-finding probability and ability to retain skills in unemployment as they become "old". As the share of workers entering the last stage of their career builds up, a decline in the average search component is produced.

For high-school graduates such an effect does not arise, because for them the conditions in the last career stage do not deteriorate so drastically. In fact for high-school graduates the endogenous distributions of offers in the last two stages do not differ much, it is approximately the same wage ladder, and the average log piece rate stays flat.

Over the first 10 years of career, search accounts for 57 percent of cumulative wage growth for high-school graduates (40 percent for college graduates), and over the life-cycle of 40 years its input is 30 percent for high-school graduates (19 percent for college graduates). Comparing these values with the results from other studies is not straightforward. First, different studies refer to different samples and different career horizons. Luckily, most studies include a subsample of high-school graduates, and regard the first 10 years of career as an important milestone. Second, not all the studies report estimates of the search input. It is very common, however, to report the estimated share of wage growth explained by the human capital component, and I will use this to infer about the role of search. The idea is that for each study, regardless of the assumptions about additional sources of wage growth, the share of human capital sets the upper bound for the combined role of all other factors of wage dynamics, including mobility. The higher the share of human capital accumulation, the lower the share that is potentially explained by search.

The table below summarizes the findings from several studies about the role of human capital accumulation in wage growth of high-school graduates in the U.S. over the first 10 years of their career, and the implied upper bound for the role of mobility (calculated as if it were the only additional factor of wage growth). The total wage growth (log points) is included, when reported by the authors, in order to highlight the comparability of different studies.

Table 3. The role of mobility in total wage growth. Comparison to other studies

HSG 10 years of career	Sample	Human Capital	Mobility (implied)	total log wage growth
Present study	**CPS**	**44%**	**56%**	**0.53**
Altonji et al. (2013)	PSID	74%	26%	0.51
Menzio et al. (2015)	SIPP	76%	24%	0.42
Schonberg (2007)	NLSY	72%	28%	0.55
Yamagouchi (2010)	NLSY	77%	23%	0.53
Buchinsky et al. (2010)	PSID	78%	22%	0.51
Bowlus and Liu (2013)	SIPP	12%	88%	not reported

Footnote after Menzio:Between 21 and 31 years.

As seen from the table, the implied input of mobility into wage growth ranges from 22 to 28 percent, apart from Bowlus and Liu (2013). My model predicts

a much higher upper bound of 56 percent, and the actual input is even higher, 57 percent, due to the negative returns to unemployment component. Below I explain the reason for this substantial input of mobility, and hypothesize about the reason of the discrepancy between my results and those (more extreme results) of Bowlus and Liu (2013).

An important factor boosting the role of mobility in search models is the low cutoff wage of the unemployed workers at the beginning of their career. When young workers start from lower wages, and are able to move job-to-job, then, other things equal, they will benefit a lot from search. This is exactly the mechanism in my model, where two complementary factors drive down the reservation cutoff of the young. The first factor is within-stage: an especially intensive human capital accumulation in the first submarket, combined with especially efficient on-the-job search,[23] make employment more valuable. The second factor is the effect of the shortening horizon, which enhances the relative attractiveness of employment even more. The workers take into account, in their value functions, that general human capital accumulated on-the-job will serve them in their entire subsequent career. From stage to stage this incentive weakens since the horizon over which the benefit of learning is reaped shortens. It is strongest in the first stage of career, making young unemployed workers accept very low offers, and this enhances the overall importance of mobility.

Though my estimate of the role of mobility is twice as high than in previous studies, it still falls short of the results of Bowlus and Liu (2013). This can be partly attributed to the endogeneity of the entire distribution of offers in my model, as opposed to the endogenous reservation cutoff in Bowlus and Liu (2013). Note that the span of increase in the search component of wage depends not only on the lowest offer from which the ascent starts, but also on the upper bound of offers. In the study of Bowlus and Liu (2013) the distribution of offers is exogenously set to be log-normal, with no upper bound, whereas in my model the upper bound is fixed and derived endogenously, based on the equilibrium constant profit condition. This may curb the role of search relative to the setting where, theoretically, the offers can be unboundedly high. That said, like the results of Bowlus and Liu (2013), my results highlight the fact that taking account of the interaction between search process and human capital process increases the estimated role of mobility for cumulative wage growth.

6.3   Human capital depreciation

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[23] λ_0^1 is also high, thereby increasing the reservation rate.
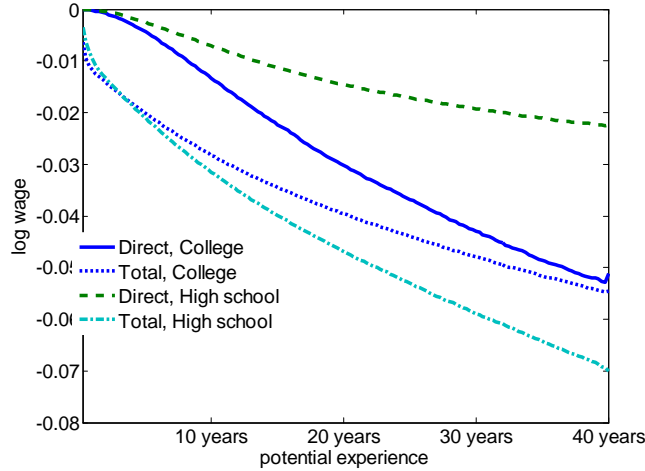
Figure 6c: losses from unemployment - loss of skills and foregone earnings

There are two sources of negative impact of unemployment on wages in the model. First, there is a direct loss due to human capital depreciation. Quantitatively, it is very small (see Figure 6c for illustration). The average direct losses amount to around 2 percent of cumulative wage growth over 40 years of career for high-school graduates and 3 percent for college graduates.[24] The direct component is more significant for college graduates, because they have a more intensive skills depreciation. Second, there is an indiret loss due to foregone human capital accumulation on-the-job. The total loss, summarizing both direct and indirect components, is non-negligible. For college graduates direct and total losses converge towards the end of career, because employment is not very different from unemployment for the "old" in terms of human capital. High-school graduates in the last stage are substantially better off when they have a job. For them indirect losses on top of skills depreciation only aggravate over the course of a career. Over 6 percent of total wage growth is lost on average for high-school graduates due to unemployment, after 40 years in the labor market (3 percent for college graduates).

Unemployment spells in the U.S. are typically short and rare, and therefore they are short and rare in the simulation as well. After 40 years of simulated career, an average college graduate will have spent 1 year and 3 months in unemployment (2 years and 1 month for a high-school graduate). These low averages conceal heterogeneity of histories: some workers accumulate much unemployment, while others accumulate almost none. By comparing average wage profiles for these different types of careers I am able to asses the cost for those workers who are unlucky enough to experience many periods off the job.

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[24]Notably, Altonji et al. (2013), the only study that estimates losses in general human capital due to cumulative unemployment spells, report losses that are -0.02 log points over the first 30 years, which is very close to my result.

Figure 7 below presents average log-wage profiles, for different types of careers, depending on how much unemployment history has been accumulated by the workers who have been in the labor market for 40 years.
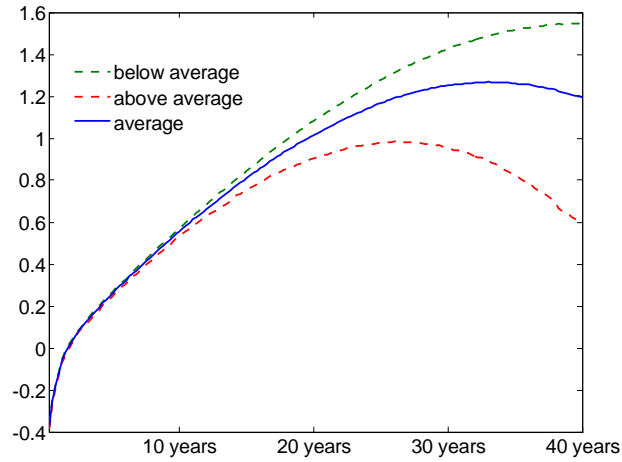

Figure 7a: College graduates: unemployment history and log-wage profiles
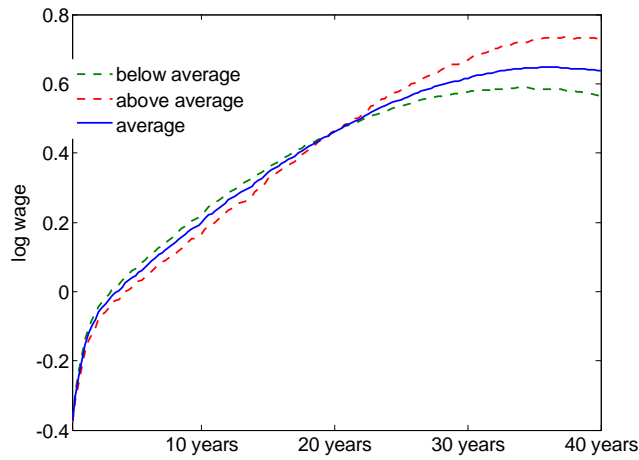
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Figure 7b: High-school graduates: unemployment history and log-wage profiles

Figure 7a shows that for college graduates with careers comprised of much and little unemployment the wage profiles look very different. The divergence

occurs in the second half of the career, where the wage profile of workers with longer unemployment histories bends down significantly relative to the average. For high-school graduates the picture is quite different - differences in unemployment histories are not translated into marked differences in average wage profiles.

To numerically evaluate the size of the damage from long unemployment histories, I perform the following simple calculation. First, I scale up the simulated hourly wage profiles so that they correspond to the actual ones not only in terms of wage growth, but also in levels. Technically it requires multiplying all wages by a constant, which can be treated in the model as a starting productivity upon entry into the labor market. The wage in the simulation is the hourly wage at the end of the quarter. I know the share of each quarter in a career that each employed worker actually spent in employment. With that information at hand, I obtain the total wage income earned by a worker over a quarter by multiplying the quarterly hourly wage by 40 (hours a week) $\times 13$(weeks per quarter)$\times$share worked. Then I discount quarterly earnings by a discount factor of $r = 0.0099$, and this gives me the present discounted value, for each worker, over 40 years of career. I do this for all the the workers who accumulate 40 years in the market. Table 3 summarizes the results of this exercise.

Table 3. Unemployment histories and lifetime earnings

Unemployment history	Share of all histories	Total, 2010 USD	% of average
College graduates			
average (1y3m)		1,659,000 (s.d. 515,130)	-
below average	61%	1,793,800 (s.d. 478,440)	+ 8.13%
above average	39%	1,443,200 (s.d. 498,060)	- 13.01%
High-School graduates			
average (2y1m)		715,000 (s.d. 102,980)	-
below average	57%	727,860 (s.d. 118,150)	+1.8%
above average	43%	698,190 (s.d. 75,640)	-2.3%

An average college graduate with 40 years of career will accumulate about 1 year and 3 months of unemployment, and his discounted income over 40 years will be around 1.6 million 2010 U.S. dollars. If he is lucky enough to be in 61 percent of his cohort who accumulate less than one year in unemployment, his discounted lifetime income will be 8 percent higher. If, on the other hand, he is one of the 39 percent of his cohort who accumulate longer than average unemployment histories over 40 years, his lifetime income will be lower by 13 percent. An average high-school graduate is expected to accumulate approximately 0.7 million 2010 dollars, and this number will not change significantly if he has a long or short unemployment history.

Life-cycle dynamics of job-finding and human capital depreciation rates stands behind these results. For college graduates, the job-finding rate is low in the last stage only, which implies that long unemployment spells are mostly

accumulated later in career, when unemployment is especially damaging for human capital. By contrast, high-school graduates have about the same chance to accumulate long unemployment spells when they are in the "young" and in the "old" stages of their life, based on their job-finding rate λ_0. Therefore, they do not necessarily accumulate much unemployment in stages when it's most damaging for their skills, and the rate of depreciation is in any case rrelatively moderate for them.

7   Conclusions

Thia paper explores the impact of job search, actual experience accumulation, and unemployment history on post-schooling wage growth of college graduates and high-school graduates in the U.S. I develop a novel approach to career, in that I regard it as a movement of workers across a number of stages that differ in terms of search and human capital technology. This approach reflects the idea that as workers age in the labor market, they face changing opportunities, in terms of mobility and ability to accumulate and retain their skills.

A central novel element in the analysis is the endogenous distribution of offers in different career stages. I am able to derive this distribution based on the assumption that workers in different stages participate in different labor submarkets. I find that this distribution changes substantially over the life-cycle. In particular, the range of offers is wide for the workers in their first career stage, then its support shrinks, and later in career the distribution shifts down. Correspondingly, I find that the role of mobility in wage growth is substantial in the first decade of a career (100 percent higher than in previous studies), and later in life, especially for college graduates, mobility becomes a negative factor of wage growth, as more and more workers move to search in the low range of offers.

The assumption of human capital depreciation in unemployment plays an important role in equilibrium through its impact on the reservation cutoff of the unemployed - in particular, it drives down reservation wages of the unemployed workers later in career. The direct impact of an average unemployment history on cumulative wage growth is very small. However, when foregone human capital accumulation is taken into account, losses become non-negligible and amount to 6.6 percent of total wage growth for high-school graduates, and 3.1 percent for college graduates. Though their average losses are smaller, and lifetime earnings are higher, college graduates' wage profiles are more sensitive to unemployment history, than those of high-school graduates, because of the differences in the typical timing and length of unemployment episodes in conjunction with the damage incurred to human capital.

The analysis in this paper highlights the fact that the role of mobility in wage growth hinges on the lowest wage in the market, namely, the level of the

reservation cutoff of the unemployed. It is only when the lowest acceptable offer is sufficiently low that mobility becomes a significant channel of life-cycle wage increase (other things held equal). The issue of low reservation wage is closely related to another aspect of the validity of search models: their ability to generate sufficient residual wage dispersion (wage differences among observationally similar workers). As noted by Hornsten et al. (2011), search models can generate enough frictional wage dispersion[25] only if there is a reason for the unemployed to lower their reservation cutoff even in the presence of the high search option. In response to the analysis in Hornsten et al. (2011), a small body of literature has emerged, looking for mechanisms that could do the job; see, e.g., Burdett et al. (2011), Carrillo-Tudela (2012), Ortego-Marti (2012), and Tjaden and Wellschmied (2014).

Jolivet et al. (2006) write that "job search models of the labor market hypothesize a very tight correspondence between the determinants of labor turnover and individual wage dynamics on one hand, and the determinants of wage dispersion on the other." The analysis above also suggests, that the ability of search mechanism to be an important factor of life-cycle wage growth, and its ability to generate sufficient wage dispersion among observationally similar workers seem to be inter-related, with the reservation wage of the unemployed being a key factor for both.

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