Intermediary Search for Suppliers in Procurement Auctions

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Abstract: In many procurement auctions, entrants determine whether to participate in auctions based on their roles as intermediaries in order to search for the best (or the cheapest) input suppliers. We build on a procurement auction model with entry, combined with intermediary search for suppliers. The model endogenously generates costs for bidders through strategic supplier pricing. We show the existence of an equilibrium with price dispersion for inputs in which costs can be heterogeneous among bidders. Interestingly, the procurement cost may rise as the number of potential bidders increases.

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In many procurement auctions, entrants determine whether to participate in auctions based on their roles as intermediaries in order to search for the best (or the cheapest) input suppliers. We build on a procurement auction model with entry, combined with intermediary search for suppliers. The model endogenously generates costs for bidders through strategic supplier pricing. We show the existence of an equilibrium with price dispersion for inputs in which costs can be heterogeneous among bidders. Interestingly, the procurement cost may rise as the number of potential bidders increases.

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1 Introduction

Today most public procurements are conducted through auctions, for which recent empirical studies have shown relevant evidence commonly observed across several countries. Similar to wages of workers (e.g. Mortensen, 2005), it is well known that prices of raw materials are widespread (e.g. Roberts and Supina, 2000), even for homogeneous goods (e.g. ready mixed concrete). As for public procurement auctions, we have observed cost heterogeneity among bidders (see, e.g., Decarolis (2013) for procurements of road construction and maintenance in northern Italy, Gugler, Weichselbaumer and Zulehner (2015) for nearly all procurements in the construction sector in Austria, and Krasnokutskaya (2011) for Michigan highway procurements.), and have known that potential bidders are capable of finding out how many potential competitors exist in an auction by submitting their proposals, causing unwanted outcomes that over half of bidders tend to drop out without bidding before the auction takes place (see, e.g., Li and Zheng (2009) for highway mowing auctions in Texas). Case studies of bidding rings or cartels in public procurement auctions held in Japan have suggested that it is significantly costly for bidders to gather supplier information used for estimating procurement costs (e.g. Kawai and Nakabayashi, 2014).[1]

This paper attempts to provide a theoretical framework incorporating the facts given above. In doing so, we consider a market with three types of agents: (i) a single buyer; (ii) intermediaries who determine whether to enter the market, where they compete to supply the final good to the buyer; (iii) suppliers who sell their homogeneous inputs to intermediaries who have entered the market. The buyer chooses a bidder (an intermediary) to procure a single unit of some good by holding an auction, where the lowest bidder wins the contract. Intermediaries are imperfectly informed about supplier charged input prices, so they must incur search costs to buy from a supplier, prior to the auction. A crucial feature is that a supplier can only earn profit if the intermediary to whom she sells wins the auction. This gives a supplier an incentive to restrain the prices she charges intermediaries. Entrants of intermediaries participate in the auction, serving as bidders to win the supply contract. The novelty of this paper is that costs of bidders are endogenously determined by their search among suppliers.

Formally, we introduce the following game for analyzing the above mentioned market. At the first stage, intermediaries decide whether to enter the market, which

[1] Typically, only a predetermined winner among cartel members incurs the cost of estimating procurement cost, while other members avoid it because of huge costs in assessing projects for details (see Kawai and Nakabayashi, 2014, footnotes 20 and 21).

is costly due to search and entry costs. The important point at this stage is that the number of potential entrants (or potential bidders) is commonly known to all agents, but every single bidder's entry decision is unobservable to all other agents. At the second stage, suppliers simultaneously charge per-unit prices of their homogeneous goods with production cost that is normalized to zero for simplicity. Together with the above stated crucial feature of the supplier-intermediary contract, another important point is that supplier prices are fixed and not negotiable afterward.[2] At the third stage, intermediaries who have entered the market (or entrants) engage in costly sequential search for acquiring supplier price information, where they pay constant cost per search to learn the price of one supplier.[3] At the fourth stage, given that all entrants have stopped searching, they simultaneously participate in a procurement auction, where the lowest bidder is chosen to supply the final good to the buyer. In the main analysis, we consider the simplest case where there are two suppliers and two intermediaries.[4] We assume that the auction is conducted through the open auction. In line with the literature on public procurement auctions, we usually consider a first price (sealed bid) auction instead of the open auction (e.g. Pesendorfer, 2000). Although the analysis in case of the first price auction gets involved, we can partially extend our analysis by deriving a closed-form (symmetric) bidding strategy that is strictly increasing in costs of bidders, which contrasts with the analysis in the standard first price auction where it is hard to obtain a closed-form bidding strategy in general.[5]

As a benchmark of this game, consider the case where there are no search and entry costs for intermediaries. In the absense of those costs, competition would force the auction-winning bid to be equal to marginal cost, because all intermediaries enter the market and can find out who is the cheapest supplier by comparing all supplier prices as in Bertrand competition, so suppliers charge (common) marginal cost, and

[2]Here we consider a posted (or uniform wholesale) price market upstream, where suppliers have *full* bargaining power against intermediaries. It is reasonable to incorporate bargaining power of intermediaries into this model, but is beyond the scope of this paper. It would be interesting in future research to address this bargaining issue.

[3]When engaging in search, intermediaries cannot observe which suppliers other intermediaries contract with. The optimal search rule used here is *sequential search* (e.g. Stahl, 1989), but it might be reasonable to consider *fixed sample size search* (Burdett and Judd, 1983) where entrants determine how many suppliers they visit prior to search, because gathering supplier information is time consuming, so that contacting multiple suppliers simultaneously may be optimal for intermediaries (Morgan and Manning, 1985). In fact, we can extend our argument under sequential search to the case of fixed sample size search. We will discuss it below for details.

[4]We also consider the case where there are multiple suppliers or multiple intermediaries for accounting for the firm number effect on market outcomes through numerical analysis.

[5]The reason we can derive a closed-form solution in our model is that the equilibrium condition for intermediary search pins down a cost distribution of bidders explicitly. Below we discuss it for more details.

then all intermediaries (bidders) have the same cost as commonly expected, and therefore a single bidder ends up winning the auction at supplier marginal cost. Taking this frictionless case as given, the purpose of this paper is to analyze the impacts of costly search and entry on market outcomes.

There are three main results. We first show that pure strategy equilibria in this game exist merely as degenerate equilibria in which the market "breaks down" – i.e., when intermediaries do not buy from suppliers and the good is not actually produced. This happens because suppliers charge a sufficiently high price, which deters entry of all intermediaries, so the auction does not take place. In line with the consumer search literature, this is the Diamond paradox outcome (Diamond, 1971; Burdett and Judd, 1983) in our framework. Secondly, and more importantly, there are other equilibria in which the good is produced through the auction. These equilibria are shown to (i) exist when the value (buyer's valuation of the good) to search cost ratio is sufficiently large; (ii) feature price dispersion whose range is increasing (resp. decreasing) with the size of the value (resp. search cost), possibly generating cost heterogeneity among bidders;[6] (iii) involve excess entry from a welfare perspective. The reason the usual Diamond paradox type outcome is not an equilibrium here is that if it were, a supplier would have an incentive to lower the price in order for her intermediary to win the auction. This incentive is unique to the procurement auction setting. Thus, in equilibria in which the final good is produced, suppliers randomize over input prices and entrants sign contracts with (possibly) different suppliers who charge different prices, therefore their bids differ and the entrant who finds the lowest priced supplier makes the lowest bid and wins the auction. Lastly, allowing for multiple suppliers or multiple intermediaries, our numerical analysis demonstrates a simple relationship between (supplier) price dispersion and the number of suppliers and intermediaries. As the number of intermediaries (resp. suppliers) increases, the average price increases (resp. decreases) and the range of price dispersion increases. The driving forces behind this are *two countervailing effects*: on the one hand, when there are more interemediaries, suppliers gain larger seller power against intermediaries and attempt to shift a price distribution upward, but an upward shifted price distribution can substantially reduce entry incentives of intermediaries. To sustain moderate entry, suppliers should keep the lower bound of a shifted price distribution low; on the other hand, when there are more suppliers, the opposite happens. Suppliers lose seller power against intermediaries and are forced to shift a price distribution downward, but entry in-

[6] In the limiting case as the search cost goes to zero given no entry cost, the supplier price distribution is concentrated on supplier marginal cost, so the market converges to the frictionless case.

creases by the price shift. Considering a possibility that all entrants visit a single supplier and stop to buy, each supplier can be a monopolist with larger power due to increased entry, thereby setting the upper bound of a shifted price distribution to be higher than before. In addtion, the numerical analysis suggests a simple relationship of procurement costs with the number of intermediaries and suppliers: the (expected) procurement cost increases as the number of intermediaries (or potential bidders), while it decreases as that of suppliers increases. We usually think that more potential bidders lead to more entry, which fosters competition among bidders, resulting in reduced procurement costs. But our result implies the opposite, indicating that the number of suppliers plays a crucial role in assessing the relationship of procurement costs with the number of potential bidders.[7]

The contributions of this paper are summerized as follows. We add intermediary search for suppliers to a procurement auction model with entry, and then analyze its impact on market outcomes. Our analysis provides two interesting points. One of them is that our model endogenously generates cost heterogeneity among bidders in auctions, and the second is a simple relationship between procurement costs (through price dispersion for inputs) and market concentration measured by the number of upstream and downstream firms. These points are mainly relevant for two strands of literature, one of which is the consumer search literature and the procurement auction literature.

The most closely related paper in the consumer search literature is García, Honda and Janssen (2015) who consider both retailer- and consumer search in the otherwise standard vertical oligopoly model. Their model is different from ours in terms of two components: (i) there is one more level of search frictions at a downstream market for a representative buyer (meaning a unit mass of consumers in their model) to acquire retail price information, together with search frictions at an upstream market for intermediaries (retailers) to acquire supplier (manufacturer) charged price information; (ii) oligopolistic price competition (with endogenous quantities) takes place at the downstream market. In contrast, in our model there is one level of search frictions at the upstream market and price competition downstream takes place through the auction, where a single buyer purchases a single good (fixed quantity). Although their model is different from ours, both models have the common feature that price dispersion emerges at both the upstream and downstream markets. This implies that suppliers generate retail cost uncertainty *endogenously* in the absence

[7]The relationship between the winnig bid and number of bidders has long been investigated theoretically and empirically in the auction literature (see, among others, Samuelson (1985), McAfee and McMillan (1987), Levin and Smith (1994), and more recently Gentry and Li (2014)), where we assume that costs of bidders are exogenously given in the absence of suppliers.

of exogenous uncertainty, which strikingly differs from the literature where we account for inflation and exchange rate or production shocks through *exogenous* cost uncertainty to investigate its impact on price dispersion at the downstream market (see, e.g., Bénabou and Gertner, 1993; Dana, 1994; Fishman, 1996; Tappata, 2009; Janssen, Pichler and Weidenholzer, 2011).

In line with the procurement auction literature, we provide two relevant points to recent empirical studies of public procurement auctions. One of them is empirically observed cost heterogeneity among bidders (Decarolis, 2013; Gugler et al., 2015; Krasnokutskaya, 2011). Our model endogenously generates cost heterogeneity through input price dispersion caused by search frictions. The second is the relationship between the procurement costs and the number of potential bidders. Li and Zheng (2009) shows that the relationship may be *non-monotonic*, because of the countervailing *entry* effect against the "competition effect".[8] By contrast, our paper provides a possible theoretical justification to complement their finding of the non-monotonic relationship, by accounting for search frictions of potential bidders in finding suitable inputs.

The rest of our paper is organized as follows. In Section 2 we introduce the model and derive the basic properties that equilibria satisfy. Our main analysis and results are contained in Sections 4. Section 5 presents an example to overview the main analytical results. Section 6 discusses the relationship between market concentration and price dispersion. Section 7 concludes.

2 Model

We consider a market with three types of agents: (i) a single buyer; (ii) $n(= 1, 2, \dots)$ intermediaries who determine whether to enter the market, where they compete to supply the final good to the buyer; (iii) $m(= 1, 2, \dots)$ suppliers who sell their homogeneous inputs to intermediaries who have entered the market. The buyer chooses a bidder (an intermediary) to procure a single unit of some good by holding an auction, where the lowest bidder wins the contract. Intermediaries are imperfectly informed about supplier charged input prices, and so must incur search costs to buy from a supplier. Entrants of intermediaries participate in the auction, serving as bidders to win the supply contract. Suppliers produce their goods at a constant

[8]The entry effect here means that a winning bidder may believe that they overestimate the intensity of entry, and it may outweigh the competition effect, resulting in a higher winning bid as the number of (potential) bidders increases. There are other representative countervailing effects known as *winner's curse* effect (Bulow and Klemperer, 2002) and *affiliation* effect (Pinkse and Tan, 2005).

unit cost that is normalized to zero.

We analyze this market by introducing the following game, where we add the role of search for inputs to bidders (as intermediaries) in a procurement auction model with entry. At the first stage, each intermediary (he) decides whether to enter the market, which is costly due to search and entry costs. The important point at this stage is that the number of potential entrants who serve as bidders is commonly known to all agents, but every single bidder's entry decision is unobservable to all other agents. At the second stage, each supplier (she) simultaneously charges per-unit prices of their homogeneous goods with production cost that is normalized to zero for simplicity.[9] Most importantly, a supplier can only earn profit if the intermediary to whom she sells wins the auction.[10] This gives a supplier an incentive to restrain the prices she charges intermediaries. Another important point is that we consider a posted price market upstream, where supplier charged prices are fixed and are not negotiable afterward. At the third stage, intermediaries who have entered the market (or entrants) engage in costly sequential search for acquiring supplier price information, where they pay constant cost per search, $s > 0$, to learn the price of one supplier. At the fourth stage, given that all entrants have stopped searching, they simultaneously participate in a procurement auction, where the lowest bidder is chosen to supply the final good to the buyer. We assume that the buyer procures a single unit good and the buyer's valuation of the good (henceforth, the value) is given by $v > 0$, which is known to all suppliers and intermediaries prior to the auction, and that the procurement auction is conducted through the open auction under which the auction begins with the known buyer's valuation v, decreasing the asking price from v to a lower price continuously, and a bidder will be a winner if he keeps participating in the auction without leaving and turns to be a single participant in the auction, and the winner pays the price determined when turning to be the single participant. If there are multiple bidders who compete in the auction, the bidder who has the lowest marginal cost wins with paying the second lowest marginal cost of bidders, whereas if the bidder is a single participant in the auction, the bidder wins with v. It is reasonable to assume that the value is larger than the search cost, that is, $v > s$, otherwise entry is not profitable for intermediaries due to costly search, so we assume that $v > s$ holds below. Here the buyer and bidders are assumed to be

[9]We assume that suppliers produce goods without fixed costs, and that there is no additional cost for intermediaries to sell purchased goods from suppliers to the buyer.

[10]We can consider that a supplier and an intermediary make their transaction based on a kick-buck policy of a contract that an intermediary can give the unsold good back to an supplier without its payment or a contract to guarantee that at any given point in time an intermediary can buy the good at a price charged by the supplier and sell it to the buyer. Thus, suppliers' profits highly depend on visiting intermediaries' sales at the downstream market.

risk neutral.

Intermediary Search

Since each intermediary does not know supplier charged prices ex-ante, he randomly visits one of the suppliers by incurring search cost s to obtain a price quotation. If he continues searching, he pays an additional search cost s to randomly visit an another supplier whom he has not visited before. We assume search with free recall, that is, after an intermediary continues to search, he can go back to the supplier whom he has visited before without any cost.[11]

An intermediary's search strategy is characterized by a reservation price strategy. The optimality of the reservation price strategy for intermediaries holds here, because intermediaries have nothing to learn in this model.[12] The reservation price strategy is basically a binary-choice strategy: there exists a reservation price ρ such that if an intermediary observes a price $w \leq \rho$, he buys immediately at that price, otherwise he continues to search for an additional price.[13] So, the reservation price strategy is defined by

$$\sigma(\rho) \quad = \quad \begin{cases} \text{buy at } w, & \text{if } w \leq \rho, \\ \text{continue to search}, & \text{otherwise.} \end{cases} \tag{1}$$

Equilibrium Concept

The equilibrium concept used in this paper is symmetric *Perfect Bayesian Equilibria* with passive beliefs (e.g. McAfee and Schwartz, 1994).[14] To formally define equi-

[11]The free recall is often assumed explicitly or implicitly in the consumer search literature. Janssen and Parakhonyak (2014) is the first among others to explore what happens if we incorporate costly revisits into a general sequential search model and show that the optimal sequential search rule can substantially differ from the reservation price rule under free recall, but they show that the reservation price equilibrium in the model of Stahl (1989) still remains even under costly revisits mainly because all consumers stop to buy at the first visiting store. In this paper, we consider the same type of the reservation price equilibrium where all intermediaries who have entered the market stop to buy at the first visiting store (supplier).

[12]See Rothschild (1974) for an observation on the (non-)optimality of the reservation price rule when the search environment is (not) stable. In an environment without learning, a reservation price strategy is optimal (Kohn and Shavell, 1974).

[13]If an intermediary visits all suppliers whose prices are above ρ, he buys at a supplier who sets the lowest price among all suppliers. Notice that by the free recall assumption, going back to a supplier whom he has previously visited is costless.

[14]Passive beliefs used in our model are out-of-equilibrium beliefs such that when observing a supplier's deviation from the equilibrium prices, the visiting intermediary believes that other suppliers still stick to charge the equilibrium prices. If we do not restrict out-of-equilibrium beliefs, most of results in the consumer search literature (e.g., the Diamond Paradox (Diamond, 1971) and price dispersion in Stahl (1989)) does not hold in general. This is because we can construct

libria, we introduce the following notation. We denote by $F(\cdot)$ a supplier adopted price distribution with compact support $[\underline{w}, \overline{w}]$ for $0 \leq \underline{w} \leq \overline{w}$ and by $\alpha \in [0,1]$ the intermediary entry probability (entry rate). The search strategy of intermediaries is determined by a reservation price strategy as discussed above. An intermediary's strategy in the open auction can be reduced to a pure bidding strategy $b(\cdot)$ as a unique undominated strategy such that (i) if intermediary $i = 1, \ldots, n$ is a single participant in the auction, he bids v; (ii) if he competes with others in the auction where his marginal cost is w_i, others' costs are w_{-i}, and the lowest cost among others is w^l_{-i}, his bid will be $\min\{w^l_{-i}, v\}$ if $w_i \leq w^l_{-i}$ and w_i otherwise. That is, a bidding strategy in the auction is reduced to

$$b(w_i, w_{-i}) \quad = \quad \begin{cases} \min\{w^l_{-i}, v\}, & \text{if } w_i \leq w^l_{-i}, \\ w_i, & \text{otherwise.} \end{cases} \tag{2}$$

Our equilibrium concept used in this paper is defined as follows.

Definition 1. A symmetric Perfect Bayesian Equilibrium is a supplier price distribution $F(\cdot)$ and an intermediary entry, search, and bidding strategy $(\alpha, \sigma(\rho), b(\cdot))$ such that $\sigma(\rho)$ is defined by (1) and $b(\cdot)$ by (2), and intermediaries have passive beliefs on supplier prices off the equilibrium path.

3 Basic Properties of Equilibria

As benchmarks of our main analysis, we provide basic properties that equilibria satisfy. First of all, we show that there is a degenerate equilibrium where all intermediaries do not enter the market, so the auction does not take place, and therefore the market would fail to exist. In doing so, suppose that there is no intermediary entry, that is, $\alpha = 0$. If suppliers charge any symmetric price $w \in [0, v - s)$, an intermediary has an incentive to unilaterally deviate to enter the market and then get a profit $v - s - w > 0$ by purchasing at w with incurring search cost s and selling at the highest possible price v in the auction due to the open auction rule. This implies that there is no equilibrium such that suppliers charge any single price below the

a parsimonious out-of-equilibrium belief instead of passive beliefs. For example, consider the Diamond Paradox set-up where multiple firms charge prices to (a continuum of) consumers who commonly incur a positive search cost to observe a single price quotation. When observing an unexpectedly higher price than the marginal cost, consumers can have a negatively correlated belief in that the other firms charges a much lower price to compensate at least the search cost, thereby continuing to search if the unexpected price is slightly higher than the search cost. Therefore, charging marginal cost can be an equilibrium price strategy. For related issues, for instance, see García, Honda and Janssen (2015) and Janssen and Shelegia (2015).

price $v - s$ and all intermediaries do not enter the market. In contrast, if suppliers charge a high price $w \geq v - s$, any intermediary has no incentive to enter the market because he cannot gain a positive profit due to costly search even when winnning at v in the auction. Also, each supplier does not have an incentive to deviate to charge a lower price than $v - s$, because charging any price does not change beliefs of intermediaries on supplier prices off the equilibrium path, and therefore all intermediaries keep staying out of the market even if the supplier deviation takes place.[15] Thus, there is an equilibrium where the suppliers charge a high price $w \geq v - s$ and all intermediaries do not enter the market.

Proposition 1. *There is an equilibrium such that all suppliers adopt a pure strategy $w \in [v - s, v]$ and all intermediaries do not enter the market.*

This implies that there exists a continuum of the market breakdown equilibria where suppliers can deter intermediary entry by charging a sufficiently high price and then the auction does not take place due to no entry. This result does not hold if the first search is costless, because a supplier can only earn profit if the intermediary to whom he sells wins the auction. In case of costless first search, there is no market breakdown equilibrium.

Next, we show that there is no equilibrium without price dispersion, except for the market breakdown equilibria shown above, First, let us consider the case where all suppliers commonly charge marginal cost, thereby getting zero profit. If a supplier deviates to charge a slightly higher price, it is profitable for her, as it can be the case that all intermediaries who have entered the market first visit her through random search and stop to buy, which occurs with positive probability. This implies that marginal cost pricing is not an equilibrium strategy for suppliers. Next, let us consider the case where all suppliers charge any symmetric price $w > 0$ above marginal cost. The key to understanding why those suppliers have an incentive to cut the price is that a supplier can only earn profit if the intermediary to whom she sells wins the auction. Suppose that a supplier slightly reduces the price to $w - \varepsilon$ for a sufficiently small $\varepsilon > 0$. If there are entrants who visit her, one of them will be a winner in the (open) auction because other entrants who visit other suppliers to buy (if any) must have higher costs, , which makes the deviating supplier's expected

[15] If we strengthen the equilibrium concept and consider a *strong* Perfect Bayesian Equilibrium, no entry cannot be part of an equilibrium. Assume to the contrary that $\alpha = 0$ and $w \in [v - s, v]$ are part of an equilibrium. Off the equilibrium path where a supplier deviates to a lower price, an intermediary has a consistent belief off the equilibrium path, thereby willing to enter the market and then get a positive payoff by visiting the deviating supplier, which results in a profitable deviation. Thus, $\alpha = 0$ and $w \in [v - s, v]$ cannot be sustained as part of a strong Perfect Bayesian Equilibrium.

profit larger than under the price w with splitting a profit among others. Competing with other suppliers is more likely to happen than being the monopolist if both intermediaries enter the market, because initially intermediaries randomly search for suppliers. More precisely, the former case happens with probability $\frac{1}{2}$, whereas the latter case with probability $\frac{1}{4}$, given that both intermediaries enter the market, which happens with probability α^2. So, competition among intermediaries gives suppliers an incentive to undercut any symmetric price above marginal cost. It turns out that charging any single price is not optimal for suppliers.[16]

Proposition 2. *There is no other equilibrium such that suppliers adopt a pure strategy profile except for a continuum of equilibria given in Proposition 1.*

This implies that "the 'law of one price' is no law at all" (Varian, 1980, p.651) and instead price dispersion necessarily emerges if there exists partial entry.

The driving force of price dispersion in our model is reminicent of the paper by Janssen and Rasmusen (2002) who consider Bertrand competition with (exogenous) uncertainty of number of entrants, whereby each firm believes by introduced uncertainty that they can be a monopolist, while competing with other potential entrants, which expresses an essential part of our mechanism to generate price dispersion. In our model, however, uncertainty of number of (intermediary) entrants is endogenously determined by strategic supplier pricing.

4 Equilibrium Analysis

In this section, we consider the case of two suppliers and two intermediaries, while in the following section we will discuss through numerical analysis the impact of the number of suppliers and intermediaries on supplier pricing and procurement costs.

Assume that suppliers adopt a price distribution F with compact support $[\underline{w}, \overline{w}]$ and intermediaries enter the market with probability $\alpha \in (0, 1]$. Since we consider an equilibrium where intermediaries use a reservation price strategy, the upper bound of the support, $\overline{w}$, should equal the reservation price ρ as an optimal strategy for suppliers.[17] To derive the expected profit of a supplier when charging any given price $w \in [\underline{w}, \overline{w}]$, consider two cases: (i) both of two intermediaries enter the market, which occurs with probability α^2; (ii) only one intermediary enters the market, which

[16] We consider only symmetric pure strategies for suppliers but we can extend Proposition 2 to all pure strategies, therefore showing that any (possibly) asymmetric pure strategy is not an equilibrium strategy for suppliers.

[17] This is a commonly used argument in the consumer search literature. For the basic arguments used in consumer search models, see, for instance, Varian (1980) and Stahl (1989). For an overview of the consumer search literature, see Baye, Morgan and Scholten (2006).

occurs with probability $2\alpha(1-\alpha)$. In a reservation price equilibrium, suppliers charge prices such that no intermediary searches more than once. In Case (i), if both of them visit the same supplier, which occurs with probability $(\frac{1}{2})^2$, she can capture the whole demand no matter how intermediaries compete in the auction; if one of them visits her and the other intermediary visits the other supplier, which occurs with probability $\frac{1}{2}$, the intermediary who visits her will win the auction if the other supplier price is above w, which occurs with probability $1 - F(w)$; if no one visits her, which occurs with probability $\frac{1}{4}$, she gets nothing. Similarly, in Case (ii) where only one intermediary enters the market, if the intermediary visits her, which occurs with probability $\frac{1}{2}$, she gains the whole demand; otherwise, she gets nothing. Since the expected profits over all prices in the support must be equal as an equilibrium strategy for suppliers, it follows

$$\Big(\alpha^2\big(\frac{1}{4} + \frac{1}{2}(1 - F(w))\big) + \alpha(1-\alpha)\Big)w \;=\; \Big(\frac{\alpha^2}{4} + \alpha(1-\alpha)\Big)\overline{w}. \tag{3}$$

Next, we consider the condition on sequential search of intermediaries as follows. First notice that in a reservation price equilibrium, the upper bound $\overline{w}$ equals the reservation price ρ and the expected gain of continuing to search when observing $\overline{w}$ at the first visiting supplier is the largest among all supplier prices, which must be below or equal to the search cost, otherwise a visiting intermediary continues to search. Suppose that an intermediary who has entered the market first visits a supplier who charges the highest price $\overline{w}$. In deriving the expected profit of that intermediary when continuing to search for a lower price, consider three cases: (i) the other intermediary enters the market and visits the same supplier, which occurs with probability $\frac{\alpha}{2}$; (ii) the other entrant visits a different supplier, which occurs with the same probability $\frac{\alpha}{2}$; (iii) the other intermediary does not enter the market, which occurs with probability $1-\alpha$. In Case (i), since the other supplier randomizes over $[\underline{w}, \overline{w}]$ through distribution F and the expected price charged by that supplier is $\int_{\underline{w}}^{\overline{w}} w dF(w)$, the gain of continuing to search is given by $\overline{w} - \int_{\underline{w}}^{\overline{w}} w dF(w)$. In Case (ii), he visits the supplier whom the other intermediary has visited and then both intermediaries compete in the auction under the same marginal cost, which means that continuing to search is not profitable. In Case (iii), since his marginal cost becomes lower to be $\int_{\underline{w}}^{\overline{w}} w dF(w)$ and there is no competition between intermediaries in the auction, the expected gain is $\overline{w} - \int_{\underline{w}}^{\overline{w}} w dF(w)$. Suppliers maximize their expected profits by exploiting intermediaries through search frictions as much as possible, thereby imposing the following condition on input prices.

$$\Big(\frac{\alpha}{2} + (1-\alpha)\Big)\Big(\rho - \int_{\underline{w}}^{\rho} w dF(w)\Big) \quad = \quad s. \tag{4}$$

Lastly, we consider the market entry decision of intermediaries when suppliers adopt a price distribution F such that the two conditions (3) and (4) hold. To do this, we first need to derive the expected gain of an intermediary when both intermediaries who have entered the market visit different suppliers and then stop to buy at those suppliers via the reservation price strategy. In this case, each intermediary gets the following expected gain in the auction.

Lemma 1. *The expected revenue in the auction is given by* $\int_{\underline{w}}^{\overline{w}}(1-F(w))F(w)dw$.

The proof of Lemma 1 is given in the Appendix. Using Lemma 1, we analyze the entry benefit and cost of each intermediary as follows. To derive the expected entry profit of an intermediary, consider the same three cases as in Condition (4). In Case (i), since the other intermediary visits the same supplier, both intermediaries have the same marginal cost and compete in the auction, resulting in zero profit due to the open auction, whereas in Case (ii), since the other intermediary visits the other supplier and marginal cost of each intermediary is considered to be randomly drawn from the distribution F, the expected gain is given by $\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw$ due to Lemma 1. In Case (iii), since there is no competition in the auction, he obtains the whole demand v under the (expected) cost of $\int_{\underline{w}}^{\rho} wdF(w)$, resulting in an expected gain of $v - \int_{\underline{w}}^{\rho} wdF(w)$. Since search cost s must equal the entry benefit under an entry rate $\alpha \in (0,1)$ in an equilibrium where intermediaries are indifferent between entering and not entering, we consider the following condition for entry.[18]

$$\frac{\alpha}{2}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw + (1-\alpha)\Big(v - \int_{\underline{w}}^{\rho} wdF(w)\Big) \quad = \quad s. \tag{5}$$

In case of full participation $\alpha = 1$, the equilibrium condition on sequential search is that the LHS of (5) is larger than or equal to the RHS.

We consider a reservation price equilibrium such that suppliers adopt the mixed strategy of $F(\cdot)$ with support $[\underline{w}, \overline{w}]$, intermediaries follow the strategy of $(\alpha, \sigma(\rho), b(\cdot))$ where $\alpha \in (0,1]$, $\sigma(\rho)$ with $\rho = \overline{w}$ is defined by (1), and $b(\cdot)$ is defined by (2), and the above considered three conditions (3), (4), and (5) simultaneously hold. By the construction of the strategies, it is optimal for both suppliers and intermediaries

[18] We can easily incorporate a common transaction fee between a supplier (or an buyer) and an intermediary or a common market entry fee by adding a corresponding term to the RHS of Condition (5).

to follow their strategies, and there is no profitable deviation for all suppliers and intermediaries.

Using the three conditions (3), (4), and (5) for some parameters (v, s) exogenously given, we derive a dispersed price equilibrium. Mechanically speaking, (3) determines the price distribution F given $(\underline{w}, \overline{w}, \alpha, s)$ where $\overline{w} = \rho$, by which (4) pins down ρ (together with $\underline{w}$) as a function of (α, s), and then (5) determines α as a function of exogenous parameters (v, s). In fact, by (3) and (4), the price distribution and the reservation price are given by

$$F(w) = \frac{4-\alpha}{2\alpha} - \frac{s}{\alpha(2-\alpha)(\frac{1}{4-3\alpha} - \frac{1}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})w} \tag{6}$$

and

$$\rho = \frac{2s}{(2-\alpha)\Big(1 - \frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)} \tag{7}$$

for which the lowest and highest prices, $\underline{w}$ and $\rho(=\overline{w})$, satisfy

$$\rho = \frac{4-\alpha}{4-3\alpha}\underline{w}, \tag{8}$$

while the equilibrium entry rate corresponds to a solution α such that Condition (5) holds. See the Appendix for details.

Non-Optimality of Full Participation

We show that it is never optimal for intermediaries to enter the market with probability one, because suppliers strategically charge prices to manipulate entry through two conditions on sequential search and entry, thereby extracting intermediary surplus appropriately.

Proposition 3. *There is no equilibrium such that all intermediaries enter the market with probability one.*

The proof of Proposition 3 is given in the Appendix. This gives us an interesting implication that it is never optimal for intermediaries to enter the market even if search cost is sufficiently small, together with no entry fee in the auction.[19] The driving force of Proposition 3 is costly first search.[20] We can extend Proposition 3 to

[19]The result holds true even if we allow for common entry fees in the auction (see the Appendix).

[20]Janssen, Moraga-González and Wildenbeest (2005) are the first to incorporate costly first search into the consumer search model of Stahl (1989), where a fraction of consumers do not participate in a market, purchasing no goods.

a more general case where there are either many suppliers given two intermediaries or many intermediaries given two suppliers (see Section 6).

Existence of Dispersed Price Equilibria

We show the existence of a dispersed price equilibrium in this model. It is of particular importance that there are two *countervailing effects* hidden in upstream and downstream competition. Suppose that the value is large enough relative to the search cost, and that entry is insufficient. Initially, as intermediaries increase their entry rate due to the high market value, downstream competition gets more intensified. Because of increased entry, suppliers are forced to charge more competitive prices by shifting a price distribution downward, which further fosters entry, while making the downstream market more competitive as the entry rate becomes higher, where the downstream competition effect tends to reduce the entry profit of an intermediary as it outweighs the upstream competition effect on supplier prices, resulting in a lower profit for entrants. It turns out there are two equilibrium entry rates if the value to search cost ratio is large enough. Thus, we can provide the following sufficient condition under which there are multiple dispersed price equilibria.

Proposition 4. *If the value to search cost ratio is sufficiently large, there exist two dispersed price equilibria.*

See the Appendix for the proof of Proposition 4. This implies that a large market value can produce two opposite market outcomes: (i) suppliers charge competitive prices to induce entry; (ii) they charge anti-competitive prices to deter entry. More generally, we can show that there are at most two dispersed price equilibria for all parameters (v, s) exogenously given, and that there is no dispersed price equilibrium if the value to search cost ratio is sufficiently small as opposed to Proposition 4, although it is tedious to prove.[21]

Two dispersed price equilibria emerge in our model without assuming heterogeneity. Interestingly, a similar pattern is generated in the consumer search model of Burdett and Judd (1983), which differs from other consumer search models (Salop and Stiglitz, 1977; Reinganum, 1979; MacMinn, 1980; Rosenthal, 1980; Varian, 1980; Stahl, 1989).[22] They consider a retail market and derive two dispersed price equilibria, provided that all retailers and consumers are identical. In their model, consumers engage in optimal *fixed sample search*, where they randomly choose how

[21] A proof is available upon request from the author.

[22] For instance, Salop and Stiglitz (1977), Varian (1980), and Stahl (1989) add different types of consumers to induce equilibrium price dispersion, whereas Reinganum (1979) and MacMinn (1980) introduce firms' cost uncertainty into their models to derive price dispersion.

many firms they visit prior to search, which causes price dispersion, and different search intensity distributions induced by different firm pricing can generate multiple equilibria. By contrast, searchers in our model engage in optimal *sequential search* with the same search intensity, and the combined effect of search frictions and auction-based competition among searchers generates price dispersion, whereas different intermediary entry rates induced by different supplier pricing can generate multiple equilibria. In fact, our model is essentially different from consumer search models mentioned above, where firms can fully exploit some fraction of consumers by charging the monopoly price due to search frictions, but in our model such an attempt of a firm (as supplier) does not work due to auction-based competition among searchers, which pushes the upper bound of prices below the monopoly price. This tells us that our model introduces a different type of search frictions, producing an unexplored mechanism of price dispersion in the literature.

The two dispersed price equilibria shown above in Proposition 4 exhibit the following property of the price distributions.

Proposition 5. *Suppose that the value to search cost ratio is sufficiently large. The equilibrium price distribution under the low entry rate first-order stochastically dominates that under the high entry rate. In addition, the range of price dispersion under the high entry rate is larger than under the low entry rate.*

See the Appendix for the proof of Proposition 5. In the following subsection, we will discuss the stability of the two equilibria regarding entry. For convenience, if there are two dispersed price equilibria, we denote the high and low entry rates by α^H and α^L, respectively, and use them below. Notice that $0 < \alpha^L < \alpha^H < 1$ holds due to Propositions 3 and 4, as long as the value to search cost ratio is sufficiently large.

Stability of Equilibria

We argue whether or not an equilibrium is stable in terms of entry by introducing the following stability concept, which reflects *free entry* of intermediaries in the long run.[23]

Definition 2. An equilibrium is *entry stable* if intermediaries who choose an entry rate close to the equilibrium rate have an incentive to change their entry rate to the

[23] Here we do not explicitly consider a dynamic model to argue whether or not an equilibrium is stable in the long run. See, for instance, Fershtman and Fishman (1992) who use a closely related stability concept to select one of multiple equilibria by explicitly introducing a dynamic version of the consumer search model of Burdett and Judd (1983). But we believe that it is natural to incorporate the stability concept used here into our static model.

equilibrium rate for equating the entry gain to the entry cost, otherwise it is *entry unstable.*

Applying the proof of Proposition 4, we can assess two dispersed price equilibria on the basis of entry stability of Definition 2 as follows.

Corollary 1. *Assume that the value to search cost ratio is sufficiently large. The equilibrium with the high entry rate is entry stable, while the other with the low entry rate is entry unstable.*

In the following, we simply call an entry (un)stable equilibrium as (un)stable equilibrium, and pay attention to the stable equilibrium.[24]

Welfare Analysis

For any given entry rate $\alpha \in [0, 1]$ and exogenous parameters (v, s), social welfare is given by

$$2\alpha(1-\alpha)(v-s) + \alpha^2(v-2s)$$

where the first (resp. second) term represents the welfare when only one intermediary enters (resp. both intermediaries enter) the market. So, the entry rate that maximizes social welfare is simply given by

$$\alpha^{\mathrm{SW}} = \frac{v-s}{v}. \tag{9}$$

Arranging Condition (5) to

$$-v(\alpha - \frac{v-s}{v}) + \frac{\alpha}{2}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw - (1-\alpha)\Big(\rho - \int_{\underline{w}}^{\rho}F(w)dw\Big),$$

one can show that the expected payoff of entry is positive when $\alpha^{\mathrm{SW}} = \frac{v-s}{v}$, because the first term is zero and the second term that takes a certain positive value due to (6)–(8) outweighs the third term that involves a sufficiently small coefficient $1-\alpha$ if $\frac{v}{s}$ is sufficiently large. We have known from Propositions 3 and 4 that the expected payoff of entry is negative when $\alpha = 1$ and that it is decreasing in α if $\frac{v}{s}$ is sufficiently large, where α is sufficiently close to one. Thus, the stable equilibrium should involve $\alpha \in (\frac{v-s}{v}, 1)$, meaning that it is inefficient due to excess entry.[25]

Proposition 6. *If the value to search cost ratio is sufficiently large, then the equilibrium is inefficient as it involves excess entry.*

[24] We can analyze the unstable equilibrium in the same way to the stable equilibrium.

[25] In this model, there is no intervention of a benevolent social planner who maximizes welfare.

This implies that competition at the downstream market exceeds an optimal level if the market value is large enough.[26]

Comparative Statics on Price Dispersion

We explore how and to what extent the value to search cost ratio influences supplier pricing. When the value increases or the search cost decreases, the (supplier) price distribution shifts downward because the upstream market becomes more competitive, leading to lower prices overall. The dependency of price dispersion on the value or the search cost primarily relies on the two countervailing effects through upstream and downstream competition. The range of price dispersion increases as the value increases, because a supplier can gain a larger power as monopolist by increased entry in case that all entrants visit her to buy goods, while sufffering from increased entry in case that an entrant visits her and the other a different supplier, causing upstream competition between suppliers. Interestingly, when considering the search cost reduction instead of the increased value, the range of price dispersion decreases. This occurs because the smaller search cost reinforces upstream competition among suppliers through less costly sequential search, lowering the upper bound substantially, which outweighs the above mentioned effect of increased entry on price dispersion in case of the increased value, and therefore suppliers are obliged to reduce the range of price dispersion. This implies that it is impossible to tell how and to what extent the increased ratio influences price dispersion, so we need to access the impact of the increased value and the decreased search cost on price dispersion, separately. This gives the following comparative statics result.

Proposition 7. *Suppose that the value to search cost ratio is sufficiently large. The upper and lower bounds of the equilibrium price distribution decrease with the ratio, and the range of price dispersion increases as v increases, while decreasing as s decreases.*

See the Appendix for the proof of Proposition 7. The first part of Proposition 7 implies that the expected surplus of suppliers (resp. entrants) decreases (resp, increases) as the value increases or the search cost decreases.

[26] By contrast, the unstable equilibrium involves *insufficient* entry, so it is socially inefficient as well as the stable equilibrium. Since suppliers charge anti-competitive prices to deter entry in the unstable equilibrium, it is much worse than under excess entry where supplier prices are competitive. Therefore, if the downstream market looks highly attractive for suppliers and intermediaries, entry becomes either insufficient or excessive and the socially optimal outcome is unobtainable in both cases.

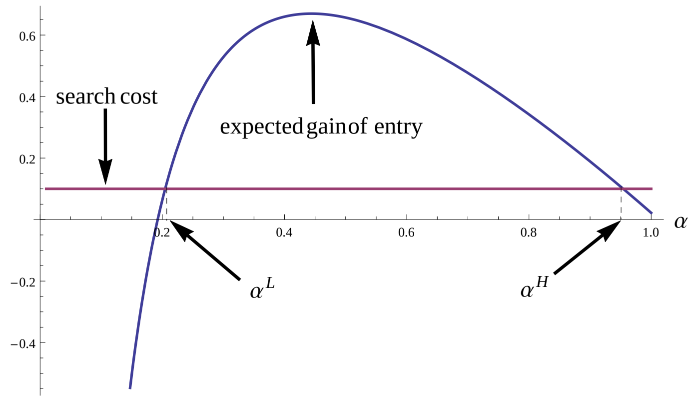

Figure 1: The expected entry gain and the search cost for Condition (5) and two equilibrium entry rates α^H and α^L when $(v, s) = (2, 0.1)$.

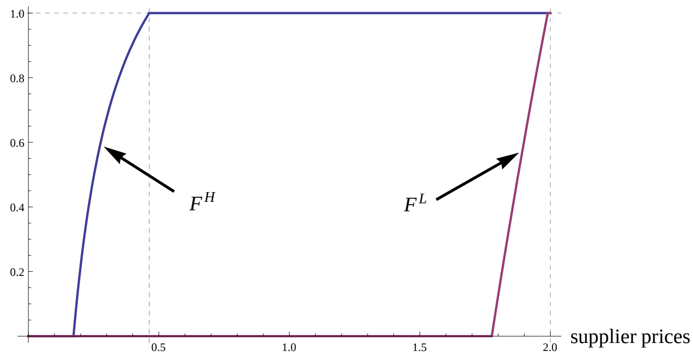

Figure 2: Equilibrium price distributions F^H and F^L when $(v, s) = (2, 0.1)$.

5 Example

We give an example to overview previous results. Suppose that the value to search cost ratio is sufficiently large. For instance, we take $(v, s) = (2, 0.1)$. Then, Condition (5) is described by Figure 1 where two equilibrium entry rates α^H and α^L are pinned down and both of them are below the full participation rate, $\alpha = 1$ (Propositions 3 and 4). These two equilibrium entry rates determine the corresponding price distributions F^H and F^L, respectively (see Figure 2). These distributions satisfy the property shown in Proposition 5. The equilibrium with the high entry rate α^H is stable and the other with the low entry rate α^L is unstable (Corollary 1). Since $\alpha^H \approx 0.954 > \alpha^{\text{SW}} = \frac{v-s}{v} = 0.95$ holds, the stable equilibrium involves excess entry, and therefore is inefficient (Proposition 6). In addition, the increased value (resp. the search cost reduction) shifts the price distribution in the (stable) equilibrium downward, while increasing (resp. decreasing) the range of price dispersion (Proposition 7). For example, consider two situations: (i) v increases from 2 to 4 given

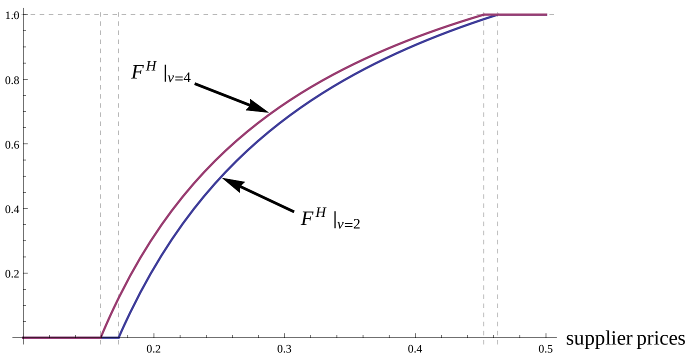

Figure 3: Equilibrium price distribution $F^H\mid_{v=4}$ (resp. $F^H\mid_{v=2}$) when $v = 4$ (resp. $v = 2$) given $s = 0.1$.

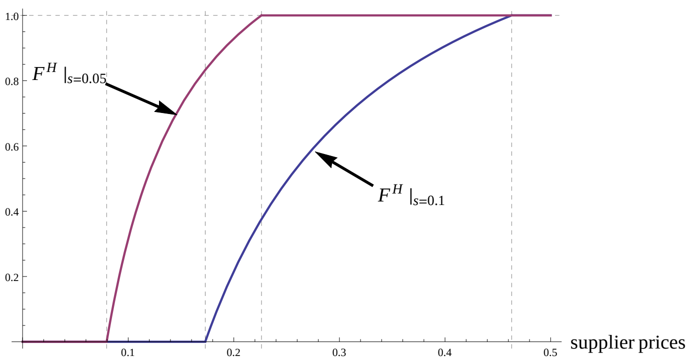

Figure 4: Equilibrium price distribution $F^H\mid_{s=0.05}$ (resp. $F^H\mid_{s=0.1}$) when $s = 0.05$ (resp. $s = 0.1$) given $v = 2$.

$s = 0.1$ and (ii) s decreases from 0.1 to 0.05 given $v = 2$, where the value to search cost ratio is the same in both cases. Then, the equilibrium prices in the two cases are illustrated by Figures 3 and 4, showing the above mentioned impact of exogenous parameters on price distributions.

6 Market Concentration and Price Dispersion

So far we have considered the case of two suppliers and two intermediaries to analyze dispersed price equilibria in the market. It would be natural to examine the impact of the number of suppliers or intermediaries on market outcomes, by adding more suppliers or more intermediaries into the market. Although the analysis contains tedius derivations of equilibrium conditions, we can briefly summarize the firm number effect as follows (see the Appendix for details).

Similar to the previous case, we can show that full participation is never optimal for intermediaries even if search cost is sufficiently small, and that there are multiple equilibria, all of which are inefficient if the value to search cost ratio is sufficiently large.

Proposition 8. *Suppose that there are multiple suppliers and two intermediaries or that two suppliers and multiple intermediaries. If the value to search cost ratio is sufficiently large, there is no equilibrium such that all intermediaries enter the market with probability one, and in addition, there are at least two dispersed price equilibria and all of them are generically inefficient.*

The proof of Proposition 8 is given in the Appendix. In general, it is hard to tell how many equilibria exist, but our numerical analysis shows that there are at most two equilibria as shown by Proposition 4 where we consider the case of two suppliers and two intermediaries.

Numerical Analysis

From now on, we provide the numerical analysis that demonstrates strikingly different features from the previous ones. First, consider the case where there are more intermediaries in the market with two suppliers. As the number of intermediaries increases, the downstream market becomes more competitive and each intermediar find less profitable to enter the market, so entry becomes insufficient. It turns out that both the average price and the range of price dispersion increase as there are more intermediaries. This happens because of the following two countervailing effects. As the number of intermediaries increases, suppliers obtain stronger seller power against intermediaries, thereby shifting the price distribution upward, but cannot increase its lower bound significantly in order to maintain moderate entry. See Figure 5 where we compare the case of ten intermediaries with the case of two intermediaries, , taking two suppliers as given, and demonstrate that both the average price and the range of price dispersion increase, whereas the lower bound of the price distribution decreases. Given that *downstream* market concentration is measured by number of intermediaries, this implies that there is a negative relationship between *downstream* market concentration and price dispersion.

Next, consider the case where there are more suppliers in the market with two intermediaries. As opposed to the case of multiple intermediaries, (in the stable equilibrium) the *average* price decreases, while the range of price dispersion increases as in the former case.[27] This occurs because of the following two countervailing effects. As the number of suppliers increases, suppliers are forced to shift the price distribution downward, lowering the average price, while increasing entry simultaneously,

[27]In contrast, the opposite happens in the unstable equilibrium.

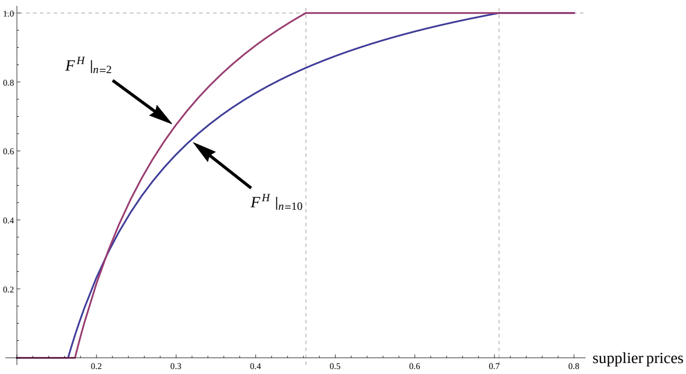

Figure 5: Equilibrium price distribution $F^H|_{n=10}$ (resp. $F^H|_{n=2}$) when $n = 10$ (resp. $n = 2$) given $(m, v, s) = (2, 2, 0.1)$.

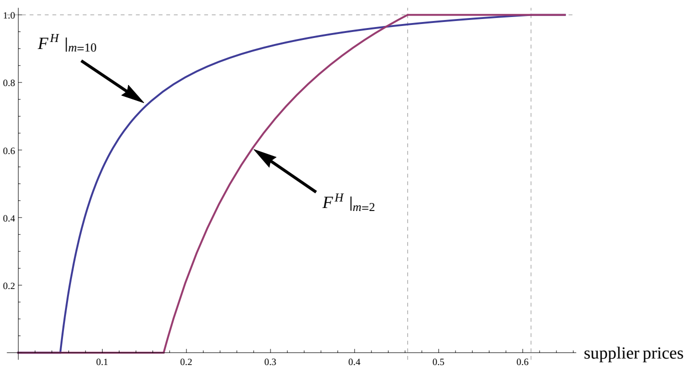

Figure 6: Equilibrium price distribution $F^H|_{m=10}$ (resp. $F^H|_{m=2}$) when $m = 10$ (resp. $m = 2$) given $(n, v, s) = (2, 2, 0.1)$.

which gives suppliers a larger payoff gap in two cases of monopoly and competition, and therefore suppliers set the upper bound of the price distribution higher than in case of two suppliers. This point is illustrated by Figure 6 where we consider the case of ten suppliers in comparison with that of two suppliers, taking two intermediaries as given, and demonstrate the above mentioned property. Considering that *upstream* market concentration is measured by number of suppliers, our numerical analysis suggests that there is a negative relationship between upstream market concentration and price dispersion, as in the former case.

A Relationship between Procurement Costs and the Number of Potential Bidders

Proposition 8 and our numerical analysis given above are of particular interest in procurement auctions to investigate the relationship between the winnig bids and the number of potential bidders, because we usually think that a larger number of

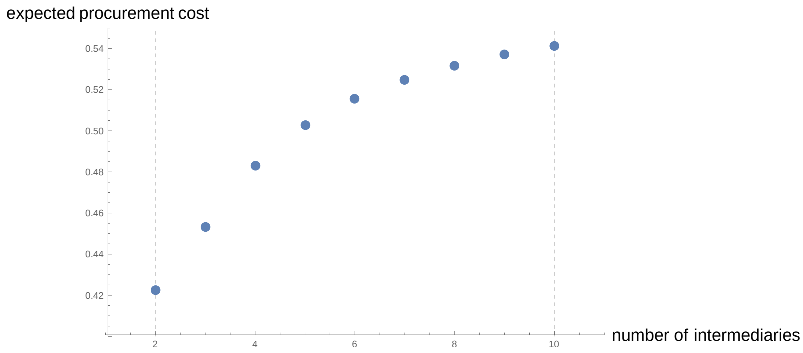

Figure 7: The expected procurement cost increases as the number of intermediaries (potential bidders) increases, taking two suppliers as given.

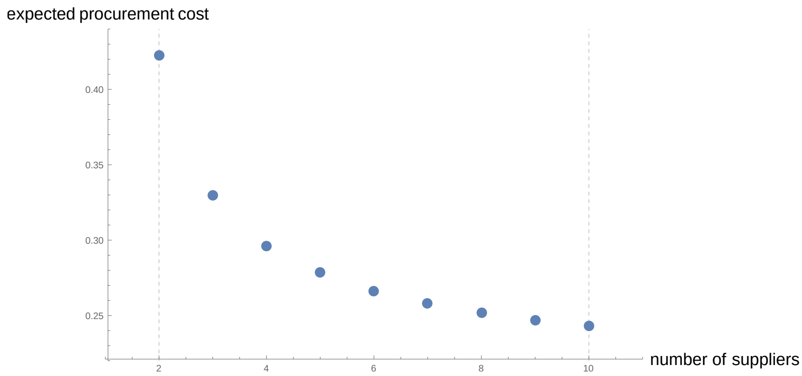

Figure 8: The expected procurement cost decreases as the number of suppliers increases, taking two intermediaries as given.

potential bidders strengthens the competition among bidders, thereby lowering the procurement cost.[28] But its relationship may be non-monotonic because of countervailing effects against the competition effect. One of the countervailing effects related to our paper is *entry* effect (Li and Zheng, 2009), meaning that a winning bidder may believe that they overestimate the intensity of entry, and it may outweigh the competition effect, resulting in a higher winning bid as there are more potential bidders.[29] By contrast, the numerical analysis in our model suggests that the expected procurement cost monotonically increases (resp. decreases) as the number of intermediaries (resp. suppliers) increases, simply because suppliers obtain larger (resp. smaller) power against intermediaries, charging higher (resp. lower) input prices to intermediaries who pass on to their bids the increased (resp. decreased) costs, which results in higher (resp. lower) procurement costs. . See Figures 7 and 8.

[28] In practice, however, "competitive bidding may lead to adverse selection" (Bajari, McMillan and Tadelis, 2009, p.379).

[29] There are other representative countervailing effects known as *winner's curse* effect (Bulow and Klemperer, 2002) and *affiliation* effect (Pinkse and Tan, 2005).

7 Discussion and Conclusion

This paper incorporated information frictions of input prices through intermediary search into a (cost-based) procurement auction model with endogenous entry. We provided a possible market mechanism of producing outcomes in accord with empirically observed features. Our model generates cost heterogeneity among bidders endogenously through supplier induced input price dispersion, and demonstrates a non-monotonic relationship of procurement costs with the number of potential bidders, depending heavily on how many suppliers exist at the upstream market. Our results, however, relies on specifications of our model concerning search, vertical relations between suppliers and intermediaries, and procurement auctions. Below, in comparison with those specifications, we discuss in detail the influence of alternatives on market outcomes in order.

We have assumed throughout the paper, that intermediaries engage in optimal sequential search. But one might argue that it seems natural to assume that intermediaries engage in optimal fixed sample search as in Burdett and Judd (1983), because intermediaries may contact multiple suppliers at the same time and then compare supplier suggested prices, as the information acquisition of supplier prices involves a time consuming process in which suppliers estimate their procurement costs, depending on projects (Morgan and Manning, 1985). In case of fixed sample size search, there are two possible cases to consider dispersed price equilibria: (i) entrants search only one supplier, where there is no active search as in sequential search; (ii) entrants search only one supplier with some probability, while searching two suppliers with the remaining probability, where active search emerges, which differs from the case of sequential search. Our model is essentially different from the model of Burdett and Judd (1983) where searchers are consumers and firms charge the monopoly price without randomizing over prices if consumers search only one firm, so the monopoly outcome emerges. By contrast, in our model searchers serve as intermediaries to compete for supplying the final good to the buyer, and suppliers do not charge the monopoly price. The key to understanding why suppliers do not charge the monopoly price here is that a supplier can only earn profit if the intermediary to whom he sells wins the auction, which strikingly differs from the model of Burdett and Judd (1983) where firms can earn profits if consumers buy their goods, no matter what consumers do afterward. Thus, a dispersed price equilibrium in our model can involve no active search as well as active search. Furthermore, there may be a different type of dispersed price equilibrium, where entrants adopt asymmetric search protocols, in other words, one engages in optimal sequential search, while the other optimal fixed sample search. Exploring details of the role of fixed sample search in our framework will be undertaken in future work.

As for vertical relations, we concern what kind of contracts suppliers use for dealing with intermediaries. Suppliers in our model are supposed to use the linear (pricing) contracts, while there is an optimal (non-linear) contract (Laffont and Tirole, 1986). In practice, however, we often observe simple supply contracts based on combinations of fixed price and cost reimbursement, because an optimal contract is too complicated to implement, whereas a simple contract are good enough to cover a large part of surplus obtained via the optimal contract (Rogerson, 2003). Considering these prevalent simple contracts, we allow for suppliers to use the two-part tariff contracts, and can show that there is at most one dispersed price equilibrium with a positive fixed fee, where suppliers *cannot* fully extract intermediary surplus due to the binding entry constraint.[30]

Besides, our model discarded buyer (intermediary) power against sellers (suppliers) by presuming a *posted* price market upstream, where suppliers have full bargaining power against intermediaries, and, therefore, intermediaries should engage in search for supplier inputs. Suppliers are obliged by the presumption of the posted price market to charge input prices simultaneously prior to search, which are fixed afterward, making them not negotiable. Although this presumption forces suppliers to set *uniform* wholesale prices, instead we can allow suppliers to use *random* pricing into our analysis with a slight modification, where each supplier charges visiting intermediaries (possibly) different input prices randomly drawn from a price distribution, so that even a single supplier induces different costs across intermediaries. Since input prices are determined by supplier-intermediary negotiations in many markets, it would be of vital importance to argue to what extent intermediaries have bargaining power against both the buyer and suppliers. For instance, the buyer (public authority) tends to allow intermediaries (winning bidders) to renegotiate as projects are more complex (e.g. Bajari, Houghton and Tadelis, 2014). If suppliers do not have full bargaining power against intermediaries in our model, it might be reasonable to consider that suppliers engage in search for selling their inputs to intermediaries.[31]

Lastly, we discuss the specification made in procurement auctions. In our model, both suppliers and intermediaries are supposed to know the buyer's valuation of the

[30] A formal argument is available from the author upon request.

[31] Apart from procurement auctions, consider a market where an intermediary who serves as a retailer has strong bargaining power against suppliers or manufacturers, because of the retailer possessed large platform in which many consumers purchase supplier produced goods, or limited supply channels to sell goods directly to consumers, which causes inventory issues to suppliers. Such examples of intermediaries are internet price comparison sites or shopbots for airline tickets and supermarkets.

object ex ante and–more importantly–the auction is the open auction. In practice, an engineer's cost estimate that accounts for a large part of the buyer's valuation is often not publicly available before the auction takes place, and in addition, a first price sealed bid auction is frequently used in public procurements, together with bidder qualifications and multiple rounds. The first point is easily incorporated into our analysis, as long as suppliers and intermediaries have a common prior about the buyer's valuation. On the other hand, the second requires a further involved argument, but this gives an interesting point that our framework allows us to explicitly derive a unique equilibrium bidding strategy that is increasing in bidder's costs, because the condition imposed on intermediary search for input prices pins down the cost distribution of bidders, which differs from the standard first price auction (e.g. Krishna, 2009). This is closely related to the study by Spulber (1995). He introduces cost uncertainty to the Bertrand competition, and shows that firms charge prices above marginal cost due to cost uncertainty exogenously given. In his model, marginal cost of each firm is independently drawn from an identical distribution. By contrast, in our model marginal costs of firms (intermediaries) depend upon which suppliers they have bought their goods from and are endogenously determined. It would be interesting in future research to examine how much our results are influenced by the first price sealed bid auction.

The model considered in this paper is limited, to a large extent, by the above mentioned points and many other possible factors, such as product quality in scoring auctions (Che, 1993; Asker and Cantillon, 2008, 2010), but the point here is to address how and to what extent asymmetric information on supplier prices caused by search frictions of intermediaries influences (cost-based) procurements, and to uncover a pricing mechanism hidden in the market, which is in accord with empirical evidence.

A Appendix

Proof of Lemma 1

Proof. Suppose that suppliers adopt a price distribution F with compact support $[\underline{w}, \overline{w}]$.[32] Consider the case where both intermediaries enter the market and visit different suppliers, purchasing their goods with possibly different input prices. In this case, the expected revenue of an intermediary in the auction is illustrated by

[32]The condition $\overline{w} = \rho \leq v$ must hold, otherwise visiting intermediaries do not stop to buy and a supplier charging a price above v gets nothing.

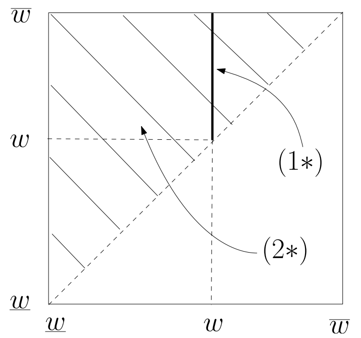

Figure 9: The term (1∗) in the equation (A.1) represents the thick vertical line from w to $\overline{w}$, whereas the term (2∗) the shaded area.

Figure 9 and given by

$$
\begin{aligned}
&\underbrace{\int_{\underline{w}}^{\overline{w}} \overbrace{\int_{w}^{\overline{w}} (w' - w) dF(w')}^{(1*)} \quad dF(w)}_{(2*)} && \text{(A.1)} \\
= \; & \int_{\underline{w}}^{\overline{w}} \Big((\overline{w} - wF(w) - \int_{w}^{\overline{w}} F(w')dw' - w(1 - F(w)) \Big) dF(w) \\
= \; & \overline{w} - \int_{\underline{w}}^{\overline{w}} w dF(w) - \int_{\underline{w}}^{\overline{w}} \Big(\int_{\underline{w}}^{w'} dF(w) \Big) F(w')dw' = \int_{\underline{w}}^{\overline{w}} (1 - F(w))F(w)dw
\end{aligned}
$$

where the first equality obtains by integrating by parts and the second equality is given by changing the order of integration. □

Derivation of Conditions (6)–(8) and a Solution

By (3), taking $(\overline{w}, \underline{w}, \alpha)$ as given, we can express F as the function of w by

$$
F(w) \;=\; 1 - \frac{4 - 3\alpha}{2\alpha} \frac{\overline{w} - w}{w} = \frac{1}{2\alpha} \Big(4 - \alpha - \big(4 - 3\alpha\big) \frac{\overline{w}}{w} \Big) \qquad \text{(A.2)}
$$

where $F(\overline{w}) = 1$ holds. Since $F(\underline{w}) = 0$ must hold, we derive (8) where the coefficient of $\underline{w}$ is increasing in α. Substituting (8) into (A.2) above, we derive (6).

Next, since $\overline{w} - \int_{\underline{w}}^{\rho} w dF(w) = \int_{\underline{w}}^{\overline{w}} F(w)dw$ and

$$
\begin{aligned}
\int_{\underline{w}}^{\overline{w}} F(w)dw \;&=\; \frac{4 - \alpha}{2\alpha} (\overline{w} - \underline{w}) - \frac{(4 - 3\alpha)\overline{w}}{2\alpha} \ln \frac{\overline{w}}{\underline{w}} \quad (\because \text{(A.2)}) \\
&=\; \Big(1 - \frac{4 - 3\alpha}{2\alpha} \ln \frac{4 - \alpha}{4 - 3\alpha} \Big) \overline{w}, \quad (\because (8)) \qquad \text{(A.3)}
\end{aligned}
$$

together with (4) where $\overline{w} = \rho$, we derive

$$\rho = \frac{2s}{(2-\alpha)\Big(1 - \frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)}. \tag{A.4}$$

and in addition, due to (8),

$$\underline{w} = \frac{2s}{(2-\alpha)(4-\alpha)\Big(\frac{1}{4-3\alpha} - \frac{1}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha}\Big)} \tag{A.5}$$

By (A.2) and (A.4), we obtain (6).

Finally, we simplify Condition (5) as follows. Since (5) is rewritten by

$$2\big((1-\alpha)v - s\big) = -\alpha\int_{\underline{w}}^{\overline{w}} F(w)dw + \alpha\int_{\underline{w}}^{\overline{w}} F^2(w)dw + 2(1-\alpha)\int_{\underline{w}}^{\overline{w}} wdF(w)$$

where

$$\int_{\underline{w}}^{\overline{w}} F^2(w)dw = \frac{1}{2\alpha^2}\Big(4\alpha(2-\alpha) - (4-\alpha)(4-3\alpha)\ln\frac{4-\alpha}{4-3\alpha}\Big)\overline{w} \tag{A.6}$$

and

$$\int_{\underline{w}}^{\overline{w}} wdF(w) = \overline{w} - \int_{\underline{w}}^{\overline{w}} F(w)dw, \tag{A.7}$$

substituting (A.3), (A.4), (A.6), and (A.7) into (5), we get

$$\frac{v}{s} = \frac{4(3-2\alpha)\alpha + (-16+16\alpha-3\alpha^2)\ln\frac{4-\alpha}{4-3\alpha}}{(1-\alpha)(2-\alpha)(2\alpha-(4-3\alpha)\ln\frac{4-\alpha}{4-3\alpha})}, \tag{A.8}$$

which determines an equilibrium entry rate for exogenously given (v, s), if any.

Proof of Proposition 3

Proof. Substituting (4) to the RHS of (5) through the search cost s, the expected profit of entry is given by

$$(1-\alpha)(v-\rho) - \frac{\alpha}{2}\int_{\underline{w}}^{\rho} F^2(w)dw,$$

which is negative when $\alpha = 1$. This implies that entry with probability one does not pay off, as no entry gives zero profit. □

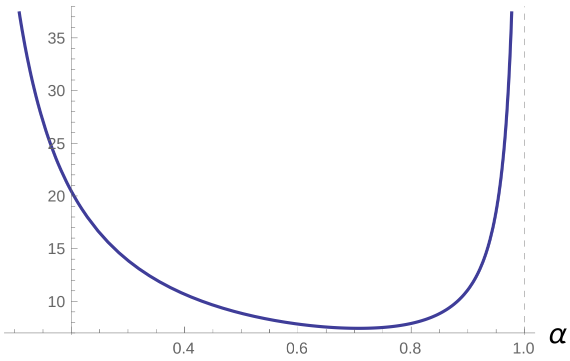

Figure 10: The equation $f(\alpha)$ of the RHS of (A.8) for $\alpha \in (0, 1)$.

Remark 1. Consider the case where there is a common entry fee in the auction. Then, by adding the fee to Condition (5), we can similarly extend the above shown result to the model with an entry fee.

Proof of Proposition 4

Proof. By Condition (A.8) that exhibits the relationship of the entry rate with the ratio $\frac{v}{s}$, we can show that the RHS of (A.8) converges to infinity as $\alpha \in (0, 1)$ goes to zero or one. In fact, we denote the RHS and its denominator and nominator by $f(\alpha)$, $f_1(\alpha)$, and $f_2(\alpha)$, respectively, whereby the RHS $= f(\alpha) = \frac{f_1(\alpha)}{f_2(\alpha)}$, and then provide two observations: (i) $\lim_{\alpha \to 0^+} \frac{f_1(\alpha)}{f_2(\alpha)} = \lim_{\alpha \to 0^+} \frac{f_1'(\alpha)}{f_2'(\alpha)} = \infty$ due to $\lim_{\alpha \to 0^+} f_1(\alpha) = \lim_{\alpha \to 0^+} f_2(\alpha) = 0$ and $\lim_{\alpha \to 0^+} f_1'(\alpha) > \lim_{\alpha \to 0^+} f_2'(\alpha) = 0$; (ii) $\lim_{\alpha \to 1^-} \frac{f_1(\alpha)}{f_2(\alpha)} = \infty$ due to $\lim_{\alpha \to 1^-} f_1(\alpha) > \lim_{\alpha \to 1^-} f_2(\alpha) = 0$. Besides, one can easily observe that $f(\alpha) \in (0, \infty)$ for all $\alpha \in (0, 1)$, and that $\lim_{\alpha \to 0^+} f'(\alpha) = -\infty$ and $\lim_{\alpha \to 1^-} f'(\alpha) = \infty$.

Taken together, there are two values of $\alpha \in (0, 1)$ such that $\frac{v}{s} = f(\alpha)$ if $\frac{v}{s}$ is sufficiently large. □

See Figure 10 for an illustration of the equation $f(\alpha)$.

Proof of Proposition 5

To prove Proposition 5, we first show that all equilibrium supplier prices are positive. Next, although dispersed price equilibria exist only for certain values of $\alpha \in (0, 1]$, assuming that the highest and lowest equilibrium prices are defined for all values of α, we derive the property of prices that both of them are decreasing in α while the range of price dispersion is increasing, which proves Proposition 5.

Lemma 2. *The highest price of (A.4) is positive for any $\alpha \in (0,1]$.*

Since the lowest price $\underline{w}$ is also positive if the highest price $\overline{w} = \rho$ is positive due to (8), Lemma 2 implies that all prices derived by the equilibrium conditions (3)–(5) are all positive for any $\alpha \in (0,1]$.

Proof. By (7), it is enough to show that $1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha} > 0$ holds for any $\alpha \in (0,1]$. Since its derivative is given by

$$2\frac{(4-\alpha)\ln\frac{4-\alpha}{4-3\alpha} - 2\alpha}{(4-\alpha)\alpha^2}$$

where both the denominator and nominator are easily shown to be positive for any $\alpha \in (0,1]$,[33] the derivative is always positive for any $\alpha \in (0,1]$. Together with $\lim_{\alpha \to 0^+}(1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha}) = 0$, this implies that $1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha} > 0$ for any $\alpha \in (0,1]$. $\square$

Lemma 3. *The highest and lowest prices of (A.4) and (A.5), $\overline{w}$ and $\underline{w}$, are strictly decreasing in $\alpha \in (0,1]$, whereas the range of price dispersion of $\overline{w} - \underline{w}$, is strictly increasing in α.*

Proof. To show that $\overline{w}$ is (strictly) decreasing in $\alpha \in (0,1]$, it is enough to show that the denominator of (A.4), $(2-\alpha)(1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha})$, is increasing in α. Together with the observation that the coefficient $\frac{4-3\alpha}{4-\alpha}$ in $\underline{w} = \frac{4-3\alpha}{4-\alpha}\overline{w}$ of (8) is decreasing in α, this implies that $\underline{w}$ is also decreasing in α. Given that $\overline{w} > 0$ for any $\alpha \in (0,1]$ by Lemma 2, the derivative of $(2-\alpha)(1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha})$ is given by

$$\frac{2\alpha(8-\alpha^2) + (32 - 8\alpha - 12\alpha^2 + 3\alpha^3)\ln\frac{4-\alpha}{4-3\alpha}}{2(4-\alpha)\alpha^2},$$

which is positive for all $\alpha \in (0,1]$, so $(2-\alpha)(1 - \frac{4-3\alpha}{2\alpha} \ln \frac{4-\alpha}{4-3\alpha})$ is increasing in α.

By (7) and (8),

$$\overline{w} - \underline{w} = \frac{2\alpha}{4-\alpha}\overline{w} = \frac{4s}{\frac{(4-\alpha)(2-\alpha)}{\alpha}(1 - \frac{4-3\alpha}{2\alpha}\ln\frac{4-\alpha}{4-3\alpha})} > 0. \tag{A.9}$$

The derivative of the denominator of (A.9) is given by

$$\frac{2\alpha(-16 + 4\alpha + \alpha^2) + (64 - 48\alpha + 3\alpha^3)\ln\frac{4-\alpha}{4-3\alpha}}{2\alpha^3},$$

[33] Since both 2α and $4-\alpha$ are linear in α and $\ln\frac{4-\alpha}{4-3\alpha}$ is strictly increasing and convex in α, the nominator of the derivative is strictly increasing and convex in $\alpha \in (0,1]$. Taking into account that the nominator takes zero at $\alpha = 0$, the nominator is positive for any $\alpha \in (0,1]$.

which is negative for all $\alpha \in (0, 1]$, thus (A.9) is decreasing in α. $\square$

Remark 2. Since $\overline{w}$ has shown above to be decreasing in α, $F(w) = \text{(A.2)} = 1 - \frac{4-3\alpha}{2\alpha}\frac{\overline{w}-w}{w}$ is increasing in α because both $\frac{4-3\alpha}{2\alpha}$ and $\frac{\overline{w}-w}{w}$ are decreasing in α. This implies that the distribution $F(\alpha^L)$ first-order stochastically dominates $F(\alpha^H)$ for low and high equilibrium entry rates $0 < \alpha^L < \alpha^H < 1$ (if any).

Proof of Proposition 7

The first part of Proposition 7 is shown by Propositions 4 and 5 where the increased value to search cost ratio leads to an increased (stable) equilibrium entry rate, lowering both the upper and lower bounds of the price distribution. Below we show the latter part of Proposition 7, the property that the range of price dispersion increases (resp. decreases) in v (resp. s).

Lemma 3 gives us three observations: (i) the highest price given by (A.4) decreases as the entry rate increases; (ii) the lowest price decreases more than the highest price as the entry rate increases; (iii) the search cost directly influences the highest and lowest equilibrium prices due to Conditions (A.4) and (A.5), thereby changing them proportionally if $\frac{v}{s}$ is sufficiently large, while the impact of the search cost on the equilibrium entry rate α^H is rather small, compared to that on the equilibrium prices (see Figure 10). By these three observations, we can show that the range of price dispersion given by (A.9) increases as v increases, while decreasing as s decreases.

Proof of Proposition 8

Multiple Suppliers and Two Intermediaries: Equilibrium Conditions and Analysis

We consider the case where there are $m(= 2, 3, \dots)$ suppliers, taking two intermediaries as given. Similar to the simple case of two suppliers, the three corresponding conditions to (3), (4), and (5) are given by

$$\begin{aligned}\Big(\alpha^2\Big(\frac{1}{m^2} + \frac{2}{m}(1 - \frac{1}{m})(1 - F(w))\Big) & \\ + \frac{2}{m}\alpha(1-\alpha)\Big)w &= \Big(\frac{\alpha^2}{m^2} + \frac{2}{m}\alpha(1-\alpha)\Big)\overline{w}, \qquad \text{(A.10)}\end{aligned}$$

$$\begin{aligned}
&\Big(\frac{\alpha}{m}+(1-\alpha)\Big)\Big(\overline{w}-\int_{\underline{w}}^{\rho} wdF(w)\Big) \\
&+\alpha(1-\frac{1}{m})(1-\frac{1}{m-1})\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw \quad = \quad s, \qquad \text{(A.11)}
\end{aligned}$$

and

$$\alpha(1-\frac{1}{m})\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw+(1-\alpha)\Big(v-\int_{\underline{w}}^{\rho} wdF(w)\Big) \quad = \quad s. \quad \text{(A.12)}$$

As in Proposition 3, we can show that full participation is not optimal for intermediaries. By (A.11) and (A.12), the expected profit per firm under full participation (as $\alpha = 1$) is rewritten by

$$\begin{aligned}
&\frac{m-1}{m}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw \\
&-\frac{1}{m}\Big(\overline{w}-\int_{\underline{w}}^{\rho} wdF(w)\Big)-\frac{m-2}{m}\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw \quad = \quad -\frac{1}{m}\int_{\underline{w}}^{\overline{w}} F^2(w) < 0,
\end{aligned}$$

which implies that the profit is negative, thus entering the market with $\alpha = 1$ is not optimal for intermediaries. In addition, if α is sufficiently close to zero ($\alpha \approx 0$), the upper and lower bounds of prices are almost the same and then the LHS of (A.12) becomes sufficiently small. Together with the observation that the expected payoff of entry is positive if α is an intermediate value in $(0, 1)$, this implies that there are *at least* two dispersed price equilibria, given that the value to search cost ratio is sufficiently large. It is not easy to show how many equilibria can exist, though a numerical analysis suggests that there are at most two equilibria. Given that there are multiple equilibria, as in Corollary 1, we can show that the equilibrium with the closest entry rate to full participation is entry stable, whereas the equilibrium with the closest entry level to no entry is entry unstable.

The expected number of entrants (intermediaries who have entered the market), social welfare, and the expected procurement cost are given in the same way as in the case of two intermediaries, which are calculated by $2\alpha(1-\alpha)+2\alpha^2 = 2\alpha$, $2\alpha(1-\alpha)(v-s)+\alpha^2(v-2s)$, and $\alpha^2\int_{\underline{w}}^{\rho} wdF(w)+2\alpha(1-\alpha)v$ respectively.

Two suppliers and Multiple Intermediaries: Equilibrium Conditions and Analysis

We consider the case where there are $n(=2,3,\dots)$ intermediaries, taking two suppliers as given. The conditions (3), (4), and (5) are modified to

$$\begin{aligned}&\Big(\sum_{k=1}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}\Big((\frac{1}{2})^k\\ &+(1-2(\frac{1}{2})^k)(1-F(w))\Big)\Big)w \;=\; \Big(\sum_{k=1}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}(\frac{1}{2})^k\Big)\overline{w}, \quad \text{(A.13)}\end{aligned}$$

$$\sum_{k=0}^{n-1}\Big(\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(\frac{1}{2})^k\Big)\Big(\overline{w}-\int_{\underline{w}}^{\rho}wdF(w)\Big) \;=\; s, \quad \text{(A.14)}$$

and

$$\begin{aligned}&\sum_{k=1}^{n-1}\Big(\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(\frac{1}{2})^k\Big)\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw\\ &\qquad +(1-\alpha)^{n-1}\Big(v-\int_{\underline{w}}^{\rho}wdF(w)\Big) \;=\; s. \quad \text{(A.15)}\end{aligned}$$

Using the binomial theorem, Conditions (A.13), (A.14), and (A.15) are simplified by

$$\Big(1-(1-\frac{\alpha}{2})^n-F(w)\big(1+(1-\alpha)^n-2(1-\frac{\alpha}{2})^n\big)\Big)w \;=\; \Big((1-\frac{\alpha}{2})^n-(1-\alpha)^n\Big)\overline{w},$$

$$(1-\frac{\alpha}{2})^{n-1}\int_{\underline{w}}^{\rho}F(w)dw \;=\; s,$$

and

$$\begin{aligned}&\Big((1-\frac{\alpha}{2})^{n-1}-(1-\alpha)^{n-1}\Big)\int_{\underline{w}}^{\rho}(1-F(w))F(w)dw\\ &\qquad +(1-\alpha)^{n-1}\Big(v-\int_{\underline{w}}^{\rho}wdF(w)\Big) \;=\; s.\end{aligned}$$

Similar to the case of multiple suppliers with two intermediaries, we can show that full participation cannot be part of an equilibrium and that there are at least two dispersed price equilibria if the value to search cost ratio is sufficiently large.

The expected number of entrants and social welfare are given by $\sum_{k=0}^{n}\binom{n}{k}\alpha^k(1-$

$\alpha)^{n-k}k = n\alpha$ and $\sum_{k=0}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}(v-ks) = v - n\alpha s$, respectively. The expected procurement cost is given by

$$n\alpha(1-\alpha)^{n-1}v + \sum_{k=2}^{n}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}\int_{\underline{w}}^{\rho} wdF(w).$$

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