The formation of partnerships in social networks

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The formation of partnerships in social networks *

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1 Introduction

1.1 The formation of partnerships in social networks

The idea that *power* and *influence* of an individual in a network depends upon his or her *position* in the network is well-established in a variety of contexts cutting across disciplines. For instance, social network analysis suggests that the power of individuals cannot be explained by the individual's characteristics alone, but must be combined with the structure of his or her relationship with others -power arises from occupying advantageous positions in networks of relationships. In particular, network exchange theories focus on *exchange rates* which essentially represent the relative "bargaining power" of one individual in his or her bilateral exchange with neighbors in the network.[1] In models of diffusion of ideas or products, it is suggested that the optimal strategy for diffusion is to target the most "central" player in the network since these players are relatively more influential in the network.

We study the impact of network structures on the pattern of bilateral exchanges both theoretically as well as by means of laboratory experiments. Our setting is one in which individuals form partnerships to exchange *favors* with one another. Favors could be small – advice on a particular issue, a small loan, help on a school project or with baby-sitting, or large – sharing one's life with another person, or forming a professional partnership with other workers . The need for such favors arises randomly for any individual at any point of time. If an individual $\boldsymbol{i}$ needs a favor at any point, he turns to one of his neighbors $\boldsymbol{j}$ in the network to request the favor. The recipient of such a request can either grant the favor at some cost c (which is strictly less than the value of the favor v) or refuse to grant the favor. In the latter case, the link between $\boldsymbol{i}$ and $\boldsymbol{j}$ is broken. Individual $\boldsymbol{i}$ can then approach another neighbor to grant him the favor. If the favor is granted, the two players enter a reciprocal agreement to grant each other the favor and leave the network. The process is repeated in the next period when some individual chosen at random needs a favor.

Since there is a cost involved in granting the favor, an individual grants a favor only because she knows that she might need a favor tomorrow. In particular, our model is purely individualistic in the sense that we ignore completely the existence of any social norm to enforce cooperative behavior. (This distinguishes our model from that of Jackson, Rodriguez-Barraquer and Tan (2012), whose work we discuss later.) The absence of societal punishments implies that typical repeated-game theoretic considerations do not apply in our model. So, any individual $\boldsymbol{i}$ who is approached with a request for a favor by individual $\boldsymbol{j}$ will only grant the favor if he does not want to break the link with $\boldsymbol{j}$. Since the cost of granting a favor is incurred in the current period, $\boldsymbol{i}$ will grant the favor if the expected discounted stream of future payoffs from maintaining the link with $\boldsymbol{j}$ exceeds c.

This is where the structure of the network comes into play. We illustrate this informally by means of a couple of examples. Suppose, for instance, that the network is a *line* on three

[1] For an overview of recent developments in network exchange theories, see Willer ( ).

individuals, with 1 and 3 being the extreme nodes while 2 is in the middle. Suppose that one of the extreme nodes, say 1, requires a favor in some period t, and requests his neighbor 2 to grant him the favor. Clearly, the only rational response is for 2 to reject the request since he does not lose anything by doing so, and saves c in the process. This is because 2 is *ensured* from any possibility of all his links being broken - once the link with 1 is broken, 2 and 3 will form a partnership which will never break up. On the other hand, suppose it is 2 who needs a favor and requests 1 to give him the favor. Notice that if 1 is sufficiently patient, then he should not refuse the request. For suppose he agrees to grant the favor to 2. In the next period, with a probability of a third, it is 3 who will need the favor and 2 will refuse leaving 1 and 2 to exchange favors indefinitely in the future. Since $v > c$, the current cost of granting the favor must be less than the future stream of discounted payoffs if $\boldsymbol{i}$ is not too impatient.

Consider another example where the initial network is a 4-person line, with 1 and 4 being the extreme nodes. Then, *two* matchings are possible. But, this requires that 2 and 3 do not form a partnership with each other since that would leave 1 and 4 unmatched. We argue that this pair will not form in equilibrium. Suppose that 2 approaches 3 with a request. Then 3 will refuse the request since she is sure that 4 will accept to grant him a favor in the future. (We will later refer to players 1 and 4 as 'captive' agents of players 2 and 3 since they will always accept the formation of the partnership with 2.)

In general, it is not difficult to see that not all requests will be granted and so the network will grow sparser over time. The long-run stationary network will be the union of *pairs* of agents; that is.a strong form of (bilateral) reciprocity results so that each agent will exchange favors with one and only one neighbor. The main focus of our paper is to examine whether the pattern of these bilateral links is *efficient* in the sense of forming the maximum number of matched pairs no matter what the initial network. [2] While this intuition is simple enough in the simple examples given above, it is far from obvious that some "wrong" pair of agents will not form a partnership, thus making the overall pattern of matches inefficient. However, we are able to show that in *all* networks with a finite number of nodes, the efficient pattern of bilateral links will indeed be established.

The intuition for this result derives from a generalization of the strategy of the players in the 4-person line, where we saw that agents 2 and 3 will never choose to form a partnership which disconnects the two other agents in the network. We establish that the unique optimal strategy of a player $\boldsymbol{i}$ is to accept the request of a player $\boldsymbol{j}$ if and only if, once the link $\boldsymbol{ij}$ is broken, player $\boldsymbol{i}$ does not belong to all maximum matchings of the graph *g* ¥ ***ij***. This characterization of the optimal strategy (which is obtained by induction on the size of the graph) relies on the identification of all maximum matchings of a graph, and of the players

[2] In our model, this is the only possible definition of efficiency since all pairs of agents generate the same surplus. As players become perfectly patient, maximizing the total number of pairs is equivalent to maximizing the sum of utilities of all agents.

who belong to all maximum matchings.[3] It immediately implies that a maximum matching is obtained in equilibrium, as players will never break a link which reduces the number of matches in the graph.

In this baseline model, the maximum number of pairs is formed because agents delay the formation of the partnership until the social network is such that there is a risk that they will not be able to find a partner in the long run. We contrast this model with another model where both agents immediately benefit from the formation of the partnership so that players rush to form partnerships. As opposed to the case of costly favors, the maximum matching in this alternative model with positive favors is not necessarily formed in equilibrium. In fact, the efficient number of pairs will be formed if and only if the initial network is *completely elementary*, in the sense that the number of matchings does not decrease by more than one whenever a pair leaves the graph.[4] We show that there are only two perfect completely elementary graphs: the complete network and the complete bipartite network with half of the players on each side of the network.

In the second part of the paper, we test whether players form efficient partnerships in social networks running a series of laboratory experiments. The experimental design mimics the game of partnership formation, but in a finite setting where, instead of receiving an expected discounted value, subjects obtain a fixed finite value when they form the partnership. [5] We consider five different settings with initial social networks of increasing complexity. We observe that a large fraction of the subjects (more than 75%) do indeed select the equilibrium action, and that the subjects' ability to compute and select the subgame perfect equilibrium action depends on the complexity of the network. We also note that, even when subjects do not employ their subject equilibrium strategy, the proportion of rounds for which the efficient maximum matching is obtained is very high – around 78% of all rounds.

We analyze the systematic departures from equilibrium behavior and discover that subjects err by accepting too often. In addition, we see that subjects who are more risk averse (as measured by a classical questionnaire on risk aversion) accept more often, in the fear of being left isolated at the end of the game. One instance where we observe that agents correctly reject the requests is when they have access to 'captive' agents who are only linked to them. We show that this 'captive agents' heuristic works very well and that players with captive agents are much more likely to play their subgame perfect equilibrium strategy. Finally, we note that the complexity of the network – and in particular the presence of cycles

[3]We call these nodes "essential" nodes of the graph. These nodes, termed "always efficient" nodes also play a role in the characterization of the equilibrium strategies in the model of non-stationary bargaining in networks of Abreu and Manea (2012). They also appear in the Edmunds Gallai decomposition of bipartite graphs which was used by Corominas-Bosch (2004) and Polansky (2007) to characterize equilibrium strategies in a model where nodes on the same side of the market make simultaneous offers.

[4]The terminology of "elementary" networks is due to Lovasz and Plummer (1986).

[5]The value is computed so that the equilibrium behavior in the finite game is equal to the equilibrium behavior in the infinite game studied in the theoretical section of the paper.

– greatly complicates the computation of the equilibrium behavior and results in subjects making more mistakes.

1.2 Relation to Literature

Our model of partnership formation in social networks is related to two different strands of the literature. First, it has close connections to models of bargaining in networks, in particular models of bargaining in non-stationary networks where agents who leave the network are not replaced.[6] Corominas-Bosch (2004) and Polansky (2007) proposed the first models of bargaining in non-stationary buyer-seller networks, but under the assumption that all players on the same side of the market make simultaneous offers. Manea (2011), and Abreu and Manea (2012a) and (2012b) consider bargaining models where a pair of players is chosen at random to make offers. While Manea (2011) analyzes the stationary situation, where players are replaced after an offer is accepted, Abreu and Manea (2012a) and (2012b) analyze the situation where the network is non-stationary. Abreu and Manea (2012a) is indeed very closely connected to our model, and some insights are common in the two papers. They study an infinite horizon bargaining game in which pairs of players connected in an exogenous network are randomly matched to bargain over a unit of surplus. If the matched pair reach agreement, then they leave the network, and so the network becomes sparser over time just as in our model. Moreover, their focus is similar to ours in the sense that they too are interested in whether the maximum number of matchings will be attained in equilibrium. Of course, this will be possible only if the “right" pairs of agents reach agreement. Abreu and Manea (2012a) point out that efficient matching cannot be attained in general in Markov equilibria, even though Markov equilibria are shown to always exist in Abreu and Manea (2012b). However, they construct an ingenious system of punishments and rewards which ensure the existence of an efficient subgame perfect non-Markovian equilibrium. The context that we are modeling is very different from theirs, leading to very different formal models and proof techniques even though some of the structural properties of nodes (i.e. the "essentiality" of nodes which belong to all maximum matchings) appear to play an important role in the proofs of both papers. The main differences are that we suppose that players do not bargain over a continuous surplus, use a different definition of the value of a partnership, assume that links are broken after a rejection and that the same player can turn to all his neighbors in sequence to form a partnership. These differences imply different conclusions. We are able to show that there is a unique Markov equilibrium which is efficient. On the other hand, there will in general be multiple equilibria in Abreu and Manea (2012), but one non-Markovian equilibrium will be efficient.

Second, because we model the value of a partnership through reciprocal exchange of

[6] See Manea (2016) for a recent survey.

favors, our model is related to the literature on favor exchange.[7] Gentzkow and Möbius (2003), Bramoullé and Kranton (2007), Bloch, Genicot and Ray (2008), Karlan, Möbius, Rozenblat and Szeidl (2009) and Ambrus, Möbius and Szeidl (2014) are all papers which share the same basic structure as our model, where agents request favors or transfers at different points in time, and favors or transfers are enforced through reciprocation in the future. The paper in the literature on favor exchange which is closest to our model is the paper by Jackson, Rodriguez-Barraquer and Tan (2012). In this paper, pairs of agents are matched randomly in any period, with one of the agents requiring a favor from the other. Contrary to our model, favors are link-specific and the agent can only obtain a favor form one of his neighbors. Pairs meet too infrequently to sustain bilateral exchange, However, the favor exchange network may be sustained through social pressures or punishments leading to possible loss of neighbors in the network. Despite the similarity in the settings, the primary focus of their model is very different from ours.

There is a growing literature on experiments in networks which is related to the experimental part of this paper.[8] To the best of our knowledge, our paper is the first to propose an experimental test of the model of partnership formation in non-stationary networks. The most closely related paper is the paper by Charness, Corominas-Bosch and Frechette (2007) who test the Corominas-Bosch bargaining model and observe that, as in our experimental study, the proportion of efficient trade is very high and players' behavior seems to conform to the equilibrium behavior predicted by the theory.

2 The Model

2.1 Partnerships

We consider a society of n agents who are organized in a social network g. The social network evolves over time, as agents will delete links and leave the network. At any discrete time $t = 1, 2, ..$, one agent is chosen with probability $\frac{1}{n}$ to request a favor from a neighbor. If the favor is granted, the agent who receives the favor obtains a flow payoff of v and the agent who grants the favor pays a flow cost c. All agents discount the factor using the same discount factor δ. We define the *value* of a *partnership* as the expected discounted payoff obtained by an agent when he has found a partner with whom he *reciprocates* favors,

$$V = \frac{v - c}{n(1 - \delta)}.$$

Partnerships are formed according to the following decentralized procedure. Suppose that an agent i needs a favor at date t. Two situations may arise:

[7]This literature is surveyed in M öbius and Rozenblat (2016).

[8]See Choi, Gallo and Kariv (2016) for an up-to-date survey of this literature

- Either agent ***i*** is already in a partnership
- Or agent ***i*** is not yet in a partnership

In the former case, the favor is offered by agent *i*'s partner. In the latter case, agent ***i*** turns to his direct neighbors in the current social network g_t and asks them sequentially for a favor. The agent needing the favor *chooses* the sequence in which to approach his neighbors for the favor. If neighbor ***j*** is approached by agent *i*, he responds by Yes or No to the offer. If agent ***j*** rejects the request from *i*, the link ***ij*** is destroyed, the new social network is g_t ¥ ***ij***, and agent ***i*** turns to the next neighbor in his chosen sequence. If all agents reject *i*'s request, the network at next period is

$$g_{t+1} = g_t \text{ ¥ } i,$$

the network obtained from g by deleting ***i*** and all his links.

If agent ***j*** responds Yes, the partnership $\{ij\}$ is formed, and the two partners leave the social network, deleting all their links. Thus, the partnership forms as soon as a favor is granted. We let

$$g_{t+1} = g_t \text{ ¥ } i, j,$$

denote the network obtained after agents ***i*** and ***j*** have left. If ***j*** accepts *i*'s request, but has not in the past approached ***i*** for a favor, then the network remains unchanged till the next period, when a possibly different agent needs a favor.

In general, a *strategy* for player ***i*** who needs a favor is to choose the sequence in which to approach his neighbors for the favor, given the history of the game, while a strategy for ***j*** who has been asked by ***i*** to grant a favor is to decide whether to grant the favor (Y) or not(N), again as a function of the history of the game. However, in what follows, we focus attention on *Markov* strategies which only depend on the current social network. More formally, let $S_i(g)$ be the set of all possible sequences over the neighbors of ***i*** in the network g. Then, if ***i*** needs a favor at time ***t*** and current network g_t, his strategy is a mapping from g_t to $S_i(g_t)$. If ***j*** is asked to grant a favor by ***i*** at network g_t, his strategy is a mapping from $(g_t, \boldsymbol{i})$ to $\{Y, N\}$.

A Markov equilibrium is a collection of Markov strategies such that all agents choose their best responses to the strategies of the others.

An outcome of the partnership formation process is a list of partnerships formed or agents leaving the network at every time period as a function of the realization of "needs"– the list of agents who request a favor at each period. Given a fixed realization of needs, an outcome is *efficient* if it maximizes the sum of payments of all agents. Given that agents are homogeneous, the sum of payments of all agents is maximized when the number of partnerships is maximized.

Definition 2.1 *A social network g* supports efficient equilibria *if and only if, for all realizations of needs, the equilibrium outcomes of the process of partnership formation starting from g are efficient.*

2.2 Matchings and bipartite graphs

In this subsection, we collect definitions in graph theory pertaining to matchings and bipartite graphs which will prove useful in our analysis.[9] Given a network g, a *matching* M is a collection of edges in g such that no pair of edges in M has a common vertex. A matching M is maximal if there is no matching $M^t \supset M$ in g. A matching M is a *maximum matching* if there is no matching M^t in g such that $|M^t| > |M|$. For any graph g, we let $\mu(g)$ denote the *matching number* of graph g, i.e. the size of any maximum matching in g. If n is even, and $\mu(g) = \frac{n}{2}$ there exists a matching covering all the vertices in g. This is called a *perfect matching* and any graph admitting a perfect matching is a *perfect graph*.

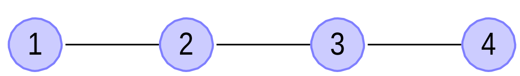
A graph g is *bipartite* if the set of vertices can be partitioned into two subsets A and B such that there is no edge among any two vertices in A and no edge among any two vertices in B. A bipartite graph is complete if all vertices in A are related to all vertices in B. If $|A| = |B|$, a partite graph is perfect if and only if it satisfies Hall's condition: for any subset $C \subseteq A$, the set of vertices in B which are connected to vertices in C, $f(C)$ satisfies $|f(C)| \geq |C|$.

3 Partnerships with costly favors

In this Section, we analyze the equilibrium of the process of partnership formation. We first illustrate the equilibrium in a simple four-player line. We then introduce the concept of essential players in the network, and prove a Lemma on the effect of link deletion on essentiality. Using this definition, we characterize the optimal behavior of agents in the partnership formation game. We then prove the main Theorem of this section, establishing that all social networks support efficient equilibria for δ sufficiently close to 1. Finally, we discuss equilibrium behavior when players are less patient in the complete network and in the line.

3.1 Equilibrium in a four player line L_4

Let $n = 4$ and suppose that $g = L_4$, the four-player line as illustrated in Figure 1.



[9]See Lovasz and Plummer (1986) for an excellent monograph on matchings and bipartite graphs.

Figure 1: The line L_4

The matching number of the line L_4 is two: the maximum number of partnerships formed is two. We claim that, when δ is sufficiently close to 1, in equilibrium, the maximum number of matchings is achieved in equilibrium. Suppose that player 1 has a need and approaches player 2. If player 2 rejects the offer, he becomes a peripheral agent in the line L_3. In L_3, if a peripheral agent requests a favor from the central agent, the central agent will decline the request, as he can form a partnership in the line L_2 and economize on the cost of giving the favor. Hence, with a positive probability, agent 2 ends up being disconnected if he rejects the request of player 1. For δ sufficiently close to 1, the cost of being disconnected exceeds the economy in the cost c, so agent 2 always accepts agent 1's request. Suppose that agent 2 has a request. If he meets agent 3, agent 3 declines the offer to form the partnership, as he can form a partnership later with agent 4 and economize on the cost c. If agent 2 meets agent 1, agent 1 always accepts the formation of the partnership.

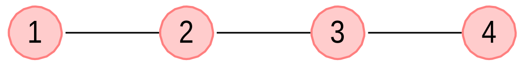
In the line L_4, we can thus characterize the optimal response of agents for δ sufficiently close to 1 as follows:

- Agent 1 (4) accepts to form a partnership with agent 2 (3)
- Agent 2 (3) accepts to form a partnership with agent 1 (4)
- Agent 2 (3) declines to form a partnership with agent 3 (2)

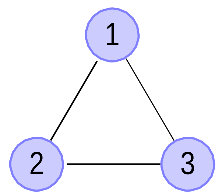
Given this equilibrium behavior, the two partnerships 12 and 34 are always formed in equilibrium: the line L_4 supports efficient equilibria. We will now show that the construction of equilibrium can be extended to any graph g, and introduce the concept of *essential nodes* to characterize equilibrium behavior.

3.2 Essential nodes

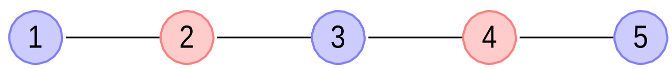
A node i in graph g is called *essential* if it belongs to *all* maximum matchings of the graph g. It is called *inessential* otherwise. Clearly, all nodes are essential in a perfect graph. As illustrated in Figure 2, all nodes are inessential in the odd cycle C_3, and in the line L_5, nodes 2 and 4 are essential, but not nodes 1, 3 and 5. In the line L_5, node 3 is the most central node according to all measures of node centrality, but is inessential. This example shows that there is no connection between centrality and essentiality of nodes in a graph.



The line L_4



The cycle C_3



The line L_5

Figure 2: Essential and inessential nodes

The next Lemma establishes properties on essential nodes which will prove useful in the characterization of equilibrium.

Lemma 3.1 *1. If* $\boldsymbol{i}$ *is an essential node in g, there exists* $\boldsymbol{ij} \in g$*, such that* $\boldsymbol{j}$ *is inessential in* $g ¥ i$*.*

2. *If* $\boldsymbol{i}$ *is not an essential node in g and* $\boldsymbol{ij} \in g$*,* $\boldsymbol{j}$ *is an essential node in* $g ¥ \boldsymbol{i}$*.*

3. *If* $\boldsymbol{i}$ *is a essential node in g and* $\boldsymbol{j}$ *is inessential in g,* $\boldsymbol{i}$ *is essential in* $g ¥ \boldsymbol{j}$*.*

4. *If* $\boldsymbol{i}$ *is an essential node in g ,* $jk \in g$*, and* $\mu(g) = \mu(g ¥ j, k) + 1$*, then* $\boldsymbol{i}$ *is essential in* $g ¥ j, k$*.*

Proof:

1. Let M be a maximum matching in g and $E_1 = (ij)$ the edge covering $\boldsymbol{i}$ in M. Then $(E_2, ...E_M)$ is a maximum matching in $g ¥ \boldsymbol{i}$ which does not contain $\boldsymbol{j}$. Hence, $\boldsymbol{j}$ is not essential in $g ¥ i$.

2. Suppose by contradiction that $\boldsymbol{j}$ is not essential in $g ¥ \boldsymbol{i}$. Then there exists a maximum matching of $g ¥ i$ with no edge covering $\boldsymbol{j}$, $M = (E_1, ...E_M)$. Consider then the matching $(M, \boldsymbol{ij})$ in g. This is a matching of cardinality $\mu(g ¥ \boldsymbol{i}) + 1$, contradicting the fact that i is not essential in g.

3. Suppose by contradiction that there exists a maximum M matching of $g \setminus \boldsymbol{j}$ where $\boldsymbol{i}$ is not covered. Because $\boldsymbol{j}$ is inessential in g, $\mu(g) = \mu(g \setminus j)$. So M has the same cardinality as a maximum matching in g and hence is a maximum matching of g, contradicting the fact that $\boldsymbol{i}$ is essential in g.
4. Suppose that $\boldsymbol{i}$ is inessential in $g \setminus \boldsymbol{j}, k$. Then there exists a maximum matching M in $g \setminus j, k$ not covering i. As $\mu(g) = \mu(g \setminus j, k) + 1$, $M^t = (M, jk)$ is a maximum matching of g not covering i, contradicting the fact that $\boldsymbol{i}$ is essential in g.

Lemma 3.1 shows that any essential node i must be connected to some node which is inessential in $g \setminus i$. On the other hand, all neighbors of an inessential node $\boldsymbol{i}$ are essential in $g \setminus i$. When an inessential agent is removed from the network, all essential agents remain essential. When a pair of agents leaves the network, without disrupting the total number of matchings, all essential agents remain essential as well.

3.3 Equilibrium behavior

With the help of Lemma 3.1, we now characterize the optimal response of agents in the game of partnership formation.

Proposition 3.2 *Let σ be a Markov equilibrium, and suppose $\boldsymbol{j}$ receives a request from $\boldsymbol{i}$ in the social network g. Then, $\boldsymbol{j}$ accepts the request* ***iff*** *$\boldsymbol{j}$ is inessential in $g \setminus \boldsymbol{i}$ or $g \setminus \boldsymbol{i}, k$ where k is the first agent if any in i's chosen sequence $s_i(g)$ to accept i's request if $\boldsymbol{j}$ refuses the request.*[10]

Proof: The proof is by induction on the number of agents in a connected component. For $n = 2$, both agents are essential and the statement is trivially satisfied. For $n = 3$, we distinguish between two cases;

- $g = L_3$
- $g = C_3$.

At $g = L_3$, the central agent rejects the request of the first peripheral agent to require a favor as he remains connected in the line L_2. On the other hand, the two peripheral agents become isolated if they reject the offer of the central agent and accept the request of the central agent. If $g = C_3$, the request of the first agent is always rejected as the other two agents remain connected and essential if that agent is removed from the network.

Suppose now that the statements are true are true for all $n^t < n$ and consider a component with n agents.

[10] The sequence $s_i(g)$ is of course part of the equilibrium strategy profile σ.

Let g^t be the component formed if $\boldsymbol{j}$ rejects i's request. So, $g^t = g ¥ i$, or $g^t = g ¥ i, k$ depending on whether there is some neighbor k of $\boldsymbol{i}$ who accepts i's request following a refusal by $\boldsymbol{j}$. In either case, $|g^t| < n$, and we use the induction hypothesis to compute the continuation payoff of agent $\boldsymbol{j}$ if he rejects i's request.

Suppose first that $\boldsymbol{j}$ is inessential in g^t. If $\boldsymbol{j}$ is chosen next period to have a request, by Lemma 3.1, (statement 2), all neighbors of $\boldsymbol{j}$ remain essential in $g^t ¥ \boldsymbol{j}$. By the induction hypothesis, the last agent in the sequence chosen by agent $\boldsymbol{j}$, $s_j(g^t)$ must reject j's request. By backward induction, the agent preceding that agent in the sequence also rejects j's request, and all agents contacted in the sequence $s_j(g^t)$ also reject the request. Hence $\boldsymbol{j}$ obtains a continuation payoff of 0 with positive probability. For δ sufficiently close to 1, agent $\boldsymbol{j}$ thus has an incentive to accept i's request.

Suppose next that $\boldsymbol{j}$ is essential in g^t. We claim that j's request will always be fulfilled. First suppose that $\boldsymbol{j}$ has a request next period, at $t + 1$, when the social network is g^t and $\boldsymbol{j}$ is essential in g^t. By Lemma 3.1 (statement 1), one of his neighbors, say k, becomes inessential in $g^t ¥ \boldsymbol{j}$. Let $\boldsymbol{j}$ choose a sequence $s_j(g^t)$ finishing with agent k. If no other agent in the sequence has accepted j's request, agent k will, by the induction hypothesis, because agent k is inessential in $g^t ¥ \boldsymbol{j}$.

Next suppose that j's first requests happens at some period $t^t > t + 1$, when the social network is $g_{t'}$. If $\boldsymbol{j}$ is essential in $g_{t'}$, then the previous argument establishes that some neighbor of $\boldsymbol{j}$ in $g_{t'}$ must accept j's request.

So, suppose instead that $\boldsymbol{j}$ is inessential in $g_{t'}$. Let g" be the first network in the sequence of networks between g^t and $g_{t'}$ where $\boldsymbol{j}$ becomes inessential. Abusing notation, denote by g^t the social network immediately preceding g^{tt} along the equilibrium path, such that $\boldsymbol{j}$ is essential in g^t but not in g^{tt} where either (i) $g^{tt} = g^t ¥ k$ or (ii) $g^{tt} = g^t ¥ k, l$ for some k, l.

Suppose that (i) holds, and that k is essential in g^t. Then, from statement (1) of Lemma 3.1 there is some l such that $k, l \in g^t$ and l is inessential in g". By the induction argument, agent l must accept k, a contradiction to k being isolated from g^t. On the other hand, if k is inessential in g^t, from statement (3) of Lemma 3.1, $\boldsymbol{j}$ must be essential in g", which is again a contradiction.

Suppose (ii) holds, so that $g" = g^t ¥ k, l$ for some $k, l \in g^t$. It cannot be that $\mu(g") = \mu(g^t)$ because we can add kl to a maximum matching in g^{tt} and obtain a matching in g^t of size $\mu(g^{tt}) + 1$. We use the following claim to show that $\mu(g") + 1 = \mu(g^t)$.

Claim 3.3 *If along the equilibrium path, at some period t, a pair* (k, l) *of agents forms a partnership, then* $\mu(g_t ¥ k, l) = \mu(g_t) - 1$.

Proof of the Claim: Suppose that agent k places the request and agent l responds. Agent k must be essential in g_t. If agent k were inessential, all his neighbors would reject his claim. By the inductive step, player l must be inessential in the graph g^t formed after his rejection. Suppose first that all agents following l reject k's request in equilibrium so that $g^t = g_t \setminus k$. As k is essential in g_t, $\mu(g_t \setminus k) = \mu(g_t) - 1$. Let M be a maximum matching of $g_t \setminus k$ not containing l. Then M is a maximum matching of $g \setminus k, l$ and $|M| = \mu(g_t) - 1$. Next suppose that there exist a sequence of agents following l who accept k's request and let $l_1, .., l_N$ denote the agents in the sequence. By the preceding argument, for the last agent in the sequence, $\mu(g_t \setminus k, l_N) = \mu(g_t) - 1$. We argue that whenever $\mu(g_t \setminus k, l_n) = \mu(g_t) - 1$, then $\mu(g_t \setminus k, l_{n-1}) = \mu(g_t) - 1$. If agent l_{n-1} accepts k's request, by the inductive step, he must be inessential in $\mu(g_t \setminus k, l_n)$. Pick a maximum matching M of $g_t \setminus k, l_n$ not containing l_{n-1}. This is a maximum matching of $g_t \setminus k, l_{n-1}$ and as $|M| = \mu(g_t) - 1$, $\mu(g_t \setminus k, l_{n-1}) = \mu(g_t) - 1$. This shows that $\mu(g_t \setminus k, l) = \mu(g_t) - 1$, concluding the proof of the Claim.

From statement (4) of lemma 3.1, if $\mu(g'') = \mu(g^t) - 1$, then j must be essential in g^{tt}. Hence, we have shown that if j is essential at g_t, j must remain essential at all social networks along the equilibrium path, and hence j has an incentive to reject i's request. ’

3.4 The main theorem

We now use the characterization of equilibrium behavior to prove our main theorem: when players are sufficiently patient, the maximum number of pairs are formed in equilibrium, and any network supports efficient equilibria.

Theorem 3.4 *There exists $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta}$, all social networks support efficient equilibria.*

Proof: For a fixed realization of needs, an equilibrium is efficient if and only if the maximum number of pairs is formed in equilibrium and no agent delays the formation of a partnership. In the equilibrium characterized in subsection 3.3, at any period t either a partnership is formed or an agent is isolated from the network. Hence, there is no delay in the formation of partnerships. Furthermore, by Claim 3.3, along the equilibrium path, whenever a pair of agents forms a partnership, it does not disrupt the formation of partnerships by the remaining agents. Hence the total number of partnerships formed in equilibrium is $\mu(g)$ the maximum number of partnerships in the original social network. D

3.5 Exact conditions for the efficient formation of partnerships

Theorem 3.4 establishes that all social networks support efficient equilibria for sufficiently large values of δ. However, the exact condition on parameters for which efficient equilibria are supported depends on the architecture of the social network. In this subsection, we explicitly compute this condition for two specific networks; the line L_n and the complete network K_n where n is an even number. Both networks are perfect so that the maximum number of matchings is equal to $\frac{n}{2}$.

3.5.1 Conditions for efficient partnership formation in the line L_n

We determine the condition for existence of an efficient equilibrium – where the maximum number of matchings is formed for the line L_n and any line L_k of length k. Let V^k denote the continuation value of a peripheral agent in a line with k agents. If k is even, $V^k = V$ as we consider an efficient equilibrium ; if k is odd, $V^k < V$ and we provide an explicit computation below. When player j receives a request from agent i, he accepts the request if and only if

$$V \geq -c + \delta V^k,$$

where k is the size of the component containing j after the link ij is severed. Clearly, if k is even, j always has an incentive to reject i's request. This guarantees that, whenever a partnership ij is formed, $\mu(g \setminus i, j) = \mu(g) - 1$ so that the total number of matchings formed in equilibrium is equal to the matching number of L_n. We now compute the continuation value of a peripheral agent, say agent 1, in the line $L - k$ where k is odd.

If an agent in $n - k$ has a need, agent 1's value is δV^k. If agent 1 has a need, his request is rejected and his value is 0. If any other inessential agent $j = 3, 5, ..., k$ has a need, this request is rejected and the component containing agent 1 becomes an even line so that agent 1's value is δV. If an essential agent $j = 2, ..., k - 1$ has a request, in an efficient equilibrium, the request will be accepted by one of his neighbors. With probability $\frac{1}{2}$ the neighbor is to the left of agent j and the component containing agent 1 becomes an even line so that agent 1's value is δV. With probability $\frac{1}{2}$ the request is accepted by an agent to the right of agent j, and agent 1 becomes a peripheral agent in an odd line of size $j - 1$. If agent 2 has a request and addresses it to agent 1, agent 1 accepts it and pays the cost c. Hence, we write the value as

$$V^k = \frac{1}{n}\left[(n-k)\delta V^k + \frac{3(k-1)\delta V}{4} - \frac{c}{2} + \sum_{j=0}^{\frac{k-1}{2}} \frac{\delta V^{2j+1}}{2}\right]. \tag{1}$$

As $V^k < V$ for all k, we observe that V^k is *increasing* in k: the value of a peripheral agent in an odd line increases with the size of the line. This implies that the condition for existence of an efficient equilibrium in the line is the most stringent when $k = n - 1$. Hence the condition for existence of an efficient equilibrium is

$$V \geq -c + \delta V^{n-1},$$

where V^{n-1} is defined recursively through equation (1)

3.5.2 Conditions for efficient partnership formation in the complete network K_n

In any complete network K_k, the continuation value is identical for all agents as they are all symmetric in the continuation network. Let ^k_kdenote the continuation value of any agent in the complete network $K - k$. We claim that, whenever ***i*** requests a need from a sequence of agents $j_1, ..j_{k-1}$, all agents but the last agent j_{k-1} are going to reject the request. If k is odd, K_{k-1} is an even complete graph, and in an efficient equilibrium, all agents obtain a value V and reject the request. If k is even, in an efficient equilibrium, the last agent accepts the request, so that the continuation graph is the even complete graph K_{k-2}. All agents preceding j_{k-1} thus have an incentive to reject the request, anticipating that the graph K_{k-2} will be formed.

An efficient equilibrium thus exists if and only if

$$V \geq -c + \delta W^k,$$

where W^k is the value of an agent, say agent 1, in an odd complete graph K_k, which is computed as follows.

If an agent in $n - k$ has a need, agent 1's value is δW^k. If agent 1 has a need, his request is rejected and he obtains a value 0. If any other agent has a need, his request is rejected and agent 1 obtains a value δV. Hence

$$W^k = \frac{1}{n}[(n - k)\delta W^k + (k - 1)\delta V].$$

We thus obtain

$$W^k = \frac{(k-1)\delta V}{n - \delta(n - k)}$$

which is increasing in k so that the most stringent equilibrium condition is

$$V \geq -c + \delta W^{n-1}.$$

Interestingly, we observe that, as $V > V^k$ for all odd k, $V^k < W^k$ for all odd k and hence $V^{n-1} < W^{n-1}$. The continuation value of an agent in an odd complete graph is always larger than in an odd line of the same cardinality. This implies that it is *easier* to sustain efficient partnership formation in the line than in the complete graph. Hence, an increase in the number of links in the social networks may be detrimental to the efficient formation of partnerships. Increasing the number of social links increases the number of potential matchings, but may also increase the continuation value of agents after a link is severed, making it more difficult to sustain the efficient formation of partnerships.

4 Partnerships with positive favors

4.1 Positive favors

We now consider a model where the formation of a partnership results in positive values for both agents. When an agent responds to a request, he obtains a positive flow payoff $w > 0$ rather than incurring a negative cost $c < 0$. The equilibrium response of an agent is obvious: every agent has an incentive to accept the formation of a partnership immediately. As opposed to the model in the previous section, where agents try to delay the formation of a partnership, when agents obtain joint values, they want to rush to form partnerships. This behavior may result in the inefficient formation of matches. For example, in the line L_4, if agent 2 approaches agent 3 with a request, agent 3 accepts immediately, and the pair (23) is the only partnership formed, short of the maximum number of matches which is equal to 2. Hence, when agents rush to form partnerships, not every social network supports efficient equilibria, and our objective in this Section is to characterize those social networks for which the maximum number of matchings is always formed in equilibrium.

4.2 Elementary social networks

Following Lovasz and Plummer (1986), we call a social network g *elementary* if any edge in g appears in some maximum matching. The line L_4 is not elementary because the edge 23 does not appear in any maximum matching. On the other hand, the line L_5 is elementary. Any cycle C_k is elementary. Any complete graph K_k is elementary. Lovasz and Plummer (1986) provide the following characterization of elementary social networks.

Lemma 4.1 *(Lovasz and Plummer) A social network g is elementary if and only if for all* $\boldsymbol{ij} \in g$, $\mu(g \setminus i, j) = \mu(g) - 1$.

Proof: Pick an edge $\boldsymbol{ij} \in g$. If g is elementary, there exists a maximum matching containing ij, $M = (ij, M^t)$. As M^t is a maximum matching of $g \setminus i, j$, $\mu(g \setminus i, j) = |M^t| = |M| - 1 = \mu(g) - 1$. Conversely, pick a maximum matching M^t of $\mu(g \setminus i, j)$, then as $\mu(g) = \mu(g \setminus i, j) + 1$, $M = (M^t, ij)$ is a maximum matching of g and thus any edge of g appears in some maximum matching.

When a network is elementary, whenever a pair of agents $i, \boldsymbol{j}$ leaves the network, the maximum number of pairs formed in $g \setminus i, \boldsymbol{j}$ is equal to the matching number of g minus one, so that the formation of the partnership (ij) does not result in the disruption of the matchings in the graph. However, this argument only works if one considers the formation of a single partnership (ij) and not of a sequence of partnerships $(ij_1)(j_1j_2)...$. Consider for example the cycle C_6. This cycle is elementary: when a partnership (ij) forms, the remaining network is the line L_4 and the matching number of the line L_4 satisfies $\mu(L_4) = 2 = \mu(C_6) - 1$. However

, as we argued earlier, the line L_4 is not elementary: the formation of the partnership (23) results in a single matching formed. Hence, in order to guarantee that the maximum number of partnerships is formed at some social network g we require that *the social network formed after any sequence of pairs have left is itself elementary*: this is a very strong property and we call graphs satisfying this condition *completely elementary*.

A social network g is *completely elementary* if after any sequence of pairs $(ij_1), (j_1, j_2), .., (j_k j)$ leaves the network, the resulting network $g \setminus i, j_1, j_2, .., j_k, \boldsymbol{j}$ is elementary. Any complete network K_k is completely elementary. The line L_5 is completely elementary because if a pair leaves, it results either in the formation of the line L_3 which is elementary or in the formation of the lines L_2 and L_1 which are elementary. The line L_7 is not completely elementary because if a pair leaves, it may result in the formation of the line L_4 which is not elementary. The cycles C_5 and C_7 are completely elementary as the formation of a partnership results in the formation of the line L_3 and L_5 which are themselves completely elementary. But the cycle C_9 which leads to the formation of the line L_7 is not completely elementary. We now state

Proposition 4.2 *When favors provide positive values to both agents, a social network g supports efficient equilibria if and only if it is completely elementary.*

4.3 Perfect networks supporting efficient equilibria

We now provide an alternative characterization of completely elementary networks when the social network g admits a perfect matching. Hence, we focus attention on networks with an even number of nodes such that $\mu(g) = \frac{n}{2}$. We will prove that a perfect network supports efficient equilibria if and only if it is either formed of components which are complete or complete bipartite graphs.

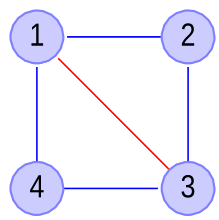
Theorem 4.3 *When favors provide values to both agents, a perfect social network g supports efficient equilibria if and only if it is the disjoint union of perfect components which are either complete or complete bipartite.*

Proof: (Sufficiency) Consider a perfect component g which is either complete bipartite or complete bipartite. and an edge $\boldsymbol{ij}$ in g. If g is the complete graph K_k, then $g \setminus i, \boldsymbol{j}$ is the complete graph K_{k-2} and hence is a perfect complete graph. If g is the perfect complete bipartite graph $B_{k,k}$, then $g \setminus i, \boldsymbol{j}$ is the perfect complete bipartite graph $B_{k-1,k-1}$. Because any perfect complete or perfect complete bipartite graph is elementary, the perfect component g is completely elementary. The full network formed of the disjoint union of perfect components is also completely elementary and hence supports efficient equilibria.

(Necessity) The proof is by induction on the number of nodes in a connected component g of the original network. If $|g| = 2$, the statement is vacuous and always satisfied. If

$|g| = 4$ and the network is completely elementary, it must either be the complete network K_4 or the cycle C_4. The cycle C_4 is equal to the complete bipartite graph $B_{2,2}$. The other two connected networks of size 4 (up to a a permutation of the players) is the line L_4 and the network formed by the cycle and one additional link. As we argued earlier, the line L_4 is not elementary (the link 23 in red does not appear in any maximum matching. The other network, illustrated below, is not elementary because one of the links (in red) does not appear in any maximum matching.

The line L_4



The cycle C_4 plus one link

Now consider a connected component g of size $2k$ which is perfect and completely elementary. For any $\boldsymbol{ij} \in g$, $g ¥ \boldsymbol{i, j}$ is completely elementary. In addition, as $\boldsymbol{ij}$ belongs to some maximum matching, $\mu(g ¥ \boldsymbol{i, j}) = \mu(g) - 1$ so that $g ¥ \boldsymbol{i, j}$ is perfect. By the induction hypothesis, $g ¥ \boldsymbol{i, j}$ is the disjoint union of components which are either complete or complete bipartite.

Claim 4.4 *If g is perfect and completely elementary, then* $g ¥ i, \boldsymbol{j}$ *is connected for all* $\boldsymbol{ij} \in g$.

Proof of the Claim: Suppose by contradiction that there exists $i, \boldsymbol{j}$ such that $g ¥ i, \boldsymbol{j}$ contains different components. Because $g ¥ i, \boldsymbol{j}$ is perfect, all components must be even. Furthermore, each agent in g must be connected to at least two other agents. Suppose that this were not the case, and some agent $\boldsymbol{j}$ is only linked to another agent $\boldsymbol{i}$. Pick any other agent k such that $\boldsymbol{ik} \in g$ (which exists since $|g| > 2$) and consider the graph $g ¥ i, k$. Agent $\boldsymbol{j}$ becomes isolated in the graph $g ¥ \boldsymbol{i}, k$ contradicting the fact that $g ¥ \boldsymbol{i}, k$ is a perfect graph. Because g is connected and $g ¥ \boldsymbol{i, j}$ contains different components, there must exist two agents k and $\boldsymbol{l}$ belonging to two different components h_1 and h_2 such that either $ik, \boldsymbol{jl} \in g$ or $\boldsymbol{il, jk} \in g$. Without loss of generality, suppose that $ik, \boldsymbol{jl} \in g$ and consider the graph $g ¥ \boldsymbol{i}, k, \boldsymbol{j}, l$. Because g is perfect and completely elementary, $g ¥ \boldsymbol{i}, k, \boldsymbol{j}, \boldsymbol{l}$ is perfect and only contains even components. But because $|h_1|$ and $|h_2|$ are even $k \in h_1, \boldsymbol{l} \in h_2$ and $\boldsymbol{i, j} \notin h_1 \cup h_2$, $g ¥ i, k, j, l$ contains two components with odd sizes $|h_1| - 1$ and $|h_2| - 1$, a contradiction.

Next, consider a link $\boldsymbol{ij} \in g$. By Claim 4.4, $g ¥ i, \boldsymbol{j}$ is connected. By the induction hypothesis, it is either a complete graph K_{2k-2} or a complete bipartite graph $B_{k-1,k-1}$. Suppose first that *g* ¥ ***i, j*** = K_{2k-2}. Because |*g*| > 4, for any *kl* ∈ *g*, the graph *g* ¥ *k, **l*** is a complete graph K_{k-2} and not a complete bipartite graph. Now we show that all agents are connected to all other agents in *g*. Pick an agent ***i***. He is connected to at least two other agents ***j*** and *k* in *g*. Consider two other nodes *l, m*. Because *g* ¥ ***i, j*** is a complete graph, *lm* ∈ *g*. So *g* ¥ *lm* is a complete graph and ***jk*** ∈ *g*. But then *g* ¥ ***jk*** is a complete graph, showing that ***i*** is connected to all agents ***l*** /= ***j,*** *k*, so that ***i*** is connected to all other agents in *g*.

Next suppose that *g* ¥ *i, **j*** is a complete bipartite graph $B_{k-1,k-1}$. The set of agents *N* ¥ *i, **j*** can thus be decomposed into two subsets *A* and *B* such that there is no edge among agents in *A* and among agents in *B*. Because |*g*| > 4, for any *k, l* ∈ *g*, the graph *g* ¥ *k, l* must be a complete bipartite graph $B_{k-1,k-1}$ and not a complete graph. Now, for *k, l* /= *i, j*, *g* ¥ *k, l* is a complete bipartite graph, so that agents ***i*** and ***j*** cannot be on the same side. We thus partition the set of players into two sets of equal cardinality *A* ∪ {*i*} and *B* ∪ {*j*} such that there is no edge among agents on the same side. The graph *g* is necessarily bipartite To prove that *g* is a complete bipartite graph, consider ***i*** and pick one agent in *A*, *k* and one agent in *B*, *l*. Because *g* ¥ *k, **j*** is a complete bipartite graph, ***i*** is connected to all agents in *B*. Because *g* ¥ *k, **l*** is a complete bipartite graph, ***i*** is connected to all agents in *B* ¥ ***l*** ∪ {*j*}. Hence ***i*** is connected to all agents in *B* ∪ {æ} completing the proof of the Theorem.

5 Experimental Design

In order to test the behavior of agents in the game of partnership formation in social networks, we design a laboratory experiment in the model with costly favors.[11] The objective of the experiment is to check whether boundedly rational agents will play equilibrium strategies when facing real incentives in real social-network interactions.

Unlike the infinite process described in Section 2, the experiment must stop in certain time. In the experiment, once agents form partnerships and leave the network, they immediately collect the value and will not request or grant favors anymore. We assume that only those participants who are not yet in a partnership are chosen with equal probability to request a favor from one of their neighbors. Therefore, the process ends in finite time when no new partnership is formed in the experiment. The value obtained by an agent in a partnership is thus either $v - c$ if the agent grants the favor or v if he requests the favor. Agents who are not in a partnership receive a value of 0. We compute the values of v and c so that, in the particular networks that we consider in the experiment, the equilibrium behavior in the finite game coincides with the equilibrium of the game of partnership formation of Section 2 when the discount factor δ converges to 1.

[11] Behavior in the model of positive favors is obvious, so we do not feel that an experiment will be helpful there.

5.1 Social Networks

We choose five social networks in the experiment, $g \in \{1, 2, 3, 4, 5\}$, which are presented in Figure 3. The number of nodes and complexity of the network structure increase from social network 1 to social network 5. The first two social networks are the line L_4 and the line L_5. The other three social networks become more complex by involving cycles with $n = 4$ in social network 3, $n = 5$ in social network 4, and $n = 7$ in social network 5, respectively.

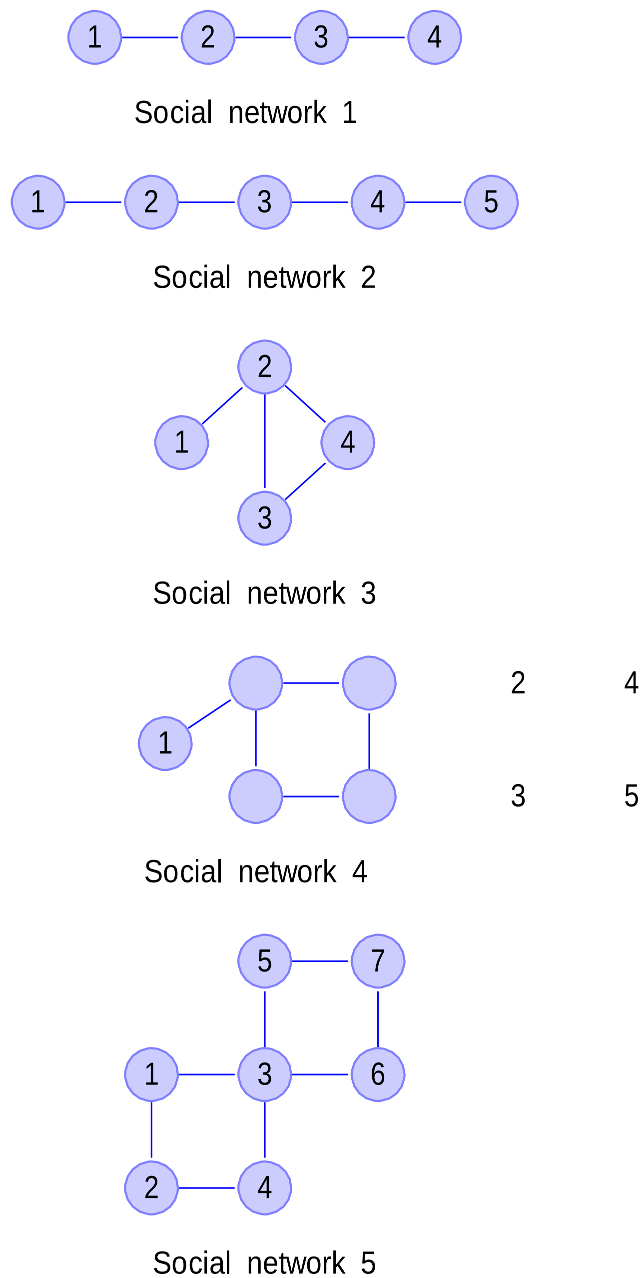


Figure 3: Social Networks 1-5 in the Experiment

In the experiment, subjects go through social networks $g = 1$ to $g = 5$ in sequence, with each social network repeated five times. Therefore, subjects play the social-network game for 25 periods in total, with 5 periods in each of 5 social networks. There are also 2 practice periods in social network 1 at the beginning of the experiment, so that subjects can learn how to play the game with practice.

There are 21 subjects in each session. At the beginning of each period, they are randomly re-matched into groups to play the game in a social network. In each period, at a given time $\boldsymbol{t}$ of graph g_t, one active subject $\boldsymbol{i}$ (i.e. a subject who is neither isolated nor in a partnership in the current network) is randomly chosen to request a favor from one of his neighbors. The sequence in which this subject approaches his neighbors is chosen at random by the computer. The subject $\boldsymbol{j}$ who receives the request then decides whether he will accept the offer or not. If the subject accepts, the partnership is formed and both of them leave the social network. The subject $\boldsymbol{i}$ who makes a request obtains a payoff of v and the subject $\boldsymbol{j}$ who grants the favor earns $v - c$. If this subject decides to reject the offer, the link between the two is destroyed. The subject $\boldsymbol{i}$ then requests a favor from his next neighbor in the sequence. If all neighbors reject his request, the subject $\boldsymbol{i}$ will be cut off from the social network and earn 0. Once decisions are made, subjects who make and receive the request are informed of results and respective payoffs, while others in their group are displayed changes of current social networks on the computer screen. The social network evolves until each agent either has a partner or is left alone.

In the experiment, we set parameters $v = 20$ and $c = 8$. Therefore, in a given partnership formation process of a social network g, a subject will obtain a payoff of 20 by requesting a favor or a payoff of 12 by granting a favor if he is in a partnership. The subject will earn 0 if he has no partner in the end. As there are 4 or 5 players per group in social networks 1 through 4, one subject will be randomly chosen not to play and be paid 10 experiment points. At the end of the experiments, 10 out of 25 periods are randomly chosen to be paid. The exchange rate is 10 experiment points for 1 Euro for all sessions. Each subject also receives a participation fee of 3 Euros. In addition, a subject could earn X points per CRT question answered correctly and the payoff resulting from the risk test.

5.2 Individual difference tests

At the end of 25 periods, subjects are then asked to answer their beliefs about others strategies, and take risk test as well as CRT test.

Belief elicitation

Subjects should make decisions in the game according to their beliefs about the rationality and the behavior of other players in the social network. We have therefore elicited a proxy for participants' beliefs about other players' behavior. In a context of a 3-person line network, participants are asked to estimate the proportion of central players who actually refuse the request from extreme nodes. This question is not incentivized. In this context central players should rationally reject the request, though even acceptance is still a rational decision for highly risk averse participant. As it is, we observe 85% of rejection from central players in a 3-person line network during the experiment.

Risk elicitation

We elicited attitudes towards risk following a procedure introduced by Eckel *et al* (2012). The procedure consists of a choice between six lotteries in the form of a coin flip that gives a low or a high payoff with equal probability. The lotteries are arrayed from a safe one with a certain payoff of 18 experiment points to a highly risky one with a high payoff of 54 points and a negative low payoff of -2 points.[12] Expected return increases along with higher variance as one moves from the safest to the riskiest lottery. The variance that a subject is willing to accept gives a proxy of his risk aversion.

Cognitive ability

In our experiment, cognitive abilities are elicited with the Cognitive Reflexion test (CRT, Frederick, 2005). This test is designed to assess an individual's ability to move from an intuitive and spontaneous wrong decision to a reflective and deliberative right one. Subjects are asked to answer three questions :

Question 1: A bat and a ball cost 11 e. The bat costs 10 emore than the ball. How much does the ball cost?

Question 2: If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?

Question 3: In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

Results of CRT test are positively related with rational thinking performance (Toplak, West and Stanovich, 2011).

5.3 Experimental Procedure

In each experimental session, subjects are randomly assigned ID numbers and seats in front of the corresponding terminal in the laboratory. The experimenter reads the instructions aloud. Subjects are given the opportunities to ask questions, which are answered in public. We check subjects' understanding of the instructions by asking them to answer incentivized review questions at their own pace. After answering one review question, each subject is shown whether his answer is correct, or as well as the right answer. After going over all review questions, subjects go through 27 periods in the social-network experiment, including 2 practice periods using social network 1. After the session, subjects are asked to report their beliefs on other subjects' behaviors in a given situation, and take the risk and CRT tests. At the end of experiment, each subject fills out a demographics survey on the computer, and is

[12] The six lotteries used for the test are 18/18, 14/26, 10/34, 6/42, 2/50 and -2/54. Values are given in experimental points.

then paid in private. Each session lasts approximately 80 minutes, with 15 minutes devoted to the instructions. The experiment is programmed in java.

Table 1: Features of Experimental Sessions

Social Networks	value of v	value of c	#Sbj. × # sessions	Total # of sbj.	Total # of obs.
1 - 5	20	8	21 × 6	126	1842

Table 1 summarizes the features of experimental sessions. All sessions were conducted in French at GATE-LAB, the Experiment Economics Laboratory in Lyon between April and September 2015. The subjects are students from an engineering department, Ecole Centrale de Lyon, a business school EM Lyon, and the University of Lyon. No one participated more than once. We ran 6 independent sessions. In total, 126 subjects participated in the experiment. The average earning (including participation fee) is 21 Euros. The English translations of the experimental instructions can be found in the Appendix.

6 Results

In this section, we analyze the results of the experiments, focussing on two main questions. First, we study individual behavior and analyze whether real players play subgame perfect equilibrium strategies, and if not, what behavioral patterns they follow. Second, we analyze whether, at the aggregate level, the social interaction among real players results in efficient outcomes in different social networks?

6.1 Individual Behavior

To check whether subjects play as the theory predicts, we consider all networks which arise during the experiment and break down behavior in these different networks, defining different situations that subjects are faced with when making decisions in each graph - e.g., player 1 or 4 requests to player 2 or 3 in social network 1. There are 36 possible graphs and 92 possible situations in total in the experiment.

For each situation, we compute the best response of the player using the characterization results of Section 2. We also calculate the expected value of acceptance, which is always 12, and the expected value of rejection for each situation. For example, in Figure 3, suppose that player 2 in social network 1 receives a request from player 3. Player 2 can make the following calculation using backward induction. The expected value is 12 by accepting the request; however, if player 2 declines the offer, the link between player 2 and 3 will be destroyed, and player 3 would make a request to player 4, who is expected to accept the offer. The network would then evolve to the L_2 where player 2 has 50% chance of earning 20 by making a request accepted by player 1, and 50% chance of earning 12 by accepting the request from player 1. The expected value is thus $0.5 \times 20 + 0.5 \times 12 = 16$. Hence player 2 should reject the offer. Generally, the difference between the expected values of rejection and acceptance is defined as follows:

$$\text{EV.difference} = \text{Expected value of rejection} - \text{Expected value of acceptance.}$$

If EV.difference > 0, the optimal behavior is to reject. If EV.difference < 0, the optimal behavior is to accept. [13]

Due to the strategy uncertainty about the behavior of other players, subjects may make decisions according to the real earning difference between rejection and acceptance, rather than the theoretical difference. For instance, in the previous example, it is possible that player 1 mistakenly rejects the offer, making the gain of rejection $0.5 \times 0 + 0.5 \times 12 = 6$ for player 2. On the other hand, he will always earn 12 for sure by accepting the offer. Taking player 1's mistake into account, it is probably best for player 2 to accept the offer. We thus decided to calculate the average gain of rejection for each situation using the real behavior of other subjects in the experiment to see if subjects make decisions based on the real earning difference. The difference between real gain of rejection and acceptance is defined as follows:

$$\text{Real.difference} = \text{Real gain of rejection} - \text{Real gain of acceptance.}$$

6.1.1 Best Response

We first examine whether subjects play their subgame perfect equilibrium strategies in the experiment. Figure 4 presents the proportions of rejection conditional on the difference between expected value of rejection and acceptance (left panel), and on the difference between real gain of rejection and acceptance (right panel). The red line corresponds to the predictions of the theory. If subjects play optimally, they will reject when EV.difference > 0, and accept when EV.difference < 0. It can be seen from Figure 4 that the proportions of rejection increase with the expected value difference as well as real earning difference. Overall, the proportion of best responses is 79.5%.

[13] We did not include any situation with indifference in the experiment.

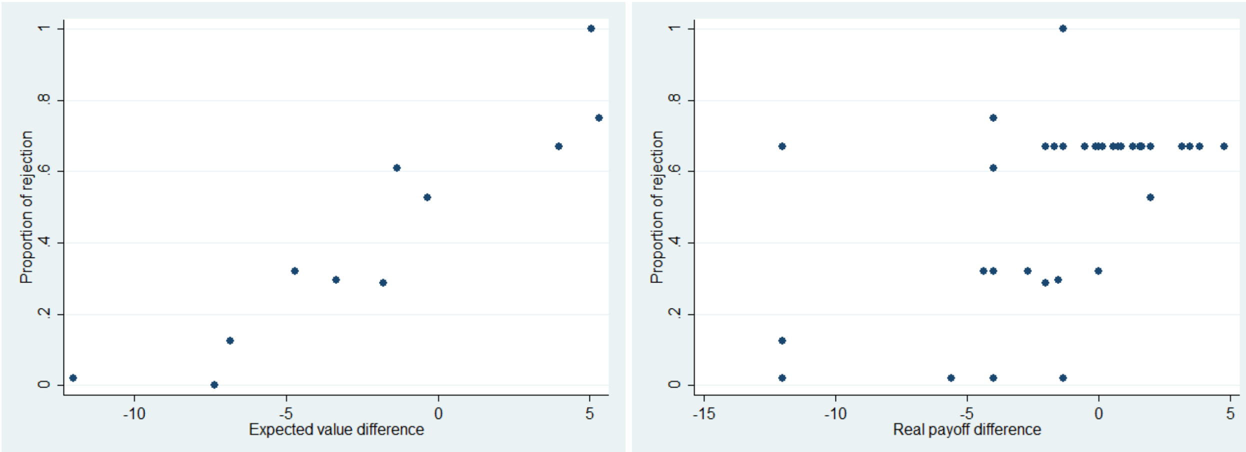


Figure 4: Proportions of Rejection with Expected Value Difference and Real Earning Difference

Table 2 presents the proportions of rejection for EV.difference > 0 (Real.difference > 0) and EV.difference < 0 (Real.difference < 0) in each session. On average, the proportion of rejection is as high as 66.9% (56.8%) when EV.difference > 0 (Real.difference > 0) and as low as 12.6% (11.3%) when EV.difference < 0 (Real.difference < 0) (the proportion of acceptance is 87.4% (88.7%) correspondingly). As previous decisions are displayed within groups and subjects are randomly re-matched in each period, observations are dependent with each other in each session. As a result, samples for EV.difference > 0 and EV.difference < 0 are related with each other, where we have only 6 independent observations at session levels for each. We perform the Wilcoxon signed-rank test at the session level to compare the proportions of rejection between EV.difference > 0 and EV.difference < 0, and between Real.difference > 0 and Real.difference < 0. As P-values are 0.139, we reject the null hypothesis in favor of H_a that subjects are more likely to reject when it is optimal to reject. This indicates that subjects in general play rationally, which is consistent with the theoretical prediction.

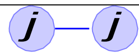
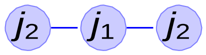
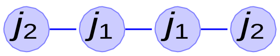
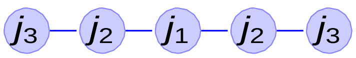
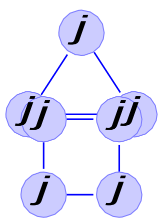
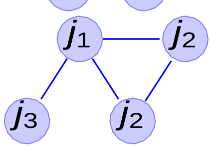
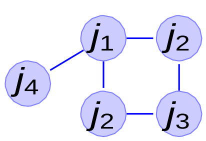
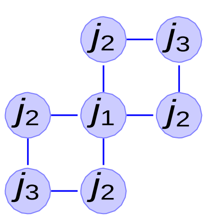
Table 2: Proportions of Rejection

	EV.diff. > 0	EV.diff. < 0	Real.diff. > 0	Real.diff. < 0
Session 1	0.691	0.106	0.564	0.088
Session 2	0.700	0.144	0.614	0.113
Session 3	0.650	0.090	0.547	0.085
Session 4	0.528	0.140	0.452	0.140
Session 5	0.759	0.140	0.642	0.129
Session 6	0.686	0.134	0.589	0.125
Average	0.669	0.126	0.568	0.113
P-values	0.139		0.139	

Note: Wilcoxon signed-rank test at the session level. H_a: (Ev.diff. > 0) > (EV.diff. < 0); H_a: (Real.diff. > 0) > (Real.diff. < 0).

Even though subjects generally obey theoretical prediction, we still observe that 20.5% of individual choices are not consistent with subgame perfect equilibrium. We now analyze in detail the departures from equilibrium choices. Table 3 presents the proportions of best response in some graphs which arise frequently during the experiment, ordered from the simplest two-player line to the most complex seven-player circle. It can be seen from Table 3 that subjects perform very well when social networks are lines. In the simplest graph where there are only two players, the proportion of best response is as high as 98.3%. However, we only have 24% and 51.3% of best responses in the circles $C3$ and $C4$. Notice that in the circle, an agent who requests a favor will then turn to his other neighbor after the link is broken. We tested the subjects' understanding of this particular aspect of the procedure through a quiz, and we are confident that the departure from equilibrium behavior in the circles is not due to subjects' misunderstanding of the game. Moreover, the proportion of best response varies greatly with positions in a given graph. For example, subjects in the two extreme nodes of the line are more likely to play optimally than those in the center. In the seven-player circle, when it is optimal for both players to reject, the proportion of best response is 80 % for j_1 but is only 41.1% for j_3. Even for the same position, subjects react quite differently when different neighbors place requests to them. For example, in the social network $C4$ plus one more node and link, it is optimal for player j_2 to accept. However, the proportion of best response is 80% when j_3 requests him, but is only 44.4% when j_1 makes a request.

Table 3: Proportions of Best Response in Selected Graphs

Graph	Role	EV.diff.	Real.diff.	Best Response	# of obs.
	j	-12	-12	0.983	478
	j_1	4	3.7	0.854	151
	j_2	-12	-12	0.948	77
	$j_2 \rightarrow j_1$	-4.7	-5.7	0.649	154
	$j_1 \rightarrow j_1$	4	3.36	0.733	60
	j_2	-12	-12	1	63
	j_1	-4.7	-2.67	0.7	20
	$j_3 \rightarrow j_2$	4	1.57	0.529	87
	$j_1 \rightarrow j_2$	4	3.2	0.769	26
	j_3	-12	-12	1	27
	j	4	2.67	0.24	25
	j	4	1	0.513	39
	$j_3 \rightarrow j_1$	-1.33	0.8	0.39	41
	$j_2 \rightarrow j_1$	4	4.17	0.889	27
	$j_2 \rightarrow j_2$	-4.7	-6.33	0.676	37
	$j_1 \rightarrow j_2$	4	4.25	0.5	32
	j_3	-12	-12	1	13
	$j_4 \rightarrow j_1$	4	0.83	0.784	37
	$j_2 \rightarrow j_1$	4	3.29	0.824	34
	$j_3 \rightarrow j_2$	-4.7	0	0.8	35
	$j_1 \rightarrow j_2$	-0.35	2	0.444	18
	j_3	4	2	0.359	39
	j_4	-12	-12	1	12
	j_1	4	4.17	0.8	30
	$j_3 \rightarrow j_2$	- 3284	-1.5	0.667	24
	$j_1 \rightarrow j_2$	-1.8	-2	0.714	7
	j_3	4	1.67	0.414	29

In order to systematically check which factors affect the subjects' strategies, we ran probit regressions. The results are presented in Table 4. The dependent variable is the proportion of best responses. Independent variables include a dummy variable " B.Reject " which is equal to 1 if it is optimal to reject, the absolute difference between expected value of rejection and acceptance "Abs_EV.diff" in specifications (1) through (3), the absolute difference between real gain of rejection and acceptance "Abs_Real.diff" in specifications (4) through (6), a dummy variable "Captive" which is equal to 1 if the subject has a captive player, a dummy variable "Cycle" which is equal to 1 if subject is in the cycle, an interaction terms between " B.Reject " and 'Cycle", the measure of risk aversion "Risk", an interaction term between " B.Reject " and "Risk", and the number of the "Period". In each specification, standard errors are clustered at session levels. We summarize the results in the table below.

Table 4: Probit Regressions: Proportion of Best Response

	(1)	(2)	(3)	(4)	(5)	(6)
B.Reject	-0.068	-0.112**	-0.151***	-0.061	-0.108	-0.157**
	(0.041)	(0.052)	(0.052)	(0.062)	(0.075)	(0.079)
Abs_EV.diff.	0.048***	0.048***	0.046***			
	(0.003)	(0.003)	(0.003)			
Abs_Real.diff.				0.036***	0.036***	0.034***
				(0.004)	(0.005)	(0.005)
Captive	0.194***	0.194***	0.21***	0.167***	0.167***	0.188***
	(0.021)	(0.021)	(0.02)	(0.026)	(0.027)	(0.025)
Cycle	0.044	0.043	-0.011	0.013	0.011	-0.056
	(0.037)	(0.036)	(0.037)	(0.054)	(0.054)	(0.06)
B.Reject × Cycle	-0.018	-0.015	0.006	0.013	0.016	0.047
	(0.062)	(0.06)	(0.056)	(0.065)	(0.063)	(0.061)
Risk		-0.007**	-0.009***		-0.007***	-0.008***
		(0.003)	(0.003)		(0.002)	(0.002)
B.Reject × Risk		0.019***	0.023***		0.02***	0.023***
		(0.007)	(0.007)		(0.007)	(0.007)
Period			0.007***			0.007***
			(0.001)			(0.001)
No. of observations	1,826	1,826	1,826	1,692	1,692	1,692

Note: standard errors in parentheses are clustered at the session; coefficients are marginal effects. *** p<0.01, ** p<0.05, * p<0.1.

We now analyze the results of the probit regression. We first observe that the absolute earning difference between rejection and acceptance significantly increases the probability of making a rational decision. We also note that subjects learn how to play the game. The coefficients of “Period” are positive and significant in both specification (3) and (6). The subject’s probability of playing a best response will increase by 0.7 percentage point for one more period of experience.

6.1.2 Heuristics

Besides studying the best responses, we also observe two heuristics that subjects tend to follow which are presented in Figure 5.

captive.png

rationrej.png

Figure 5: Heuristic Behaviors in the Experiment

First, the position in a given network plays an important role on the decision to accept or to reject. We find that subjects are more likely to reject the offer when they have captive players. We find that 81.9% of requests are optimally rejected when subjects have captive players who do not make the request, compared to 23.1% of rejection by subjects who do not have such captive players. We then focus attention on those situations where it is optimal to reject. We observe from Figure 5 (left panel) that subjects who have captive players are more likely to reject when it is optimal to reject, with 81.9% of rejection with captive players compared to 53% without captive players. On the contrary, when subjects are captive players, the proportion of acceptance is as high as 98%. Coming back to the probit regression, we note that the coefficients of the dummy ‘Captive” In Table 4are positive (from 0.167 to 0.21) and significant ($p < 0.01$) in all specifications.

Second, we observe that subjects are more likely to accept than to reject. On average, 65.6% of requests are accepted by subjects. As the higher acceptance rate is probably because optimal acceptance is more likely to happen, we further examine proportions of best response when it is optimal to accept and when it is optimal to reject, respectively. These are presented on the right panel of Figure 5. We still find a higher rate of rational acceptance, which is 87.4%, compared to 66.9% for rational rejection. The probit regression also shows this tendency to accept. In Table 4, the coefficients of " B.Reject " are negative in all specifications. We do not find a significant effect in specification (1). However, the effect of rational rejection becomes significant when controlling for risk preference in specification (2) ($p < 0.05$), controlling for risk preference and period variables in specification (3) ($p < 0.01$), and controlling for absolute real earning difference, risk preference and period variables in specification (6) ($p < 0.05$). Furthermore, the magnitude of this effect becomes larger when controlling for absolute real earning difference, risk preference and period variables.

The main reason for subjects to accept rather than reject is risk aversion – subjects prefer to earn 12 points for sure by accepting the offer immediately than to take risk of earning more in the future. So a highly risk averse subject should be more likely to accept the offer although it is optimal for him to reject it. To test this hypothesis, we used the test eliciting risk attitude from the subjects. Based on this test, subjects are divided into two groups: subjects who choose 0 and 1 in the risk test are in the highly risk averse group, and subjects who choose between 2 and 5 form the low risk averse group. Figure **??** presents the proportion of best response for each group when best response is to accept and when best response is to reject. Subjects in highly risk averse group are less likely to reject when is best to reject, which is 60.4% compared to 72.3% in low risk averse group. On the other side, when it is best to accept, the proportion of acceptance is 87.2% in the highly risk averse group and 87.6% in low risk averse group.

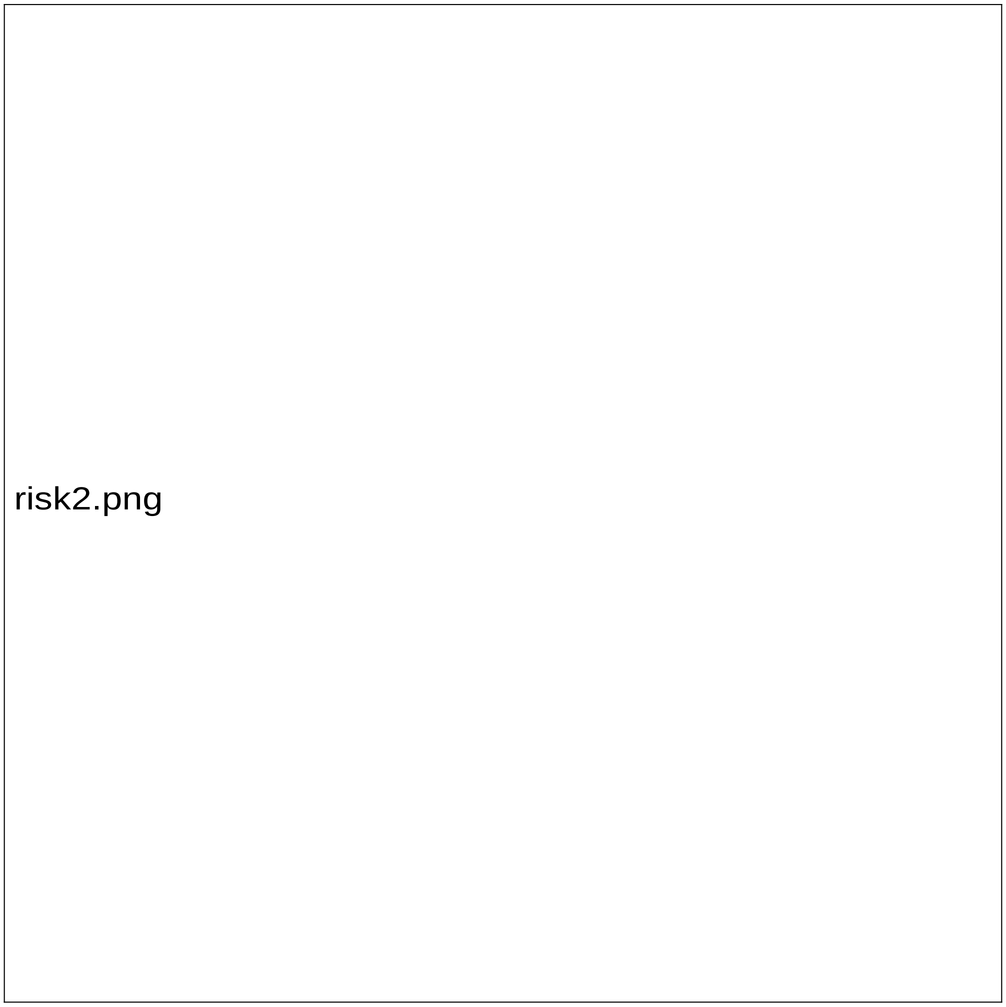


Figure 6: Risk in the Experiment

The fact that highly risk averse subjects are more likely to accept when it is optimal to reject is also borne by the results of the probit regression. In Table 4, the coefficients of “Risk” are negative (from -0.007 to -0.009) and significant ($p < 0.001$) in all specifications, indicating that subjects who are more risk loving are less likely to accept the offer when it is best to accept. I don’t think I understand. On the other hand, the coefficients of “B.Reject × Risk” are positive (from 0.019 to 0.023) and significant ($p < 0.001$) in all specifications, indicating that subjects who are more risk averse are less likely to reject the offer when it is best to reject .

6.2 Aggregate outcomes

In this Subsection, we analyze whether aggregate behavior leads to efficient outcomes in the experiment. We compute an efficiency index as follows:

$$\text{Efficiency index} = \frac{\text{Real number of matched pairs}}{\text{Maximum number of matched pairs}}.$$

Table 5 presents the efficiency index for each of the five social networks in each period. On average, the efficiency index is as high as 0.903. We also find that 78% of times (493 out of 630 total outcomes) social networks achieve their maximum number of matched pairs. More interestingly, in the most complex seven-player social network, all groups achieve efficient outcomes in the last period. These results indicate that we can obtain a high level of efficient outcomes with real subjects' interaction in our experimental setting. In addition, the efficiency index is 0.837 and 0.783 in networks 1 and 3, lower than those in social network 2, 4 and 5. We also calculate the proportions of best response in each social network, which are presented in Table 5. The lower efficiency index in social network 3 is likely due to the fact that subjects are less likely to play their best response in this network. What is interesting to see is that although the proportion of best responses is low in social network 5, its outcome still reaches a high level of efficiency. On the contrary, although the proportion of best responses in social network 1 is higher than in other social networks, it has a lower efficiency index. This is probably because social network 1 is more sensitive to individual behaviors as all players in social network 1 are essential.

Table 5: Outcome Efficiency for Social Networks

Period	Network 1	Network 2	Network 3	Network 4	Network 5
1	0.817	0.979	0.767	0.979	0.926
2	0.867	0.958	0.767	0.938	0.963
3	0.800	0.979	0.817	0.979	0.981
4	0.817	0.938	0.767	0.979	0.963
5	0.883	0.938	0.783	0.979	1.000
Average	0.837	0.958	0.780	0.971	0.967
Best Response	0.828	0.821	0.756	0.804	0.774

We also compute the number of rounds for the partnership formation process to achieve equilibrium outcome. Table 6 presents the number of rounds to equilibrium for each of five social networks. It can be seen from Table 6 that social network 1 only needs 2.1 rounds on average to reach equilibrium, but it takes 4.7 rounds on average for social network 5. These results indicate that when we have more number of players in social network 2, 4 and 5, it will take more rounds to reach the final outcomes. The more links a given social network has, the more rounds it takes to achieve the equilibrium outcome.

Table 6: Number of Rounds to Equilibrium for Social Networks

Period	Network 1	Network 2	Network 3	Network 4	Network 5
1	2.1	2.9	2.5	3.3	5.0
2	2.1	2.8	2.5	3.3	4.8
3	2.1	2.8	2.6	3.0	4.6
4	2.0	2.9	2.2	3.3	4.8
5	2.3	3.1	2.2	3.3	4.6
Average	2.1	2.9	2.4	3.2	4.7

7 Conclusion

This paper analyzes the formation of partnerships in social networks. Agents randomly request favors and turn to their neighbors to form a partnership where they commit to provide the favor when requested. If favors are costly, agents have an incentive to delay the formation of the partnership. In that case, we show that for any initial social network, the unique Markov Perfect equilibrium results in the formation of the maximum number of partnerships when players become infinitely patient. If favors provide benefits, agents rush to form partnerships at the cost of disconnecting other agents and the only perfect initial networks for which the maximum number of partnerships are formed are the complete and complete bipartite networks. The theoretical model is tested in the lab. Experimental results show that a large fraction of the subjects (75%) play according to their subgame perfect equilibrium strategy and reveals that the efficient maximum matching is formed over 78% of the times. When subjects deviate from their best responses, they accept to form partnerships too early. The incentive to accept when it is optimal to reject is positively correlated with subjects' risk aversion, and players employ simple heuristics – like the presence of a captive partner – to decide whether they should accept or reject the formation of a partnership. We are aware of a number of limitations of our model and experimental study and would like to focus our attention to two important questions in future work. First, we would like to extend the model to the study of partnerships of more than two agents. While this extension does not pose any conceptual difficulty, it requires to define generalized matchings of more than two agents, and requires to use more complex tools from graph theory. The second extension is to allow for heterogeneity in the value of partnerships, letting the value of the partnership depend on the pair ij, v_{ij}. Computing the optimal behavior of agents in non0stationary networks with heterogeneous values is probably a very complex task, but we are hopeful that it is tractable and that our model and experimental results could be generalized to a model with heterogeneous players.

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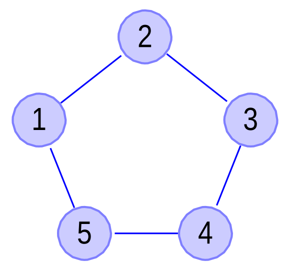
A Experimental Instructions

We would like to thank you for having agreed to participate in this economics experiment. During this experiment, you will earn a certain sum of money. Your earnings are stated in experimental currency unit (ECU). At the end of the session they will be converted to euros using the following rate of conversion :

1 ECU = 0,1 Euros

So **10 ECU= 1 Euros**

Besides the earnings you will make during the experiment, you will receive a 3 Euros participation fee. Your earnings will be paid using a bank transfer during a maximum of 4 weeks . All the decisions which you will take during this experiment are anonymous. You will never have to identify yourself on the computer.
The experiment consists of several periods. At the beginning of every period the groups of players are randomly formed. The links between the members of the same group are represented in the form of a graph. In a graph a player can form a pair with his direct neighbors but not with the other players. The number of players and the structure of the graph change every five periods. The first two periods of the first sequence are trial periods which are not taken into account to determine your earnings. This experiment contains a total of 27 periods.
Example 1 :



In this group of 5 players, player #1 can form a pair with players #2 and #5 but not with players #3 and #4.
A player is chosen randomly among every group to be the claimant. All the players in the graph have an equal chance to be chosen. A neighbor chosen randomly among the neighbors of the claimant is requested to form a pair with the claimant. All the neighbors of the claimant have an equal chance to be chosen.
If this chosen neighbor accepts to form a pair with the claimant, then:

- The pair leaves the graph: all the links that linked the pair to the rest of the graph are deleted. The period ends for the two players of the pair.

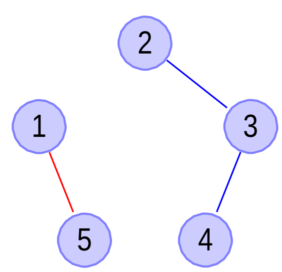
- The claimant earns 20 ECU.
- The neighbor that accepted to form the pair with the claimant earns 20 - 8 ECU, so 12 ECU.
- If there is another possibility of forming a new pair in the remaining graph, another player is chosen randomly among the remaining players to be the new claimant.

If this chosen neighbor refuses to form a pair with the claimant, then:

- The link that linked the claimant to this neighbor is deleted.
- A neighbor is chosen randomly among the remaining neighbors to form a pair with the claimant.
- If the claimant has no remaining neighbors, then if there is another possibility of forming a new pair in the remaining graph, another player is chosen among the remaining players to be the new claimant.

Example 2 : In the graph of example 1. We suppose that player #5 is chosen to be the claimant. We suppose that among the neighbors of player #5 (in this case player #1 and #4), player #1 is chosen to form a pair with player #5.
If player #1 accepts to form the pair with player #5.

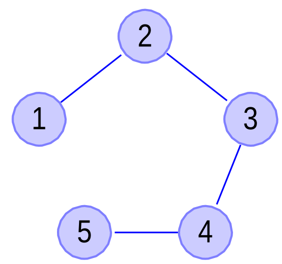
- Players #1 and #5 are no longer linked to the remaining graph.
- Player #5 earns 20 ECU for this period.
- Player #1 earns 12 ECU for this period.
- A new claimant is randomly chosen among the players of the remaining graph formed by players #2, #3 and #4.



If player #1 refuses to form a pair with player #5.

- Players #1 and #5 are no longer linked in the graph.

- Player #4 has the opportunity to form a pair with player #5.



In the end, a claimant that forms a pair earns 20 ECU. A neighbor player who was chosen to form a pair and he accepts, earns 12 ECU. A player that doesn't belong to any pair at the end of a period, earns 0 ECU.
After the last period of the last sequence, 10 periods will be drawn randomly among the periods except the trial periods. The earnings obtained for these 10 periods will determine your earnings for this experiment. Every period has an equal chance to be drawn.
You are 21 participants in the room. When the number of players in the group is 4 or 5, there is then a participant that is randomly chosen, that won't be able to participate during one period. In this case, his earning for this period is 10 ECU.
It is totally forbidden to communicate between each other during the experiment. Any communication may cause the exclusion of the participant from the experiment without compensation. We kindly ask you to reread carefully these instructions and answer the questionnaire which is going to appear on your screens. Every correct answer to this questionnaire will yield a profit of 2 ECU . If you have questions - now or during the experiment, kindly call us by pressing your call button. We shall come to answer you in private.
A series of questions will be given to you after the 27 periods of the experiment. Some of these questions will allow you to win additional earnings