Social Learning and the Design of New Experience Goods

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Abstract: Consumers often consult the reviews of their peers when deciding whether to purchase a new experience good; however, their initial quality expectations are typically set by the product's observable attributes. This paper focuses on the implications of social learning for a monopolist firm's choice of product design. In our model, the firm's design choice determines the product's ex ante expected quality, and designs associated with (stochastically) higher quality incur higher costs of production. Consumers are forward-looking social learners, and may choose to strategically delay their purchase in anticipation of product reviews. We demonstrate that the endogenous nature of social learning gives rise to a complex tradeoff between accelerating consumer learning (through a design of higher expected quality) and appropriating consumer learning (through a design of lower expected quality). We show that this tradeoff results in an interesting phenomenon: in the presence of SL, the firm opts for a design of lower expected quality. Moreover, we find that the relationship between social learning and product design holds significant implications for both firm profit and consumer surplus: contrary to conventional knowledge, we show that social learning can have a net negative impact on the firm's ex ante profit, in particular when the consumers are sufficiently forward-looking; conversely, we establish that unless the consumers are sufficiently forward-looking, the net impact of social learning on the consumers' ex ante surplus cannot be positive.

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Consumers often consult the reviews of their peers when deciding whether to purchase a new experience good; however, their initial quality expectations are typically set by the product's observable attributes. This paper focuses on the implications of social learning for a monopolist firm's choice of product design. In our model, the firm's design choice determines the product's ex ante expected quality, and designs associated with (stochastically) higher quality incur higher costs of production. Consumers are forward-looking social learners, and may choose to strategically delay their purchase in anticipation of product reviews. We demonstrate that the endogenous nature of social learning gives rise to a complex tradeoff between *accelerating* consumer learning (through a design of higher expected quality) and *appropriating* consumer learning (through a design of lower expected quality). We show that this tradeoff results in an interesting phenomenon: in the presence of SL, the firm opts for a design of *lower* expected quality. Moreover, we find that the relationship between social learning and product design holds significant implications for both firm profit and consumer surplus: contrary to conventional knowledge, we show that social learning can have a net negative impact on the firm's ex ante profit, in particular when the consumers are sufficiently forward-looking; conversely, we establish that unless the consumers are sufficiently forward-looking, the net impact of social learning on the consumers' ex ante surplus cannot be positive.

Key words: Bayesian social learning, endogenous product quality, product design, strategic consumer behavior, applied game theory

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1. Introduction

The influence of consumer reviews on the purchase decisions of potential product buyers has grown dramatically in the last decade, with recent surveys suggesting that up to 69% of consumers now consult peer reviews before deciding whether to purchase a product (e.g., Hinckley 2015, Mintel.com 2015). This figure can only be expected to increase in the future: on one hand, rapid innovation and technological advancements are rendering more and more products (e.g., consumer electronics such as smartphones, media items such as movies, digital goods such as software, etc.)

increasingly complex and difficult to evaluate before purchase; on the other, the proliferation of online forums and platforms hosting product reviews (e.g., *Amazon*, *TripAdvisor*, *Yelp*, etc.) is providing consumers with unprecedented ease-of-access to the post-purchase opinions of their peers.

From the firm's perspective, these trends translate into increasing pressure to understand how various product policies interact with review-based *social learning* (SL) and to optimize these policies accordingly (e.g., McKinsey 2010). In working towards such an understanding, a recent stream of literature has adapted the classic theory of "experience goods" (Nelson 1970) to allow for the exchange of post-purchase consumer opinions through product reviews.[1] So far, efforts in this area of research have focused primarily on developing insights regarding optimal pricing policies, implicitly assuming that a firm is endowed with a product whose attributes are specified exogenously (e.g., Crapis et al. 2015, Papanastasiou and Savva 2016, Yu et al. 2015). This paper takes the natural next step of recognizing the firm's role in choosing these attributes – a decision we refer to as "product design" – and investigating how this decision interacts with the process of SL.

We focus, in particular, on product-design choices pertaining to ex ante observable "quality attributes" (i.e., attributes which are valued by all consumers in a "more-is-better" fashion); for example, in the design of a new smartphone, these attributes may include the processor speed, screen definition, and memory capacity. Thus, product design in the context of this paper exhibits the familiar tradeoff between adding or enhancing product attributes that increase the product's perceived quality (so as to increase the consumers' willingness-to-pay), and avoiding higher costs of production (so as to maintain higher profit margins). Although this tradeoff has been previously studied from numerous perspectives (e.g., Villas-Boas 1998, Netessine and Taylor 2007, Jerath et al. 2015), existing work has by-and-large treated products as "search goods," in that, conditional on the product design, there is no quality uncertainty or, equivalently, no opportunity for consumers to interact to resolve such uncertainty.[2] As a result, the implications of SL for product design remain as-of-yet unclear.

Taking the above into account, in this paper we consider two main research questions. First, how should a firm incorporate review-based SL into its choice of product design and, by extension, how does SL affect product quality? Second, how does the interaction between SL and product design impact the firm's profit and the consumers' surplus? To investigate these questions, we develop a stylized two-period model that captures the interactions between a monopolistic firm

[1] The term "experience good" refers to a product whose quality is difficult to assess before purchase.

[2] In reality, experience goods can exhibit significant quality uncertainty even when their design is perfectly observable (e.g., owing to the complex interaction between a product's components); recent notable examples include Apple's "bending" iPhone 6 (Forbes 2015) and Samsung's "exploding" Galaxy Note 7 (Cnet.com 2016).

and a population of strategic (i.e., forward-looking) rational consumers. At the beginning of the selling season, the firm chooses the product's design, which in our model determines the product's quality only up to the expected value. Designs associated with higher expected quality incur higher per unit production costs. Consumers are Bayesian social learners, and those who choose to defer their purchasing decision to the second period can benefit by observing the reviews of first-period buyers, thereby reducing their uncertainty over product quality. Our equilibrium analysis of this model yields insights along three dimensions, which can be summarized as follows:

(i) **Product Design.** We show that the presence of SL results in a product of *lower* expected quality. To explain this phenomenon, we demonstrate that SL influences the firm's design choice through two opposing effects: on one hand, designs of higher expected quality help the firm *accelerate* consumer learning, but on the other, designs of lower expected quality allow the firm to better *appropriate* consumer learning. Our analysis indicates that the latter effect generally dominates, causing the firm to opt for a design of lower expected quality.

(ii) **Firm Profit.** We illustrate that the relationship between the firm's ex ante profit and the SL process critically depends on the level of strategic behavior in the consumer population. As the fraction of buyers who write reviews increases (thus increasing the pervasiveness of SL among consumers), we find that the consumers' increased tendency to delay purchase in anticipation of reviews may in fact *reverse* the benefits of increased consumer learning for the firm. Thus, depending on the level of strategic consumer behavior, we observe that the firm's ex ante profit may be maximized when all (low degree), some (intermediate degree), or none (high degree) of the consumers engage in writing reviews.

(iii) **Consumer Surplus.** We highlight that the presence of SL is often detrimental in terms of the consumers' ex ante surplus. Although the exchange of information through product reviews generates value for consumers by allowing them to make better-informed purchasing decisions, the detrimental impact of SL on the firm's design choice (and thus on the product's expected quality) may overshadow this benefit. In particular, we show that the net impact of SL on the consumers' surplus can only be positive provided that they are sufficiently strategic.

2. Related Literature

The literature on SL has its origins in economics and the seminal papers by Banerjee (1992) and Bikhchandani et al. (1992), which illustrate how agents endowed with private information on some unobservable state of the world can learn from each other through observation of each others' actions. While earlier work focused on learning outcomes in homogeneous societies (i.e., agents with homogeneous preferences) and simple social structures (i.e., sequential actions and observations), more recent work has expanded the scope of study to heterogeneous societies and more complicated social networks (e.g., Acemoglu et al. 2011, Jadbabaie et al. 2012, Lobel and Sadler 2015, Manshadi and Misra 2016). With the recent proliferation of online opinion forums and review platforms where consumers exchange information on products and services, SL has also received significant attention in the fields of Operations Management and Marketing: Chen and Xie (2010) identify online reviews as a new element in the marketing communications mix, while Debo and Veeraraghavan (2009) highlight the importance of consumers-to-consumer learning in shaping firms' operational strategies. Several recent papers build on the latter notion, studying operational decisions in environments where consumers' purchasing decisions are influenced by the actions and opinions of their peers. Papanastasiou et al. (2016) illustrate how a review platform can employ information obfuscation to promote exploration in a multi-product setting; Hu et al. (2015) analyze inventory decisions when the consumers' choices between two substitutable products are affected by the choice of their predecessors; Debo and Van Ryzin (2009) and Papanastasiou et al. (2015) explore the use of strategic stockouts as a means to increase demand through SL; Crapis et al. (2015) point out the benefits of pricing policies that take the presence of SL into account. Closest in spirit to our work are two recent papers by Papanastasiou and Savva (2016) and Yu et al. (2015) that consider optimal pricing policies in the presence of review-based SL and strategic consumer behavior.[3] In these papers, the firm sells a product whose quality attributes are exogenously specified; by contrast, the main goal of our paper is to understand how SL interacts with the firm's (costly) choice of such attributes.

The firm's choice of quality attributes is a central decision in another stream of research, which is primarily concerned with "search goods." In single-product settings, Xu (2009) highlights the implications of the distribution-channel structure on a firm's quality and pricing decisions, while Shi et al. (2013) focus on the impact of different types of consumer heterogeneity (i.e., vertical and horizontal). Jerath et al. (2015) consider how quality decisions are affected by demand uncertainty and inventory risk. In multi-product settings, the literature on product-line design focuses on how multiple products of different qualities may be offered in order to segment consumers based on their heterogeneity in willingness-to-pay for quality (e.g., Mussa and Rosen 1978, Moorthy 1984). More recent work in this area considers product-line decisions in the context of competition (Desai 2001), development-intensive products (Krishnan and Zhu 2006), different distribution-channel structures (Villas-Boas 1998), and different production technologies (Chen et al. 2013, Netessine and Taylor 2007), among others. A recent paper by Godes (2016) considers quality choice in a setting where persuasive word-of-mouth communication between consumers occurs exogenously to the firm's policy, and argues that product quality should increase in the intensity of such communications.

[3] For operational implications of strategic consumer behavior in other contexts, see Su (2007), Swinney (2011), Cachon and Feldman (2015), and references therein.

Among other differences with this work, a central feature of our model is the endogenous nature of word-of-mouth communications, both in terms of their intensity and in terms of their content; once this endogeneity is taken into account, we show that optimal product quality decreases with SL. Significantly, in all of the aforementioned papers, product quality is perfectly known conditional on the firm's design choice (i.e., in this work, design and quality are synonymous). By contrast, in this paper we focus on the design of "experience goods," which are characterized by ex ante quality uncertainty despite their design being perfectly observable.

3. Model Description

We consider a monopolist firm selling a new experience good over a single selling season. The product's quality, $\hat{q}$, is the sum of two components, q and ϵ. Component q is observable and represents the *product design*, which is chosen by the firm; by contrast, component ϵ is an unobservable quality shock, which is drawn by nature from the Normal distribution $N(0, \sigma_p^2)$. The product design q is a summary measure of all observable product attributes which consumers value in a "more-is-better" manner (e.g., for a new smartphone, these may include the processor speed, screen definition, memory capacity, etc.). Since in our model the firm's design choice determines the product's ex ante expected quality, we occasionally refer to q directly as the firm's "quality choice." Consistent with the literature on endogenous product quality, we assume that a product of expected quality q incurs a per unit production cost $c(q)$, where $c(\cdot)$ is twice-differentiable, increasing and convex (e.g., Desai 2001, Moorthy 1984). On the other hand, nature's shock ϵ captures the ex ante uncertainty associated with the product's quality, which may be attributed to product characteristics which are ex ante unobservable, such as the product's usability, durability and usefulness, and/or to how the product's various components interact with each other (see also Papanastasiou and Savva 2016, Yu et al. 2015).

The market consists of a continuum of consumers of total mass normalized to one, and sales of the product occur in two representative periods, indexed by $t \in \{1, 2\}$. Each consumer demands at most one unit of the product throughout the selling season, and a consumer who purchases the product in period t derives a net utility $u_{it} = \delta^{t-1}(x_i q_i - p)$ (e.g., Desai 2001, Jerath et al. 2015). In the consumer's utility function, x_i represents the consumer's willingness-to-pay for quality (referred to as the consumer's "type"), q_i is the quality experienced by the consumer after purchase, p is the product's price, and δ is a discount factor that applies to second-period purchases. We assume that consumers are heterogeneous in their type, with x_i components distributed on the positive interval $[x_l, x_h]$ according to the probability density function $g(\cdot)$. Consumers also differ in their ex post quality perceptions: conditional on the product's underlying quality $\hat{q}$, we assume that a consumer's post-purchase quality perception is a random draw from the distribution $q_i \sim N(\hat{q}, \sigma_q^2)$, where σ_q^2 measures the extent of heterogeneity in consumer-specific taste (e.g., Papanastasiou and Savva 2016).[4] The discount factor $\delta \in [0,1)$ captures the rate at which the product's perceived value declines, but may also be interpreted as the degree of the consumers' patience or the level of strategic consumer behavior in the consumer population (e.g., Cachon and Swinney 2009).

In the first period, the consumers enter the market, observe the product's design q, and arrive at a rational prior belief over $\hat{q}$, which we denote by $\tilde{q}_p$; since $\hat{q} = q + \epsilon$ and $\epsilon \sim N(0, \sigma_p^2)$, this prior belief is Normally distributed $\tilde{q}_p \sim N(q, \sigma_p^2)$.[5] From the consumers who choose to purchase a unit in the first period, we assume that a fraction $\beta \in [0,1]$ write a product review, with each review consisting of the consumer's ex post perception of quality q_i.[6] Parameter β is referred to throughout as the consumers' "review propensity" and plays a central role in our analysis, as it essentially determines the pervasiveness of SL in the consumer population; the extreme case of $\beta = 0$ represents settings where SL is absent.[7]

In the second period, consumers remaining in the market engage in the review-based SL process described as follows. Let n denote the mass (henceforth the "number") of customers who purchase in the first period, so that the number of reviews available in the second period is βn. Customers remaining in the market observe the reviews and update their belief over $\hat{q}$ from $\tilde{q}_p$ to $\tilde{q}_u$ via Bayes' rule. Given the above specifications of the consumers' prior belief and the review-generating process, the updated belief is $\tilde{q}_u \sim N(q_u, \sigma_u^2)$, where

$$q_u = \frac{1}{n\beta\gamma + 1} q + \frac{n\beta\gamma}{n\beta\gamma + 1} R \text{ and } \sigma_u^2 = \frac{\sigma_p^2}{n\beta\gamma + 1}, \tag{1}$$

with R representing the average of first-period reviews and $\gamma := \frac{\sigma_p^2}{\sigma_q^2}$.[8] The updated quality expectation q_u is a convex combination of the prior expectation q and the average rating R. The weight placed by consumers on the average rating R increases with the number of product reviews, and with parameter γ, which is the ratio of prior quality uncertainty relative to review noise.

All of the above are common knowledge; in addition, each consumer has private knowledge of her own type, x_i. At the beginning of the selling season, the firm chooses the product design q to maximize its expected profit. We conduct our main analysis under the assumption of an exogenous

[4] We note that this implies that a consumer's type is independent of her ex post quality perception; this can be relaxed without loss, provided any association between the two is common knowledge.

[5] Note that this is also the firm's belief over $\hat{q}$ at the beginning of the selling season; there is no information asymmetry between the firm and the consumers at any point in our model.

[6] The analysis is unchanged if we assume, alternatively, that a review consists of the consumer's ex post net or gross utility.

[7] We note that as in Papanastasiou et al. (2016) and Yu et al. (2015), our model implicitly assumes that all consumers are ex ante aware of the product's existence. We discuss the potential implications of this assumption in §8.

[8] Formally, since the purchase of each consumer is infinitesimally small, each review is only infinitesimally informative; thus, learning occurs on the aggregate of all available reviews (see Bergemann and Välimäki (1997) for more details).

price p (e.g., Swinney 2011), which allows us to illustrate the drivers underlying the firm's design choice and is relevant to settings where price is constrained by market norms/expectations.[9] We subsequently extend our analysis to the case of endogenous pricing and demonstrate that our main insights continue to hold qualitatively (see §7). For simplicity, we assume that the firm operates in the absence of any binding capacity constraints (e.g., Papanastasiou and Savva 2016, Yu et al. 2015). The consumers make adoption decisions to maximize their expected utility; this implies, in particular, that a consumer will purchase the product in the first period only if the expected utility from doing so is nonnegative and greater than the expected utility of delaying the purchase decision.

Throughout our analysis, we make the following two technical assumptions.

Assumption 1. *$g(\cdot)$ is the probability density function of an inverse-uniformly distributed random variable with parameters (a,b), where $0 < a < b$.*

Assumption 2. *The firm's cost function satisfies $c\left(\frac{p}{x_h}\right) < p < c\left(\frac{p}{x_l}\right)$.*

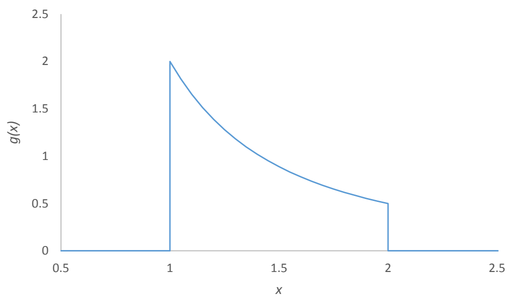
The first assumption implies that consumer types are supported on $[b^{-1}, a^{-1}]$ (i.e., $x_l = b^{-1}$, $x_h = a^{-1}$) with a probability density function (pdf) $g(x) = \frac{x^{-2}}{(b-a)}$, and a cumulative distribution function (cdf) $G(x) = \frac{b-x^{-1}}{(b-a)}$ (see Figure 1 for an example of the pdf and cdf of an inverse-uniform random variable). This distributional assumption simplifies analysis and exposition, and does not appear to affect our results qualitatively; in Appendix B, we demonstrate that our main results extend to the more general class of distributions with increasing hazard ratio. The second assumption is made in order to avoid extreme solutions to the firm's problem. In particular, $c\left(\frac{p}{x_h}\right) < p$ implies that it is profitable for the firm to launch the product (i.e., there exist at least some quality choices which result in positive firm profit), while $c\left(\frac{p}{x_l}\right) > p$ ensures that the firm's cost function is sufficiently steep so that it cannot be optimal to design a product that covers the entire market.

Our analysis of the game between the firm and the consumers proceeds as follows. First, we consider the subgame played between the consumers under an arbitrary product design and establish the consumers' purchasing strategy. Then, we analyze the firm's optimal design choice and investigate the implications of social learning for firm profit and consumer surplus. Throughout our analysis, we focus on equilibria in pure strategies.

4. Consumers' Purchasing Strategy

In the model described in §3, the presence of SL influences the consumers' decision-making process in two respects. First, depending on the consumers' level of patience δ, the anticipation of

[9] For example, new iPhone models have been historically introduced at the same price, despite differences in their technological capabilities (Fleishman 2016).



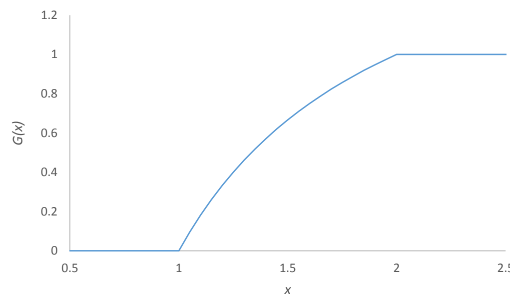


Figure 1 **The probability density function (left) and cumulative distribution function (right) of an inverse uniform random variable with shape parameters $a = 0.5$ and $b = 1$.**

product reviews may affect their decision of *when* to make the purchasing decision; second, for consumers who remain in the market in the second period, the information contained in product reviews may affect their decision of *whether* to purchase the product. We begin our analysis with a characterization of these two effects, for an arbitrary firm choice q.

Consider first an individual consumer's decision of *when* to make her purchasing decision. In our two-period model, each consumer decides in the first period whether to buy a unit or delay her purchasing decision until the second period. According to the prior belief $\tilde{q}_p$, for a customer of type x_i the expected utility from purchasing in the first period is $x_i q - p$. On the other hand, if the consumer delays the decision, her expected utility from purchase in the second period will be $x_i q_u - p$, where q_u is the updated mean belief after observing the reviews of her peers (thus, for the purposes of our model, q_u may be described as the "outcome of the SL process"). In deciding whether to delay her purchase decision until the second period, the consumer takes an expectation of her second-period utility with respect to the SL outcome, which is viewed in the first period as a random variable. In particular,

Lemma 1. *Suppose that m reviews become available to consumers remaining in the market in the second period. Then, in the first period, the expected utility from delaying purchase for a customer of type x_i is*

$$E[u_{i2}] = \delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) h(s; q, m) ds, \tag{2}$$

where $h(\cdot; q, m)$ is a Normal density function with mean q and standard deviation $\sigma_p \sqrt{\frac{m\gamma}{m\gamma+1}}$. Furthermore, $E[u_{i2}]$ is strictly increasing in m.

All proofs are provided in Appendix A. Lemma 1 suggests that $h(\cdot; q, m)$ (i.e., the density of the ex post mean belief) is the density of a Normal random variable, whose mean is equal to the prior mean q, and whose variance *increases* with the number of reviews m. To see why this occurs, consider first the extreme case $m = 0$. Since in this case no reviews become available in the second period, the consumer's mean belief must remain unchanged from the first to the second period, so that $q_u = q$. By contrast, as the number of reviews m increases, the posterior mean belief becomes increasingly likely to depart from q, and this is captured through an increase in the variance of q_u. The ex ante distribution of the ex post mean belief q_u is often referred to in decision theory as the "preposterior" distribution of q_u, a term we will adopt in the remainder of our analysis.

For the consumer, expression (2) captures the cost and benefit of waiting until the second period. The cost is simply the discount δ which is applied to second-period utility, while the benefit is the option value of delaying the decision, whereby the consumer's uncertainty will be (partially) resolved through observation of the reviews, and the consumer will purchase only if this resolution is sufficiently favorable (hence the lower limit of the integral in (2)). The last part of Lemma 1 verifies an intuitive feature of the consumers' decision process: the larger the amount of information available in the second period, the higher the value of delaying the purchase decision (in our model, this feature arises naturally as a consequence of an increased preposterior variance).

Although Lemma 1 treats the amount of information available in the second period as exogenous, an important aspect of our model is that the amount of information available in the second period (i.e., the number of reviews m) is endogenous to the adoption decisions of the consumers in the first period. The equilibrium in the consumers' adoption game is thus characterized by two opposing forces: on one hand, the higher the number of reviews consumers expect to be available in the second period, the higher the number of consumers who will choose to delay their purchasing decision; on the other, the higher the number of consumers who choose to delay their purchasing decision, the smaller the number of reviews available in the second period. Proposition 1 below describes the unique threshold-type pure-strategy equilibrium that results. We use $\phi(\cdot)$ to denote the standard Normal pdf, with the corresponding cdf denoted by $\Phi(\cdot)$ and $\bar{\Phi}(\cdot) := 1 - \Phi(\cdot)$.

Proposition 1. *For any given $q \in \left[\frac{p}{x_h}, \frac{p}{x_l}\right]$, a consumer of type x_i will purchase a unit in the first period if and only if $x_i \geq z(q)$, where $z(q) \in \left[\frac{p}{q}, x_h\right]$ is the unique solution to the implicit equation*

$$Y + \delta \int_Y^\infty \bar{\Phi}(s) ds = 0,$$

with $Y = \frac{\frac{p}{z} - q}{\sigma_z}$ and $\sigma_z = \sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma + 1}}$. Furthermore, z is increasing in the review propensity β and in the consumers' patience level δ, and decreasing in the firm's design choice q.

According to Proposition 1, consumers with relatively higher types purchase in the first period in order to avoid discounted utility, while lower-type consumers prefer to wait until they are more informed. Notice that the purchasing threshold $z(q)$ is higher than $\frac{p}{q}$, which suggests informational free-riding: at least some consumers whose expected utility from purchase is positive in the first period nevertheless choose to delay adoption in anticipation of the reviews of their peers. The properties associated with the threshold strategy are intuitive: if the first-period buyers' tendency to write reviews is higher, a relatively larger number of consumers decide to delay their purchasing decision as a result of an increased amount of information available in the second period; on the other hand, if the product's ex ante expected quality is higher, fewer consumers find the wait for more information to be worthwhile.

In the second period, the decision of consumers remaining in the market is straightforward: they simply observe the reviews of first-period buyers, update their belief over product quality to $\tilde{q}_u$, and purchase a unit provided $x_i q_u - p \geq 0$.

5. Firm's Design Choice and Profit

In the presence of SL, the firm's optimal design choice depends on the interaction between production costs, potential outcomes of the SL process, and the consumers' strategic purchasing behavior. Settings characterized by different combinations of product and consumer characteristics may call for emphasis on different aspects of this interaction. To gain an understanding of the various effects at play, our analysis of the firm's design choice builds progressively as follows. We first establish a benchmark where there is no SL and no strategic purchasing delays. Next, we consider the case where SL occurs in the second period, but where consumers are sufficiently impatient ("myopic") so that no consumer strategically delays purchase until product reviews become available. Then, we analyze the more general setting which combines SL and (at least some degree of) strategic consumer behavior.

To facilitate exposition of the analysis in this section, we introduce the following notation.

Definition 1. $f(\cdot;\mu,\nu)$ denotes a Normal pdf with mean μ and standard deviation $\sigma_\nu = \sigma_p\sqrt{\frac{\bar{G}(\nu)\beta\gamma}{\bar{G}(\nu)\beta\gamma+1}}$.

Thus, $f(\cdot;\mu,\nu)$ will be used below to describe the density of the preposterior distribution when the firm's design choice is μ and the consumers' purchasing threshold (given in Proposition 1) is ν.

5.1. Benchmark: No Social Learning

The benchmark case where there is no SL is retrieved from the general model by setting $\beta = 0$ (i.e., so that no consumers write reviews). In the absence of SL, the firm chooses q and any residual quality uncertainty can only be resolved by consumers if they purchase and experience the product themselves. Since there is no opportunity to learn from their peers, there is also no rationale for consumers to delay their purchase decision until the second period (irrespective of their patience level δ). Therefore, the model collapses to a single-period model where, given the firm's design choice, consumers with expected utility $x_i q - p \geq 0$ purchase a unit (i.e., the purchasing threshold of Proposition 1 in this case is $z = \frac{p}{q}$). The firm's profit, as a function of its quality choice q, is then given by

$$\pi_0(q) = (p - c(q)) \int_{\frac{p}{q}}^{x_h} g(x)dx. \tag{3}$$

In identifying the optimal product design, the firm faces a tradeoff involving the profit margin $(p - c(q))$, which is decreasing in the design choice q, and the volume of sales achieved $\int_{\frac{p}{q}}^{x_h} g(x)dx$, which is increasing in q. The resolution of this tradeoff is characterized in the following proposition.

PROPOSITION 2. *In the absence of SL, the optimal design choice q_0^* is given by the unique solution to the implicit equation*

$$\frac{p - c(q)}{c'(q)} = q - ap. \tag{4}$$

5.2. Social Learning with Myopic Consumers

We next consider the case where consumers are myopic (i.e., no consumer delays purchase strategically), but consumers who did not purchase a unit initially may later be persuaded to purchase through the reviews of their peers (i.e., $\delta = 0$, $\beta > 0$). In this case, a consumer will purchase a unit in the first period provided $x_i q - p \geq 0$, while in the second period, a consumer remaining in the market will purchase provided $x_i q_u - p \geq 0$, where q_u is the updated mean belief after observing the available reviews. Using Definition 1, we may express the firm's expected profit as

$$\pi_m(q) = \pi_0(q) + (p - c(q)) \int_q^{\infty} \int_{\max\{\frac{p}{s}, x_l\}}^{\frac{p}{q}} g(x) f\left(s; q, \frac{p}{q}\right) dx ds. \tag{5}$$

The firm's first-period profit at design choice q is equivalent to the firm's profit in the benchmark model in §5.1 (i.e., $\pi_0(q)$ is as given in (3)). In the second period, any sales are contingent on the content of product reviews: no sales occur if the updated mean belief, denoted by s in (5), is lower than the prior q (i.e., when the outcome of SL is "negative"), while in the opposite case (i.e., when the outcome is "positive") the number of sales achieved increases with s up until the entire market is captured.

Under Assumption 1, the firm's profit function can be shown to be concave in the design choice q, leading to the result of Proposition 3.

PROPOSITION 3. *When consumers are myopic, the optimal design choice q_m^* is the unique solution to the implicit equation*

$$\frac{p - c(q)}{c'(q)} = \frac{(b - a)p - \sigma_z \int_0^V \Phi(s)ds}{\Phi(V) + \frac{d\sigma_z}{dq}(\phi(0) - \phi(V))}, \tag{6}$$

where $V = \frac{bp-q}{\sigma_z}$ and $z = \frac{p}{q}$. Furthermore, the product's expected quality in the presence of SL is strictly lower than that in its absence; that is, $q_m^* < q_0^*$.

The latter part of the proposition suggests a striking phenomenon: the SL process provides an incentive for the firm to *lower* the product's expected quality. The significant implication of this finding is that SL may inadvertently result in the production of experience goods whose quality is lower on average. To develop an understanding of this result, it is instructive to first identify how SL affects the "margin-vs-sales" tradeoff faced by the firm, and in particular how it affects the firm's expected sales (since the profit margin $(p-c(q))$ is independent of consumer learning). We do so by first considering a simpler setting where consumer learning occurs *exogenously* in the second period.

EXAMPLE 1. Suppose that $\beta=0$, but that a proportion $\theta\in[0,1]$ of the ex ante quality uncertainty is resolved exogenously in the second period. Then (i) for any fixed q, the firm's expected sales (and therefore its expected profit) are strictly increasing in θ, and (ii) the firm's optimal design choice q_m^* is strictly decreasing in θ.

Example 1 constructs a setting where the *extent* of consumer learning (captured by θ) does not depend on the early sales volume, and therefore does not depend on the firm's quality choice. Thus, in this example, the firm's profit can be expressed as

$$\pi_e(q)=\pi_0(q)+(p-c(q))\int_q^\infty\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)f\left(s;q,k_\theta\right)dxds,$$

for some fixed k_θ which is positively related to the parameter θ. This exogenous-learning paradigm reveals two significant effects, which are helpful in developing intuition regarding the firm's problem when learning occurs endogenously through product reviews.

First, observe that that the firm's expected sales and profit increase with θ, even if the design choice q is held constant. In particular, note that the firm's first-period sales are independent of θ, but that the firm's expected second-period sales are strictly increasing in θ. To see why the latter occurs, note that an increased extent of consumer learning renders both more positive and more negative second-period learning outcomes ex ante more likely (since the preposterior variance increases); however, the effect of this increased variability on the firm's expected sales is asymmetrically positive: while the firm benefits as more positive outcomes become more likely, it does not suffer as more negative outcomes become more likely, since all negative outcomes result in equivalent (i.e., zero) second-period profit. As a result, the more the consumers are able to learn about the product in the second period, the higher the firm's expected sales.

Second, notice that consumer learning causes the firm to decrease the product's expected quality q_m^*. This observation is the result of an intertemporal sales-cannibalization effect that takes place in the presence of consumer learning: the higher the number of consumers who purchase in the first period, the lower the number of consumers remaining in the market in the second period, and therefore the lower the potential to generate sales through consumer learning – put simply, while the firm's first-period sales increase in q, its second-period sales decrease in q. This has the effect of tilting the balance between the firm's profit margin and its total expected sales towards a lower-expected-quality product, which better allows the firm to generate sales from the consumer learning process.

Now let us return to the case where consumer learning occurs *endogenously* through product reviews. In this case, the two effects identified in Example 1 become conflicting with regards to their impact on the firm's design choice. On one hand, the firm would like to increase the extent of consumer learning so as to increase its expected profit; when learning is endogenous, this is achieved through a higher number of first-period sales and reviews, which implies a higher investment in the product's design. On the other hand, as Example 1 suggests, the best way to take advantage of consumer learning is to decrease the design investment so as to reduce intertemporal sales cannibalization. This tension between *accelerating* and *appropriating* the SL process is what determines the firm's optimal design choice. As β increases from zero, Proposition 3 establishes that the appropriation effect generally dominates; as a result, the firm opts for a design of *lower* expected quality.

Thus, what we observe here is that, instead of placing pressure on the firm to increase the quality of its product, the SL process in fact acts a substitute for the firm's quality investment. In the absence of SL, the firm relies exclusively on the product's observable design in order to generate sales, and is therefore forced to invest more in the product's quality attributes. By contrast, when the product's unobservable attributes become observable to the consumers through the SL process (and despite the fact that the outcome of this process is equally likely to be favorable or unfavorable), the firm finds itself with less of an incentive to invest in a high-quality design.

To conclude our discussion of the myopic-consumer case, we consider how the SL process affects the firm's expected profit.

PROPOSITION 4. *When the consumers are myopic, the firm's optimal expected profit is monotonically increasing in the consumers' review propensity β.*

In order to draw the conclusion of Proposition 4, an argument based on a fixed product design q will suffice. For any fixed q, as the consumers' review propensity β increases, the firm's profit in the first period remains unchanged, but the extent of consumer learning in the second period increases (this manifests as an increase in the preposterior variance in (5); see also Definition 1). As discussed in the context of Example 1 above, this has an asymmetrically positive effect on the firm's expected second-period sales, thus resulting in higher expected profit. It follows that profit can only increase further if the firm chooses q optimally.

5.3. Social Learning with Strategic Consumers

We have seen in §5.2 that in the absence of strategic purchasing delays, SL (i) induces the firm to lower the product's quality relative to the case where there is no SL, and (ii) results in an increase in expected firm profit. In this section, we focus on the impact of strategic consumer behavior on the firm's design choice and profit. When the consumers are strategic, the firm's expected profit is given by

$$\pi_s(q) = (p - c(q)) \int_{z(q)}^{x_h} g(x)dx + (p - c(q)) \int_{\frac{p}{z(q)}}^{\infty} \int_{\max\{\frac{p}{s}, x_l\}}^{z(q)} g(x) f\left(s; q, z(q)\right) dxds, \tag{7}$$

where $z(q)$ is the first-period purchasing threshold described in Proposition 1. In order to isolate the effects of strategic consumer behavior, it is instructive to think of the firm's profit along the lines of Lemma 2.

LEMMA 2. *In the presence of SL, the firm's expected profit can be expressed as*

$$\pi_s(q) = \pi_m(q) - (p - c(q)) \left(S_D + S_L\right), \tag{8}$$

where $\pi_m(q)$ is the firm's profit in the absence of strategic consumer behavior (given in (5)), while $S_D > 0$ and $S_L > 0$ are given by

$$S_D = \int_{-\infty}^{\frac{p}{z(q)}} \int_{\frac{p}{q}}^{z(q)} g(x) f\left(s; q, z(q)\right) dxds + \int_{\frac{p}{z(q)}}^{q} \int_{\frac{p}{q}}^{\frac{p}{s}} g(x) f\left(s; q, z(q)\right) dxds,$$

$$S_L = \int_{q}^{\infty} \int_{\max\{\frac{p}{s}, x_l\}}^{\frac{p}{q}} g(x)[f\left(s; q, pq^{-1}\right) - f\left(s; q, z(q)\right)]dxds.$$

Lemma 2 illustrates that strategic consumer behavior negatively impacts the firm's expected profit through two channels. The first is the "delay" channel (corresponding to term S_D), which represents the potential loss of sales from consumers who, despite having a positive expected utility from purchase in the first period, decide to delay their decision until the second period (i.e., consumers of type $\frac{p}{q} < x_i \le z$). All such sales are retained by the firm only if the updated quality belief s is higher than the prior q (in which case $S_D = 0$); if the opposite occurs, either all (significantly lower s) or a fraction (moderately lower s) of these sales are lost.

The second channel, which we refer to as the "learning" channel (corresponding to term S_L), is more subtle but equally significant. This channel captures the expected loss of sales that occurs as a result of a weaker SL effect. In particular, recall that the firm benefits in the second period from a higher extent of consumer learning (i.e., from a higher preposterior variance); however, as a result of strategic purchasing delays in the first period, fewer reviews are available in the second period, resulting in a decrease in consumer learning. In turn, this negatively affects sales generated from consumers who would consider purchasing in the second period (i.e., consumers of type $x_i < \frac{p}{q}$).

Thus, strategic purchasing delays effectively cause "market shrinkage" both in the first period, through intertemporal purchase-decision shifts, and in the second period, through a weaker SL effect. Taking into account these two detrimental effects, the firm's adjusts its design choice as described in Proposition 5.

PROPOSITION 5. *The optimal design choice in the presence of SL, q_s^*, satisfies the implicit equation*

$$\frac{p-c(q)}{c'(q)} = \frac{(b-a)p - \sigma_z \int_Y^V \Phi(s)ds}{\Phi(V) + \frac{d\sigma_z}{dq}\left(\phi(Y) + Y\Phi(Y) - \phi(V)\right)}, \tag{9}$$

where $V = \frac{bp-q}{\sigma_z}$, $Y = \frac{\frac{p}{z}-q}{\sigma_z}$, and z is specified in Proposition 1. Furthermore, there exist thresholds $\Delta_1, \Delta_2, \Delta_3 \in [0,1)$ such that (i) $q_s^* > q_m^*$ for any $\delta \geq \Delta_1$, and (ii) $q_s^* < q_0^*$ for any $\Delta_2 \leq \delta \leq \Delta_3$.

Although the values of the thresholds $\Delta_1, \Delta_2, \Delta_3$ appear difficult to establish analytically, our numerical experiments suggest that the properties of the optimal design choice q_s^* stated in Proposition 5 hold in general, so that $q_m^* < q_s^* < q_0^*$ holds across all values of δ (e.g., see Figure 2).

The left-hand side of the last inequality (i.e., $q_m^* < q_s^*$) represents an intuitive measure taken by the firm to combat the consumers' tendency to strategically delay their purchase. By increasing the product's expected quality, the firm attempts to contain the two detrimental effects of strategic consumer behavior identified in Lemma 2 (recall from Proposition 1 that the number if first-period buyers increases in q). The right-hand side (i.e., $q_s^* < q_0^*$), however, indicates that although strategic consumer behavior forces the firm to increase the product's expected quality, the optimal design choice still remains lower than that in the absence of SL. Thus, in terms of the acceleration-versus-appropriation tradeoff described in §5.2, Proposition 5 suggests that strategic consumer behavior forces the firm to place relatively more emphasis on accelerating consumer learning (leading to a relative increase in the product's expected quality), but that appropriation nevertheless remains the dominant force in the firm's design choice.

We next consider the impact of strategic consumer behavior on the firm's expected profit. First, note that since both S_D and S_L in (8) are positive, it follows that strategic consumer behavior always results in a decrease in the firm's profit relative to the case where consumers are myopic (i.e., for any q, we have $\pi_s(q) < \pi_m(q)$). Our focus here is on whether the magnitude of this decrease has a significant impact on the firm's profit relative to the case where SL is absent; that is, whether the firm, once strategic consumer behavior is taken into account, remains better off (ex ante) in the presence of SL. Proposition 6 suggests that this is not necessarily the case.

PROPOSITION 6. *Suppose that the firm's design choice in the absence of SL, as given in* (4), *satisfies $q_0^* > \frac{(a+b)p}{2}$. Then, for sufficiently high consumer patience δ, the firm's optimal profit is monotonically decreasing in the consumers' review propensity β.*

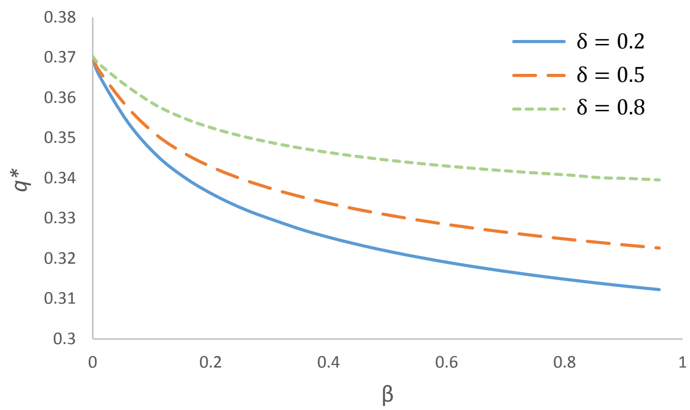


Figure 2 Optimal quality choice as a function of the review propensity β, at different values of consumer patience δ. Parameter values: $a = 0.5$, $\beta = 2$, $\sigma_p = 1$, $\gamma = 1$, $p = 0.3$. Cost function: $c(q) = q^2$.

The condition of Proposition 6 is essentially a condition on the firm's cost function, which requires that in the absence of SL the firm finds it optimal to serve a large enough portion of the consumer population (we note that the condition is sufficient, but not necessary, for the result to hold). Recall that when consumers are myopic, the firm benefits the more consumers engage in writing reviews (see Proposition 4); by contrast, at the other extreme where consumers are highly strategic, Proposition 6 suggests that this observation can be reversed, implying that the negative effects of strategic consumer behavior can be strong enough to render the presence of SL detrimental.

More interesting still is the manner in which the transition occurs from cases where SL is beneficial for the firm to cases where SL becomes detrimental. To illustrate, we conduct the numerical experiments of Figure 3, where we plot the firm's optimal profit as a function of the consumers' review propensity β. Observe that for intermediate values of δ, the firm's profit is nonmonotone in β, initially increasing and then decreasing. From Proposition 4, we know that in the absence of strategic purchasing delays, the firm's profit increases with β; however, note that according to Proposition 1 the consumers' tendency to strategically delay purchase also increases with β. The numerical experiments of Figure 3 illustrate that the positive effects of consumer learning tend to dominate the negative effects of strategic consumer behavior at low values of β, but that the opposite is true for higher values of β. As the consumers' patience δ increases, the range of values of β for which the firm benefits from SL shrinks from above and eventually vanishes, leading to the result of Proposition 6.

6. Consumer Surplus

So far, our analysis has focused on the implications of SL for the firm. In this section, attention is focused on the consumers' perspective, which becomes especially interesting in light of the

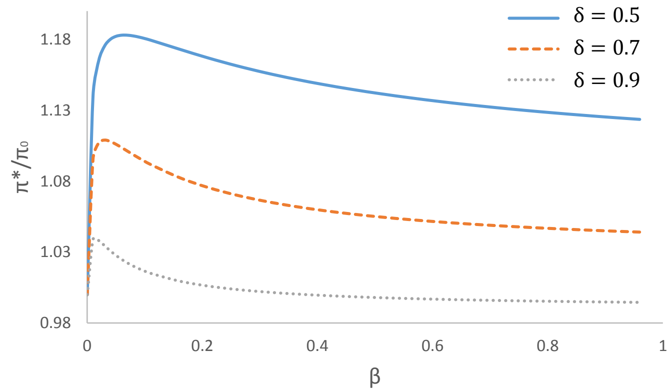


Figure 3 **Optimal firm profit π^* as a function of review propensity β, at different values of discount factor δ. Parameter values:** $a=0.5$, $b=2$, $\sigma_p=1$, $\gamma=2$, $p=0.25$. **Cost function** $c(q)=q^2$.

preceding analysis. In particular, the net impact of SL on the consumers' surplus is determined by two opposing effects. On one hand, SL allows consumers to learn from the experiences of their peers, and as a result, to make better-informed purchasing decisions (it is straightforward to show that, all else being equal, SL results in an increase in expected consumer surplus). On the other hand, however, the analysis of §5 suggests that as a result of the consumers' ability to learn from their peers, the firm elects for a product of lower expected quality. Furthermore, the magnitude of either effect is complicated by the discount factor δ applied to second-period utility and the associated strategic purchasing delays this generates.

We begin our analysis of consumer surplus by highlighting the following pattern that emerges as a result of the firm's decision to lower the product's quality.

Lemma 3. *For any firm design choice $q_s < q_0^*$, there exist threshold types $\psi_1 \geq \psi_2$ with $\psi_1, \psi_2 \in \left(\frac{p}{q_0^*}, z(q_s)\right)$, such that every consumer of type $x_i \geq \psi_1$ ($x_i < \psi_2$) has strictly lower (strictly higher) expected surplus in the presence of SL.*

According to Lemma 3, low-type consumers benefit from the SL process, but this benefit comes at the expense of high-type consumers. Consider, for instance, two consumers, one of the very highest type (i.e., $x_i = a^{-1}$) and another of the very lowest type (i.e., $x_i = b^{-1}$). The high-type consumer purchases the product in the first period both in the presence of SL and in its absence; however, her expected surplus under SL is lower than that in its absence (i.e., $\frac{q_s}{a} - p < \frac{q_0^*}{a} - p$). By contrast, the low-type consumer would never purchase the product in the absence of SL, but may do so in the presence of SL in the second period, provided she learns that the product's quality is much higher than expected; thus, in the absence of SL her expected surplus is zero, while in its presence it is strictly positive (in particular, her surplus under SL is $\delta \int_{bp}^{\infty} (\frac{w}{b} - p) f(w; q_s, z(q_s)) dw > 0$).

Lemma 3 therefore suggests that the main beneficiaries of SL are consumers who delay adoption until the second period (i.e., those who benefit from the reviews of early buyers), while those consumers who choose to purchase in the first period tend to suffer a loss in expected surplus. This observation, along with the result of Proposition 3 (i.e., that product quality is lower in the presence of SL), is sufficient to establish Proposition 7 below.

Proposition 7. *For sufficiently low consumer patience δ, SL results in a decrease in expected consumer surplus.*

Since in the case of low δ the utility of consumers who delay purchase until the second period (i.e., the main beneficiaries of SL) is highly discounted, the surplus loss of consumers who purchase early dominates, leading to an aggregate loss in expected consumer surplus. In the opposite case (i.e., for high δ), our numerical experiments suggest that consumers can benefit from SL. In Figure 4, we plot an example of the dependence of consumer surplus on parameters β and δ (we note that Figure 2 illustrates the firm's corresponding quality choices for the same parameter values). At low values of δ, consumer surplus decreases in β as (i) the firm decreases product quality aggressively and (ii) the utility of second-period consumers is highly discounted so that the benefits of learning in terms of surplus generated are limited. By contrast, for high values of δ, the firm's tendency to decrease quality is tempered, and at the same time the benefits of learning increase as the utility of second-period consumers is more highly valued.

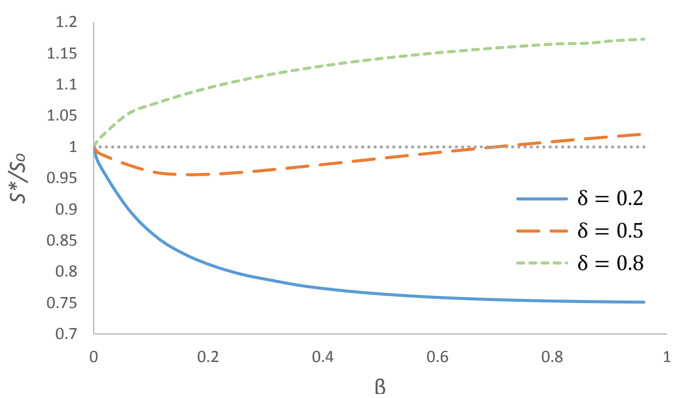


Figure 4 Expected consumer surplus as a function of review propensity β, at different values of consumer patience δ. Parameter values: $a=0.5$, $\beta=2$, $\sigma_p=1$, $\gamma=1$, $p=0.3$. Cost function: $c(q)=q^2$.

7. Endogenous Pricing

In this section, we consider an extension of our model where the firm, in addition to its design choice q, also chooses the product's price p. A comprehensive analytical treatment of pricing decisions in the presence of SL is provided by Papanastasiou and Savva (2016) and Yu et al. (2015), who cover both pre-announced (i.e., fixed ex ante) and subgame-perfect (i.e., dynamic) pricing policies. Here, our focus is on identifying how endogenous pricing affects the insights of the preceding analysis.

Under a pricing-and-design policy (p,q), the firm's expected profit is given by

$$\pi(p,q) = (p - c(q)) \int_{z(p,q)}^{x_h} g(x)dx + (p - c(q)) \int_{\frac{p}{z(p,q)}}^{\infty} \int_{\max\{\frac{p}{s}, x_l\}}^{z(p,q)} g(x) f\left(s; q, z(p,q)\right) dx ds, \tag{10}$$

where we note that the first-period purchasing threshold z now also depends on the firm's pricing decision. With respect to the firm's optimal policy, recall that in the exogenous-price analysis of §5, we cast the impact of SL on the firm's decision as one that introduces a tradeoff between accelerating and appropriating consumer learning; we then established that appropriation generally dominates the firm's design choice, resulting in an optimal design whose expected quality is lower than that in the absence of SL. In the case of endogenous pricing, it is of interest to check whether these findings extend both in terms of the *drivers* underlying the firm's decisions (i.e., whether appropriation of SL continues to be the firm's focus), as well as in terms of the *outcome* of the firm's decisions (i.e., whether the optimal policy involves a product of lower expected quality than in the absence of SL). While the latter can be deduced in a straightforward manner, in order to establish the former we point out that the tradeoff between acceleration and appropriation may also be viewed as one between increasing and decreasing, respectively, the volume of first-period sales (note, however, that under endogenous pricing appropriation does not necessarily imply a product of lower expected quality, since the firm can now control the first-period sales volume by simultaneously modulating price and design).

The properties of the firm's optimal price-and-design policy appear difficult to establish analytically, in part because of the difficulty in establishing uniqueness of an optimal policy (we note that this issue is commonly encountered even in simpler two-dimensional problems that involve stochasticity; see also Lu and Simchi-Levi (2012)). To make progress, we proceed by first providing a partial analytical characterization of the optimal policy in Proposition 8 below; we then use this result to investigate the properties of the firm's optimal policy through numerical experiments.

Proposition 8. *Define the sets* $C_1 = \{(p,q) : c(q) \le p \le \frac{q}{a}\}$ *and* $C_2 = \{(p,q) : q \le \bar{q}(p)\}$, *where*

$$\bar{q}(p) = \left\{ \min q : x_i q - p \ge \delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) f(s; q, b) ds, \ \forall x_i \in [b^{-1}, a^{-1}] \right\}.$$

The optimal policy (p^*, q^*) *lies in the set* $C = C_1 \cap C_2$. *Furthermore:*

(i) If the optimal policy lies in the interior of C, then it satisfies the equation

$$\frac{\partial}{\partial p}\left[\sigma_z \int_Y^V \Phi(s)ds\right] + \frac{1}{c'(q)}\frac{\partial}{\partial q}\left[\sigma_z \int_Y^V \Phi(s)ds\right] - (b-a) = 0, \tag{11}$$

where $V = \frac{bp-q}{\sigma_z}$ and $Y = \frac{\frac{p}{z}-q}{\sigma_z}$.

(ii) If the optimal policy lies on the boundary of C, then it satisfies $q = \bar{q}(p)$.

In Proposition 8, C_1 is the convex set of policies where the firm's expected profit is nonnegative: $p > c(q)$ ensures a positive profit margin, while $p < \frac{q}{a}$ ensures a positive number of sales. Furthermore, within C_1, any policy that involves $q > \bar{q}(p)$ (where $\bar{q}(p)$ is the lowest expected quality at which all consumers choose to purchase in the first period when the product's price is p) increases the firm's production costs without increasing its revenues, and therefore cannot be optimal. Thus, the search for an optimal policy can be restricted to the set C. Next, we note that the firm's profit function is smooth and continuous over the set C, positive-valued in its interior, and zero-valued at all boundaries except boundaries involving policies with $q = \bar{q}(p)$, if such boundaries exist. This implies that an optimal policy can be found either in the interior of C, in which case it satisfies the first-order necessary conditions $\frac{\partial \pi}{\partial p} = \frac{\partial \pi}{\partial p} = 0$ (these reduce to equation (11)), or at its exterior, in which case it satisfies $q = \bar{q}(p)$.

Using the result of Proposition 8, we conduct numerical experiments to investigate (i) whether the firm's pricing-and-design policy remains geared towards appropriating, rather than accelerating, the SL process, and (ii) whether the product's expected quality remains lower in the presence of SL, as compared to that in its absence. Across our experiments, we observe that equation (11) generally specifies the firm's optimal policy (i.e., the solution to the firm's problem lies in the interior of the set C) and that a solution to this equation is unique; we provide an example contour plot of the firm's expected profit as a function of price and expected quality in Figure 5.

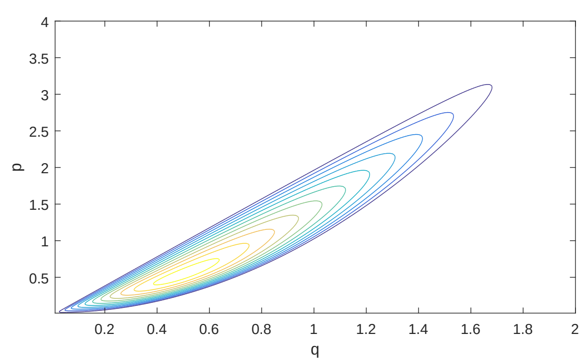


Figure 5 Contour plot of expected firm profit as a function of price p and expected quality q. Parameter values: $a = 0.5$, $b = 2$, $\sigma_p = 1$, $\gamma = 1$, $\beta = 1$, $\delta = 0.5$. Cost function: $c(q) = q^2$.

In the top row of Figure 6, we examine whether the firm's chosen policy increases or decreases the first-period sales volume, by plotting the first-period purchasing threshold as a function of the consumers' review propensity at low (left) and high (right) levels of consumer patience. We observe that this threshold is typically higher than that in the absence of SL (i.e., case $\beta = 0$ in the plots), with the exception of cases where the consumers' review propensity and the consumers' patience are simultaneously very low. Therefore, as in the analysis of §5, we find that the firm's optimal policy is typically geared towards appropriating, rather than accelerating, the SL process. In the middle row of Figure 6, we plot the firm's optimal price-and-design policy. Although the firm's optimal policy is generally nonmonotonic, we observe that the optimal design choice (i.e., the product's expected quality) is lower than that in the absence of SL, with the exception of cases where the consumers' patience and the review propensity are simultaneously very high. Our experiments therefore indicate that, in most cases, the policy which the firm employs in order to achieve appropriation of the SL process involves a design of lower expected quality, as was the case in our preceding analysis.

We consider next the impact of endogenous pricing on the firm's ex ante profit. Here, it is possible to extend analytically the results of §5 which refer to the two extreme levels of consumer patience. We start with the case of myopic consumers.

Proposition 9. *Suppose that the the firm's optimal policy in the absence of SL satisfies* $\frac{q_0^*}{p_0^*} < b$. *Then, when the consumers are myopic (i.e.,* $\delta = 0$*), the firm's optimal expected profit is monotonically increasing in the consumers' review propensity* β.

As we show in the proof of the proposition, the sufficient condition $\frac{q_0^*}{p_0^*} < b$ guarantees that the solution to the firm's problem lies in the interior of the set C for all $\beta \in [0,1]$ (the corresponding property of the optimal design choice was ensured in §5 by Assumption 2); once this is ensured, the result, as well as the underlying intuition, mirror that of Proposition 4 in §5. Thus, when the consumers are myopic, the firm continues to benefit from a higher review propensity β. By contrast, consistent with our analysis of the exogenous-price case, the same is not necessarily true in cases where the consumers exhibit high levels of strategic consumer behavior.

Proposition 10. *Suppose that the the firm's optimal policy in the absence of SL satisfies* $\frac{q_0^*}{p_0^*} > \frac{(a+b)}{2}$. *Then, for sufficiently high consumer patience* δ*, the firm's optimal profit is monotonically decreasing in the consumers' review propensity* β.

The underlying intuition parallels that discussed in the preceding analysis for Proposition 6, and is therefore omitted here for brevity.

In the bottom row of Figure 6, we plot the firm's expected profit along with the consumers' expected surplus, again for low (left) and high (right) levels of consumer patience. As in our preceding analysis, we observe that consumers benefit from a higher review propensity when they are sufficiently patient, and are harmed by SL when they are impatient (except for cases of very low β). Finally, we find that the firm typically benefits as the review propensity increases, although relatively low values of propensity are sufficient to achieve the majority of this benefit; moreover, the firm's benefit from SL decreases as consumers become more patient.

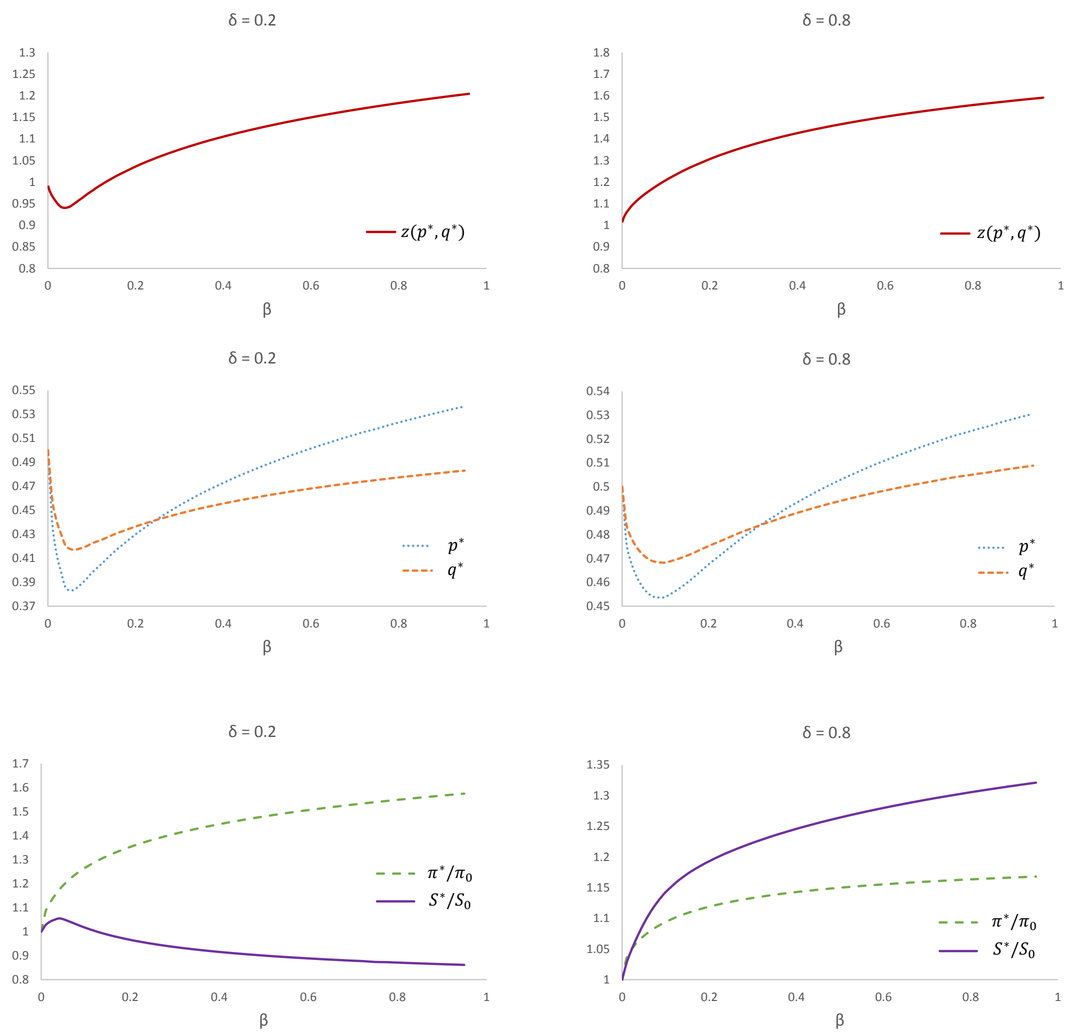


Figure 6 **Top row: Optimal first-period purchase threshold as a function of review propensity β.**
Middle row: Optimal price and quality as a function of review propensity β.
Bottom row: Expected firm profit and consumer surplus as a function of review propensity β.
Parameter values: $a = 0.5$, $b = 2$, $\sigma_p = 1$, $\gamma = 1$. Cost function: $c(q) = q^2$.

8. Discussion

This paper studies the implications of review-based SL for a monopolist firm's choice of product design. In our model, the product's design determines its quality up to the expected value, and designs associated with higher expected quality incur higher per unit production costs. Consumers are modelled as strategic (i.e., forward-looking) Bayesian social learners, who may choose to delay their purchase decisions in order to benefit from the reviews of their peers.

In the described setting, we find that a monopolist firm operating in the presence of SL should opt for a product of *lower* expected quality, a finding that challenges the commonly-held notion that the increased influence of social learning on modern-day consumers should push firms to invest more in the quality of their product.[10] In particular, our analysis suggests that although selecting an inferior product design leads to a decrease in early adoption, a lower volume of consumer reviews, and a less favorable (on average) review content, this nevertheless puts the firm in a better position to appropriate consumer learning in the subsequent stages of the selling season. It is important to note here that our model focuses on the role of SL in resolving consumers' uncertainty over product quality, assuming that consumers are ex ante aware of the product's existence. If this is not the case, then SL may also serve to raise product awareness (e.g., Bass 1969). Including the latter role of SL in our model would have obvious implications: assuming that product awareness is positively related to the volume of early adoption, the firm will now have an incentive to choose a relatively higher-quality design in order to entice more consumers to purchase early; whether the awareness role of SL is powerful enough to overturn the firm's decision to reduce product quality will then depend on its relative importance.

In terms of the firm's ability to extract profit from the SL process, we find that depending on the degree of the consumers' forward-looking behavior, the firm's expected profit may range from monotone increasing (low degree), to non-monotone (intermediate degree), to monotone decreasing (high degree) in the pervasiveness of SL in the consumer population (this is measured in our model by the fraction of consumers who engage in writing reviews). Recent trends in online commerce suggest a widespread belief that "more reviews are better," with high-profile firms including *Apple*, *Dell*, and *Samsung* encouraging the exchange of consumer opinions by soliciting and hosting product reviews on their online portals. However, our analysis highlights the importance of understanding how the availability and anticipation of product reviews interacts with the purchase behavior of a firm's consumers. For example, our results suggest that in the case of innovative

[10] In the words of *Amazon.com* CEO Jeff Bezos: "The individual is getting empowered. And [...] the right way to respond to this if you're a company is to say, 'OK, I'm going to put the vast majority of my energy, attention and dollars into building a great product or service [...] because I know if I build a great product or service, my customers will tell each other' " (Rose 2010).

technology goods which are characterized by high performance uncertainty and short life cycles, a high pervasiveness of SL may do more harm than good for the firm, with the positive effects of consumer learning being overshadowed by the consumers' increased tendency to delay their purchasing decisions.

Taking the consumer's perspective, we find that the ability to exchange information through product reviews is not necessarily beneficial ex ante for the consumers, owing to its detrimental effects on the firm's product design choice. In particular, we find that consumers tend to benefit from a higher SL pervasiveness provided they are sufficiently forward-looking, but are typically harmed by it when the opposite is true. Noting that the firm's profit exhibits the opposite pattern (i.e., higher SL pervasiveness benefits the firm when consumers are *not* too forward-looking), this result may have implications for platforms such as *Amazon.com* and *Yelp.com*, which mediate firm-consumer interactions and host product reviews. For instance, assuming that such platforms are interested in maintaining positive surplus for both sides of the market, the appropriate balance may be struck by modulating the volume of product reviews which is made publicly available.

Appendix

A. Proofs

Proof of Lemma 1 If m product reviews become available in the second period and their mean is R, then by Bayes' rule, the posterior belief $\tilde{q}_u$ is Normally distributed with a mean of

$$q_u = \frac{\sigma_q^2}{m\sigma_p^2+\sigma_q^2}q + \frac{m\sigma_p^2}{m\sigma_p^2+\sigma_q^2}R = \frac{1}{m\gamma+1}q + \frac{m\gamma}{m\gamma+1}R,$$

where q is the mean of the prior belief and $\gamma = \frac{\sigma_p^2}{\sigma_q^2}$. The posterior mean belief q_u is viewed in the first period as a random variable (r.v.), since it depends on the unobservable realization of quality $\hat{q}$, as well as the (noisy) average of first period reviews. Conditional on product quality $\hat{q}$, the sample mean of m (i.i.d. Normal) reviews, R, follows $R \sim N(\hat{q}, \frac{\sigma_q^2}{m})$ and the updated mean belief follows $q_u \mid \hat{q} \sim N\left(\frac{1}{m\gamma+1}q + \frac{m\gamma}{m\gamma+1}\hat{q}, \left(\frac{m\gamma}{m\gamma+1}\right)^2 \frac{\sigma_q^2}{m}\right)$. Next, as $\hat{q}$ is an ex ante Normal r.v. ($\hat{q} \sim N(q, \sigma_p^2)$), we have

$$\begin{aligned} E[q_u] &= E(E[q_u \mid \hat{q}]) = E\left(\frac{1}{m\gamma+1}q + \frac{m\gamma}{m\gamma+1}\hat{q}\right) = q, \text{ and} \\ \mathrm{Var}[q_u] &= E(\mathrm{Var}[q_u \mid \hat{q}]) + \mathrm{Var}(E[q_u \mid \hat{q}]) \\ &= \left(\frac{m\gamma}{m\gamma+1}\right)^2 \left(\frac{\sigma_q^2}{m} + \sigma_p^2\right) = \left(\frac{m\gamma}{m\gamma+1}\right)^2 \left(\frac{\sigma_q^2(m\gamma+1)}{m}\right) \\ &= \frac{m\gamma}{m\gamma+1}\sigma_p^2. \end{aligned}$$

Therefore, the updated mean belief q_u, upon which the consumer bases her purchase decision in the second period, is viewed in the first period as a Normal r.v. with mean q and variance $\frac{m\gamma}{m\gamma+1}\sigma_p^2$; let $h(\cdot; q, m)$ denote the corresponding pdf. In the second period, a consumer of type x_i will purchase only if her updated expected utility from purchase is positive. Thus, in the first period her expected utility from delaying purchase is

$$E[u_{i2}] = \delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) h(s; q, m) ds.$$

Now, note that an increase in m increases the variance of the preposterior distribution of q_u, without changing its mean. Since (i) the consumer's second-period expected utility is nonnegative, increasing and convex in q_u and (ii) q_u is a Normal r.v., it follows from second-order stochastic dominance that the consumer's utility from delaying purchase is increasing in m.

Proof of Proposition 1 We show in turn that a threshold-type equilibrium exists and is unique. We then establish the properties of the purchasing threshold.

(i) Existence. If it exists, a threshold-type equilibrium requires that consumers with type $x_i \geq z$ $(x_i < z)$ purchase a unit in the first period (delay purchase), for some $z \in \left[\frac{p}{q}, x_h\right]$. In turn, this implies that a mass of $\beta \bar{G}(z)$ reviews are made available in the second period. Therefore, the equilibrium is specified by the following indifference equation (note f is defined in Definition 1)

$$\begin{aligned} zq - p &= \delta \int_{\frac{p}{z}}^{\infty} (zs - p) f(s; q, z) ds \\ zq - p &= \delta \left(zq \bar{\Phi}\left(\frac{\frac{p}{z} - q}{\sigma_z}\right) + z\sigma_z \phi\left(\frac{\frac{p}{z} - q}{\sigma_z}\right) - p\bar{\Phi}\left(\frac{\frac{p}{z} - q}{\sigma_z}\right)\right) \end{aligned}$$

Let rhs (lhs) denote the right-hand side (left-hand side) of the last equation. Now, note that for $z \to x_h$, $\sigma_z \to 0$ and therefore $lhs \to x_h q - p$ and $rhs \to 0$; that is $lhs > rhs$. By contrast for $z \to \frac{p}{q}$, we have $lhs \to 0$ and $rhs \to k$ for some $k > 0$; that is, $rhs > lhs$. Continuity then implies that there exists some $z \in \left[\frac{p}{q}, x_h\right]$ that satisfies the indifference equation. Furthermore, from the last expression, we have for $Y := \frac{\frac{p}{z} - q}{\sigma_z}$,

$$-Y = \delta \left(\phi(Y) - Y\bar{\Phi}(Y)\right), \tag{12}$$

which is equivalent to $Y + \delta \int_Y^\infty \bar{\Phi}(s) ds = 0$.

(ii) Uniqueness. Rearranging expression (12), we have

$$\frac{Y}{Y\bar{\Phi}(Y) - \phi(Y)} = \delta, \tag{13}$$

Note that $\frac{Y}{Y(1-\Phi(Y)) - \phi(Y)} = \frac{1}{1 - \Phi(Y) - \frac{\phi(Y)}{Y}}$ is decreasing in Y (since $\frac{d}{dY}\left(1 - \Phi(Y) - \frac{\phi(Y)}{Y}\right) = \phi(Y)\frac{1}{Y^2} > 0$), with $\lim_{Y\to-\infty}\left(\frac{Y}{Y(1-\Phi(Y))-\phi(Y)}\right) = 1$ and $\lim_{Y\to 0}\left(\frac{Y}{Y(1-\Phi(Y))-\phi(Y)}\right) = 0$. Therefore, there exists a unique value $Y < 0$ that solves (12).

We next establish that there exists a unique $z \in \left[\frac{p}{q}, x_h\right]$ that solves $Y = \frac{p - zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}$. First note that for $z = \frac{p}{q}$ we have $\frac{p-zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}} = 0$ and that $\lim_{z\to x_h}\left(\frac{p-zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}\right) = -\infty$ (since $\lim_{z\to x_h}\left(\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}\right) = 0$). Therefore, by continuity there exists at least one $z \in \left[\frac{p}{q}, x_h\right]$ that solves the equation. Moreover, to see that such a solution is unique, note that

$$\frac{d}{dz}\left(\frac{p - zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}\right) = \frac{\sigma_p\beta\gamma\left((p - zq)\, zg(z) - 2p\bar{G}(z)\left(\bar{G}(z)\beta\gamma + 1\right)\right)}{2\left(\bar{G}(z)\beta\gamma + 1\right)^2 z^2\sigma_p^2\left(\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}\right)^{\frac{3}{2}}} < 0,$$

where the last inequality holds because for $z \in \left[\frac{p}{q}, x_h\right]$ we have $p - zq < 0$.

(iii) Properties. (1) As δ increases, Y decreases and therefore z increases. (2) As β increases, the function $\frac{p-zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}$ increases (since $\frac{d}{d\beta}\left(\frac{p-zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}\right) = -\frac{p-zq}{2z\sigma_p\left(\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}\right)\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}\frac{\bar{G}(z)\gamma}{(\bar{G}(z)\beta\gamma+1)^2} > 0$) and therefore z increases. (3) As q increases, the function $\frac{p-zq}{z\sigma_p\sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}$ decreases and therefore z decreases.

Proof of Proposition 2 In the absence of SL, the firm's profit function is given by

$$\pi_0(q) = (p - c(q))\int_{\frac{p}{q}}^{x_h} g(x)dx = (p - c(q))\bar{G}\left(\frac{p}{q}\right),$$

which under Assumption 1 is equivalent to

$$\pi_0(q) = (p - c(q))\left(\frac{\frac{q}{p} - a}{b - a}\right).$$

By Assumption 2, to maximize $\pi_0(q)$ it suffices to consider values of $q \in [ap, bp]$. It is straightforward to deduce that (i) $\pi_0'(ap) > 0$, (ii) $\pi_0'(bp) < 0$, and (iii) $\pi_0(q)$ is strictly concave. Therefore, the first-order condition $\frac{p-c(q)}{c'(q)} = q - ap$ is necessary and sufficient for optimality.

Proof of Proposition 3 The proof consists of two parts. In the first part, we establish that the firm's profit function is concave, and derive the first-order necessary and sufficient condition that specifies q_m^*. In the second part, we show that the optimal design choice is lower in the presence of SL, i.e., $q_m^* < q_0^*$.

Part (i). When the consumers are myopic, the firm's profit function is

$$\pi(q) = (p - c(q))\int_{\frac{p}{q}}^{x_h} g(x)dx + (p - c(q))\int_{q}^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)f(s;q,z)\,dxds,$$

for $z = \frac{p}{q}$. Under Assumption 1, the above is equivalent to

$$\begin{aligned}
\pi(q) &= p - c(q)\left(\frac{\frac{q}{p} - a}{b - a} + \int_{q}^{bp}\left(\frac{\frac{s}{p} - \frac{q}{p}}{b - a}\right)f(s;q,z)\,ds + \int_{bp}^{\infty}\left(\frac{b - \frac{q}{p}}{b - a}\right)f(s;q,z)\,ds\right)\\
&= \frac{p - c(q)}{b - a}\left(\frac{q}{p} - a + \frac{\sigma_z}{p}\int_0^{\frac{bp-q}{\sigma_z}} y\phi(y)\,dy + \left(b - \frac{q}{p}\right)\int_{\frac{bp-q}{\sigma_z}}^{\infty}\phi(y)\,dy\right)\\
&= \frac{p - c(q)}{(b - a)p}\left(q - ap + \sigma_z(\phi(0) - \phi(V)) + (bp - q)\bar{\Phi}(V)\right),
\end{aligned}$$

for $V = \frac{bp-q}{\sigma_z} > 0$. Defining

$$M(q) = \frac{p - c(q)}{(b - a)p} \text{ and } S(q) = \left(q - ap + \sigma_z(\phi(0) - \phi(V)) + (bp - q)\bar{\Phi}(V)\right),$$

we have

$$\pi = MS, \quad \pi' = M'S + MS', \quad \pi'' = M''S + 2M'S' + MS''.$$

Now, note that that by Assumption 2, $(p - c(q))$ is positive for $q \in (ap, bp)$, and that $c(\cdot)$ is convex and increasing so that $M > 0$, $M' < 0$, and $M'' < 0$. Since $S > 0$ for $q \in (ap, bp)$, the profit function is concave (i.e., $\pi'' < 0$) provided $S' > 0$ and $S'' < 0$; we show each of the two in turn. First, we have

$$\begin{aligned}
S' &= 1 + \sigma_z'(\phi(0) - \phi(V)) + \sigma_z(V\phi(V)V') - \bar{\Phi}(V) - (bp - q)\phi(V)V'\\
&= \Phi(V) + \sigma_z'(\phi(0) - \phi(V)),
\end{aligned}$$

since $\sigma_z V = (bp-q)$. Therefore, if $\sigma_z' > 0$, then $S' > 0$. Indeed,

$$\sigma_z' = \frac{d}{dq}\left(\sigma_p\sqrt{\frac{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma}{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma+1}}\right) = \frac{1}{2}p\beta\gamma\sigma_p\frac{b-a}{\sqrt{\frac{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma}{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma+1}}\left(\left(q-ap\right)\beta\gamma+\left(b-a\right)p\right)^2}$$

$$= \frac{1}{2}\frac{\left(b-a\right)p\sigma_z}{\left(q-ap\right)\left(\left(q-ap\right)\beta\gamma+\left(b-a\right)p\right)} > 0. \tag{14}$$

We next show that $S'' < 0$. We have

$$S'' = \phi(V)V' + \sigma_z''\left(\phi(0)-\phi(V)\right) + \sigma_z'\left(V\phi(V)V'\right).$$

Therefore, if $V' < 0$ and $\sigma_z'' < 0$, then $S'' < 0$. Indeed,

$$V' = -\left(\frac{1}{\sigma_z} + \frac{bp-q}{\sigma_z^2}\frac{d\sigma_z}{dq}\right) = -\frac{1}{\sigma_z}\left(1+V\sigma_z'\right) < 0,$$

and

$$\sigma_z'' = \frac{d}{dq}\left(\frac{1}{2}p\beta\gamma\sigma_p\frac{b-a}{\sqrt{\frac{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma}{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma+1}}\left(\left(q-ap\right)\beta\gamma+\left(b-a\right)p\right)^2}\right)$$

$$= -\frac{1}{4}p\sigma_p\frac{b-a}{\left(q-ap\right)^2}\frac{\sqrt{\frac{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma}{\left(\frac{\frac{q}{p}-a}{b-a}\right)\beta\gamma+1}}}{\left(\left(q-ap\right)\beta\gamma+\left(b-a\right)p\right)^2}\left(\left(b-a\right)p+4\beta\gamma\left(q-ap\right)\right)$$

$$= -\frac{1}{4}p\sigma_z\frac{b-a}{\left(q-ap\right)^2}\frac{\left(\left(b-a\right)p+4\beta\gamma\left(q-ap\right)\right)}{\left(\left(q-ap\right)\beta\gamma+\left(b-a\right)p\right)^2} < 0.$$

We have thus shown that the profit function is concave, so that the condition $\pi' = 0$ is necessary and sufficient for optimality. Accordingly, we have

$$\pi' = \frac{-c'(q)}{(b-a)p}\left(q-ap+\sigma_z\left(\phi(0)-\phi(V)\right)+(bp-q)\bar{\Phi}(V)\right) + \frac{p-c(q)}{(b-a)p}\left(\Phi(V)+\sigma_z'\left(\phi(0)-\phi(V)\right)\right) = 0,$$

which reduces to

$$\begin{aligned}\frac{p-c(q)}{c'(q)} &= \frac{q-ap+\sigma_z(\phi(0)-\phi(V)+V\bar{\Phi}(V))}{\Phi(V)+\frac{d\sigma_z}{dq}(\phi(0)-\phi(V))} \\ &= \frac{q-ap+\sigma_z\int_0^V\bar{\Phi}(s)ds}{\Phi(V)+\frac{d\sigma_z}{dq}(\phi(0)-\phi(V))} \\ &= \frac{(b-a)p-\sigma_z\int_0^V\Phi(s)ds}{\Phi(V)+\frac{d\sigma_z}{dq}(\phi(0)-\phi(V))}\end{aligned}$$

Part (ii). Since the profit function is concave it suffices to show that $\pi'(q_0^*) < 0$. Recall from Proposition 2 that q_0^* is defined implicitly via

$$\frac{p-c(q)}{c'(q)} = q-ap.$$

For $V_0 = \frac{bp-q_0^*}{\sigma_z}$, we have

$$\pi'(q_0^*) = \frac{1}{(b-a)p}\left(-c'(q_0^*)\left(q_0^* - ap + \sigma_z\left(\phi(0)-\phi(V_0)\right) + (bp-q_0^*)\bar{\Phi}(V_0)\right) + (p-c(q_0^*))\left(\Phi(V_0) + \sigma_z'\left(\phi(0)-\phi(V_0)\right)\right)\right)$$
$$= \frac{p-c(q_0^*)}{(b-a)p}\left(-\frac{\sigma_z}{q_0^*-ap}\left(\frac{q_0^*-ap}{\sigma_z} + \left(\phi(0)-\phi(V_0)\right) + V_0\bar{\Phi}(V_0)\right) + \Phi(V_0) + \sigma_z'\left(\phi(0)-\phi(V_0)\right)\right)$$
$$= \frac{p-c(q_0^*)}{(b-a)p}\left(\left(\sigma_z' - \frac{\sigma_z}{q_0^*-ap}\right)\left(\phi(0)-\phi(V_0)\right) + \left(\frac{ap-bp}{q_0^*-ap}\right)\bar{\Phi}(V_0)\right)$$

Thus, if $\left(\sigma_z' - \frac{\sigma_z}{q_0^*-ap}\right) < 0$ then $\pi'(q_0^*) < 0$ as required. Indeed,

$$\sigma_z' - \frac{\sigma_z}{q_0^*-ap} = \frac{1}{2}\frac{(b-a)\,p\sigma_z}{(q_0^*-ap)\left((q_0^*-ap)\,\beta\gamma + (b-a)\,p\right)} - \frac{\sigma_z}{q_0^*-ap} = -\frac{1}{2}\sigma_z\frac{2\beta\gamma\left(q_0^*-ap\right) + (b-a)\,p}{(q_0^*-ap)\left((q_0^*-ap)\,\beta\gamma + (b-a)\,p\right)} < 0.$$

Proof of Example 1 From the proof of Proposition 3, the firm's profit function can be written as

$$\pi(q) = \frac{p-c(q)}{b-a}\left(\frac{q}{p} - a + \frac{\sigma_z}{p}\left(\phi\left(0\right) - \phi\left(V\right)\right) + \left(b - \frac{q}{p}\right)\left(1 - \Phi\left(V\right)\right)\right),$$

for $V = \frac{bp-q}{\sigma_z}$. Now, fix $\sigma_z = \theta\sigma_p$. We first show that the firm's optimal profit is increasing in θ. Using the envelope theorem, we have

$$\frac{d\pi^*}{d\theta} = \frac{\partial\pi^*}{\partial\theta} = \frac{p-c(q)}{b-a}\left(\frac{\sigma_p}{p}\left(\phi\left(0\right) - \phi\left(V\right)\right)\right) > 0.$$

We next show that the optimal design q_m^* is strictly decreasing in θ. Noting that the profit function is concave, we consider the first-order condition $\pi'(q) = 0$. We have

$$\pi'(q) = \frac{p-c(q)}{b-a}\left(\frac{1}{p} + \frac{\theta\sigma_p}{p}V\phi(V)V' - \left(b - \frac{q}{p}\right)\phi(V)V' - \frac{1}{p}\bar{\Phi}(V)\right)$$
$$- \frac{c'(q)}{b-a}\left(\frac{q}{p} - a + \frac{\theta\sigma_p}{p}\left(\phi\left(0\right) - \phi\left(V\right)\right) + \left(b - \frac{q}{p}\right)\left(1 - \Phi\left(V\right)\right)\right)$$
$$= \frac{p-c(q)}{b-a}\left(\frac{1}{p}\Phi(V)\right) - \frac{c'(q)}{b-a}\left(\frac{q}{p} - a + \frac{\theta\sigma_p}{p}\left(\phi\left(0\right) - \phi\left(V\right)\right) + \left(b - \frac{q}{p}\right)\left(1 - \Phi\left(V\right)\right)\right).$$

Therefore, the optimal quality choice q_m^* is given implicitly via

$$\frac{p-c(q)}{c'(q)} = \frac{\frac{q}{p} - a + \frac{\theta\sigma_p}{p}\left(\phi\left(0\right) - \phi\left(V\right)\right) + \left(b - \frac{q}{p}\right)\left(1 - \Phi\left(V\right)\right)}{\frac{1}{p}\Phi(V)}.$$

Now, note that the *lhs* of the latter equation is independent of θ, while in the *rhs* the denominator is decreasing in θ (since V is decreasing in θ) and the numerator is increasing in θ (since its derivative w.r.t. θ is $\frac{\sigma_p}{p}\left(\phi\left(0\right) - \phi\left(V\right)\right) > 0$). It follows that q_m^* is decreasing in θ.

Proof of Proposition 4 Under Assumption 1 (see also the proof of Proposition 3), we first note that the firm's profit function can be expressed as

$$\pi(q) = \pi_0(q) + \frac{p-c(q)}{b-a}\left(\int_q^{bp}\left(\frac{s}{p} - \frac{q}{p}\right)f(s;q,\frac{p}{q})ds + \int_{pb}^{\infty}\left(b - \frac{q}{p}\right)f(s;q,\frac{p}{q})ds\right)$$
$$= \pi_0(q) + \frac{p-c(q)}{b-a}\left(\frac{1}{p}\int_q^{bp}sf(s;q,\frac{p}{q})ds - \frac{q}{p}\left(\bar{\Phi}\left(0\right) - \bar{\Phi}\left(\frac{bp-q}{\sigma_z}\right)\right) + \left(b - \frac{q}{p}\right)\bar{\Phi}\left(\frac{bp-q}{\sigma_z}\right)\right)$$
$$= \pi_0(q) + \frac{p-c(q)}{b-a}\left(\frac{q}{p}\left(\bar{\Phi}\left(0\right) - \bar{\Phi}\left(\frac{bp-q}{\sigma_z}\right)\right) + \frac{\sigma_z}{p}\left(\phi\left(0\right) - \phi\left(\frac{bp-q}{\sigma_z}\right)\right) - \frac{q}{p}\frac{1}{2} + b\bar{\Phi}\left(\frac{bp-q}{\sigma_z}\right)\right)$$

Invoking the envelope theorem, we have $\frac{d\pi^*}{d\beta} = \frac{\partial \pi^*}{\partial \beta}$ and

$$\begin{aligned}\frac{\partial \pi^*}{\partial \beta} =& \frac{p-c(q)}{b-a}\left(\frac{q}{p}\phi\left(\frac{bp-q}{\sigma_z}\right)\frac{\partial}{\partial \beta}\left(\frac{bp-q}{\sigma_z}\right)+\frac{1}{p}\frac{\partial \sigma_z}{\partial \beta}\left(\phi(0)-\phi\left(\frac{bp-q}{\sigma_z}\right)\right)\right.\\ &\left.+\frac{\sigma_z}{p}\left(\frac{bp-q}{\sigma_z}\right)\phi\left(\frac{bp-q}{\sigma_z}\right)\frac{\partial}{\partial \beta}\left(\frac{bp-q}{\sigma_z}\right)-b\phi\left(\frac{bp-q}{\sigma_z}\right)\frac{\partial}{\partial \beta}\left(\frac{bp-q}{\sigma_z}\right)\right)\\ =&\left(\frac{p-c(q)}{b-a}\right)\frac{1}{p}\frac{\partial \sigma_z}{\partial \beta}\left(\phi(0)-\phi\left(\frac{bp-q}{\sigma_z}\right)\right)>0,\end{aligned}$$

where the last inequality holds because $\frac{\partial \sigma_z}{\partial \beta} > 0$.

Proof of Lemma 2 When consumers are strategic, the profit function is:

$$\begin{aligned}\pi_s(q) &= (p-c(q))\left(\int_z^{x_h} g(x)dx + \int_{\frac{p}{z}}^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{z} g(x)f(s;q,z)\,dxds\right)\\ &= (p-c(q))\left(\int_z^{x_h} g(x)dx + \int_{\frac{p}{z}}^{q}\int_{\frac{p}{s}}^{z} g(x)f(s;q,z)\,dxds\right.\\ &\qquad\left.+\int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)f(s;q,z)\,dxds+\int_q^{\infty}\int_{\frac{p}{q}}^{z} g(x)f(s;q,z)\,dxds\right)\end{aligned}$$

When consumers are myopic, the profit function is:

$$\pi_m(q) = (p-c(q))\left(\int_{\frac{p}{q}}^{x_h} g(x)dx + \int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)f\left(s;q,pq^{-1}\right)dxds\right)$$

Let $\pi_s(q) = \pi_m(q) - K(q)$, so that $K(q) = \pi_m(q) - \pi_s(q)$. Then

$$\begin{aligned}K(q) =& (p-c(q))\left(\int_{\frac{p}{q}}^{x_h} g(x)dx + \int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)f\left(s;q,pq^{-1}\right)dxds\right.\\ &\left.-\int_z^{x_h} g(x)dx - \int_{\frac{p}{z}}^{q}\int_{\frac{p}{s}}^{z} g(x)f(s;q,z)\,dxds - \int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)f(s;q,z)\,dxds - \int_q^{\infty}\int_{\frac{p}{q}}^{z} g(x)f(s;q,z)\,dxds\right)\\ =&(p-c(q))\left(\int_{\frac{p}{q}}^{z} g(x)dx + \int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)[f\left(s;q,pq^{-1}\right)-f(s;q,z)]dxds\right.\\ &\left.-\int_{\frac{p}{z}}^{q}\int_{\frac{p}{s}}^{z} g(x)f(s;q,z)\,dxds - \int_q^{\infty}\int_{\frac{p}{q}}^{z} g(x)f(s;q,z)\,dxds\right)\\ =&(p-c(q))\left(\int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)[f\left(s;q,pq^{-1}\right)-f(s;q,z)]dxds\right.\\ &\left.-\int_{\frac{p}{z}}^{q}\int_{\frac{p}{s}}^{z} g(x)f\left(s;q,pq^{-1}\right)dxds + \int_{-\infty}^{q}\int_{\frac{p}{q}}^{z} g(x)f(s;q,z)\,dxds\right)\\ =&(p-c(q))\left(\int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)[f\left(s;q,pq^{-1}\right)-f(s;q,z)]dxds\right.\\ &\left.+\int_{\frac{p}{z}}^{q}\left(\int_{\frac{p}{q}}^{z} g(x)f(s;q,z) - \int_{\frac{p}{s}}^{z} g(x)f(s;q,z)\,dx\right)ds + \int_{-\infty}^{\frac{p}{z}}\int_{\frac{p}{q}}^{z} g(x)f(s;q,z)\,dxds\right)\\ =&(p-c(q))\left(\int_q^{\infty}\int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)[f\left(s;q,pq^{-1}\right)-f(s;q,z)]dxds\right.\\ &\left.+\int_{\frac{p}{z}}^{q}\int_{\frac{p}{q}}^{\frac{p}{s}} g(x)f(s;q,z)\,dxds + \int_{-\infty}^{\frac{p}{z}}\int_{\frac{p}{q}}^{z} g(x)f(s;q,z)\,dxds\right)\end{aligned}$$

To complete the proof, define

$$S_D = \int_{-\infty}^{\frac{p}{z}} \int_{\frac{p}{q}}^{z} g(x) f(s;q,z)\,dxds + \int_{\frac{p}{z}}^{q} \int_{\frac{p}{q}}^{\frac{p}{s}} g(x) f(s;q,z)\,dxds$$
$$S_L = \int_{q}^{\infty} \int_{\max\{\frac{p}{s},x_l\}}^{\frac{p}{q}} g(x)[f(s;q,pq^{-1}) - f(s;q,z)]dxds.$$

Proof of Proposition 5 In the first part of the proof, we derive the necessary condition that must be satisfied by the optimal design choice q_s^*. In the second part, we show that there exist thresholds $\Delta_1, \Delta_2, \Delta_3 \in [0,1)$ such that (i) $q_s^* > q_m^*$ for any $\delta \geq \Delta_1$, and (ii) $q_s^* < q_0^*$ for any $\Delta_2 \leq \delta \leq \Delta_3$.

Part (i). When consumers are strategic, the firm's profit function is

$$\pi(q) = (p - c(q)) \int_{z}^{x_h} g(x)dx + (p - c(q)) \int_{\frac{p}{z}}^{\infty} \int_{\max\{\frac{p}{s},x_l\}}^{z} g(x) f(s;q,z)\,dxds,$$

for z specified in Proposition 1. Under Assumption 1, the above is equivalent to

$$\begin{aligned}
\pi(q) &= \frac{p - c(q)}{b-a}\left(z^{-1} - a + \int_{\frac{p}{z}}^{bp}\left(\frac{s}{p} - z^{-1}\right) f(s)\,ds + \int_{bp}^{\infty}\left(b - z^{-1}\right) f(s)\,ds\right) \\
&= \frac{p-c(q)}{b-a}\left(z^{-1} - a + \frac{\sigma_z}{p}(\phi(Y) - \phi(V)) + \left(\frac{q}{p} - z^{-1}\right)(\Phi(V) - \Phi(Y)) + (b - z^{-1})\bar{\Phi}(V)\right) \\
&= \frac{p-c(q)}{(b-a)p}\left(-ap + \sigma_z(\phi(Y) - \phi(V)) + q\Phi(V) - q\Phi(Y) + \frac{p}{z}\Phi(Y) + bp\bar{\Phi}(V)\right) \\
&= \frac{p-c(q)}{(b-a)p}\left((b-a)p + \sigma_z(\phi(Y) + Y\Phi(Y) - \phi(V) - V\Phi(V))\right),
\end{aligned}$$

for $V = \frac{bp-q}{\sigma_z}$ and $Y = \frac{\frac{p}{z}-q}{\sigma_z}$. Note that the above profit function is smooth and continuous, and by Assumption 2 we have $\pi(q) \leq 0$ for $q \in (0, ap] \cup [bp, \infty)$ and $\pi(q) > 0$ for $q \in (ap, bp)$. Therefore, a profit-maximizing quality $q^* \in (ap, bp)$ exists and satisfies the first-order condition $\pi'(q) = 0$. Defining

$$M(q) = \frac{p - c(q)}{(b-a)p} \text{ and } S(q) = (b-a)p + \sigma_z(\phi(Y) + Y\Phi(Y) - \phi(V) - V\Phi(V)),$$

we have

$$\pi = MS, \quad \pi' = M'S + MS'$$

where

$$\begin{aligned}
S' &= \sigma_z'(\phi(Y) + Y\Phi(Y) - \phi(V) - V\Phi(V)) \\
&\qquad + \sigma_z(-Y\phi(Y)Y' + Y'\Phi(Y) + Y\phi(Y)Y' + V\phi(V)V' - V'\Phi(V) - V\phi(V)V') \\
&= \sigma_z'(\phi(Y) + Y\Phi(Y) - \phi(V) - V\Phi(V)) + \sigma_z(Y'\Phi(Y) - V'\Phi(V)) \\
&= \sigma_z'(\phi(Y) + Y\Phi(Y) - \phi(V) - V\Phi(V)) + \sigma_z\left(\frac{1}{\sigma_z}\Phi(V) + \frac{V}{\sigma_z}\sigma_z'\Phi(V)\right) \\
&= \sigma_z'(\phi(Y) + Y\Phi(Y) - \phi(V)) + \Phi(V),
\end{aligned}$$

where we have used from (12) that Y depends only on δ (i.e., $Y' = 0$). Then, $\pi'(q) = 0$ implies $-\frac{M}{M'} = \frac{S}{S'}$, or

$$\frac{p - c(q)}{c'(q)} = \frac{(b-a)p + \sigma_z(\phi(Y) + Y\Phi(Y) - \phi(V) - V\Phi(V))}{\Phi(V) + \sigma_z'(\phi(Y) + Y\Phi(Y) - \phi(V))}, \text{ or}$$

$$\frac{p-c(q)}{c'(q)}=\frac{(b-a)p-\sigma_z\int_Y^V\Phi(s)ds}{\Phi(V)+\frac{d\sigma_z}{dq}\left(\phi(Y)+Y\Phi(Y)-\phi(V)\right)}. \tag{15}$$

Part (ii). By continuity of q_s^* in δ, to prove the existence of the describe thresholds it suffices to show that $\lim_{\delta\to 1} q_s^* > q_m^*$. Since $q_0^* > q_m^*$ (see Proposition 3), this is ensured by the following result, which we state as a lemma for use in subsequent proofs.

LEMMA 4. *Let q_s^* (q_0^*) denote the optimal quality choice in the presence (absence) of SL. Then* $\lim_{\delta\to 1} q_s^* = q_0^*$.

Proof. Let $\pi_s(q)$ ($\pi_0(q)$) denote the firm's profit function in the presence (absence) of SL. Note that q_s^* must satisfy the first-order condition $\pi_s'(q)=0$ and that, by Proposition 2, $\pi_0'(q)=0$ has a unique solution q_0^*. Therefore, to prove the claim, it suffices to show that $\lim_{\delta\to 1}\pi_s'(q)=\pi_0'(q)$. For $S(q)=b-a-\frac{\sigma_z}{p}\left(\phi(V)+V\Phi(V)-\phi(Y)-Y\Phi(Y)\right)$, we have

$$\begin{aligned}\pi_s(q) &= \frac{p-c(q)}{b-a}S(q)\\ \pi_s'(q) &= \frac{p-c(q)}{b-a}S'(q)-\frac{c'(q)}{b-a}S(q)\end{aligned}$$

Then

$$\lim_{\delta\to 1}\pi_s'(q)=\frac{p-c(q)}{b-a}\lim_{\delta\to 1}S'(q)-\frac{c'(q)}{b-a}\lim_{\delta\to 1}S(q). \tag{16}$$

Before calculating (16), we note that from Proposition 1 we have $\lim_{\delta\to 1} z = x_h = a^{-1}$, $\lim_{\delta\to 1}\sigma_z=0$, $\lim_{\delta\to 1} V=\infty$, and $\lim_{\delta\to 1} Y=-\infty$. Furthermore, it follows from the properties of the Normal distribution that $\lim_{\delta\to 1}\phi(V)=\lim_{\delta\to 1}\phi(Y)=\lim_{\delta\to 1} Y\Phi(Y)=0$ (where the last can be shown using L'Hopital's rule).

In addition, we show here that $\lim_{\delta\to 1}\sigma_z'=0$. We have $\frac{d\sigma_z}{dq}=\frac{d\sigma_z}{dz}\frac{dz}{dq}$ and

$$\frac{d\sigma_z}{dz}=\frac{d}{dz}\left(\sigma_p\sqrt{\frac{\left(\frac{1}{z}-a\right)\beta\gamma}{\left(\frac{1}{z}-a\right)\beta\gamma+(b-a)}}\right)=-\frac{1}{2}\frac{(b-a)\beta\gamma\sigma_p}{z^2\sqrt{\frac{\left(\frac{1}{z}-a\right)\beta\gamma}{\left(\frac{1}{z}-a\right)\beta\gamma+(b-a)}}\left(\left(\frac{1}{z}-a\right)\beta\gamma+(b-a)\right)^2},$$

which implies that $\lim_{\delta\to 1}\frac{d\sigma_z}{dz}=-\infty$. Next, recall that z is defined implicitly (see proof of Proposition 1) via $-Y-\delta\left(\phi\left(Y\right)-Y\left(1-\Phi\left(Y\right)\right)\right)=: g\left(q,z\right)=0$. Therefore,

$$\begin{aligned}\frac{dz}{dq} &= -\frac{\frac{\partial g}{\partial q}}{\frac{\partial g}{\partial z}}=-\frac{\frac{1}{\sigma_z}-\delta\left(\frac{1}{\sigma_z}Y\phi\left(Y\right)+\frac{1}{\sigma_z}\left(1-\Phi\left(Y\right)\right)-\frac{1}{\sigma_z}Y\phi\left(Y\right)\right)}{\left(-\frac{\partial Y}{\partial z}-\delta\left(-\frac{\partial Y}{\partial z}Y\phi\left(Y\right)-\frac{\partial Y}{\partial z}\left(1-\Phi\left(Y\right)\right)+\frac{\partial Y}{\partial z}Y\phi\left(Y\right)\right)\right)}\\ &= \frac{\frac{1}{\sigma_z}\left(1-\delta\left(1-\Phi\left(Y\right)\right)\right)}{\frac{\partial Y}{\partial z}\left(1-\delta\left(1-\Phi\left(Y\right)\right)\right)}=\frac{\frac{1}{\sigma_z}}{\frac{\partial Y}{\partial z}}=-\frac{1}{\left(\frac{p}{z^2}+Y\frac{d\sigma_z}{dz}\right)}\end{aligned} \tag{17}$$

so that

$$\lim_{\delta\to 1}\frac{d\sigma_z}{dq}=\lim_{\delta\to 1}\left(-\frac{\frac{d\sigma_z}{dz}}{\left(\frac{p}{z^2}+Y\frac{d\sigma_z}{dz}\right)}\right)=\lim_{\delta\to 1}\left(-\frac{1}{\left(\frac{\frac{p}{z^2}}{\frac{d\sigma_z}{dz}}+Y\right)}\right)=0.$$

Note that (17) also implies that $\lim_{\delta\to 1} z'=0$.

Using the above limits, we now calculate (16).

$$\lim_{\delta\to 1}S(q)=b-a-\frac{bp-q}{p}=\frac{q}{p}-a$$

$$\begin{aligned}
\lim_{\delta\to 1} S'(q) &= \lim_{\delta\to 1}\left(-\frac{\sigma_z'}{p}\left(\phi(V)+V\Phi(V)-\phi(Y)-Y\Phi(Y)\right)-\frac{\sigma_z}{p}\left(V'\Phi(V)-Y'\Phi(Y)\right)\right)\\
&= \lim_{\delta\to 1}\left(-\frac{\sigma_z'}{p}V\Phi(V)-\frac{\sigma_z}{p}\left(V'\Phi(V)-Y'\Phi(Y)\right)\right)\\
&\qquad\qquad\qquad\qquad (\text{note: } V' = -\frac{1}{\sigma_z}-\frac{bp-q}{\sigma_z^2}\sigma_z',\ Y' = -\frac{\frac{p}{z^2}z'+1}{\sigma_z}-\frac{\frac{p}{z}-q}{\sigma_z^2}\sigma_z')\\
&= \lim_{\delta\to 1}\left(-\frac{\sigma_z'}{p}V\Phi(V)-\frac{\sigma_z}{p}\left(\left(-\frac{1}{\sigma_z}-\frac{bp-q}{\sigma_z^2}\sigma_z'\right)\Phi(V)-\left(-\frac{\frac{p}{z^2}z'+1}{\sigma_z}-\frac{\frac{p}{z}-q}{\sigma_z^2}\sigma_z'\right)\Phi(Y)\right)\right)\\
&= \lim_{\delta\to 1}\left(-\frac{\sigma_z'}{p}V\Phi(V)-\frac{1}{p}\left(\left(-1-\frac{bp-q}{\sigma_z}\sigma_z'\right)\Phi(V)-\left(-\left(\frac{p}{z^2}z'+1\right)-\frac{\frac{p}{z}-q}{\sigma_z}\sigma_z'\right)\Phi(Y)\right)\right)\\
&= \lim_{\delta\to 1}\left(-\frac{\sigma_z'}{p}V-\frac{1}{p}\left(-1-V\sigma_z'+\left(\frac{p}{z^2}z'+1\right)\Phi(Y)+Y\sigma_z'\Phi(Y)\right)\right)\\
&= \frac{1}{p}\lim_{\delta\to 1}\left(-\sigma_z'V+1+V\sigma_z'-\sigma_z'Y\Phi(Y)\right) = \frac{1}{p}-\lim_{\delta\to 1}\sigma_z'Y\Phi(Y) = \frac{1}{p}.
\end{aligned}$$

It follows from the proof of Proposition 2 that $\lim_{\delta\to 1}\pi_s'(q) = \pi_0'(q)$ and the proof of the lemma is complete.

Proof of Proposition 6 Under Assumption 1, the firm's profit function can be written as (see proof of Proposition 5)

$$\begin{aligned}
\pi(q) &= \frac{p-c(q)}{(b-a)p}\left(q-ap+\sigma_z\left(\phi(Y)-\phi(V)+V\bar{\Phi}(V)+Y\Phi(Y)\right)\right)\\
&= \frac{p-c(q)}{(b-a)}\left(\frac{1}{z}-a+\frac{\sigma_z}{p}\left(\phi(Y)-\phi(V)+V\bar{\Phi}(V)-Y\bar{\Phi}(Y)\right)\right),
\end{aligned}$$

for $V = \frac{bp-q}{\sigma_z}$ and $Y = \frac{\frac{p}{z}-q}{\sigma_z}$. Let π^* denote the firm's optimal profit. Using the envelope theorem, we have $\frac{d\pi^*}{d\beta} = \frac{\partial\pi^*}{\partial\beta}$, and

$$\begin{aligned}
\frac{\partial\pi}{\partial\beta} &= -\frac{1}{z^2}\frac{\partial z}{\partial\beta}+\frac{1}{p}\frac{\partial\sigma_z}{\partial\beta}\left(\phi(Y)-\phi(V)+V\bar{\Phi}(V)-Y\bar{\Phi}(Y)\right)+\frac{\sigma_z}{p}\left(\frac{\partial V}{\partial\beta}\bar{\Phi}(V)-\frac{\partial Y}{\partial\beta}\bar{\Phi}(Y)\right)\\
&= -\frac{1}{z^2}\frac{\partial z}{\partial\beta}+\frac{1}{p}\frac{\partial\sigma_z}{\partial\beta}\left(\phi(Y)-\phi(V)\right)+\frac{\sigma_z}{p}\left(\frac{p}{z^2\sigma_z}\frac{\partial z}{\partial\beta}\bar{\Phi}(Y)\right)\\
&= \frac{1}{z^2}\frac{\partial z}{\partial\beta}\left(\bar{\Phi}(Y)-1\right)+\frac{1}{p}\frac{\partial\sigma_z}{\partial\beta}\left(\phi(Y)-\phi(V)\right)
\end{aligned}$$

where we have used $\frac{\partial V}{\partial\beta} = -\frac{V}{\sigma_z}\frac{\partial\sigma_z}{\partial\beta}$ and $\frac{\partial Y}{\partial\beta} = -\frac{Y}{\sigma_z}\frac{\partial\sigma_z}{\partial\beta}-\frac{p}{z^2\sigma_z}\frac{\partial z}{\partial\beta}$. Now, note that from Proposition 1 we have

$$\frac{\partial z}{\partial\beta} = -\frac{Y\frac{\partial\sigma_z}{\partial\beta}}{\frac{p}{z^2}+Y\frac{\partial\sigma_z}{\partial z}} > 0$$

and that

$$\frac{\partial\sigma_z}{\partial\beta} = -\frac{d\sigma_z}{dz}\frac{Y\frac{\partial\sigma_z}{\partial\beta}}{\left(\frac{p}{z^2}+Y\frac{d\sigma_z}{dz}\right)}+\frac{\partial\sigma_z}{\partial\beta} = \frac{-Y\frac{\partial\sigma_z}{\partial\beta}\frac{d\sigma_z}{dz}+\frac{\partial\sigma_z}{\partial\beta}\left(\frac{p}{z^2}+Y\frac{d\sigma_z}{dz}\right)}{\left(\frac{p}{z^2}+Y\frac{d\sigma_z}{dz}\right)} = \frac{\frac{\partial\sigma_z}{\partial\beta}\frac{p}{z^2}}{\left(\frac{p}{z^2}+Y\frac{d\sigma_z}{dz}\right)} > 0.$$

Therefore, it follows that if $\phi(Y) < \phi(V)$, then $\frac{\partial\pi}{\partial\beta} < 0$. Since $Y < 0$ and $V > 0$, this occurs provided $q^* - \frac{p}{z} > bp - q^*$ or, equivalently, provided $z > \frac{p}{2q^*-bp}$. Now, recall from Proposition 1 that $z \in [\frac{p}{q^*}, x_h]$, with $\frac{dz}{d\delta} > 0$ and $\lim_{\delta\to 1} z = x_h$. Therefore, for sufficiently high δ, we have $z > \frac{p}{2q^*-bp}$ provided that $x_h = \frac{1}{a} > \frac{p}{2q^*-bp}$, or equivalently, provided $q^* > \frac{(a+b)p}{2}$. Suppose that $q_0^* > \frac{(a+b)p}{2}$; then, by Lemma 4, we have $\lim_{\delta\to 1} q^* = q_0^* > \frac{(a+b)p}{2}$ which, since q^* is continuous in δ, implies that for sufficiently large δ, we have $q^* > \frac{(a+b)p}{2}$.

Proof of Lemma 3 In the absence of SL, the firm chooses q_0^*, and consumers with $x_i > \frac{p}{q_0^*}$ purchase a unit and derive expected utility $x_i q_0^* - p$, while customers with $x_i \le \frac{p}{q_0^*}$ do not purchase and derive zero utility. In the presence of SL, if the firm chooses $q_s < q_0^*$ then customers with $x_i > z(q_s)$ purchase in the first period and derive expected utility $x_i q_s - p$. Since by Proposition 1 we have $z(q_s) \in \left[\frac{p}{q_s}, a^{-1}\right]$, this implies that customers with $x_i > \frac{p}{q_s}$ would purchase the product either in the presence or in the absence of SL; however, the expected utility of these customers is strictly lower in the presence of SL, since $x_i q_0^* - p > x_i q_s - p$. Next, note that in the presence of SL, customers with $x_i \le \frac{p}{q_0^*}$ have a strictly positive expected surplus equal to $\delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) f(s; q_s, z(q_s)) ds$ while their expected utility in the absence of SL is zero. The above imply the existence of the threshold types $\psi_1, \psi_2 \in (\frac{p}{q_0}, z(q_s))$ as stated in the lemma.

Proof of Proposition 7 The consumers' expected surplus is given by

$$S(q) = \int_{z(q)}^{x_h} (x_i q - p) g(x) dx + \delta \int_{\frac{p}{z(q)}}^{\infty} \int_{\max\{\frac{p}{s}, x_l\}}^{z(q)} (x_i s - p) g(x) f(s; q, z(q)) dx ds.$$

In the extreme case of $\delta = 0$, we have $z(q) = \frac{p}{q}$ and the above expression becomes $S(q) = \int_{\frac{p}{q}}^{x_h} (x_i q - p) g(x) dx$. Now, recall from Proposition 3 that for $\delta = 0$ the firm chooses q_m^* such that $q_m^* < q_0^*$. Furthermore, note from Lemma 3 that any customer with $x_i > z(q)$ has strictly lower expected utility in the presence of SL. Therefore, for $\delta = 0$ the expected consumer surplus is lower in the presence of SL. Continuity of $S(q)$ and q_s^* in δ then imply the existence of a threshold k such that consumer surplus is lower under SL for any $\delta < k$.

Proof of Proposition 8 Note first that set C_1 covers all policies (p, q) that result in nonnegative expected profit. Let $D = \{(p, q) : (p, q) \in C_1, q > \bar{q}(p)\}$, and note that any policy $(p, q) \in D$ cannot be optimal, because it is dominated by policies of the type $(p, q) = (p, \bar{q}(p))$ (i.e., the firm already achieves maximum sales at a lower, and therefore cheaper, design choice). Thus, the solution to the firm's problem is either of the type $(p, \bar{q}(p))$ if the solution is "exterior" (i.e., all customers buy in the first period), or it satisfies the first-order conditions if it is "interior" (i.e., not all customers buy in the first period).

We next establish the equation that characterizes interior solutions. Using the simplification of equation (15), the profit function can be written

$$\pi(p, q) = \frac{p - c(q)}{b - a} \left((b - a)p - \sigma_z \int_Y^V \Phi(s) ds \right),$$

where $V = \frac{bp - q}{\sigma_z}$ and $Y = \frac{zp - q}{\sigma_z}$. Let $M = \sigma_z \int_Y^V \Phi(s) ds$. We have

$$\frac{\partial \pi}{\partial q} = \frac{p - c(q)}{b - a} \left(-\frac{\partial M}{\partial q} \right) - \frac{c'(q)}{b - a} \left((b - a)p - M \right)$$
$$\frac{\partial \pi}{\partial p} = \frac{p - c(q)}{b - a} \left((b - a) - \frac{\partial M}{\partial p} \right) + \frac{1}{b - a} \left((b - a)p - M \right).$$

Next, if $\frac{\partial \pi}{\partial q} = \frac{\partial \pi}{\partial p} = 0$, we have

$$\frac{1}{c'(q)} \left(-\frac{\partial M}{\partial q} \right) - ((b - a)p - M) = -\left((b - a) - \frac{\partial M}{\partial p} \right) - ((b - a)p - M)$$
$$\frac{1}{c'(q)} \left(-\frac{\partial M}{\partial q} \right) = -\left((b - a) - \frac{\partial M}{\partial p} \right)$$
$$c'(q) \frac{\partial M}{\partial p} + \frac{\partial M}{\partial q} - c'(q)(b - a) = 0,$$

and substituting M we get

$$c'(q)\frac{\partial}{\partial p}\left[\sigma_z\int_Y^V \Phi(s)ds\right]+\frac{\partial}{\partial q}\left[\sigma_z\int_Y^V \Phi(s)ds\right]-c'(q)(b-a)=0$$
$$\frac{\partial}{\partial p}\left[\sigma_z\int_Y^V \Phi(s)ds\right]+\frac{1}{c'(q)}\frac{\partial}{\partial q}\left[\sigma_z\int_Y^V \Phi(s)ds\right]-(b-a)=0.$$

Proof of Proposition 9 We first show that $\frac{q_0^*}{p_0^*}<b$ ensures that for any $\beta\in[0,1]$ it can never be optimal to sell to all customers in the first period, which (by Proposition 8) is equivalent to showing that the optimal policy lies in the interior of the set C for all $\beta\in[0,1]$. Let $\pi_0^*:=\max_{\{p,q\}}\pi_0(p,q)$ and $\pi_m^*:=\max_{\{p,q\}}\pi_m(p,q)$ denote the firm's optimal profit in the absence ($\beta=0$) and presence ($\beta>0$) of SL, respectively. Let $\pi_{a0}^*=\max_{\{p,q:\,\frac{q}{p}\geq b\}}\pi_0(p,q)$ and $\pi_{am}^*=\max_{\{p,q:\,\frac{q}{p}\geq b\}}\pi_m(p,q)$ denote the firm's optimal profit when the firm's policy induces all customers to purchase in the first period (when the consumers are myopic, this occurs for any $\beta\in[0,1]$ provided $\frac{q}{p}\geq b$), in the absence and presence of SL, respectively. Next, notice that for any $\frac{q}{p}\geq b$ we have $\pi_m(p,q)=\pi_0(p,q)$ (because all customers purchase in the first period), while for any $\frac{q}{p}<b$ we have $\pi_m(p,q)>\pi_0(p,q)$ (see (5)). Now, $\frac{q_0^*}{p_0^*}<b$ implies that $\pi_0^*>\pi_{a0}^*$. In turn, this means that $\pi_m^*\geq\pi_m(p_0^*,q_0^*)>\pi_0(p_0^*,q_0^*)=\pi_0^*>\pi_{a0}^*=\pi_{am}^*$ which implies that $\frac{q_m^*}{p_m^*}<b$ so that it can never be optimal to sell to all customers in the first period. Therefore, we have shown that the solution to the firm's problem lies in the interior of the set C for all $\beta\in[0,1]$; monotonicity of the firm's optimal expected profit in β then follows from the application of the envelope theorem provided in the proof of Proposition 4.

Proof of Proposition 10 First note that for sufficiently high δ, there does not exist any policy that induces all customers to purchase in the first period, so that a solution to the firm's problem must lie in the interior of the set C. Next, note that Lemma 2 extends to the case of joint pricing-and-design in a straightforward manner given existence of an optimal solution in the benchmark case without SL; that is, we have $\lim_{\delta\to 1}\{p_s^*,q_s^*\}=\{p_0^*,q_0^*\}$. It follows that the application of the envelope theorem used in the proof of Proposition 6 along with the sufficient condition therein extend readily to the case of endogenous pricing.

B. General Distribution of Consumer Types

In our main analysis, we have assumed that the consumers' types (i.e., willingness-to-pay for quality) are inverse-uniformly distributed. Here, we consider how our main results extend to the more general class of distributions with increasing hazard ratio $h(\cdot)$, where $h(\cdot):=\frac{g(\cdot)}{\bar{G}(\cdot)}$; this class of distributions includes, among several others, the uniform distribution, which is commonly assumed in existing literature (e.g., Biyalogorsky and Koenigsberg 2014, Moorthy 1984, Shi et al. 2013).

To begin, we note that Proposition 1 of our main analysis does not make any use of Assumption 1, so that the result, which establishes existence and uniqueness of a threshold-type purchasing equilibrium, holds unaltered. In the benchmark case where there is no SL, the optimal quality choice q_0^* which is characterized in Proposition 2 now satisfies the necessary and sufficient (under the increasing-hazard-rate assumption) condition

$$\frac{p-c(q)}{c'(q)}=\frac{1}{h\left(\frac{p}{q}\right)}\frac{q^2}{p}.$$

Proposition 3 then states that, in the case of myopic consumers, the optimal design choice in the presence of SL is lower than that in its absence. This is a central result in our analysis, which extends to the class of distributions with increasing hazard rate as follows.

Proposition 11. *When the consumers are myopic, the optimal design choice q_m^* satisfies the implicit equation*

$$\frac{p-c(q)}{c'(q)}=\frac{1-\int_{x_l}^{\frac{p}{q}}\Phi\left(V\right)g\left(x\right)dx}{\frac{p}{2q^2}g\left(\frac{p}{q}\right)-\int_{x_l}^{\frac{p}{q}}\phi\left(V\right)\frac{dV}{dq}g\left(x\right)dx} \tag{18}$$

where $V=\frac{bp-q}{\sigma_z}$ and $z=\frac{p}{q}$. Furthermore, for sufficiently large σ_p, the product's expected quality in the presence of SL is strictly lower than that in its absence; that is, $q_m^*<q_0^*$.

Proof. The profit function can be written as

$$\begin{aligned}\pi_m\left(q\right)&=\left(p-c\left(q\right)\right)\left(\int_{\frac{p}{q}}^{x_h}g\left(x\right)dx+\int_{x_l}^{\frac{p}{q}}\left(\int_{\frac{p}{x}}^{\infty}f\left(s;q,\frac{p}{q}\right)ds\right)g\left(x\right)dx\right)\\&=\pi_0\left(q\right)+\left(p-c\left(q\right)\right)\left(\int_{x_l}^{\frac{p}{q}}\left(\int_{\frac{p}{x}}^{\infty}f\left(s;q,\frac{p}{q}\right)ds\right)g\left(x\right)dx\right)\end{aligned}$$

Now, let $\sigma_z=\sigma_p\sqrt{\frac{\bar{G}\left(\frac{p}{q}\right)\beta\gamma}{\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1}}$ and $y=\frac{s-q}{\sigma_z}$. Then

$$\begin{aligned}\int_{x_l}^{\frac{p}{q}}\left(\int_{\frac{p}{x}}^{\infty}h\left(s;q,\frac{p}{q}\right)ds\right)g\left(x\right)dx&=\int_{x_l}^{\frac{p}{q}}\left(\int_{\frac{\frac{p}{x}-q}{\sigma_z}}^{\infty}\phi\left(y\right)dy\right)g\left(x\right)dx=\int_{x_l}^{\frac{p}{q}}\left(1-\Phi\left(\frac{\frac{p}{x}-q}{\sigma_z}\right)\right)g\left(x\right)dx\\&=G\left(\frac{p}{q}\right)-\int_{x_l}^{\frac{p}{q}}\Phi\left(\frac{\frac{p}{x}-q}{\sigma_z}\right)g\left(x\right)dx,\end{aligned}$$

so that

$$\begin{aligned}\pi_m\left(q\right)&=\left(p-c\left(q\right)\right)\left(1-\int_{x_l}^{\frac{p}{q}}\Phi\left(\frac{\frac{p}{x}-q}{\sigma_z}\right)g\left(x\right)dx\right)\\&=\left(p-c\left(q\right)\right)\left(1-\int_{x_l}^{\frac{p}{q}}\Phi\left(V\right)g\left(x\right)dx\right)\end{aligned}$$

where $V=\frac{p-qx}{x\sigma_z}\geq 0$. By Assumption 2, an optimal design choice $q_m^*\in\left[\frac{p}{x_h},\frac{p}{x_l}\right]$ exists and satisfies the necessary first-order condition $\pi_m'(q)=0$ or, equivalently,

$$\frac{p-c(q)}{c'(q)}=\frac{1-\int_{x_l}^{\frac{p}{q}}\Phi\left(V\right)g\left(x\right)dx}{\frac{p}{2q^2}g\left(\frac{p}{q}\right)-\int_{x_l}^{\frac{p}{q}}\phi\left(V\right)\frac{dV}{dq}g\left(x\right)dx}$$

We next show that for sufficiently large σ_p, we have $q_m^*<q_0^*$. Recall that q_0^* is uniquely defined by

$$\frac{p-c(q)}{c'(q)}=\frac{\bar{G}\left(\frac{p}{q}\right)}{\frac{p}{q^2}g\left(\frac{p}{q}\right)},$$

and note that the *lhs* is strictly decreasing in q while the *rhs* is strictly increasing in q. Therefore, to show that $q_m^*<q_0^*$ it suffices to show that for all $q\in\left[\frac{p}{x_h},\frac{p}{x_l}\right]$

$$\frac{\bar{G}\left(\frac{p}{q}\right)}{\frac{p}{q^2}g\left(\frac{p}{q}\right)}<\frac{1-\int_{x_l}^{\frac{p}{q}}\Phi\left(V\right)g\left(x\right)dx}{\frac{p}{2q^2}g\left(\frac{p}{q}\right)-\int_{x_l}^{\frac{p}{q}}\phi\left(V\right)\frac{dV}{dq}g\left(x\right)dx}.$$

Consider first the numerators, and note that $1-\int_{x_l}^{\frac{p}{q}}\Phi(V)\,g(x)\,dx=\bar{G}\left(\frac{p}{q}\right)+G\left(\frac{p}{q}\right)-\int_{x_l}^{\frac{p}{q}}\Phi\left(\frac{\frac{p}{x}-q}{\sigma_z}\right)g(x)\,dx=\bar{G}\left(\frac{p}{q}\right)+\int_{x_l}^{\frac{p}{q}}\left(1-\Phi\left(\frac{\frac{p}{x}-q}{\sigma_z}\right)\right)g(x)\,dx>\bar{G}\left(\frac{p}{q}\right)$. Therefore, a sufficient condition for the above inequality to hold is that the denominators satisfy

$$\frac{p}{q^2}g\left(\frac{p}{q}\right)>\frac{p}{2q^2}g\left(\frac{p}{q}\right)-\int_{x_l}^{\frac{p}{q}}\phi(V)\,\frac{dV}{dq}g(x)\,dx$$

$$\frac{p}{2q^2}g\left(\frac{p}{q}\right)>-\int_{x_l}^{\frac{p}{q}}\phi(V)\,\frac{dV}{dq}g(x)\,dx. \tag{19}$$

Now, consider the integrand of the *rhs*, and note that $V\geq 0$, $\frac{dV}{dq}=\frac{-x^2\sigma_z-(p-qx)x\frac{d\sigma_z}{dq}}{x^2\sigma_z^2}=-\frac{1}{\sigma_z}\left(1+V\frac{d\sigma_z}{dq}\right)$, and

$$\frac{d\sigma_z}{dq}=\sigma_p\frac{1}{2\sqrt{\frac{\bar{G}\left(\frac{p}{q}\right)\beta\gamma}{\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1}}}\frac{g\left(\frac{p}{q}\right)\frac{p}{q^2}\beta\gamma\left(\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1\right)-\bar{G}\left(\frac{p}{q}\right)\beta\gamma g\left(\frac{p}{q}\right)\frac{p}{q^2}\beta\gamma}{\left(\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1\right)^2}$$

$$=\sigma_p\frac{1}{2\sqrt{\frac{\bar{G}\left(\frac{p}{q}\right)\beta\gamma}{\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1}}}\frac{g\left(\frac{p}{q}\right)\frac{p}{q^2}\beta\gamma}{\left(\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1\right)^2}=\sigma_z\frac{1}{2\bar{G}\left(\frac{p}{q}\right)}\frac{g\left(\frac{p}{q}\right)\frac{p}{q^2}}{\left(\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1\right)}>0,$$

so that $\frac{dV}{dq}<0$ and the *rhs* of (19) is positive. However, we also have

$$I:=\phi(V)\,\frac{dV}{dq}g(x)=-\phi(V)\,\frac{1}{\sigma_z}\left(1+V\frac{d\sigma_z}{dq}\right)g(x)$$

$$=-\phi\left(\frac{p-qx}{x\sigma_z}\right)\frac{1}{\sigma_z}\left(1+\frac{p-qx}{x}\frac{1}{2\bar{G}\left(\frac{p}{q}\right)}\frac{g\left(\frac{p}{q}\right)\frac{p}{q^2}}{\left(\bar{G}\left(\frac{p}{q}\right)\beta\gamma+1\right)}\right)g(x).$$

To see that (19) holds for sufficiently large σ_p, note that $\lim_{\sigma_p\to\infty}I=0$, because $\lim_{\sigma_p\to\infty}\sigma_z=\infty$, $\lim_{\sigma_p\to\infty}\gamma=\infty$, and $\lim_{\sigma_p\to\infty}\phi\left(\frac{p-qx}{x\sigma_z}\right)=\phi(0)$. $\square$

Thus, the main result that $q_m^*<q_0^*$ can be shown to hold under the (sufficient but not necessary) condition that σ_p (i.e., the ex ante uncertainty over the product's quality) is sufficiently large. Proposition 4, which states that the firm's expected profit increases in β, carries through unchanged. In the case of strategic consumers, Proposition 5, which compares q_s^* with q_m^*, extends in a similar manner as Proposition 3 above, and under the same sufficient condition that σ_p is sufficiently large. The result of Proposition 6, which establishes that the firm can be hurt by the SL process when consumers are strategic, appears to hold for at least some distributions (e.g., inverse uniform, uniform), but not necessarily for the entire class of distributions with increasing hazard rate. Finally, Proposition 7, which states that consumers surplus is lower under SL for sufficiently low δ, continues to hold under the aforementioned sufficient condition.

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