Should Capital and Housing Be Taxed Differently?

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Abstract

This paper studies taxation of different types of wealth when credit markets are incomplete and investment abilities are heterogeneous. Wealth is divided into two types: capital and housing. The key is that, due to market incompleteness, returns on capital are positively linked to its owner's ability to invest it. In contrast, returns on housing are independent of ownership. I calibrate my model to U.S. data and run simulations in which the current U.S. capital income tax is replaced with a wealth tax. I find that it is optimal to tax housing at a positive and higher rate than capital. On the one hand, the higher housing tax raises housing costs across the board because housing demand is inelastic. On the other hand, the lower capital tax mostly encourages households with higher investment skills to accumulate more capital and increase their capital investments. This, in turn, raises wages paid to labor because wages are positively related to productivity. Since wages rise more rapidly than housing costs, overall welfare rises.

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I Introduction

Wealth can be separated into two broad categories: capital and housing. The key distinction between them is the following: when markets are incomplete and the ability to invest capital varies across households, returns on capital depend heavily on its owner's ability to invest it.

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In contrast, returns on housing are independent of ownership[1]. Since housing accounts for one quarter of the net wealth of U.S. households, the policy on capital and housing taxation may matter a great deal for various outcomes, such as wages, GDP, welfare, etc[2]. In this paper, I will examine how capital and housing should be taxed in order to maximize welfare.

To answer this question I combine several features from previous research into one framework. I begin by using the framework in Guvenen et al. (2017) to model rate-of-return heterogeneity. This feature in the model allows me to mimic the empirical wealth distribution in the U.S.. To model the housing market, I follow the convention in the literature and allow households to rent or own a house (Sommer et al. (2013), Kaplan el al. (2016), and others). Consistent with the aforementioned papers, rental units in my model are smaller than owner-occupied units. The rental market structure is taken from Kaplan et al. (2016), in which rental units are owned by a competitive firm.

My model integrates six inter-related components: overlapping generations of households, competitive mortgage lenders, a competitive rental firm, a final goods producer, a housing construction sector, and a government. Moreover, there are two types of wealth in the model: capital and housing. Capital is used to produce output, while housing only generates utility flow for households.

Throughout their lives, households differ in their ability to invest capital. To increase their capital investments and capital income, households can borrow capital from each other. However, financial markets are incomplete, which means that households can only borrow up to a certain fraction of their net wealth. Given households' own and borrowed capital, they produce intermediate goods, which are sold to a final goods producer for a price (to be discussed below). During their labor force participation period, households face labor income risk. At a certain age, households are forced to retire from the labor force, at which point they receive social security benefits. The latter two features are added to the model in order to capture the standard savings motives in heterogeneous macroeconomic models that deal with capital taxation (Conesa et al. (2009), Imrohoroglu (1998), Aiyagari (1995)). Upon their death, households' net wealth is inherited by their offspring (inheritance includes the house and mortgage debt)[3]. As well, when households die they imperfectly transmit their investment ability to their offspring. However, the inter-generational transmission of investment ability is mean-reverting across a household's lineage. Therefore, sometimes, significant wealth is held by unproductive investors. Since markets are incomplete, some of this wealth is not being put to its most productive use.

To account for the presence of housing, my model features a housing market. Since in reality housing cannot be bought in very small amounts, I model housing as a lumpy asset. Households can be either renters or homeowners. Following the convention in the literature, my model has

[1]Returns on housing, meaning rents, imputed rents, and house price appreciation, mainly depend on market forces and not on ownership.

[2]According to data from the Survey of Consumer Finances (SCF), between 1988-2012, on average, 23% of total net wealth was tied up in households' primary residences. Net housing wealth tied up in primary residences as a fraction of total net wealth has been quite stable in the past 25 years (with a slight downward trend). Its lowest was in 2012 (20% of total net wealth) and its highest was in 1988 (26% of total net wealth).

[3]There is no estate tax in my model. Furthermore, a household that inherits a positive amount of capital and housing is free from paying a wealth tax on these assets in the first period of their life.

owner-occupied units that are larger than rental units[4]. To simplify the analysis, all owner-occupied units come in one size and all rental units come in one, smaller, size. To become homeowners, households must pay at least the minimum down-payment and, if necessary, take on mortgage debt, on which interest is paid, to cover the remainder of the purchase. Homeowners who have mortgage debt can deduct their mortgage interest payments from their taxable income.

Mortgage lenders in the model are exogenous. They lend perfectly elastically at the market going mortgage interest rate.

Households who choose to be renters, pay a rental price to a competitive rental company. The rental company collects rents and incurs management and upkeep costs on their rental units.

The final goods producer is competitive and hires labor inputs from households (for a wage) and buys intermediate inputs from households (for a price). Using these inputs, the final goods producer produces a final good that is sold to households.

The housing supply is determined by the construction sector, in which a construction company chooses a level of housing investment each period. Housing investment is sold to households and the rental company for the market-going housing price. Profits in the construction sector are distributed back to households. Households who own more capital receive more of the construction sector profits.

The government raises revenues by taxing labor income, non-durable consumption, and capital. Government revenues are then used to pay social security benefits to retired households and to provide public goods. As is standard in the Ramsey literature, the government in the model must meet a minimum revenue requirement in order to provide an exogenous amount of public goods. Public goods, however, do not affect private consumption nor enhance future production.

I calibrate the model to U.S. data from the Survey of Consumer Finances collected between 1989-2013. The main target of the calibration is the fraction of wealth owned by the top 1% of the wealth distribution. I use it to pin down the variability in investment ability. This target is important because the more variability in investment ability there is in my model, the more misallocation there is for the government to correct, and the more wages and welfare can be enhanced. To understand how housing affects the calibrated value of investment ability variability, I calibrate my model first with housing and then without housing. I find that housing depresses wealth inequality because it is more equally distributed than capital. Therefore, when housing is included in the model, more dispersion in capital is required to generate the wealth concentration at the top 1% of the wealth distribution observed in the data. This means that, larger variability in investment ability is required when housing is accounted for.

I start by running an experiment in which I replace the currently prevailing capital income tax with a capital tax, leaving housing un-taxed. The capital tax is imposed on the stock rather than on the flow of income generated from it. I do so because Guvenen et al. (2017) show that wealth taxation is superior to capital income taxation when markets are incomplete, investment abilities are heterogeneous, and the transmission of investment ability is imperfect. I compare the results of this exercise with results of the same exercise performed with the same model but without housing. The comparison of the two models highlights the importance of housing when

[4]There is also ample empirical evidence which show that rents renters pay are lower than homeowners imputed rents.

considering introducing a wealth tax. This exercise also tests whether the intuition in Guvenen et al. regarding the superiority of wealth taxation over capital income taxation survives the inclusion of housing. In both exercises, I keep the taxes on labor income and non-durable consumption constant. Government revenue neutrality is preserved. The results reported are for the stationary equilibrium.

My results indicate that when housing is not included, the tax on capital is 0.95%. This tax scheme reduces aggregate capital by 1.3%, but increases both GDP and wages by 7.4%. Thus, welfare rises by 2.8%. These results are a direct consequence of wealth taxation increasing the tax burden on wealthy, but unproductive investor-households, and lessening the burden on productive investor-households. The resulting re-allocation of resources increases the efficiency of capital, which increases GDP, wages, and ultimately welfare.

In contrast, when housing is added into the model, households shift their resources to housing in response to the introduction of the capital tax. As a result, the tax on capital is 1.08%, 13.6% higher than in the model without housing, and aggregate capital declines by 1.5%. Furthermore, housing prices rise by 19.5%, the home-ownership rate increases from 66% to 74.2%, and construction sector profits go up by an amount equal to 0.6% of the benchmark GDP. As well, GDP rises by 9.2%, wages by 8%, and rental prices increase by an amount equivalent to 0.2% of the benchmark wage. As a result, welfare rises by 3.6%.

The reason aggregate capital does not decline more substantially than it does in the model without housing is that households in the model with housing are motivated to save capital to obtain the higher profits in the construction sector. Furthermore, GDP, wages, and welfare rise by more than they do in the model without housing even though capital declines only slightly more in the model with housing. This is because in the model without housing, housing is unaccounted for, making capital 30% more abundant in the model without housing. As such, in the model without housing, the wealth tax corrects a lower inefficiency in the allocation of capital.

In my second experiment I replace the currently prevailing capital income tax with the optimal combination of capital and housing tax rates. To be clear, the taxes on capital and housing are imposed on the stock of capital and the stock of housing, respectively. I explore tax policies on the stock rather than the flow because, as I mentioned earlier, Guvenen et al. show that wealth taxation is superior to capital income taxation in an environment similar to mine[5]. The housing tax is also imposed on the rental company. The optimum is achieved when the ex-ante expected welfare of a newly born household is maximized. As in the first experiment, I keep taxes on labor income and non-durable consumption constant, and maintain government revenue neutrality. The results reported are for the stationary equilibrium. For obvious reasons, this exercise is not compared against the model without housing.

The optimum is achieved when housing is taxed at a rate of 11% and capital is subsidized at a rate of 2%. The optimal tax system generates a 32% decrease in housing prices, a decline in construction sector profitability equivalent to 0.8% of the benchmark GDP, a decline in the home-ownership rate from 66% to 33%, and a rise rental prices that is equivalent to 7.7% of the

[5]Since my environment includes two types of wealth, welfare may be farther enhanced if taxes were imposed on the housing stock and capital income rather than on the housing stock and capital.

benchmark wage. GDP rises by 21.5%, wages by 24.7%, and as a result welfare increases by 7.8%.

On the one hand, the higher housing tax raises housing costs across the board because the housing demand is inelastic. On the other hand, the lower capital tax mostly encourages households with higher investment skills to accumulate more capital and increase their capital investments. This is because their returns on capital investments are high, but since they face incomplete markets they must accumulate more capital in order to benefit from capital investments. This, in turn, raises wages paid to labor because wages are positively related to productivity. Since wages rise more rapidly than housing costs, overall welfare rises.

This tax scheme generates winners and losers. The housing tax hurts everyone. It causes some households in the middle and bottom of the wealth distribution to become renters, and thus, live in smaller houses. Since rental prices go up, these households also pay higher rents. The tax also increases the cost of living in a house, a cost that is mostly incurred by the top of the wealth distribution because they are mostly the ones who remain home-owners. Furthermore, since most of the profits in the construction sector go to the top of the wealth distribution, the reduced profitability in the construction sector hurts them as well. In contrast to the housing tax, the subsidy on capital benefits almost everyone. Since it increases the accumulation of productive capital, wages increase. This benefit is quite high and is accrued to most of the economy. Since the top of the wealth distribution holds most of their portfolio in capital, the subsidy benefits them greatly. Moreover, since many households in the bottom of the wealth distribution become renters, their portfolio shifts heavily towards capital, which means that the subsidy on capital benefits them too. The main losers in this economy are retired households who are at the bottom of the wealth distribution. These households are renters and do not benefit from the higher wages or the capital subsidy. However, they incur the cost of higher rents, making them worse off.

II Related Literature

Early work on capital taxation studied models with infinite lives, no labor income risk, and complete financial markets (Judd (1985), Chamley (1986)). The main result of these assumptions is that capital is elastic, and therefore, should be left un-taxed in the long-run.

In their seminal papers, Huggett (1993) and Aiyagari (1994) incorporate more realistic assumptions into the Chamley-Judd class of models, such as finite lives, labor income risk, and incomplete financial markets. Due to precautionary savings motives, retirement savings motives, and incomplete markets capital responds less to taxation in these environments, and thus, the optimal capital income tax is positive and large even in the log-run (Conesa et al. (2009), Aiyagari (1995), Imrohoroglu (1998)). However, this class of models does not account for rate-of-return heterogeneity. Therefore, it can neither be used to study whether it is more efficient to tax capital income or wealth nor can it generate the realistic wealth distributions needed for proper quantitative work on capital taxation[6].

A newer strand of literature has shown that including rate-of-return heterogeneity enables

[6]This class of models fails at mimicking empirical wealth distributions because precautionary savings motives taper off quickly as wealth increases. As a result, this class of models generates wealth distributions with thin tails.

models to generate more realistic wealth distributions (Quadrini (2000), Cagetti & De Nardi (2006, 2009), Benhabib et al. (2011, 2015, 2016)). This occurs due to households having production savings motives in addition to savings motives that were considered in previous models. However, this line of research has neither accounted for housing nor has it attempted to answer the question of whether capital income or wealth should be taxed.

Recently, empirical justification for modeling rate-of-return heterogeneity has been put forth by Fagereng et al. (2016). Using 20 years of Norwegian tax record data, they show that there are vast differences in the rates of return earned across different individuals. These differences have a strong permanent component to them, are correlated with wealth, and are also present within asset classes. They also show that rate-of-return heterogeneity persists across generations, although it does mean revert.

Guvenen et al. (2017) are the first to study whether capital income or wealth should be taxed. They find that wealth taxation is superior to capital income taxation when markets are incomplete, investment abilities are heterogeneous across households, and households imperfectly transmit their investment ability to their children. This is because under capital income taxation, households who are more productive generate more income and pay higher taxes. In contrast, under wealth taxation, households with similar wealth levels pay similar taxes, independent of their productivity. In this environment, the tax burden shifts from productive households to unproductive ones when capital income tax is replaced with a wealth tax, generating positive aggregate outcomes for wages and welfare. A shortcoming of their work is that, in their environment, returns on all types of wealth heavily depend on which households own it. In practice, returns on some types of wealth, such as housing, are independent of ownership. As such, their framework cannot be used to study whether the tax treatment of capital and housing should be different.

Lastly, the housing literature has neither looked at weather capital income or wealth should be taxed nor has it incorporated heterogeneity in investment ability (Gervais (2002), Englund (2003), Chambers et al. (2009), Sommer et al. (2013), Arslan et al. (2015), Kaplan et al. (2016)). As such, to the best of my knowledge, this paper is the first to have a housing market and a realistic wealth distribution in one model.

III Model

I Environment

Households:

There are overlapping generations of households. A newly born cohort of households has a mass of 1. Each household can live up to J periods. Households are forced to retire from the labor force for the last R periods of their lives. The age of a household is denoted by o. The probability of a household who is o years old, to survive to age $o+1$, is denoted by ϕ_o. Households care about non-durable consumption good and a durable housing consumption good. Housing is heterogeneous and binary. Households cannot own more than one house. Households pay a consumption tax τ_c, on their non-durable consumption.

While in the labor force, households receive a wage ω for every efficiency unit they supply in the labor market. There is a deterministic life-cycle component to labor market efficiency ζ_o[7]. Labor market efficiency also has a random component, e. The random component can be thought of as a shock to employment. A household is employed full-time when the random component is e_{hi} and is partially employed when the random component is e_{lo}. The transition probability matrix between employment states is $\Pi_e = \begin{bmatrix} \pi_{e_{hi,hi}} & \pi_{e_{hi,lo}} \\ \pi_{e_{lo,hi}} & \pi_{e_{lo,lo}} \end{bmatrix}$. $\pi_{e_{ij}}$ is the probability of moving from state e_i, $i \in \{hi, lo\}$, this year, to state e_j, $j \in \{hi, lo\}$, next year. At birth, the unconditional probability of drawing e_{hi} is $\pi_{1_{hi}}$, and the unconditional probability of drawing e_{lo} is $\pi_{1_{lo}}$. There is a proportional labor income tax rate τ_l. Thus, households who are in the labor force, with employment status e_i and deterministic labor market efficiency ζ_o, have an after-tax labor income equal to $\omega\zeta_o e_i(1-\tau_l)$. Retired households receive social security benefits denoted by b. b is a function of the average before-tax labor income in the economy, $b = \Phi\bar{E}$. Φ is a constant (a number) and $\bar{E}$ is the average before-tax labor income in the economy.

A household also has an ability to invest capital z. Investment ability is unchanging throughout a household's life-time. When a household dies it imperfectly transmits their investment ability to their child. This is represented by the mean reverting process below:

$$\log(z^{\text{child}}) = \rho_z \log(z^{\text{parent}}) + \epsilon_z$$

Where $\rho_z \in (0,1)$ and $\epsilon_z \sim \mathcal{N}(0, \sigma^2_{\epsilon_z})$.

Households have the choice of renting or owning a house. Following Sommer et al. (2013), in my model there is no utility advantage to home-owning. Instead, as is standard in the housing literature, rental units are smaller than owner-occupied units. Specifically, $\bar{h} > \underline{h}$, where $\bar{h}$ is the size of an own-occupied unit and $\underline{h}$ is the size of a rental unit. All owner-occupied units are of the same size, and all rental units are of the same, smaller, size. Household utility is given by $u(c,h) = \frac{(c^{1-\gamma}h^{\gamma})^{1-\sigma_c}}{1-\sigma_c}$, where γ is the relative importance of housing consumption in the utility function, and σ_c is the relative risk aversion of the household.

Households who choose to be renters pay the rental price $p_r\underline{h}$[8]. Households who choose to be homeowners can obtain a mortgage m', that is no larger than a certain fraction of the price of a house (for future reference, the terms mortgage, mortgage debt, and HSD, which is an acronym for home secured debt, will be used interchangeably). In particular, $m' \leq (1-\lambda_p)p\bar{h}$, where $\lambda_p p\bar{h}$ is the minimum down-payment on a house and $p\bar{h}$ is the price of an owner-occupied house. Households are not allowed to take on mortgage debt during retirement. The interest rate charged on mortgage debt is $r_m = r_f + \Delta r_m$, where Δr_m is a premium paid on mortgage debt. Households

[7]In my calibration I set this parameter to equal to 1 for all ages.

[8]Households whose after-tax wealth less rental payments is lower than W_{min}, are assisted by the government. That is, if a renter-household's after-tax wealth (which includes all incomes earned) less the rental price, does not reach a certain minimum threshold level of wealth W_{min}, the government provides the household a subsidy to bring the households to minimum wealth level. However, in equilibrium and all counter-factual simulations this state of the world never materializes.

can deduct a portion δ_m, of their mortgage interest payments from their taxable labor income[9]. To mimic the costs associated with buying and selling a house, such as Realtor fees, search costs, home inspection, and other costs, households who purchase a house must pay $p\bar{h}\chi_b$ above the house price and households who choose to sell their house receive $p\bar{h}\chi_s$ below the house price. Furthermore, each period, homeowners must incur maintenance costs M_h. These costs are paid to the housing construction company (to be discussed later), and are proportional to the housing price $M_h = \delta_p p\bar{h}$, where δ_h is the depreciation rate of housing.

Households can accumulate capital, $a \geq 0$, and can borrow and lend capital, denoted by d, at the risk-free interest rate r_f. Borrowing can be at most a multiple λ_a of a household's net wealth $(a + p\bar{h} - m)$. Lending can be no larger than a household's capital a. This amounts to the following constraints: $-a \leq d \leq (a + p\bar{h} - m)\lambda_a$. Capital used in production depreciates at rate δ_a. Household i with capital a_i, who borrowed or lent capital d_i, and who has investment ability z_i, can produce intermediate output $x_i = k_i z_i$, where $k_i = a_i + d_i$. For its intermediate output x_i, a household receives a price q_{x_i}, from the final goods producer (to be discussed). Furthermore, a household's own capital a, also determines the amount of profits it gets from the housing construction company. In particular, a household with capital a_i obtains dividends from the construction company that are equal to $\pi_h \frac{a_i}{\bar{a}}$, where $\bar{a}$ denotes the average capital in the economy and π_h denotes the per-household economic profit of the construction company. I drop the subscript i to save on notation.

Given the above, I can write a household's before-tax, and after-production wealth as follows:

$$\max_{0 \leq k \leq (a+p\bar{h}-m)(1+\lambda_a)} \left\{ a\pi_h \frac{1}{\bar{a}} + (1-\delta_a)k + q_{(x)}x - (1+r_f)(k-a) \right\} =$$

$$a(1 + r_f + \pi_h \frac{1}{\bar{a}}) + \max_{0 \leq k \leq (a+p\bar{h}-m)(1+\lambda_a)} \left\{ q_{(x)}x - (r_f + \delta_a)k \right\} =$$

$$a + (r_f + \pi_h \frac{1}{\bar{a}})a + \pi^*(a, z)$$

$(r_f + \pi_h \frac{1}{\bar{a}})a + \pi^*(a, z)$ is capital income earned when k is chosen optimally given the constraints placed on it. Households are also faced with a proportional capital income tax rate τ_k, and a proportional capital tax rate τ_a. Therefore, a household's wealth after production and after taxes are paid is as follows:

$$a(1 - \tau_a) + ((r_f + \pi_h \frac{1}{\bar{a}})a + \pi^*(a, z))(1 - \tau_k)$$

Let the density and cumulative distributions of households across age, investment ability, employment status, capital, mortgage debt, and housing status be denoted by $\psi(o, z, e, a, m, h)$ and $\Psi(o, z, e, a, m, \kappa)$, respectively. $\kappa = 1$ represents households who are homeowners (own $\bar{h}$), whereas $\kappa = 0$ represents households are renters (renting $\underline{h}$). Denote the tax rate on capital income τ_k. The tax on wealth is divided into two: a tax rate on capital τ_a, and a tax rate on housing τ_h. Thus, a household who is o years old (before retirement), with deterministic labor

[9]In the model mortgage interest payments cannot be deducted in excess of a household's total taxable labor income.

market efficiency ζ_o, employment status e, capital a, mortgage debt m, and housing status κ, has the following after-tax net wealth:

$$W = \underbrace{\omega\zeta_o e}_{1} - \underbrace{\tau_l(\omega\zeta_o e - \delta_m \kappa m r_m)}_{2} + \underbrace{((r_f + \pi_h \frac{1}{\bar{a}})a + \pi^*(a,z))}_{3} -$$

$$\underbrace{\tau_k((r_f + \pi_h \frac{1}{\bar{a}})a + \pi^*(a,z))}_{4} + \underbrace{a(1-\tau_a)}_{5} + \underbrace{\kappa(p\bar{h}(1-\tau_h) - m)}_{6} -$$

$$\underbrace{\kappa m r_m}_{7}$$

Term 1 on the right-hand-side in the above equation is gross labor income. Term 2 is labor income tax paid after subtracting the deductions allowed for mortgage interest payments. Term 3 is capital income. Term 4 is the tax paid on capital income[10]. Term 5 is capital net of capital taxes paid. Term 6 is net housing wealth net of housing taxes paid. Term 7 is mortgage interest payments. Together these terms make up the after-tax net wealth of a household. When a household retires, term 1 plus term 2 are equal to social security benefits b. Furthermore, when I introduce run counter-factual simulations with the wealth tax, I assume that households are exempted from paying the wealth tax on inherited assets in the first period of their lives.

A household has the following dynamic problem:

$$V(o,z,e,a,m,\kappa;\Psi) = \max_{c,a',m',\kappa'\in\{0,1\}} \left\{ \frac{(h^\gamma c^{1-\gamma})^{1-\sigma_c}}{1-\sigma_c} + \beta\phi_o \mathbb{E}V(o+1,z,e',a',m',\kappa';\Psi'|e;\Psi) \right\}$$

subject to

(1) $(1+\tau_c)c + a' + p_r\underline{h}(1-\kappa') + p\bar{h}(1+\chi_b)\kappa' = W + \kappa' m'$, if $\kappa = 0$

(2) $(1+\tau_c)c + a' + p\bar{h}\kappa' + (p_r\underline{h} + p\bar{h}\chi_s)(1-\kappa') + M_h = W + \kappa' m'$, if $\kappa = 1$

(3) $m' = 0$, if $\kappa' = 0$ or if $o \geq J - R$

(4) $0 \leq m' \leq p\bar{h}(1-\lambda_p)$, if $\kappa' = 1$

(5) $a' \geq 0$

(6) $\psi(o',z',e',a',m',\kappa') = \int_\psi \pi_{ee'} \mathbb{1}(o,z,e,a,m,\kappa;\Psi) \quad \forall o',z',e',a',m',\kappa'$

$\mathbb{1}(o,z,e,a,m,\kappa;\Psi)$ is an indicator function. It indicates that for a given Ψ, households with state variables o,z,e,a,m, and κ, optimally choose a', m', and κ'.

Constraint (1) is for a household who is a renter because $\kappa = 0$. For a renter who chooses to become a homeowner ($\kappa' = 1$), the constraint says that after-tax non-durable consumption expenditures plus capital savings plus the cost of purchasing a house must equal to after-tax net wealth plus mortgage debt. For a renter who chooses to stay a renter ($\kappa' = 0$), the constraint implies that after-tax non-durable consumption expenditures plus capital savings plus the rental price must equal to after-tax net wealth. Constraint (2) is for a household is who a homeowner

[10]For tax purposes, taxes paid on capital income cannot be negative, i.e. there are no subsidies.

since $\kappa = 1$. For a homeowner who chooses to remain a homeowner ($\kappa' = 1$), the constraint says that after-tax non-durable consumption expenditures plus capital savings plus the price of the house plus housing upkeep costs are equal to after-tax net wealth plus the new mortgage debt. For a homeowner who becomes a renter ($\kappa' = 0$), the constraint implies that after-tax non-durable consumption expenditures plus capital savings plus the rental price plus the cost of selling the house plus upkeep costs are equal to after-tax net wealth. Constraint (3) says that a household who chooses to become a renter, $\kappa' = 0$, or is in retirement cannot get a mortgage. Constraint (4) indicates that mortgage debt cannot be negative and that a household cannot obtain mortgage debt that is larger than $(1 - \lambda_p)$ fraction of the house price. The next constraint imposes that a household cannot hold negative capital. The last constraint indicates that households know the economy's law of motion.

Rental Market:

I take the rental market structure from Kaplan et al. (2016). The rental company owns a stock of housing H_r, which it rents out to households. For each rental unit, the rental company obtains the rental price p_r, and incurs management costs c_m, upkeep costs $\delta_p p\underline{h}$, and housing tax costs $\tau_h p\underline{h}$. In my counter-factual simulations, I impose the housing tax on the rental company.

The rental company can frictionlessly buy housing units at the housing price p. Each period, the rental company decides its next period housing stock $H_r^{'}$. Thus, the rental company has the following problem:

$$V(H_r, \Psi) = \max_{H_r^{'}} \left\{ (p_r - c_m)H_r^{'} - p(H_r^{'} - (1 - \delta_h - \tau_h)H_r) + \frac{1}{1 + r_f} \mathbb{E}V(H_r^{'}, \Psi') \right\}$$

Ψ describes the distribution of households across age, investment ability, employment status, capital, mortgage debt, and housing status. Using the fact that in the stationary equilibrium $\Psi = \Psi'$ and the housing price is constant, optimization implies the following relationship between the rental price and the housing price[11]:

$$p_r = c_m + \frac{p}{1 + r_f}(r_f + \delta_h + \tau_h) \tag{1}$$

Equation 1 shows that the rental price increases whenever the housing price, depreciation rate, management costs, housing tax, or risk-free rate increase.

Final Goods Producer:

I use the final goods producer set up in Guvenen et al. (2017). The final goods producer takes as given intermediate goods prices $\{q_{x_i}\}_i$, for $\{x_i\}_i$, as well as the wage rate ω, paid for 1 unit of labor market efficiency. Intermediate outputs are combined with labor to produce the final good Y.

[11] In my calibration, δ_h is chosen based on data. I pick p_r as a fraction of p, jointly with other parameters, to match certain moments in the data. Then c_m is selected to maintain the equality in equation 1. In counter-factual simulations, I hold δ_h and c_m fixed. Then, for different value of τ_h, and housing price, the rental price is determined by satisfying equation 1.

The production function is $Y = AQ^{\alpha}L^{1-\alpha}$, where A is aggregate productivity, $Q = (\int_i x_i^{\nu})^{\frac{1}{\nu}}$ aggregates intermediate outputs to produce the final good, and L is aggregate labor market efficiency units.

The final goods producer has the following problem:

$$\max_{\{x_j\}_{j=i},L} \left\{ A(\int_i x_i^{\nu})^{\frac{\alpha}{\nu}} L^{1-\alpha} - \int_i q_i x_i - \omega L \right\}$$

For a given wage rate ω, labor demand is determined by the first order condition with respect to L:

$$\omega = (1-\alpha)AQ^{\alpha}L^{-\alpha}$$

For given prices of intermediate outputs $\{q_{x_i}\}_i$, the demand for intermediate output x_i, is determined by the first order condition with respect to x_i:

$$q_{x_i} = \alpha A x_i^{\nu-1} Q^{\alpha-\nu} L^{1-\alpha} \tag{2}$$

Housing Supply:

The model has a construction company. Following Topel, & Rosen (1988), construction costs are convex, that is, marginal costs are increasing. This assumption is supported by empirical evidence (Glaeser, Gyourko, & Saks (2004)).

Each period, the construction company generates a mass of housing I_h, which is added to the depreciated housing stock. For each housing unit, the construction company receives the housing price p. To generate I_h, the construction company incurs a cost. Furthermore, up to a certain cut-off of housing investment a minimum and constant marginal cost of construction must be incurred. The latter enables the model to have an implied land value to house price ratio that is consistent with the land value to house price ratio observed in the data. Denote the cut-off level by I_h^c. The above translates to following cost structure:

$$TC(I_h) = c_{\text{min}}\min\{I_h^c, I_h\} + \max\{0, \frac{1}{\frac{1}{\epsilon_p}+1}[(I_h)^{\frac{1}{\epsilon_p}+1} - (I_h^c)^{\frac{1}{\epsilon_p}+1}]\} \tag{3}$$

As I will show below, ϵ_p is the price elasticity of the housing stock. Given this cost structure, the problem of the construction company is to choose investment to maximize profits:

$$\Pi_h = \max_{I_h}\{pI_h - TC_h\}$$

When $I_h > I_h^c$, the first order condition gives the following relationship between housing investment and the price of housing:

$$I_h = p^{\epsilon_p}$$

Let the total stock of housing supplied be denoted by H^s[12]. In the stationary equilibrium,

[12] In equilibrium $H^s = H_r + H_o$, where H_r is the total stock of housing available for rent and H_o is the total stock of housing owned by all households who are homeowners (in other words, it is the owner-occupied housing stock).

the total housing stock H^s, is constant. Thus, $I_h = \delta_h H^s$. And so, when $I_h > I_h^c$, the latter gives the following housing supply in the stationary equilibrium:

$$H^s = \frac{1}{\delta_h} p^{\epsilon_p} \tag{4}$$

Since $\frac{\partial H}{\partial p}\frac{p}{H} = \epsilon_p$, the price elasticity of the housing supply is simply ϵ_p. To be clear, residential investment is paid for by households and the rental company (a cost that is passed down to households through rents).

Government:

Gross government revenues in the model are denoted by G_g and SSC denotes the sum of social security benefits paid to all retired households. As such, net government revenues are denoted by $G_n = G_g - SSC$.

In my simulations I replace τ_k with τ_a and τ_h. I then require the government to raise G_n and provide the same SSC as in the benchmark. This set up exactly preserves government revenue neutrality. In all simulations I keep the tax rates on labor income and non-durable consumption, at their benchmark levels.

II Equilibrium

A recursive competitive equilibrium is a housing price $p(\Psi)$, a rental price $p_r(\Psi)$, a wage rate $\omega(\Psi)$, intermediate outputs prices $\{q_{x_i}(\Psi)\}_i$, a housing construction total cost function $TC_h(I_h)$, per-household construction sector profits $\pi_h(\Psi)$, a risk-free interest rate on borrowing and lending in the capital market $r_f(\Psi)$, an interest rate $r_m(\Psi)$, on mortgage debt, tax rates on labor income, non-durable consumption, capital income, capital and housing, τ_l, τ_c, τ_k, τ_a, τ_h, respectively, a density distribution of households across age, investment ability, employment status, capital, mortgage debt, and housing status, $\psi(o, z, e, a, m, \kappa)$, a value function $V(o, z, e, a, m, \kappa; \Psi)$, policy functions $g_j(o, z, e, a, m, \kappa; \Psi)$ $j \in \{k, c, a', m', \kappa'\}$, and indicator for the household and an indicator function $\mathbb{1}(o, z, e, a, m, \kappa; \Psi)$, for the household, a competitive rental company, labor and intermediate outputs demands $L(\Psi)$ and $\{x_i(\Psi)\}_i$, respectively, for the final goods producer, generating $Y(\Psi)$, an exogenous net government revenue constraint G_n, housing investment $I_h(\Psi)$, for the construction sector, an exogenous rental unit management cost c_m, and an exogenous economy-wide productivity A, such that:

1) Given $p(\Psi)$, $p_r(\Psi)$, $\omega(\Psi)$, $\{q_i(\Psi)\}_i$, $r_f(\Psi)$, $r_m(\Psi)$, and $\pi_h(\Psi)$, $V(o, z, e, a, m, \kappa; \Psi)$ solves the household's problem, the policy functions $g_j(o, z, e, a, m, \kappa; \Psi)$ $j \in \{k, c, a', m', \kappa'\}$ are optimal, and $\mathbb{1}(o, z, e, a, m, \kappa; \Psi)$ indicates that for a given distribution Ψ, households with state variables (o, z, e, a, m, κ) optimally choose (k, a', m', κ').

2) Given $\omega(\Psi)$, $\{q_{x_i}(\Psi)\}_i$, and A, $L(\Psi)$ and $\{x_i(\Psi)\}_i$ solve the firm's problem.

3) Given the tax system and the optimal decisions in (1) and (2) above, the government satisfies its net revenue constraint, G_n.

4) Given $p(\Psi)$ and $TC_h(I_h)$, $I_h(\Psi)$ solves the construction company's optimization problem.

5) Markets clear: i) $\omega(\Psi)$ clears the labor market: $\int_\psi \zeta_h e d\psi = L(\Psi)$

ii) $\{q_{x_i}(\Psi)\}_i$ clear the intermediate outputs market: $\int_\psi g_k(o,z,e,a,m,\kappa;\Psi)zd\psi = \int_i x_i(\Psi)di$
iii) $p(\Psi)$ clears the housing market: $\int_\psi (g_\kappa(o,z,e,a,m,\kappa;\Psi)d\psi\bar{h} + (1-g_\kappa(o,z,e,a,m,\kappa;\Psi))d\psi\underline{h}) = H^s(\Psi)$.
iv) Residential construction sector clears: $\int_\psi (\kappa d\psi p\delta_h\bar{h} + (1-g_\kappa(o,z,e,a,m,\kappa;\Psi))d\psi p\delta_h\underline{h}) = TC(I_h) + \Pi_h(\Psi)$
v) $p_r(\Psi)$ satisfies the rental company's problem (equation 1 on page 10).
vi) $r_f(\Psi)$ clears the capital market: $\int_\psi (g_k(o,z,e,a,m,\kappa;\Psi) - a)d\psi = 0$
vii) The goods market clears: $\int_\psi g_c(o,z,e,a,m,\kappa;\Psi)d\psi + \int_\psi g_{a'}(o,z,e,a,m,\kappa;\Psi)d\psi + \int_\psi g_{\kappa'}(o,z,e,a,m,0;\Psi)d\psi(1+p\chi_b\bar{h}) + \int_\psi g_{\kappa'}(o,z,e,a,m,1;\Psi)d\psi p\bar{h} + \int_\psi (1-g_{\kappa'}(o,z,e,a,m,1;\Psi))d\psi p\chi_s\bar{h} + \int_\psi (1+r_f(\Psi))md\psi + TC(I_h) + G_g = Y(\Psi) + \Pi_h(\Psi) + \int_\psi g_{m'}(o,z,e,a,m,\kappa;\Psi))d\psi$

6) Consistency:
i) $\psi(o',z',e',a',m',\kappa') = \int_\psi \pi_{ee'}\mathbb{1}(o,z,e,a,m,\kappa;\Psi)d\psi \quad \forall o,e,a,m,\kappa$

IV Calibration

In this section I choose the values of the model's parameters. First, I choose certain parameters externally, without solving the equilibrium of the model. I separate the externally calibrated parameters into non-housing market parameters and housing market parameters. Second, given the externally chosen parameters' values, I choose other parameters' values internally, so that the model's stationary equilibrium generates certain statistics that are aligned with the data.

External Calibration of Non-Housing Market Parameters:

J is set equal to 81 to have households live from age 20 to age 100, inclusive. I pick R equal to 35 to have households start retirement at age 66. Survival probabilities $\{\phi_o\}_{o=1}^J$ are chosen to equal to the ones provided in the article "United States Life Tables, 2010" (Arias (2014)), published in the National Vital Statistics Report, which is produced by the U.S. Department of Health and Human Services. I follow the standard in the literature and pick the risk aversion parameter σ_c, to equal 2. Following Huggett (1993), I set full employment $e_{hi} = 1$, and unemployment $e_{lo} = 0.1$. The averages of Shimer's (2005) estimates of the annual job separation rate and job finding rate for 1952-2004 are used to set $\pi_{e_{hi,hi}} = 0.86$ and $\pi_{e_{lo,lo}} = 0.034$. I choose the unconditional probability of employment at birth $\pi_{1_{hi}} = 0.916$ to reflect the fact that, according to the Bureau of Labor Statistics (BLS), in 2016, 91.6% of labor force persons aged 20-24 were employed. To calibrate Φ, I follow the U.S. social security benefits system. According the Social Security Administration website, social security benefits replace 40% of a person's average labor income during their work period. Thus, I set $\Phi = 0.4$. I use McDaniel's (2007) estimates for the U.S. of the average labor income tax rate, average non-durable consumption tax rate, and average capital income tax rate, and set $\tau_l = 0.224$, $\tau_c = 0.075$, and $\tau_k = 0.25$. ν, which affects the curvature of the intermediate goods production function, is chosen to equal 0.95 (see footnote for more discussion of this parameter)[13]. I pick the depreciation rate of productive capital δ_a, to equal to 0.05 as in

[13] The reason for this choice is the following: Guvenen et al. (2017) show that to match the Pareto tail of the wealth distribution, ν needs to be larger than 0.9. Furthermore, $\nu = 0.95$ generates a risk-free interest rate closer to the data

Guvenen et al. (2017). I set the persistency of investment ability ρ_z, equal to 0.1. This choice is consistent with the estimate of Fagereng et al. (2016) of the inter-generational persistency of the permanent component of investment ability. I normalize the economy-wide productivity $A = 1$, and consistent with Guvenen et al. (2017), I set $\alpha = 0.6$ so that the share of labor income in production is 60%. I show the parameters and their values in table 1 on page 14.

Table 1: External Calibration of Non-Housing Market Parameters

Variable	Value	Description
J	81	100 Years of Life
R	35	Retirement Starting at Age 66
$\{\phi_o\}_{o=1}^{J}$	Varies	Probability of Death (United States Life Tables)
σ_c	2	Risk Aversion (Standard)
e_{hi}	1	Full Employment (Huggett (1993))
e_{lo}	0.1	Unemployment (Huggett (1993))
$\pi_{e_{hi,hi}}$	0.86	U.S. Average Annual Separation Rate (Shimer (2005))
$\pi_{e_{lo,lo}}$	0.034	U.S. Average Annual Job Finding Rate (Shimer (2005))
$\pi_{1_{hi}}$	0.916	U.S. Employment Rate of Workforce Persons Aged 20-24 (BLS)
Φ	0.4	U.S. Social Security Benetfits as Fraction of Average Wage (Social Security Administration)
τ_l	0.224	Labor Income Tax Rate (McDaniel (2007))
τ_c	0.075	Non-Durable Consumption Tax Rate (McDaniel (2007))
τ_k	0.25	Capital Income Tax Rate (McDaniel (2007))
ν	0.95	Curvature of Intermediate Goods Production (Guvenen et al. (2017))
δ_a	0.05	Productive Capital Depreciation Rate (Guvenen et al. (2017))
ρ_z	0.1	Persistency of Inter-Generational Investment Ability (Fagereng et al. (2016))
A	1	Aggregate Productivity (Normalization)
α	0.6	Share of Labor Income in Output (Guvenen et al. (2017))

External Calibration of Housing Market Parameters:

I normalize the owner-occupied unit size to 1, i.e. $\bar{h} = 1$. $\lambda_p = 0.2$ so that the minimum down-payment is 20%. I follow Sommer et al. (2013) and set the interest rate premium on mortgage debt $\Delta r_m = 0.015$. I choose the deductible portion of mortgage interest payments from taxable income $\delta_m = 1$, to be in line with the U.S. tax code. Consistent with the estimates in Gruber, & Martin (2003), I set the cost of selling a house $p\chi_s = 0.07p$ and the cost of buying a house $p\chi_b = 0.025p$. Consistent with estimates in Harding, Rosenthal, & Sirmans (2007) for the depreciation rate of housing, I set $\delta_h = 0.02$. Lastly, I choose ϵ_p equal to 0.08, so that the calibrated model's housing stock elasticity is consistent with the long-run U.S. housing stock price elasticity estimated by Mayer, & Somerville (2000). I present these variables in table 2 on page 15.

Internal Calibration:

My internal calibration is performed for the stationary equilibrium[14]. I detail the solution

than $\nu = 0.9$. Hence, $\nu = 0.95$ is a reasonable choice for my model.

[14]In my calibration I allow households to save up to a multiple of 60,000 of before-tax annual labor income of an employed household, which gives an upper bound on savings that is equivalent to about 4.5 billion dollars in 2016.

Table 2: External Calibration of Housing Market Parameters

Variable	Value	Description
$\bar{h}$	1	Owner-Occupied Unit Size (Normalization)
λ_p	0.2	Minimum Downpayment (Data)
Δr_m	0.015	Mortgage Rate Premium (Sommer et al. (2013))
δ_m	1	Mortgage Deductibility (U.S. Tax Code)
χ_s	0.07	Cost of Selling (Gruber, & Martin (2003))
χ_b	0.025	Cost of Buying (Gruber, & Martin (2003))
δ_h	0.02	Housing Depreciation (Harding et al. (2007))
ϵ_p	0.08	Price Elasticity of Housing Stock (Mayer, & Somerville (2000))

method in the appendix on page 23. The calibrated parameters, their values, and statistics targeted are shown at the top of table 3 on page 16. I pick $\lambda_a = 0.72$ to generate a debt-to-assets ratio of 0.31 (this debt does not include mortgages). This is consistent with the statistic estimated by Asker, Farre-Mensa, & Ljungqvist (2011) for private firms' debt-to-assets ratio[15]. I set $\beta = 0.986$ so that the ratio of net wealth to GDP in the model equals the standard 3. To calibrate the next set of the parameters, I use averages of the statistics of interest from the 1989-2013 Surveys of Consumer Finances. I choose $\sigma_{\epsilon_z} = 0.56$ to have the top 1% of the wealth distribution own 33% of all the net wealth[16]. Next, I jointly choose the size of the rental unit $\underline{h}$, the rental price to housing price ratio $\frac{p_r}{p}$, and the share of housing consumption in the utility function γ, to target a net housing wealth to net wealth ratio of 0.23, a 66% home-ownership rate, and a 95% home-ownership rate at the top 10% of the wealth distribution. The values of these parameters are $\underline{h} = 0.833$, $\frac{p_r}{p} = 0.028$, and $\gamma = 0.1$[17]. Lastly, to pin down cut-off housing investment I_h^c (above which the marginal cost of housing investment is increasing), I use the fact that when $I_h = I_h^c$, $c_{\text{min}} = (I_h)^{\epsilon_p}$. I choose the ratio of the cut-off housing investment to actual investment $\frac{I_h^c}{I_h}$, to have the implicit value of land in the model equal to 31% of the house price[18]. This is consistent with the estimates in Case (2007), which show that in the U.S., between 1995-2005, on average, the ratio of total land value of all residential real estate to total value of all residential real estate was approximately 0.31[19]. The value of I_h^c implies that the minimum marginal cost of construction c_{min}, is 68.1% of the equilibrium housing price.

To learn about the impact of housing on the calibrated model's parameters, I shut down the housing market and re-calibrate the model. I show the values of the borrowing limit λ_a, discount factor β, and variability of investment ability σ_{ϵ_z}, in the calibrated model without housing, at

[15]Asker et al. (2011) estimate that public firms' debt-to-assets ratio is 0.2. This implies that my model allows generous borrowing, which means I will be estimating a lower bound for welfare gains from wealth taxation.

[16]In the model there are 9 investment abilities. I approximate the log-normal investment ability transmission process using the Tauchen method.

[17]Given the choices of the rental price p_r, and the equilibrium housing price p, c_m is chosen to keep equation 1 on page 10 true.

[18]The value of land is the value of the housing investment less the cost of construction, divided by the cost of construction.

[19]During that period, on average, the ratio of total land value of all owner-occupied housing to total value of all owner-occupied housing was very similar, equaling 0.32.

Table 3: Internal Calibration of Model with Housing and Model without Housing

Variable	Value	Target	Model	Data
With Housing				
λ_a	0.72	Debt to Assets Ratio	0.31	0.31 (Asker et al. (2011))
β	0.986	Net Wealth To GDP Ratio	3	3 (Standard)
σ_{ϵ_z}	0.56	Fraction of Wealth Owned by Top 1%	0.33	0.33 (SCF)
$\underline{h}$	0.833	Home-ownership Rate of Top 10%	0.94	0.95 (SCF)
$\frac{p_r}{p}$	0.028	Owner-Occupied Net Housing Wealth to Net Wealth	0.23	0.23 (SCF)
γ	0.1	66% Home-ownership Rate	0.66	0.66 (SCF)
$\frac{I_h^c}{I_h}$	0.97	Land Value to Housing Value	0.31	0.31 (Case (2007))
Without Housing				
λ_a	0.49	Debt to Assets Ratio	0.31	0.31 (Asker et al. (2011))
β	0.989	Net Wealth To GDP Ratio	3	3 (Standard)
σ_{ϵ_z}	0.541	Fraction of Wealth Owned by Top 1%	0.33	0.33 (SCF)

the bottom of table 3 on page 16[20]. Notice that when I deactivate the housing market, both the fraction a household can borrow against their net wealth and the value of the variation in investment-ability parameter are lower, but the value of the discount factor is larger.

The value of λ_a is larger when the housing market is included in the model because the ratio of capital to GDP is lower relative to when the housing market is not included. Therefore, to have the same ratio of aggregate capital debt to aggregate assets, households must be less borrowing constrained.

To explain why the value of σ_{ϵ_z} is larger in the model with housing note that when housing is present, the dispersion in wealth comes from both the dispersion in net housing wealth and the dispersion in capital wealth. Since net housing wealth is more equally distributed along the wealth distribution than capital wealth, more dispersion in capital is required to generate the observed fraction of wealth held by the top 1%. Thus, in the model with housing there is larger variability in investment abilities.

To understand why the value of β is lower when housing is added into the model note that households have five savings motives: a precautionary motive, a production motive, a rent motive, a home-ownership motive, and a motive to receive greater dividends from the construction company. The precautionary motive is present in both models. It propels households to save capital to smooth non-durable consumption during unemployment spells and during retirement. The production motive is also present in both models. It induces households to save capital in order to increase their future capital income, and thus their future non-durable consumption. The rent and home-ownership motives are only present when housing is added into the model. The rent motive induces households to accumulate more capital so that they can afford more non-durable consumption once rent is paid. The home-ownership motive drives households to accumulate capital in order to buy a house, as home-owning generates greater utility flow from non-durable consumption than renting does. Home-owning is further desired because it enables households to take on mortgage debt and increase their after-tax labor income due to mortgage debt payments being tax deductible. The motive to accumulate capital in order to receive more of the construction company's profits is also only present in the model with housing. The last

[20] All other parameters are unchanged.

three motives discussed account for the lower calibrated value of the discount factor in the model with housing relative its value in the calibrated model without housing.

Table 4: Statistics Not Targeted in Model with Housing and Model without Housing

Statistic	Housing	No Housing	Data (from SCF)
Risk-Free Rate	0.45%	-0.72%	0.8% (Mehra, & Prescott (1993))
Average Before-Tax Return	7.04%	5.56%	7.06% (Piketty (2014))
Capital Income Tax Revenue as % of Government Revenues	19%	19%	27% (OECD (2011))
Gini Coefficient	0.63	0.73	0.82
Fraction of Net Wealth Owned by Top 10%	0.56	0.62	0.70
Fraction of Net Wealth Owned by Top 20%	0.66	0.73	0.82
Fraction of Net Wealth Owned by Bottom 50%	0.14	0.08	0.03
Home-ownership Rate of Top 1%	0.94	N/A	0.96
Home-ownership Rate of Top 20%	0.85	N/A	0.83
Home-ownership Rate of Bottom 50%	0.51	N/A	0.40
Home Secured Debt to Gross Owner-Occupied Housing Wealth	0.21	N/A	0.35
Fraction of Homeowners With Home Secured Debt	0.40	N/A	0.66
Net Housing Wealth Fraction in Wealth of Top 1%	0.01	N/A	0.07
Net Housing Wealth Fraction in Wealth of Top 10%	0.05	N/A	0.14
Net Housing Wealth Fraction in Wealth of Top 20%	0.09	N/A	0.17
Net Housing Wealth Fraction in Wealth of Bottom 50%	0.58	N/A	0.52

Statistics Not Targeted:

In table 4 on page 17 I summarize the performance of the model with housing, relative to the model without housing, and relative to the data, on dimensions not targeted. As you can see, the model with housing is better at predicting the risk-free rate and the average returns to capital. In contrast, the model without housing is better at predicting wealth inequality. That aside, the main takeaway message from table 4 is that although the model with housing has limitations, it quite reasonably mimics the wealth distribution, the portfolio composition, and home-ownership rates across the different percentiles of the wealth distribution.

The difference in the risk-free rate and average before-tax returns between the two models is due to the following reasons: whereas in the model without housing all wealth is productive, in the model with housing, capital only accounts for 77% of all net wealth. Therefore, there is more capital in the model without housing than in the model with housing. Given that productive capital exhibits diminishing returns, it generates higher returns in the model with housing. Furthermore, due to the higher returns on capital in the model with housing, it is in higher demand, causing the risk-free rate to rise.

In contrast, the model with housing under-predicts wealth inequality because housing depresses wealth inequality as it is more equally distributed than capital.

There are some dimensions on which only the model with housing can be evaluated against the data. It does reasonably well at predicting the home-ownership rates at the top 1%, 20%, and bottom 50% of the wealth distribution. However, the model does less well at predicting the ratio of home secured debt to gross owner-occupied housing wealth and the percentage of homeowners with mortgage debt. This is because in my model there are not enough incentives for households to take on mortgage debt and in larger amounts. Adding multiple home sizes to the model could help match these statistics better, but such addition comes with high computational costs.

The model also falls short in generating equivalent statistics for the fraction of net housing wealth in total wealth owned by the top percentiles of the wealth distribution. My model predicts a lower fraction of net housing wealth in net wealth owned by the top 1%, 10% and 20%. The reason my calibrated model is unable to get these statistics more accurately is that households cannot purchase very large homes in my model while in reality they can[21]. This implies that their net housing wealth must be a smaller fraction of their net wealth relative to the data.

V Replacing Capital Income Taxation with Wealth Taxation

In this section I run counter-factual simulations in which I examine the effects of different capital taxation systems on the economy in the stationary equilibrium. I begin by looking at a tax system where only capital taxation is used. I then search for the optimal combination of capital and housing tax rates. In my counter-factual simulations, the rental price is found by satisfying equation 1 on page 10. In all simulations of all versions, I keep labor income tax rate, non-durable consumption tax rate, management costs, minimum cost of construction, and cut-off housing investment at their benchmark levels.

In each simulation, I pick the capital tax rate τ_a (and set $\tau_k = \tau_a$) in order to generate the same net government revenues G_n, as in the respective benchmark economy.

I Capital Taxation Only:

Model without Housing:

In order to understand the basic mechanism through which wealth taxation increases welfare, I start by studying a version of my model without housing.

In this version, the capital tax rate that satisfies G_n is 0.95%. It generates a 2.78% increase in welfare for the youngest cohort and a 0.01% increase in welfare for the average household. It also increases both GDP and the wage rate by 7.37%, even as aggregate capital in the economy decreases by 1.36%. Furthermore, the Gini coefficient increases by 6.7%. These results are shown in second column from the left in table 5 on page 19.

Capital taxation achieves welfare gains by shifting the tax burden from productive investor-households to unproductive investor-households. This enables productive investor-households to accumulate more wealth and erodes the wealth of wealthy unproductive investor-households, which causes aggregate capital in the model economy to decrease and wealth inequality to rise (the Gini coefficient goes up from 0.73 to 0.77). However, the re-allocation of capital from

[21] In the data, for example, in 2012, the average homeowner at the top 1% of the wealth distribution had a principle residence worth $1,740,722 (16.4% of which was HSD), while the average homeowner at the bottom 50% of the wealth distribution had a principle residence worth $108,863 (85.5% of which was HSD). These numbers indicate that home-owners at the top 1% of the wealth distribution have houses that are 16 times more expensive than those owned by homeowners at the bottom 50% of the wealth distribution. To generate such large differences more heterogeneity in house size is needed. This, of course, would come at high computational costs.

Table 5: Effects of Pure Capital Taxation in Model without Housing and in Model with Housing (Compared to Benchmark)

Variable	No Housing	Housing
Capital Tax Rate	0.95%	1.08%
	%Δ	%Δ
Average Welfare of Youngest	+2.78	+3.58
Average Welfare	+0.01	+0.96
GDP	+7.37	+9.20
Wage	+7.37	+8.05
Gini Coefficient	+6.70	+6.26
Aggregate Capital	-1.36	-1.41
Effective Aggregate Capital $((\int_i x_i^{\nu})^{\frac{1}{\nu}})$	+19.45	+21.37
Housing Price	N/A	+19.50
$\frac{\Delta \text{Rental Price}}{\text{Wage}_{\text{Bench}}}$	N/A	+0.29
Home-ownership Rate	N/A	+12.06
$\frac{\Delta \text{Construction Sector Profits}}{\text{GDP}_{\text{Bench}}}$	N/A	+0.58

unproductive to productive investor-households means that capital is more efficiently used, which raises effective aggregate capital. The latter generates positive aggregate outcomes for GDP, the wage rate, and ultimately, welfare.

Average welfare gains are lower than average welfare gains of the youngest cohort due to the following reason: the youngest cohort of households is born with little wealth (wealth inherited from households who died). As such, their real life-time tax liability is low relative to older cohorts, who have had some time to accumulate wealth over their life-cycle. The youngest cohort also benefits from higher real life-time wages compared to older cohorts who are in or close to retirement. As such, younger cohorts experience larger benefits and smaller costs relative to older cohorts, resulting in higher welfare gains.

Table 6 on page 20 shows how welfare gains are distributed across age and productivity. z_1 denotes the lowest productivity and z_9 denotes the highest productivity. The reform benefits highly productive investor-households since it increases their wage rate and lessens their tax liability. The reform also benefits low productivity households which occupy the bottom of the wealth distribution. The higher wage rate more than compensates them for the introduction of the wealth tax, and thus, their welfare rises too. In contrast, the reform does not benefit investor-households which are in the middle of the productivity distribution to the same extent as it does low and high productivity households. Households in the middle of the productivity distribution tend to accumulate moderate amounts of capital for production. However, returns on their wealth are not high enough to compensate them for their tax liability following the reform. Thus, their wealth erodes and they experience milder, and at times even negative, welfare gains. Unproductive investor-households who inherit large amounts of wealth also lose from this reform, because the increase in their tax liability is greater than the increase in their wage rate. Welfare gains are higher for younger households relative to older households. This is because, as discussed, younger households enjoy a higher real life-time income and have low real life-time tax liability since they have little wealth. The opposite is true for older households, who are generally wealthy, and experience milder increases in their real life-time income.

Table 6: Percentage Change in Welfare by Age and Productivity in Model with No Housing Market

Prod. \ Ages	$20-29$	$30-39$	$40-49$	$50-59$	$60-69$	$70-79$	$80-89$	$90-100$
z_1 (lowest)	+3.1%	+2.3%	+1.4%	+0.1%	-1.6%	-2.0%	-0.7%	+0.0%
z_2	+3.1%	+2.2%	+1.3%	+0.0%	-1.6%	-2.1%	-0.7%	+0.0%
z_3	+3.0%	+2.2%	+1.3%	+0.0%	-1.7%	-2.1%	-0.7%	-0.1%
z_4	+2.9%	+2.1%	+1.2%	-0.1%	-1.8%	-2.2%	-0.7%	-0.1%
z_5	+2.7%	+1.9%	+1.0%	-0.3%	-2.0%	-2.5%	-0.9%	-0.1%
z_6	+0.5%	-1.8%	-4.3%	-6.8%	-9.3%	-11.6%	-10.7%	-4.5%
z_7	+7.6%	+8.8%	+9.5%	+10.0%	+10.4%	+10.4%	+10.9%	+12.4%
z_8	+17.2%	+26.4%	+33.9%	+39.8%	+44.0%	+47.3%	+50.3%	+52.1%
z_9 (highest)	+22.2%	+40.6%	+50.1%	+56.8%	+60.6%	+59.2%	+46.4%	+37.1%

Model with Housing:

Next, I introduce housing into the model and examine how it interacts with a pure capital tax. As in the model without housing, I replace capital income taxation with pure capital taxation. I keep $\tau_h = 0$, set $\tau_k = \tau_a$, and then pick $\tau_a = 0.0108$ to generate the same net government revenues G_n, as in the benchmark. Results are shown in the third column from the left in table 5 on page 19.

The introduction of a pure capital tax increases GDP by 9.2% and the wage rate by 8.05%, even though aggregate capital decreases by 1.41%. Furthermore, it causes the price of housing to increase by a 19.5%, the rental price to rise by an amount equivalent to 0.29% of the benchmark wage, the Gini coefficient to rise by 6.26%, the average welfare of the youngest cohort to rise by 3.58%, and average welfare to go up by 0.96%.

The same mechanisms that increase GDP, the wage rate, and welfare in the model without housing also increase them in the model with housing. However, there are two main differences between the model without housing and the model with housing. First, since in the model with housing households can divert resources to housing in response to the introduction of the pure wealth tax, housing prices increase, construction sector profits rise, rental prices increase, but aggregate capital declines slightly more substantially. Second, GDP, wages, and welfare rise by more than they do in the model without housing even though capital declines more in the model with housing. This is because in the model with housing capital 30% less abundant. As such, in the model with housing, the wealth tax corrects a more severe inefficiency in the allocation of capital.

II Optimal Wealth Taxation:

To find the optimal wealth tax system, I fix τ_h, and choose τ_a to generate net government revenues as in the benchmark economy. As mentioned earlier, the optimal system is the one that maximizes the ex-ante expected welfare of a newly born household.

As you can see in table 7 on page 21, the optimal tax on housing is 11% and it is optimal to subsidize capital at a rate of 1.94%.

Table 7: Different Combinations of Capital and Housing Tax Rates (Compared to Benchmark)

τ_h	τ_a	$\%\Delta p$	%ΔHome-ownership Rate	$\frac{\Delta \pi_h}{\text{GDP}_{\text{Bench}}}$	$\frac{\Delta p_r}{\omega_{\text{Bench}}}$	$\%\Delta\omega$	%ΔWelfare
0%	1.08%	+19.5	+12.1	+0.58	+0.3	+8.05	+3.58
1%	0.30%	+3.3	+2.3	+0.14	+0.8	+10.77	+5.14
2%	-0.17%	-6.9	-5.1	-0.14	+1.3	+12.86	+5.92
5%	-0.82%	-31.9	-55.9	-0.81	+2.3	+17.00	+7.33
10%	-1.77%	-31.9	-62.5	-0.81	+6.8	+23.44	+7.68
11%	**-1.94%**	**-31.9**	**-50.0**	**-0.81**	**+7.8**	**+24.78**	**+7.84**
12%	-2.07%	-31.9	-50.2	-0.81	+8.7	+25.85	+7.83
15%	-2.40%	-31.9	-47.9	-0.81	+11.6	+28.92	+7.52
20%	-2.81%	-31.9	-46.6	-0.81	+16.6	32.56	+5.62

The intuition for the result is as follows: the tax on housing generates government revenues, and thus, enables the government to reduce the tax on capital. The costs and benefits can be described as follows: on the one hand, the housing tax increases housing costs across the board and causes some households to become renters and live in smaller units. On the other hand, the lower capital tax mostly encourages the more productive households accumulate more of capital and increase their capital investments. This, in turn, raises wages since wages are positively linked to productivity of the owners of the capital stock. The more productive households in the economy are responsive to the tax on capital because they can generate high returns on capital investments. However, because markets are incomplete they must accumulate more capital if they wish to increase their future income from capital investments. Therefore, the subsidy for capital increases their future income from their capital investments, and thus, they save more. Quantitatively, the wages rise more rapidly then housing costs, and thus, overall welfare rises.

The tax scheme generates winners and losers. The housing tax hurts every household in the economy. First, it causes some households in the middle and the bottom of the wealth distribution to become renters. Since rental prices go up, these households lose both from living in smaller houses and from having to pay higher rents. The tax on housing also increases the cost of home-owning. This cost is mostly imposed on households who occupy the top of the wealth distribution, since they are mostly the ones who remain home-owners. Furthermore, since in my model most of the construction sector profits end up going to the top of the wealth distribution, the reduced profitability in that sector hurts mostly the top of the wealth distribution as well. In contrast, the subsidy on capital benefits almost everyone. Since it increases the accumulation of productive capital, wages increase. This benefit is quite high and it is accrued to most of the economy (other than the unemployed and retired households). Since the top of the wealth distribution holds most of their portfolio in capital, the subsidy benefits them greatly. Moreover, since households at the bottom of the wealth distribution switch to renting, their portfolio is geared more heavily towards capital also. As a result the subsidy on capital benefits them too, although less so. The main losers in this economy are retired households who are at the bottom of the wealth distribution. These households are renters and do not benefit from the higher wages or the capital subsidy. In contrast, they incur the cost of higher rents, making them worse off. This is evident in table 8 on page 22.

Table 8: Percentage Change in Welfare by Productivity and Age with Optimal Tax Scheme

z \ Age	$20-29$	$30-39$	$40-49$	$50-59$	$60-69$	$70-79$	$80-89$	$90-100$
z_1	+4.6%	+2.2%	-0.7%	-4.3%	-8.2%	-12.3%	-17.9%	-22.3%
z_2	+4.8%	+2.3%	-0.6%	-4.2%	-8.1%	-12.2%	-17.8%	-22.3%
z_3	+4.9%	+2.3%	-0.5%	-4.1%	-8.0%	-12.0%	-17.7%	-22.3%
z_4	+5.0%	+2.4%	-0.4%	-4.0%	-7.9%	-11.9%	-17.6%	-22.3%
z_5	+5.2%	+2.6%	-0.3%	-3.8%	-7.7%	-11.7%	-17.3%	-22.2%
z_6	+14.5%	+15.7%	+17.1%	+18.4%	+19.4%	+19.1%	+14.7%	-7.1%
z_7	+22.4%	+24.8%	+27.0%	+29.0%	+31.0%	+32.1%	+34.3%	+35.6%
z_8	+22.6%	+25.4%	+27.8%	+29.5%	+31.0%	+32.3%	+33.6%	+34.8%
z_9	+23.7%	+25.6%	+26.8%	+27.8%	+28.1%	+25.5%	+20.2%	+19.6%

VI Conclusion

To the best of my knowledge, this is the first paper to study how different types of wealth should be taxed in a model with credit market incompleteness and rate-of-return heterogeneity. I divide wealth into two types: capital and housing. Due to credit market incompleteness and rate-of-return heterogeneity, returns on capital are tightly linked to its owner's ability to invest it. In contrast, the returns on housing are independent of ownership. The model also includes labor income risk, imperfect inter-generational transmission of investment ability, and a housing market. The model is calibrated to U.S. data from the Survey of Consumer Finances and can reasonably replicate both the empirical distribution wealth and the portfolio allocation along the wealth distribution. To my knowledge, this is the first paper to do so with a housing market.

I find that it is optimal to tax housing at a rate of 11% and to subsidized capital at a rate of 2%. On the one hand, the higher housing tax raises housing costs across the board because the housing demand is inelastic. On the other hand, the lower capital tax mostly encourages households with higher investment skills to accumulate more capital and increase their capital investments. This is because their returns on capital investments are high, but since they face incomplete markets they must accumulate more capital in order to increase their future income from capital investments. This in turn, raises wages paid to labor because wages are positively related to productivity. Since wages rise more rapidly than housing costs, overall welfare rises. My results suggest that welfare can most likely be further enhanced if housing were to be taxed and capital income were to be subsidized rather than capital. This is because capital income is a more accurate measure of productivity than capital is.

As a result of the optimal tax scheme, productive households accumulate more capital. This increase wages by 25%, which is the main source through which welfare gains are achieved. The housing tax reduces the home-ownership rate from 66% to 33%, increases rents by an amount equivalent to 7.7% of the benchmark wage, and reduces construction sector profits. These costs, however, are small relative to the significant benefits accrued to the whole economy through higher wages. This tax scheme raises the average welfare of all newly born households regardless of their productivity type. The main losers from this scheme are retired households with little capital. These households do not benefit from the higher wages or the subsidy provided for capital. However, they end up having to either pay a higher housing tax or higher rents.

A Solution Algorithm

To solve for the stationary equilibrium in the model with no housing I go through the following steps:

1. I generate a grid for capital with N_1 points. Each point corresponds to a multiple of before-tax labor income of an employed household. The grid has more points placed at lower levels of wealth than at higher levels of wealth.

2. Using the regular value function backward iteration method I find the optimal capital choice given the N_1 grid points. Denote the index of the optimal capital choice by x^*.

3. Given x^* from the previous step, I generate an internal grid of size N_2 between $x^* - 1$ and $x^* + 1$. Let grid points on this grid be denoted by y. Furthermore, if the capital value associate with a point y is lower than the capital value associated with x^* then denote the point y as y^{-1}. If the capital value associate with a point y is lower than the capital value associated with x^* then denote the point y as y^{+1}

4. For each feasible y^{-1}, evaluate the utility generated from it. The utility generated by y^{-1} is the utility flow from the choice of y^{-1} and a weighted average between the value function associated with x^* and the value function associated with $x^* - 1$. The weight given to the value function associated with $x^* - 1$ is distance between the capital value associated with x^* and the capital value associated with y^{-1} divided by the distance between the capital value associated with x^* and the capital value associated with $x^* - 1$. The weight given to the value function associated with x^* is 1 minus the weight given to the value function associated with $x^* - 1$. I do the same procedure for y^{+1}. I pick the point y that maximizes utility. Denote the optimal y by y^*.

5. To converge on the stationary distribution I do the following:

a. If capital value associated with y^* is below the capital value associated with x^*, then I move a fraction of households to the density where x^* is the optimal choice and a fraction of households to the density where $x^* - 1$ is the optimal choice. The fraction moved to the density where x^* is the optimal choice is the weight given to the value function associated with x^* and the fraction of households moved to the density where $x^* - 1$ is the optimal choice is the weight given to the value function associated with $x^* - 1$.

b. If capital value associated with y^* is above the capital value associated with x^*, then I move a fraction of households to the density where x^* is the optimal choice and a fraction of households to the density where $x^* + 1$ is the optimal choice. The fraction moved to the density where x^* is the optimal choice is the weight given to the value function associated with x^* and the fraction of households moved to the density where $x^* + 1$ is the optimal choice is the weight given to the value function associated with $x^* - 1$.

To solve for the stationary equilibrium in the model with housing I follow similar steps to the ones above but for higher dimensions. In particular, I generate internal grids for both capital and mortgage debt. The weights used for the value functions and fractions that are moving to different densities are calculated using the relative vector lengths of each of the internal grid points from x^*.

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