Deserving Poor and the Desirability of a Minimum Wage ♣

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This paper provides a novel justification for using a minimum wage to supplement an optimal tax-and-transfer system. We demonstrate that if labor supply decisions are concentrated along the intensive margin and employment is efficiently rationed, a minimum wage can be socially beneficial by serving as a screening device that targets benefits to the deserving poor. We also show that with a minimum wage in place, a negative marginal tax rate may not be optimal.

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1. Introduction

Minimum wages are used in most OECD countries as a redistributive tool for the benefit of low-skilled workers. However, they are highly controversial due to their adverse effect on employment, the magnitude of which has been the subject of intense empirical debate.[1] Furthermore, the possibility of levying a negative marginal tax rate (e.g., the Earned Income Tax Credit in the US) as part of an optimal tax-and-transfer system raises a fundamental normative question regarding the social desirability of a minimum wage as a redistributive tool.

Only a small strand of the literature has investigated whether a minimum wage can be a desirable supplement to an optimal tax-and-transfer system in a competitive labor market environment.[2] The early studies of Allen (1987) and Guesnerie and Roberts (1987) focus on the intensive-margin choice of working hours and assume that a minimum wage results in an involuntary reduction in working hours of the low-skilled workers. They conclude that the minimum wage cannot be a useful supplement to an optimal tax-and-transfer system. However, other papers have questioned this conclusion. In particular, Boadway and Cuff (2001), also employing an intensive-margin setting, assume that a minimum wage results in involuntary unemployment. They demonstrate that a minimum wage can serve to distinguish between involuntarily and voluntarily unemployed workers

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[1] See Neumark and Wascher (2007) for a survey. The federal minimum wage in the US has been $7.25 per hour since July 2009 (reflecting an increase of 40 percent over the years 2007-2009). Some states and cities have set minimum wages exceeding the federal level, the highest being $9.32 per hour in the state of Washington (as of January 2014) and $15.00 in the city of Seattle (not yet fully implemented).

[2] The literature also considers the efficiency-enhancing role of a minimum wage in the presence of labor market imperfections such as monopsonistic competition [Manning (2003), Cahuc and Laroque (2014)]; efficiency wages [Jones (1987), Rebitzer and Taylor (1995)]; bargaining models [Cahuc et al. (2001)]; signaling models [Lang (1987), Blumkin and Sadka (2005)]; and search models [Flinn (2006), Hungerbühler and Lehmann (2009)].

and find that this would make a minimum wage a warranted supplement to an optimal tax-and-transfer system. More recently, Danziger and Danziger (in press) show that in an intensive-margin setting a Pareto improvement can be achieved by supplementing an optimal tax-and-transfer system with a graduated (rather than a constant) minimum wage. Finally, Lee and Saez (2012), focusing on the extensive-margin choice in an occupational-choice model with fixed working hours, show that if rationing is efficient, namely, the involuntary unemployment triggered by a minimum wage will hit the workers with the strongest taste for leisure first, then a minimum wage can serve as a desirable supplement to an optimal tax-and-transfer system.[3]

In this paper, we offer a novel justification for the use of a minimum wage to supplement an optimal tax-and-transfer system. Central to our argument is the distinction between the deserving and the undeserving poor, where the former refers to individuals who are either willing to work hard or who are truly disabled, and the latter to individuals who are perceived to be lazy. In our model, workers differ in their earning abilities and work-leisure preferences, and make choices along the intensive margin.[4] It is assumed that the government maximizes a social welfare function that exhibits a bias against the undeserving poor and, furthermore, that employment is rationed efficiently. An important implication is that the extra transfers offered by the government to the low-skilled

[3] The above literature, by and large, assumes that the skill distribution is given. However, the tax-and-transfer system and the minimum wage may affect human capital formation and thereby the skill distribution. For a recent analysis of the redistributive role of minimum wage as a supplement to the tax-and-transfer system in the presence of endogenous human capital formation, see Gerritsen and Jacobs (2014).

[4] With fixed working hours, there is no difference between wage and income. Hence, a minimum wage can be replicated by taxing all incomes below a certain threshold at a confiscatory 100 percent rate. An intensive-margin model, in contrast, captures the difference between wage and income, and therefore provides a more natural framework for examining the social desirability of a minimum wage.

workers will be targeted toward the deserving poor rather than being accorded to all the poor across the board. By relying on the screening of workers through the efficient rationing of employment, the government overcomes its inability to identify the deserving poor directly. Consequently, if the government is sufficiently biased against the undeserving poor, a minimum wage becomes a desirable supplement to an optimal tax-and-transfer system.[5] We further show that if, in addition, the government is not too inequality averse, in the absence of a minimum wage a negative marginal tax rate for the deserving poor will be part of an optimal tax-and-transfer system. We demonstrate that this would not be the case with a minimum wage in place. Thus, the minimum wage dominates a negative marginal tax rate as a means to direct benefits to the deserving poor.

The notion of welfare deservedness has attracted much attention in recent years and has become a key issue in the public discourse about the role of the welfare system. Whereas abundant evidence shows that society is generally sympathetic toward the unfortunate disabled, generosity is often conditioned on the poor either working hard or being truly disabled. For instance, Gilens (1999) reports that people are more concerned about the conditions determining which recipients should benefit from social security programs than about the cost of the programs, the main question for taxpayers being not so much “who gets what?” but rather “who deserves what?” In other words, it is not the government support for the truly needy that sparks considerable public resentment, but

[5] The traditional assumption in the optimal taxation literature is that the government is unable to observe wages and therefore conditions transfers and taxes on observable income levels. As acknowledged by previous studies, this informational assumption is somewhat inconsistent with the common practice of simultaneously imposing an income tax and a minimum wage. However, we follow the reasoning in Lee and Saez (2012) who argue that this simultaneous use can be enforced by a combination of whistle blowing by underpaid workers and ex-post costly verification of wages by the government.

rather the perception that most people receiving welfare are un-deserving.[6] These trends are reflected in the 1996 welfare reform in the US and the shift from the Aid to Families with Dependent Children program to the Temporary Assistance for Needy Families program with its emphasis on the work requirement, as well as the significant expansion in recent years of the Earned Income Tax Credit program that conditions welfare on labor market participation.[7]

Several previous papers have distinguished between the deserving and the undeserving poor to provide a normative foundation for commonly used policy tools such as Earned Income Tax Credit and workfare to target benefits to the deserving poor. For instance, Besley and Coate (1992, 1995) assume that the government objective is to alleviate poverty rather than to maximize social welfare. Effectively, this eliminates disutility from work from the government objective and may be interpreted to reflect the conservative view that high disutility from work indicates a socially unacceptable laziness. In particular, they show that workfare can be an effective supplementary screening tool to means testing. Relatedly, Kanbur et al. (1994) establish the case for levying a negative marginal tax rate on the working poor when the government aims to minimize an income-based poverty index. Cuff (2000) employs a framework where individuals differ along the skill dimension and in their work-leisure preferences. She demonstrates that if the government objective is to maximize the well-being of the deserving poor, work requirements can be a desirable supplement to an optimal tax-and-transfer system. Saez

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[6] See also Heclo (1986), Farkas and Robinson (1996), Gallop Organization (1998), Miller (1999), and Fong (2001). According to one poll cited in Gilens (1999), 74% of the public agrees that the criteria for welfare are not strong enough but only 3% reports that they would oppose a 1% sales tax increase aimed at funding help to the poor.

[7] See Salanié (2011). The Earned Income Tax Credit program, initially adopted in 1975 and extended considerably over the 90's, benefits 25 million families in the US at a total cost to the federal government of $61 billion [Tax Policy Center (2013)].

(2002) discusses the possibility of assigning a relatively low marginal social weight to unemployed low-skilled workers and shows that this would reinforce the case for an Earned Income Tax Credit. Fleurbaey and Maniquet (2006) examine the optimal income tax in a setting where individuals differ both in skill levels and in work preferences. Assuming that redistribution should not be driven by heterogeneity in preferences, they demonstrate that the optimal policy is to grant the greatest subsidy to the hardworking (i.e., deserving) poor. Finally, Blumkin et al. (in press) demonstrate that statistical stigma can be an effective welfare ordeal mechanism to sort out the undeserving claimants.

2. The Model

We consider a simple setup with just the key ingredients necessary to demonstrate our point. There are low- and high-skilled workers in the economy that produce a single consumption good the price of which is unity. The mass of each skill group is unity. The output *X* of the productive sector is given by

(1) $$X = F(N^l, N^h),$$

where N^l and N^h denote the total working hours of the low- and high-skilled workers, respectively. The function F is increasing, has constant returns to scale, and exhibits diminishing marginal productivity in the input of each skill level.

Let *c* denote consumption and *n* working hours. The utility of the high-skilled workers (indexed by superscript h) is given by $u^h \equiv c^h - g(n^h)$, where $g(0) = 0$, $g' > 0$, $g'' > 0$ and $\lim_{n \to 0} g'(n) = 0$. The utility of the low-skilled workers depends on their taste for leisure. For a fraction $\alpha \in (0,1)$ of the low-skilled workers (indexed by superscript *d*) the utility is given by $u^d \equiv c^d - g(n^d)$. For the remaining $1-\alpha$ of the low-skilled workers (indexed by superscript *u*) the utility is given by $u^u \equiv c^u - kg(n^u)$, where k>1. That is, type-u low-skilled workers incur a higher disutility (both total and marginal) from work relative to their type-d low-skilled counterparts for the same working hours supplied.

The higher disutility from work incurred by type-u workers may either be associated with physical and mental disabilities or be attributed to laziness. In light of the common perception that a significant share of type-u workers are lazy and choose to decrease their working hours by their own volition, rather than being forced to do so by a true disability, we will henceforth often refer to type-d workers as "deserving poor" and to type-u workers as "undeserving poor".

The total labor supply of the high-skilled workers is given by $N^h = n^h$, and the total labor supply of the low-skilled workers by $N^l = \alpha n^d + (1-\alpha)n^u$. Assuming a competitive labor market, each worker is paid the value of his marginal product. Therefore, $w^h \equiv \partial F(N^l, N^h)/\partial N^h$ is the wage for high-skilled workers and $w^l \equiv \partial F(N^l, N^h)/\partial N^l$ is the wage for low-skilled workers. We assume that $w^h > w^l$.

We follow Mirrlees (1971) in assuming that worker types (in our model, skill levels and preferences) are private information unobserved by the government. Accordingly, the government is confined to second-best redistributive policies.

3. The Government Problem

The social welfare is given by

(2) $$W \equiv V(u^h) + \alpha V(u^d) + \beta(1-\alpha)V(u^u),$$

where $V(0) = 0$, $V' > 0$, $V'' < 0$, and $\beta \in [0,1]$. The strict concavity of V reflects the inequality-aversion exhibited by the government and the parameter β measures the extent to which welfare-deservedness matters. The conventional case is captured by $\beta = 1$ where each group is weighted according to its size in the population. Having $\beta < 1$ implies that the government assigns the undeserving poor a lower weight than their size in the population. This bias against the undeserving poor reflects public resentment of individuals who seem unwilling to exert a socially acceptable level of effort in the labor market and instead choose to rely on the generosity of the welfare system. In the extreme case where $\beta = 0$ the government completely "launders out" the undeserving poor from the welfare calculus. Assuming that the type-u group is comprised of two subgroups, the truly disabled and the lazy, a simple interpretation is that β measures the fraction of truly disabled workers within the type-u group and that society is completely intolerant to lazy workers and hence assigns them zero weight in the welfare function.

The government is interested in reducing its support of the lazy poor but unable to distinguish between the truly disabled and those in the type-u group that are just lazy (both types make identical labor-leisure choices since they share the same preferences). The government is therefore faced with a difficult screening problem and will have to rely on policy rules that adversely affect the entire u-type group.[8] As will be shown below, such "tagging" of the u-type group will be socially desirable when the fraction of

[8] For instance, applicants for disability benefits commonly claim back pain and mental problems, complaints that are hard to disprove.

lazy individuals within the *u*-type group is sufficiently large (that is, when β is sufficiently small).

4. The Benchmark Regime: No Minimum Wage

We start by analyzing the benchmark case with no minimum wage in place. As is customary, we represent a nonlinear tax-and-transfer system as a triplet of consumption-work bundles (c^i, n^i), where $i = h, d, u$. The government maximizes welfare (2) subject to both the revenue constraint

(3) $$F(N^l, N^h) \geq c^h + \alpha c^d + (1-\alpha)c^u$$

and the six incentive-compatibility constraints IC^{ij} which state that for each pair of worker types (i, j), where $i, j = h, d, u$ and $i \neq j$, a worker of type *i* has no incentive to mimic a worker of type *j*, i.e.,

(4) $$c^i - k^i g(n^i) \geq c^j - k^i g\left(\frac{n^j w^j}{w^i}\right),$$

where $k^h = k^d = 1$, $k^u = k$ and $w^d = w^u = w^l$.

We assume that the optimal solution is separating so that each type of worker receives a distinct consumption-work bundle. Focusing on the case where the government assigns a low weight to those considered un-deserving, the following lemma summarizes important properties of the optimal solution:

Lemma 1: *If β is sufficiently small, then*

(i) *The incentive-compatibility constraint IC^{uh} is non-binding;*

(ii) *The incentive-compatibility constraint IC^{ud} is binding;*

(iii) $n^d > n^u$.

Proof: See Appendix A.

Part (i) of the lemma shows that the upward incentive-compatibility constraint IC^{uh} is non-binding. This property accords with standard optimal tax models where the direction of redistribution goes from high to low earners. Hence, only downward incentive-compatibility constraints are binding; that is, high earners are indifferent between choosing their intended bundle and mimicking low earners. However, parts (ii) and (iii) of the lemma state that the upward incentive-compatibility constraint IC^{ud} is binding, with the undeserving poor working less and hence earning less than their deserving counterparts. This unusual feature derives from the bias of the government against the undeserving poor. It implies that the undeserving poor are indifferent between choosing their intended bundle and working more in order to mimic the deserving poor, thereby becoming eligible for more generous transfers.

The unusual feature that IC^{ud} is binding may be reflected in the sign of the optimal marginal tax rate levied on the deserving poor. Whereas in standard optimal tax models (where heterogeneity derives solely from the variation in earning abilities) marginal tax rates would be non-negative, in our framework a negative marginal tax rate on the deserving poor may be optimal.[9] Indeed, we will now show that if the government's inequality aversion is not too large, then the deserving poor will be subject to a negative marginal tax rate.

For concreteness, suppose that the V-function in the social welfare given in equation (2) exhibits constant relative inequality aversion z; that is, $V(u) = u^{1-z}/(1-z)$, where $z > 0, \neq 1$. Then:

[9] A negative marginal tax rate at the bottom may arise in the presence of heterogeneity in work preferences [see for example Chone and Laroque (2010)].

Lemma 2: *If β and z are sufficiently small, then the optimal marginal tax rate on the deserving poor is negative.*

Proof: See Appendix A.

If the relative inequality aversion were large, redistributing from the high-skilled workers to the deserving poor would yield a substantial welfare gain. In such a case, the incentive-compatibility constraint IC^{hd} would be binding. Thus, a positive marginal tax rate on the deserving poor would reduce their labor supply, making it less attractive for the high-skilled workers to mimic and thereby mitigating IC^{hd}.[10] However, the desirability of distorting the deserving poor's labor supply downward in order to mitigate IC^{hd} is countered by the gain that can be obtained by distorting their labor supply upward so as to mitigate IC^{ud}, which is binding by Lemma 1. To mitigate IC^{ud} would require a negative marginal tax rate that makes mimicking less attractive for the underserving poor.[11] Consequently, the sign of the deserving poor's optimal marginal tax rate is generally ambiguous. However, Lemma 2 shows that if the relative inequality aversion is sufficiently small, then the benefit from redistributing from the high-skilled workers to the deserving poor is not strong enough to make IC^{hd} binding. It follows then that the optimal marginal tax rate on the deserving poor is negative.[12] In other words, if the

[10] The desirability of taxing the deserving poor at a positive marginal tax rate in order to mitigate the high-skilled workers' mimicking incentive also holds in the case of perfect substitutability across skill levels but is reinforced when skills are complements. A positive marginal tax rate induces the deserving poor to reduce their labor supply, which decreases the high-skilled workers' marginal product and hence their wage. This renders mimicking more costly for the high-skilled workers who need to work longer hours in order to earn the same income as the deserving poor.

[11] The same logic underlies the finding in Cuff (2000) that welfare may be improved by an unproductive workfare program that serves to separate between the deserving and undeserving poor.

[12] By continuity considerations, the optimal marginal tax rate on the deserving poor will be negative even with a binding IC^{hd}, provided that the relative inequality aversion is sufficiently small.

relative inequality aversion is small, it is socially optimal to tax the deserving poor at a negative marginal tax rate in order to target benefits to them.

5. The Desirability of a Minimum Wage

Since a binding minimum wage sets a lower bound for the wage that can be paid to the low-skilled workers, it effectively determines a binding upper bound for their working hours. This entails an excess supply of low-skilled workers and therefore necessitates some form of rationing. We follow Lee and Saez (2012) by assuming that the rationing is efficient in that the low-skilled workers who are forced to involuntarily reduce their working hours are those with the least surplus from working.[13] We now show that the government can enhance welfare by introducing a minimum wage as a supplement to the tax-and-transfer system:

Proposition 1: *If β is sufficiently small and employment is efficiently rationed, then a minimum wage is an optimal supplement to the tax-and-transfer system.*

Proof: See Appendix B.

The rationale for the desirability of the minimum wage is as follows. Lemma 1 shows that in the absence of a minimum wage, the incentive-compatibility constraint IC^{ud} associated with the undeserving poor would be binding. This limits the government's

[13] Efficient rationing may be obtained in practice, as workers with a stronger taste for leisure are more prone to quit through natural attrition (see also footnote 14 below), or, alternatively, assuming turnover is costly, would be the first to be laid-off by the firm (being perceived as less stable). For direct evidence of efficient rationing of employment, see Luttmer (2007) who shows that an increase in the minimum wage does not cause workers with higher reservation wages to displace equally skilled workers with lower reservation wages. Thus, the workers who value their job the least are those who tend to lose their jobs due to a minimum wage increase. See also Neumark and Wascher (2007) who show that the employment effect of a minimum wage is strongest amongst those who are likely to have the highest reservation wage. In our framework, efficient rationing entails underemployment rather than unemployment (see the proof of Proposition 1 in Appendix B).

redistributive capacity as increasing the transfer to the deserving poor would violate the underserving poor's incentive-compatibility constraint. However, introducing a minimum wage, given efficient rationing, blocks this undesirable supply-side response so that the entire incidence of the induced involuntary underemployment falls on the undeserving poor. Namely, the underserving poor will be forced to work less than they would prefer given the tax-and-transfer schedule. Effectively, the minimum wage relaxes the incentive-compatibility constraint IC^{ud}, allowing the government to offer more generous transfers to the deserving poor. Thus, the minimum wage plays a screening role that ensures that the extra transfers are targeted to those considered deserving, rather than being accorded to all low-skilled workers.

The assumption of efficient rationing essentially tags the underserving poor and ensures that they bear the full burden of the involuntary underemployment triggered by the minimum wage. However, in order for tagging to be optimal it requires only that the correlation between the observed tag and being a member of the targeted group be sufficiently large. Thus, for a minimum wage to be a desirable supplement to an optimal tax-and-transfer system, it would suffice that a large enough share of the involuntary underemployment falls on the undeserving poor. Loosely speaking, a minimum wage would be a desirable supplement to the tax-and-transfer system when rationing is "sufficiently efficient" in the sense that most of the burden of involuntary underemployment would be borne by the undeserving poor who have a stronger taste for leisure. [14]

[14] More formally, suppose that low-skilled workers can be assigned to either *d*–jobs or *u*-jobs, the respective measures of which are given by α and (1- α), with all low-skilled workers preferring the *d*-jobs to the *u*-jobs. Jobs are assigned as follows: a fraction $0<q<1$ of the *d*-jobs (*u*-jobs) is assigned to the deserving (undeserving) poor, respectively, whereas the remaining jobs are assigned randomly. Notice that

It is worth noting the reason for the difference between our finding that a minimum wage is desirable and the negative result in Allen (1987) and Guesnerie and Roberts (1987) that a minimum wage cannot be a useful supplement to an optimal tax-and-transfer system. In these two studies the government's redistributive policy is constrained by the high-skilled workers' binding downward incentive-compatibility constraint, which makes them indifferent between whether or not to mimic the low-skilled workers. In such a case, imposing a minimum wage is useless since it does not make mimicking harder for the high-skilled workers. In contrast, in our setting, the government's redistributive policy is constrained by the undeserving poor's binding upward incentive-compatibility constraint, which makes them indifferent between whether or not to mimic the deserving poor. Since the undeserving poor would have to increase their working hours in order to mimic the deserving poor, an effective upper bound on the undeserving poor's working hours would be desirable. The latter is achieved by the minimum wage which sets an upper bound on the working hours of all low-skilled workers that, given the efficient rationing of

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when $q \to 1$ rationing is efficient, whereas, rationing is random when $q \to 0$. Let the utility of deserving poor assigned to a d–job (u–job) be denoted by u^{dd} (u^{du}), and the utility of an undeserving poor assigned to a d–job (u–job) be denoted by u^{ud} (u^{uu}). The social welfare is then given by: $\alpha[q+(1-q)\alpha]V(u^{dd}) + \alpha(1-q)(1-\alpha)V(u^{du}) + \beta(1-\alpha)[q+(1-q)(1-\alpha)]V(u^{uu}) + \beta(1-\alpha)(1-q)\alpha V(u^{ud}) + V(u^h)$. By continuity considerations, supplementing the tax-and-transfer system with a binding minimum wage is desirable if rationing is "sufficiently efficient", i.e., if q is sufficiently large.

Assuming efficient rationing of employment abstracts from the fact that any rationing process is likely to entail some costs associated with rent-seeking efforts. However, one can sketch the following positive foundation for virtually costless efficient rationing. Suppose that the more attractive low-skilled jobs, i.e., those where working hours are rationed, are allocated by a time-consuming queuing process. Workers that do not participate in the queuing process are given the less attractive low-skilled jobs, i.e., those where working hours are not rationed. Since the time cost of queuing is lower for the deserving than the undeserving poor, if k is sufficiently large, the queuing time supporting an equilibrium in which all deserving poor participate whereas all undeserving poor opt out, could be very short and therefore not very costly.

employment, translates into an upper bound on only the working hours of the undeserving poor.

6. The Optimality of a Positive Marginal Tax Rate for the Deserving Poor

A minimum wage and a negative marginal tax rate are two widely used policy tools for providing low-income support and their relative importance is subject to an ongoing debate. We next show that our model implies that a binding minimum wage may obviate the need to levy a negative marginal tax rate on the deserving poor.

Proposition 2: *If β and z are sufficiently small and employment is efficiently rationed, with a minimum wage the optimal marginal tax rate on the deserving poor is positive.*

Proof: See Appendix B.

The rationale of Proposition 2 is as follows. If the relative inequality aversion is sufficiently small and there is no minimum wage, we know from Lemma 2 that a negative marginal tax rate for the deserving poor may serve to enhance the screening efficiency of the tax-and-transfer system and target benefits to the deserving poor. The ensuing upward distortion of the deserving poor's labor supply entailed by the negative marginal tax rate would be justified as it mitigates the underserving poor's mimicking incentive. However, a binding minimum wage with employment being efficiently rationed blocks the underserving poor from mimicking and hence obviates the need to distort upward the deserving poor's labor supply. As shown by Proposition 2, under the same assumptions as in Lemma 2 but with a minimum wage, it is then optimal to levy a positive marginal tax rate on the deserving poor thereby distorting downward their labor supply in order to mitigate the high-skilled workers' mimicking incentive.

7. Conclusion

In this paper we show that if the government is sufficiently biased against the undeserving poor, a minimum wage is a desirable supplement to an optimal tax-and-transfer system when the labor market rationing is sufficiently efficient. If, in addition, the government's inequality aversion is moderate, we also show that a minimum wage is a more efficient screening device than a negative marginal tax rate levied on the deserving poor. In other words, if the government is sufficiently biased against the undeserving poor but not too inequality averse, the deserving poor's optimal marginal tax rate is negative in the absence of a minimum wage, but positive in the presence of minimum wage.

Our model is designed to capture the public's prevalent perception that the welfare system should target the truly deserving. This perception is reflected in the growing popularity of the Earned Income Tax Credit and the use of workfare as an eligibility condition for welfare transfers. Previous theoretical papers have provided a normative justification for the use of these tools to direct benefits to the deserving poor. The current paper adds to this literature by highlighting the role that the minimum wage can play as a screening device that redistributes income toward those considered deserving.

Appendix A

Proof of the Lemma 1

We let $\beta = 0$. Our argument will extend to the case of sufficiently small values of β by continuity considerations.

Part (i): The incentive-compatibility constraint IC^{uh} is non-binding.

Proof: Suppose, by way of contradiction, that IC^{uh} is binding. Thus,

(A1) $c^u - kg(n^u) = c^h - kg\left(\frac{n^h w^h}{w^l}\right)$.

By virtue of IC^{ud} it follows that

(A2) $c^u - kg(n^u) \geq c^d - kg(n^d)$.

Substituting (A1) into (A2) yields

(A3) $c^h - kg\left(\frac{n^h w^h}{w^l}\right) \geq c^d - kg(n^d)$.

By virtue of IC^{dh} it follows that

(A4) $c^d - g(n^d) \geq c^h - g\left(\frac{n^h w^h}{w^l}\right)$.

After rearrangement, (A3) and (A4) yield

(A5) $(k-1)[g\left(\frac{n^h w^h}{w^l}\right) - g(n^d)] \leq 0$.

As *k*>1 and *g* is increasing, (A5) implies that $n^h w^h / w^l \leq n^d$. By the assumption that the equilibrium is separating it follows that

(A6) $\frac{n^h w^h}{w^l} < n^d$.

By virtue of IC^{hd} it follows that

(A7) $c^h - g(n^h) \geq c^d - g\left(\frac{n^d w^l}{w^h}\right)$.

After rearrangement, (A7) and (A4) yield

(A8) $g\left(\frac{n^h w^h}{w^l}\right) - g(n^h) \geq g(n^d) - g\left(\frac{n^d w^l}{w^h}\right)$

$\Leftrightarrow H\left(\frac{n^h w^h}{w^l}\right) \geq H(n^d),$

where $H(n) \equiv g(n) - g(\frac{nw^l}{w^h})$. Differentiation of H with respect to *n* yields

(A9) $H' = g'(n) - g'\left(\frac{nw^l}{w^h}\right)\frac{w^l}{w^h} > 0,$

where the inequality follows from the strict convexity of *g* and the fact that $w^h > w^l$. It follows from (A9) that $n^h w^h / w^l \geq n^d$, which violates (A6).

Part (ii): The incentive-compatibility constraint IC^{ud} is binding.

Proof: Suppose, by way of contradiction, that IC^{ud} holds as a strict inequality. Consider the following small perturbation to the presumed optimal solution:

$\tilde{c}^h = c^h + \varepsilon,$

$\tilde{c}^d = c^d + \varepsilon,$

$\tilde{c}^u = c^u - \delta,$

where $\varepsilon, \delta > 0$ and $(1-\alpha)\delta = (1+\alpha)\varepsilon$.

By continuity considerations, IC^{ud} is maintained. Moreover, by continuity considerations and by virtue of part (i), IC^{uh} is also maintained. Neither the revenue constraint nor any of the other incentive-compatibility constraints is violated. The suggested perturbation thus yields an increase in social welfare, since no weight is assigned to the undeserving workers whose level of consumption is reduced. We thus obtain the desired contradiction.

Part (iii): $n^d > n^u$.

Proof: By virtue of IC^{du} it follows that

(A10) $c^d - g(n^d) \geq c^u - g(n^u).$

By virtue of part (ii), the constraint IC^{ud} is binding; hence

(A11) $c^u - kg(n^u) = c^d - kg(n^d)$.

Subtracting (A11) from (A10) yields upon rearrangement

(A12) $(k-1)[g(n^d) - g(n^u)] \geq 0$.

As g is increasing and k>1, it follows that $n^d \geq n^u$. By the assumption of a separating equilibrium, it then follows that $n^d > n^u$. This completes the proof.

Proof of Lemma 2

We let $\beta = 0$. Our argument will extend to the case of sufficiently small values of β by continuity considerations.

For convenience, the proof will be arranged as a set of claims.

Claim 1: If z is sufficiently small, the incentive-compatibility constraint IC^{dh} is binding.

Proof: We let $z = 0$. Our result will extend to the case of sufficiently small values of z by continuity considerations. Suppose, by way of contradiction, that IC^{dh} holds as a strict inequality. Consider the following small perturbation to the presumed optimal solution:

$\tilde{c}^h = c^h + \varepsilon$,

$\tilde{c}^d = c^d - \varepsilon$,

$\tilde{c}^u = c^u - \varepsilon$,

where $\varepsilon > 0$.

By continuity considerations, IC^{dh} is maintained. Moreover, by continuity considerations and by virtue of part (i) of Lemma 1, IC^{uh} is also maintained. By construction of the perturbation, neither the revenue constraint nor any of the other incentive-compatibility constraints is violated. The suggested perturbation yields an increase in social welfare, as

$z = 0$ and hence the social marginal utility of income associated with the high-skilled workers (=1) strictly exceeds the social marginal utility of income associated with the deserving poor (=α). The reduction in the consumption level of the undeserving poor does not affect the welfare level since the social marginal utility of their income is zero. We thus obtain the desired contradiction.

Claim 2: If z is sufficiently small, the incentive-compatibility constraint IC^{hd} is slack.

Proof: Suppose by way of contradiction that IC^{hd} is binding. It follows that

(A13) $c^h - g(n^h) = c^d - g\left(\frac{n^d w^l}{w^h}\right)$.

By virtue of claim 1 the incentive-compatibility constraint IC^{dh} is binding; hence

(A14) $c^d - g(n^d) = c^h - g\left(\frac{n^h w^h}{w^l}\right)$.

Subtracting (A14) from (A13) upon rearrangement yields

(A15) $g\left(\frac{n^h w^h}{w^l}\right) - g(n^h) = g(n^d) - g\left(\frac{n^d w^l}{w^h}\right)$

$\Leftrightarrow H\left(\frac{n^h w^h}{w^l}\right) = H(n^d)$,

where $H(n) \equiv g(n) - g(\frac{n w^l}{w^h})$. Differentiation of H with respect to n yields

(A16) $H' = g'(n) - g'\left(\frac{n w^l}{w^h}\right)\frac{w^l}{w^h} > 0$,

where the inequality follows from the strict convexity of g and the fact that $w^h > w^l$. It follows from (A15) that $\frac{n^h w^h}{w^l} = n^d$, which violates our assumption of a separating equilibrium.

Claim 3: If z is sufficiently small, the incentive-compatibility constraint IC^{du} is slack.

Proof: Suppose by way of contradiction that IC^{du} is binding. It follows that

(A17) $c^d - g(n^d) = c^u - g(n^u)$.

By virtue of part (ii) of Lemma 1, the constraint IC^{ud} is binding; hence

(A18) $c^u - kg(n^u) = c^d - kg(n^d)$.

Subtracting (A18) from (A17) upon rearrangement yields

(A19) $(k-1)[g(n^d) - g(n^u)] = 0$.

As g is increasing and k>1 it follows that $n^d = n^u$, which violates our assumption that the equilibrium is separating.

Claim 4: If z is sufficiently small, the incentive-compatibility constraint IC^{hu} is slack.

Proof: Suppose by way of contradiction that IC^{hu} is binding. It follows that

(A20) $c^h - g(n^h) = c^u - g\left(\frac{n^u w^l}{w^h}\right)$.

By virtue of claim 2, the incentive-compatibility constraint IC^{hd} is slack; hence

(A21) $c^h - g(n^h) > c^d - g\left(\frac{n^d w^l}{w^h}\right)$.

Substituting (A20) into (A21) yields

(A22) $c^u - g\left(\frac{n^u w^l}{w^h}\right) > c^d - g\left(\frac{n^d w^l}{w^h}\right)$.

By virtue of part (ii) of Lemma 1, the incentive-compatibility constraint IC^{ud} is binding; hence

(A23) $c^u - kg(n^u) = c^d - kg(n^d)$.

Subtracting (A23) from (A22) upon rearrangement yields

(A24) $kg(n^u) - g\left(\frac{n^u w^l}{w^h}\right) > kg(n^d) - g\left(\frac{n^d w^l}{w^h}\right) \Leftrightarrow H(n^u) > H(n^d)$,

where $H(n) \equiv kg(n) - g(\frac{nw^l}{w^h})$. Differentiation of H with respect to n yields

(A25) $H' = kg'(n) - g'\left(\frac{nw^l}{w^h}\right)\frac{w^l}{w^h} > 0$,

where the inequality follows from the strict convexity of g and the fact that $w^h > w^l$ and k>1. It follows from (A24) that $n^u > n^d$, which violates part (iii) of Lemma 1.

Claim 5: If z is sufficiently small, the marginal tax rate levied on the deserving poor is negative.

Proof: By virtue of parts (i) and (ii) of Lemma 1 and claims 1-4, the only binding incentive-compatibility constraints are IC^{dh} and IC^{ud}. Formulating the Lagrangean for the optimization problem yields

(A26) $$L \equiv max_{(c^i,n^i);i=u,d,h} V[c^h - g(n^u)] + \alpha V[c^d - g(n^d)] + \lambda[F(N^l, N^h) - c^h - \alpha c^d - (1-\alpha)c^u] + \mu[c^d - g(n^d) - c^h + g\left(\frac{n^h w^h}{w^l}\right)] + \eta[c^u - kg(n^u) - c^d + kg(n^d)],$$

where $\lambda, \mu,$ and η are the Lagrange multipliers associated with the revenue constraint and the two binding incentive-compatibility constraints, IC^{dh} and IC^{ud}.

The first-order condition associated with the optimal consumption level of the deserving poor is

(A27) $$\frac{\partial L}{\partial c^d} = \alpha V'[c^d - g(n^d)] - \alpha\lambda + \mu - \eta = 0.$$

The first-order condition associated with the optimal labor supply choice of the deserving poor is

(A28) $$\frac{\partial L}{\partial n^d} = -\alpha V'[c^d - g(n^d)]g'(n^d) + \lambda w^l - \mu\left[g'(n^d) - g'\left(\frac{n^h w^h}{w^l}\right) n^h \left(\frac{\frac{\partial w^h}{\partial n^d} w^l - \frac{\partial w^l}{\partial n^d} w^h}{w^{l^2}}\right)\right] + \eta k g'(n^d) = 0.$$

Substituting for $\alpha V'[c^d - g(n^d)]$ from (A27) into (A28) and rearranging yields

(A29) $$\lambda\alpha[w^l - g'(n^d)] = -\eta(k-1)g'(n^d) - \mu\left[g'\left(\frac{n^h w^h}{w^l}\right) n^h \left(\frac{\frac{\partial w^h}{\partial n^d} w^l - \frac{\partial w^l}{\partial n^d} w^h}{w^{l^2}}\right)\right] < 0,$$

where the inequality sign follows since k>1, g'>0, $\frac{\partial w^l}{\partial n^d} < 0$, by virtue of the diminishing marginal productivity property, and $\frac{\partial w^h}{\partial n^d} > 0$.[15]

The marginal tax rate levied on the deserving poor is given by

(A30) $MTR^d \equiv 1 - \frac{g'(n^d)}{w^l} < 0,$

where the inequality sign follows since $g'(n^d) > w^l$ by virtue of (A29). This concludes the proof.

Appendix B

Proof of Proposition 1

We let $\beta = 0$. Our argument will extend to the case of sufficiently small values of β by continuity considerations. Suppose that there is no minimum wage and let the triplet (c_*^i, n_*^i), where i= h, d, u, denote the optimal tax-and-transfer schedule that maximizes welfare (2) subject to the revenue constraint (3) and the incentive-compatibility constraints (4). Consider the following small perturbation to the optimal solution:

$\tilde{c}^h = c_*^h + \varepsilon,$

$\tilde{c}^d = c_*^d + \varepsilon,$

$\tilde{c}^u = c_*^u - \delta,$

where $\varepsilon, \delta > 0$ and $(1-\alpha)\delta = (1+\alpha)\varepsilon$.

In addition, suppose that the government sets a minimum wage at the level of the equilibrium low-skilled wage under an optimal income tax-and-transfer schedule in the

[15] The latter is due to the constant returns to scale and Euler's Theorem. Differentiating the production function with respect to N^l and re-arranging yield $N^l \partial w^l / \partial N^l = -N^h \partial w^h / \partial N^l$, which is negative by virtue of the diminishing marginal productivity property.

absence of a minimum wage. Formally, let $\overline{w} = \partial F(\alpha n_*^d + (1-\alpha)n_*^u, n_*^h)/\partial N^l$ denote the minimum wage.

By construction, neither the revenue constraint nor any of the incentive-compatibility constraints are violated. Furthermore, by virtue of part (i) of Lemma 1, IC^{uh} is non-binding and hence remains satisfied by continuity. On the other hand, IC^{ud}, which by virtue of part (ii) of Lemma 1 is binding under an optimal tax-and-transfer regime, is violated by the suggested perturbation, since the undeserving poor would want to mimic the deserving poor. However, we will now demonstrate that the binding minimum wage blocks such mimicking.

By virtue of the incentive-compatibility constraints IC^{du} and IC^{ud}, the introduction of a binding minimum wage results in involuntary underemployment/unemployment. To see this, notice that the deserving and undeserving poor are willing to work n_*^d hours since both types prefer the bundle (c_*^d, n_*^d) to any other bundle. This implies that the total labor supply of the low-skilled workers is given by n_*^d. However, the total labor demand for the low-skilled workers is given by $\alpha n_*^d + (1-\alpha)n_*^u < n_*^d$, where the inequality sign follows from part (iii) of Lemma 1.

Efficient rationing implies that the entire incidence of underemployment will fall on the undeserving poor. That is, the undeserving poor will become underemployed and only work n_*^u hours, whereas the deserving poor will continue to work n_*^d hours. To see this, notice that by virtue of the quasi-linear utility functions, a necessary and sufficient condition for a rationing rule to be efficient is that it maximizes the total surplus (S) of the low-skilled workers

(B1) $S \equiv [(x^d + x^u)\tilde{c}^u + (z^d + z^u - x^d - x^u)\tilde{c}^d - (z^d - x^d)g(n_*^d) - x^d g(n_*^u) - (z^u - x^u)kg(n_*^d) - x^u kg(n_*^u)]$

subject to the constraint

(B2) $(z^d + z^u - x^d - x^u)n_*^d + (x^d + x^u)n_*^u = \alpha n_*^d + (1-\alpha)n_*^u,$

where $0 \leq x^d \leq z^d \leq \alpha$ and, $0 \leq x^u \leq z^u \leq 1-\alpha$.

Several remarks are in order. First, maximizing the sum of utilities is a sufficient condition for attaining a Pareto efficient allocation under any utility specification. Quasi-linearity implies that this is also a necessary condition due to the linearity of the frontier of the utility possibility set. Thus, the solution to the maximization of (B1) subject to the constraint (B2) characterizes the unique efficient rationing rule. Second, we consider the most general rationing rule that allows each type of low-skilled worker to be underemployed ($x^i \leq z^i; i = d, u$) and/or unemployed ($z^d \leq \alpha, z^u \leq 1-\alpha$). Third, the formulation of the surplus in (B1) accounts for the fact that the reservation utility of unemployed workers of both types is zero. Finally, we assume that the utility levels under the optimal tax-and-transfer regime (hence, by continuity, also under the perturbed tax-and-transfer regime) are bounded away from zero for both types of low-skilled workers; hence both types of low-skilled workers will have positive working hours.

Rearranging (B2) yields

(B2') $x^d + x^u = \Delta,$

where $\Delta \equiv (1-\alpha) - \frac{(1-z^d-z^u)n_*^d}{n_*^d - n_*^u}$.

Substituting for x^u from (B2') into (B1) and rearranging yields

(B3) $S = [(z^d + z^u)\tilde{c}^d + \Delta(\tilde{c}^u - \tilde{c}^d) - (z^d - x^d)g(n_*^d) - x^d g(n_*^u) - (x^d + z^u - \Delta)kg(n_*^d) - (\Delta - x^d)kg(n_*^u)].$

Differentiating (B3) with respect to x^d and rearranging yields

(B4) $\frac{\partial S}{\partial x^d} = -(k-1)[g(n_*^d) - g(n_*^u)] < 0,$

where the inequality follows since $n_*^d > n_*^u$, g is increasing, and k>1. We conclude that $x^d = 0$ and, by virtue of (B2'), that $x^u = \Delta$.

Differentiating (B3) with respect to z^d upon rearrangement yields

(B5) $\frac{\partial S}{\partial z^d} = \frac{\{n_*^d(k-1)g(n_*^d)+n_*^d[\tilde{c}^u-kg(n_*^u)]-n_*^u[\tilde{c}^d-g(n_*^d)]\}}{(n_*^d-n_*^u)}.$

As $n_*^d > n_*^u$ and k>1, it follows by substituting n_*^u for n_*^d in the first term of the numerator of right-hand-side expression of (B5) that

(B6) $\frac{\partial S}{\partial z^d} > \frac{\{n_*^u(k-1)g(n_*^d)+n_*^d[\tilde{c}^u-kg(n_*^u)]-n_*^u[\tilde{c}^d-g(n_*^d)]\}}{(n_*^d-n_*^u)},$

which, after rearrangement, yields

(B6') $\frac{\partial S}{\partial z^d} > \frac{\{n_*^d[\tilde{c}^u-kg(n_*^u)]-n_*^u[\tilde{c}^d-kg(n_*^d)]\}}{(n_*^d-n_*^u)}.$

As $n_*^d > n_*^u$, for $\frac{\partial S}{\partial z^d} > 0$ it suffices to show that the numerator of the right-hand-side of (B6') is positive; that is

(B7) $n_*^d[\tilde{c}^u - kg(n_*^u)] - n_*^u[\tilde{c}^d - kg(n_*^d)]$>0.

By virtue of the binding incentive-compatibility constraint IC^{ud}, under the optimal unperturbed tax-and-transfer regime

(B8) $\lim_{\varepsilon\to 0}[\tilde{c}^d - kg(n_*^d)] = \lim_{\delta\to 0}[\tilde{c}^u - kg(n_*^u)] \equiv B > 0,$

where the inequality sign follows from our assumption that the utilities derived under the optimal tax-and-transfer regime are positive. Therefore,

(B9) $\lim_{\varepsilon\to 0,\delta\to 0} n_*^d[\tilde{c}^u - kg(n_*^u)] - n_*^u[\tilde{c}^d - kg(n_*^d)] = B(n_*^d - n_*^u) > 0,$

where the inequality sign follows from (B8) and $n_*^d > n_*^u$. Thus, by continuity considerations, for sufficiently small ε and δ, the inequality (B7) holds. We thus conclude that $\frac{\partial S}{\partial z^d} > 0$. Hence, $z^d = \alpha$.

Differentiating (B3) with respect to z^u upon rearrangement yields

(B10) $$\frac{\partial S}{\partial z^u} = \frac{\{n_*^d[\tilde{c}^u - kg(n_*^u)] - n_*^u[\tilde{c}^d - kg(n_*^d)]\}}{(n_*^d - n_*^u)}.$$

Noting that the expression on the right-hand-side of (B10) is identical to the expression on the right-hand-side of (B6'), by repeating the arguments used to establish the positive sign of $\frac{\partial S}{\partial z^d}$, it follows that $\frac{\partial S}{\partial z^u}>0$. Hence, $z^u = 1 - \alpha$.

We conclude that under efficient rationing none of the low-skilled workers are forced into unemployment. Moreover, the entire incidence of underemployment falls on the undeserving poor who are unable to mimic the deserving poor. The suggested perturbation therefore yields an increase in social welfare. This completes the proof.

Proof of Proposition 2

We let $\beta = 0$. Our argument will extend to the case of sufficiently small values of β by continuity considerations. Consider the following relaxed optimization program. The government seeks to maximize social welfare subject to: (i) the revenue constraint, (ii) the incentive-compatibility constraint IC^{hd}, and (iii) the participation constraint associated with the undeserving poor, according to which their utility weakly exceeds their reservation utility (which is zero).[16] The Lagrangean associated with the relaxed government problem is

(B11) $$L \equiv max_{(c^i,n^i);i=u,d,h} V[c^h - g(n^u)] + \alpha V[c^d - g(n^d)] + \lambda[F(N^l, N^h) - c^h - \alpha c^d - (1-\alpha)c^u] + \mu[c^h - g(n^h) - c^d + g\left(\frac{n^d w^l}{w^h}\right)] + \eta[c^u - kg(n^u)],$$

where λ, μ, and η are the Lagrange multipliers associated with the revenue constraint, the incentive-compatibility constraint IC^{hd}, and the participation constraint for the undeserving poor.

The structure of the proof will be as follows. We first characterize the optimal solution to the relaxed program and show that the marginal tax rate levied on the deserving poor is positive. We then show that the allocation associated with the optimal solution to the relaxed program can be implemented by a tax-and-transfer system supplemented by a binding minimum wage. For convenience, the proof will be arranged as a set of claims.

Claim 1: In the optimal solution to the relaxed program the revenue constraint is binding.

Proof: Suppose by way of contradiction that the revenue constraint is slack. The resulting fiscal surplus can then be rebated as a uniform lump-sum transfer without affecting any of the constraints. We thus obtain a contradiction to the presumed optimality.

Claim 2: In the optimal solution to the relaxed program the participation constraint is binding.

Proof: Suppose by way of contradiction that the participation constraint is slack. Then one can slightly reduce the consumption level of the undeserving poor, hereby violating neither the participation constraint nor the incentive-compatibility constraint IC^{hd}, and create a fiscal surplus. We thus obtain a contradiction to optimality by virtue of claim 1.

[16] In the proof of Proposition 1 we assumed that with no minimum wage the participation constraint was non-binding.

Claim 3: In the optimal solution to the relaxed program the incentive-compatibility constraint IC^{hd} is binding.

Proof: Suppose by contradiction that IC^{hd} is slack and consider the following small perturbation to the presumed optimal solution:

$\tilde{c}^h = c^h - \varepsilon$,

$\tilde{c}^d = c^d + \varepsilon/\alpha$,

where $\varepsilon > 0$.

By continuity considerations, IC^{hd} remains slack. By construction of the perturbation neither the revenue constraint nor the participation constraint is violated. Invoking a first-order approximation, the total effect of the suggested perturbation on welfare is given by $\Delta W \simeq \varepsilon(V'[c^d - g(n^d)] - V'[c^h - g(n^u)]) > 0$, where the inequality sign follows from the strict concavity of V and the fact that $c^h - g(n^h) > c^d - g\left(\frac{n^d w^l}{w^h}\right) > c^d - g(n^d)$, where the first inequality follows from the slack IC^{hd} constraint and the second inequality from w^h>w^l and $g' > 0$. We thus obtain the desired contradiction.

Claim 4: The welfare level associated with the optimal solution to the relaxed government problem provides an upper bound for the welfare level associated with the extended government problem that takes into account all incentive-compatibility constraints and allows for the use of minimum wage.

Proof: The solution for the extended government problem has to satisfy the revenue constraint, the undeserving poor' participation constraint, and the incentive-compatibility constraint IC^{hd}.

Although setting a binding minimum wage may serve to mitigate some of the additional incentive-compatibility constraints that are not considered explicitly in the relaxed problem (as demonstrated in Proposition 1), a minimum wage cannot serve to mitigate the high-skilled workers' binding incentive-compatibility constraint [as already shown by Allen (1987)], nor can it mitigate the binding revenue and participation constraints. Thus, the relaxed problem is properly nested into the extended problem. This concludes the proof.

Claim 5: In the optimal solution to the relaxed government problem the marginal tax rate levied on the deserving poor is positive.

Proof: The first-order conditions with respect to n^d and c^d for the Lagrangean given in (B11) are

(B12) $$\frac{\partial L}{\partial n^d} = -\alpha V'[c^d - g(n^d)]g'(n^d) + \lambda w^l \alpha$$

$$+\mu g'\left(\frac{n^d w^l}{w^h}\right)\left[\frac{w^l}{w^h} + n^d\left(\frac{\frac{\partial w^l}{\partial n^d}w^h - \frac{\partial w^h}{\partial n^d}w^l}{{w^h}^2}\right)\right] = 0,$$

(B13) $$\frac{\partial L}{\partial c^d} = \alpha V'[c^d - g(n^d)] - \alpha\lambda - \mu = 0.$$

Substituting for $\alpha V'[c^d - g(n^d)]$ from (B13) into (B12) and rearranging yields

(B14) $$\lambda\alpha[w^l - g'(n^d)]$$

$$=\mu\left[\left[g'(n^d) - g'\left(\frac{n^d w^l}{w^h}\right)\frac{w^l}{w^h}\right] - g'\left(\frac{n^d w^l}{w^h}\right)n^d\left(\frac{\frac{\partial w^l}{\partial n^d}w^h - \frac{\partial w^h}{\partial n^d}w^l}{{w^h}^2}\right)\right] > 0,$$

where the inequality sign follows from $g'' > 0$, $w^h > w^l$, $\frac{\partial w^l}{\partial n^d} < 0$ by virtue of the diminishing marginal productivity, and $\frac{\partial w^h}{\partial n^d} > 0$ by virtue of the constant returns-to-scale assumption (see footnote 15).

The marginal tax rate on the deserving poor is given by

(B15) $$MTR^d \equiv 1 - \frac{g'(n^d)}{w^l} > 0,$$

where the inequality sign follows since $g'(n^d) < w^l$ by virtue of (B14).

This concludes the proof.

Claim 6: In the optimal solution to the relaxed government problem the marginal tax rate on the undeserving poor is zero.

Proof: The first-order conditions with respect to n^u and c^u for the Lagrangean given in (B11) are

(B16) $\frac{\partial L}{\partial n^u} = \lambda w^l(1-\alpha) - \eta k g'(n^u) = 0,$

(B17) $\frac{\partial L}{\partial c^u} = -\lambda(1-\alpha) + \eta = 0.$

Substituting for $\lambda(1-\alpha)$ from (B17) into (B16) and rearranging yield

(B18) $w^l = kg'(n^u).$

It follows that the marginal tax rate on the undeserving poor is given by

(B19) $MTR^u \equiv 1 - \frac{kg'(n^u)}{w^l} = 0.$

This concludes the proof.

Claim 7: In the optimal solution to the relaxed government problem the marginal tax rate on the high-skilled workers is negative.[17]

Proof: Formulating the first-order conditions with respect to n^h and c^h for the Lagrangean given in (B11) yields, respectively:

(B20) $\frac{\partial L}{\partial n^h} = -V'[c^h - g(n^h)]g'(n^h) + \lambda w^h$

$$-\mu\left[g'(n^u) - g'\left(\frac{n^d w^l}{w^h}\right)n^d\left(\frac{\frac{\partial w^l}{\partial n^h}w^h - \frac{\partial w^h}{\partial n^h}w^l}{{w^h}^2}\right)\right] = 0,$$

(B21) $\frac{\partial L}{\partial c^h} = V'[c^h - g(n^h)] - \lambda + \mu = 0.$

[17] This result was first shown by Stiglitz (1982).

Substituting for $V'[c^h - g(n^h)]$ from (B21) into (B20) and rearranging yields

(B22) $\lambda[w^h - g'(n^h)] = -\mu g'\left(\frac{n^d w^l}{w^h}\right) n^d \left(\frac{\frac{\partial w^l}{\partial n^h}w^h - \frac{\partial w^h}{\partial n^h}w^l}{{w^h}^2}\right) < 0,$

where the inequality sign follows since $g' > 0$, $\frac{\partial w^h}{\partial n^h} < 0$ by virtue of the diminishing marginal productivity property, and $\frac{\partial w^l}{\partial n^h} > 0$ by virtue of the constant returns-to-scale assumption (see footnote 15).

The marginal tax rate levied on high-skilled workers is

(B23) $MTR^h \equiv 1 - \frac{g'(n^h)}{w^h} < 0,$

where the inequality sign follows since $g'(n^h) > w^h$ by virtue of (B22).

This concludes the proof.

Claim 8: In the optimal solution to the relaxed program the incentive-compatibility constraint IC^{dh} is slack.

Proof: Suppose by way of contradiction that IC^{dh} is non-slack. Formally,

(B24) $c^d - g(n^d) \leq c^h - g\left(\frac{n^h w^h}{w^l}\right).$

By virtue of claim 3, IC^{hd} is binding. Namely,

(B25) $c^h - g(n^h) = c^d - g\left(\frac{n^d w^l}{w^h}\right).$

Subtracting (B24) from (B25) yields

(B26) $g\left(\frac{n^h w^h}{w^l}\right) - g(n^h) \leq g(n^d) - g\left(\frac{n^d w^l}{w^h}\right)$

$\Leftrightarrow H\left(\frac{n^h w^h}{w^l}\right) \leq H(n^d),$

where $H(n) \equiv g(n) - g(\frac{nw^l}{w^h})$. Differentiation of H with respect to n yields

(B27) $H' = g'(n) - g'\left(\frac{nw^l}{w^h}\right)\frac{w^l}{w^h} > 0,$

where the inequality follows from the strict convexity of g and the fact that $w^h > w^l$. It follows from (B26) that $n^h w^h / w^l \leq n^d$, which implies that $n^h \leq n^d$.

By virtue of (B15) and (B23) it follows that $g'(n^d) < w^l < w^h < g'(n^h)$, which implies that $n^h > n^d$ by virtue of the strict convexity of g. We therefore obtain the desired contradiction.

Claim 9: If z is sufficiently small, in the optimal solution to the relaxed program the incentive-compatibility constraint IC^{du} is slack.

Proof: Let $(\hat{c}^i, \hat{n}^i); i = u, d, h$ denote the laissez-faire allocation. Denote by $\hat{w}^l$ and $\hat{w}^h$ the wage rates associated with the low-skilled and high-skilled workers, respectively, under the *laissez-faire* allocation. We first prove that the utility level associated with the deserving poor's bundle in the optimal solution to the relaxed program weakly exceeds his laissez-faire level of utility. We will consider the limiting case where $z \to 0$, in which $V' \to 1$. Substituting for *V'* into (B13) and (B21) implies

(B28) $\alpha\lambda + \mu \to \alpha$,

(B29) $\lambda - \mu \to 1$.

Adding (B28) and (B29) implies that $\lambda \to 1$ and $\mu \to 0$. Substituting into (B14) and (B22) and employing (B18) implies that

(B30) $\lim_{z \to 0} n^i = \hat{n}^i; i = u, d, h;$ and $\lim_{z \to 0} w^i = \hat{w}^i; i = l, h.$

Suppose by way of contradiction that

(B31) $\lim_{z \to 0} [c^d - g(n^d)] < \hat{c}^d - g(\hat{n}^d).$

By construction, the welfare level associated with the optimal solution to the relaxed program weakly exceeds that associated with the laissez-faire allocation. Recalling that the weight assigned to the undeserving poor in the social welfare function is zero, it follows by virtue of (B31) that

(B32) $\lim_{z\to 0}[c^h - g(n^h)] > \hat{c}^h - g(\hat{n}^h)$.

By virtue of (B30) and (B31) it follows that

(B33) $\lim_{z\to 0}[c^d - g\left(\frac{n^d w^l}{w^h}\right)] < \hat{c}^d - g\left(\frac{\hat{n}^d \hat{w}^l}{\hat{w}^h}\right)$.

By the strict convexity of *g*, it follows that under the laissez-faire regime the high-skilled workers strictly prefer their bundle to the bundle associated with the deserving poor. Formally,

(B34) $\hat{c}^h - g(\hat{n}^h) > \hat{c}^d - g\left(\frac{\hat{n}^d \hat{w}^l}{\hat{w}^h}\right)$.

Combining (B32)-(B34) yields

(B35) $\lim_{z\to 0}[c^h - g(n^h)] > \lim_{z\to 0}[c^d - g\left(\frac{n^d w^l}{w^h}\right)]$.

The inequality condition in (B35) implies that IC^{hd} is slack which contradicts claim 3.

We therefore obtain a contradiction to (B31) implying that

(B36) $\lim_{z\to 0}[c^d - g(n^d)] \geq \hat{c}^d - g(\hat{n}^d)$.

By the strict convexity of *g*, it follows that under the laissez-faire regime the deserving poor strictly prefer their bundle to the bundle associated with the undeserving poor. Formally,

(B37) $\hat{c}^d - g(\hat{n}^d) > \hat{c}^u - g(\hat{n}^u)$.

By virtue of claim 2 and the fact that *g*(0)=0 the participation constraint associated with the undeserving poor is binding and they receive their reservation utility given by zero:

(B38) $\lim_{z\to 0}[c^u - kg(n^u)] = 0 < \hat{c}^u - kg(\hat{n}^u)$, where the inequality sign follows from the fact that $\lim_{n\to 0} g'(n) = 0$, implying that the undeserving poor' utility associated with their laissez-faire allocation is bounded away from zero.

By virtue of (B30) and (B38) it follows that

(B39) $\lim_{z\to 0}[c^u - g(n^u)] < \hat{c}^u - g(\hat{n}^u).$

Combining the inequality conditions given in (B36), (B37), and (B39) implies that

(B40) $\lim_{z\to 0}[c^d - g(n^d)] > \lim_{z\to 0}[c^u - g(n^u)].$

Thus, IC^{du} is slack, which completes the proof.

Claim 10: If z is sufficiently small, the optimal solution to the relaxed program has $n^d > n^u$.

Proof: Consider the limiting case where $z \to 0$. The claim follows immediately from (B30), noting that by virtue of the convexity of g and the fact that k>1, the deserving poor will work more than the undeserving poor under the laissez-faire allocation.

Claim 11: If z is sufficiently small, in the optimal solution to the relaxed program the incentive-compatibility constraint IC^{hu} is slack.

Proof: Consider the case where z is sufficiently close to zero and suppose by way of contradiction that

(B41) $c^h - g(n^h) \leq c^u - g\left(\frac{n^u w^l}{w^h}\right).$

By virtue of claim 3, the incentive-compatibility constraint IC^{hd} is binding, hence

(B42) $c^h - g(n^h) = c^d - g\left(\frac{n^d w^l}{w^h}\right).$

Moreover, by virtue of claim 9 the incentive-compatibility constraint IC^{du} is slack; hence

(B43) $c^d - g(n^d) > c^u - g(n^u).$

Substituting for $c^h - g(n^h)$ from (B42) into (B41) yields

(B44) $c^d - g\left(\frac{n^d w^l}{w^h}\right) \le c^u - g\left(\frac{n^u w^l}{w^h}\right)$.

Subtracting (B43) from (B44) yields upon rearrangement

(B45) $g(n^d) - g\left(\frac{n^d w^l}{w^h}\right) < g(n^u) - g\left(\frac{n^u w^l}{w^h}\right) \Leftrightarrow H(n^d) < H(n^u)$,

where $H(n) \equiv g(n) - g(\frac{n w^l}{w^h})$ and $H' > 0$ by virtue of (B27).

Thus, (B45) implies that $n^u > n^d$. However, this contradicts claim 10.

Claim 12: If z is sufficiently small, in the optimal solution to the relaxed program the incentive-compatibility constraint IC^{ud} is violated.

Proof: We first show that in the optimal solution to the relaxed program the undeserving poor strictly prefer the bundle associated with the deserving poor to that associated with high-skilled workers. Suppose, hence, by way of contradiction the following:

(B46) $c^d - kg(n^d) \le c^h - kg\left(\frac{n^h w^h}{w^l}\right)$.

By virtue of claim 8, IC^{dh} is slack; hence

(B47) $c^d - g(n^d) > c^h - g\left(\frac{n^h w^h}{w^l}\right)$.

Subtracting (B46) from (B47) and rearranging yield

(B48) $(k-1)\left[g(n^d) - g\left(\frac{n^h w^h}{w^l}\right)\right] > 0$.

Since k>1, g'>0, and $w^h > w^l$, condition (B48) implies that $n^d > n^h$. However, by virtue of claims 5 and 7, $n^h > n^d$. Hence, we obtain a contradiction. It follows that

(B49) $c^d - kg(n^d) > c^h - kg\left(\frac{n^h w^h}{w^l}\right)$.

Let the welfare associated with the relaxed program, the extended program with no minimum wage, and the extended program with a binding minimum wage be denoted by

W^{relax}, W^{tax_only}, $W^{tax\_\min}$. Suppose now, by way of contradiction, that IC^{ud} is not violated, namely

(B50) $c^u - kg(n^u) \geq c^d - kg(n^d)$.

Combining (B49) and (B50) implies that IC^{uh} is slack. Thus, by virtue of claims 8, 9, and 11, the relaxed program violates none of the incentive-compatibility constraints of the extended program. Thus, the optimal solution to the relaxed program is a feasible solution for the extended program without a minimum wage. This implies that

(B51) $W^{tax\_only} \geq W^{relax}$.

By virtue of Proposition 1, supplementing the optimal tax-and-transfer system with a binding minimum wage is welfare enhancing. Thus,

(B52) $W^{tax\_\min} > W^{tax\_only}$.

Combining (B51) and (B52) implies that

(B53) $W^{tax\_\min} > W^{relax}$.

However, by virtue of claim 4, the welfare associated with the optimal solution to the relaxed program constitutes an upper bound for the welfare level associated with the optimal extended program where the minimum wage is an available supplementary tool to the tax-and-transfer system. It follows that

(B54) $W^{relax} \geq W^{tax\_\min}$.

Comparing (B53) and (B54) yields the desired contradiction. This completes the proof.

Claim 13: If z is sufficiently small, the optimal solution to the relaxed program can be implemented by a tax-and-transfer system supplemented by a binding minimum wage.

Proof: Let the triplet (c_*^i, n_*^i), where i= h, d, u, denote the optimal solution to the relaxed program. By virtue of claims 8, 9, and 11 and by construction of the optimal solution to the relaxed program, the allocation satisfies the revenue constraint, the undeserving poor' participation constraint, and the four incentive-compatibility constraints $IC^{du}, IC^{dh}, IC^{hu}$, and IC^{hd}. However, by virtue of claim 12, the undeserving poor strictly prefer the bundle associated with the deserving poor to any other bundle. Hence, the resulting allocation is not incentive compatible.

Suppose that the government sets a minimum wage equal to the low-skilled workers' marginal product associated with the optimal solution to the relaxed program. Formally, let $\overline{w} = \partial F(\alpha n_*^d + (1-\alpha) n_*^u, n_*^h)/\partial N^l$ denote the minimum wage. The introduction of the minimum wage results in involuntary underemployment. To see this, notice that both the deserving and the undeserving poor are willing to work n_*^d hours since, by claims 8, 9 and 12, both types strictly prefer the bundle (c_*^d, n_*^d) to any other bundle. This implies that the total labor supply of the low-skilled workers is given by n_*^d. However, the total labor demand for low-skilled workers is given by $\alpha n_*^d + (1-\alpha) n_*^u < n_*^d$, where the inequality sign follows from claim 10.

As demonstrated in Proposition 1, efficient rationing implies that the entire incidence of involuntary underemployment will fall on the undeserving poor. That is, the undeserving poor will become underemployed and only work n_*^u hours, whereas the deserving poor will continue to work n_*^d hours. Thus, the introduction of the binding minimum wage renders it infeasible for the undeserving poor to mimic the deserving poor. We conclude that the optimal solution to the relaxed program can be implemented with a tax-and-transfer system supplemented with a binding minimum wage. This concludes the proof.

Claim 14: The optimal marginal tax rate levied on the deserving poor in the optimal extended program with a minimum wage is strictly positive.

Proof: First, by claim 13, the optimal solution to the relaxed program is a feasible solution for the extended program with a minimum wage. Thus,

(B55) $W^{tax\_\min} \geq W^{relax}$.

However, by claim 4, the welfare associated with the optimal solution to the relaxed program constitutes an upper bound for the welfare associated with the optimal extended program when a minimum wage is an available supplementary tool to the tax-and-transfer system. It follows that

(B56) $W^{relax} \geq W^{tax\_\min}$.

Combining (B55) and (B56) implies that $W^{tax\_\min} = W^{relax}$. Hence, the optimal extended program with a minimum wage coincides with the optimal relaxed program. The proof is then established by recalling claim 5, which states that with the optimal relaxed program the marginal tax rate for the deserving poor is positive. This completes the proof.

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