Dynamic Asset Sales with Information Externalities*

Sivan Frenkel†

Job-Market Paper (December 8, 2013)

Abstract

I analyze a dynamic model of a public firm in which a transaction of an asset sale conveys information about the firm's value. The information release affects a manager whose compensation is sensitive to the stock price. The model is based on having, in every period, a potential buyer that, with some probability, engages in symmetric information bargaining with the manager. I examine how the information externality affects the timing of the transaction, the prices at which these assets are sold, and the pattern of stock prices before and after the sale. I then analyze how the division of bargaining power between the buyer and the seller affects the above. I then consider the case in which the firm sells only some of its assets, and therefore a transaction affects the market perception of the value of the remaining assets.

JEL Classification: G12, G14, D82, C73.

Keywords: asset sales, information externalities, asset prices.

---

*I thank Ilan Kremer for many helpful discussions. I gratefully acknowledge financial support from ERC grant number 263305.

†Center for the Study of Rationality and the School of Business Administration, Hebrew University of Jerusalem. E-mail: frenkel.sivan@mail.huji.ac.il; Web page: https://sites.google.com/site/sivanfrenkel/.

1 Introduction

Real actions made by firms are often observable, and as such enable market participants to learn more about the value of those firms. At the same time, managers' compensation in public firms is often tied to the stock price and is therefore affected by these actions. In this paper I seek to investigate how stock-sensitive compensation affects the real decision taken by firms. Much attention in the finance literature is paid to the behavior of market participants with superior information. The majority of these papers, however, deals with traders.[1] Empirical evidence shows that managers, too, take into account the information their actions disclose to the market. For example, Agarwal and Kolev (2013) show that managers of public firms prefer to conduct mass layoffs in a recession month more than managers in private companies. They conjecture the reason is that during such periods layoffs signal less about the firm's relative condition. Several papers (Sekine, Kobayashi, and Saita, 2003; Peek and Rosengren, 2005) have found evidence that banks in Japan continued to lend to severely impaired borrowers during the 1990s in order to avoid a realization of losses on their own balance sheets.[2] In this paper, I analyze a model where a manager takes into account the information that her real decisions reveal. I specifically examine a dynamic model of asset sales. A few examples include (a) a company that sells one of its divisions or operations, (b) a venture capital that sells one of the firms in its portfolio, and (c) a financial institution that sells assets from its balance sheet. The last case is most relevant in light of the recent financial crisis. During the crisis, the market volume in structured assets such as CDOs and MBSs has significantly declined. It was suggested that banks have avoided trade in these assets, because a trade would have forced them to write down the value of their inventory, which could lead to insolvency.[3]

A sale is usually preceded by a valuation process by the buyer, and therefore the sale

[1] Ever since Kyle (1985) we know that insiders with superior knowledge take into account how their information enters the price, and therefore they do not trade too aggressively on their information.

[2] Another example is earnings management using real activities such as temporary R&D decrease, price reduction, etc. (Roychowdhury, 2006; Cohen and Zarowin, 2010; Gunny, 2010; Zang, 2011). In this case the outcome is observed, whereas the actions are not.

[3] An example is an op-ed by By Kenneth E. Scott and John B. Taylor published July 21, 2009 in the *Wall Street Journal.* The authors write "In September 2008 credit spreads skyrocketed and credit markets froze. By then it was clear that the problem was not liquidity, but rather the insolvency risks of counterparties with large holdings of toxic assets on their books."

price reflects the information that is acquired in the process. An asset sale is profitable if the price is higher than what the asset is worth for the company. The reaction of the stock, on the other hand, depends on whether the price is above or below the market beliefs about the value of the assets. Managers with overvalued assets may therefore prefer not to sell their assets in order to maintain a high stock price, even when a sale would be profitable. When outsiders are rational, they should gradually infer that assets are overpriced if they do not see a sale for a long period of time. I am therefore interested in the dynamics of asset sales: how do prices change? Who sells first? Under what conditions does it take longer to sell an asset?

I explore these questions in a model where a manager has to decide when to sell a firm's assets whose value he privately knows. The manager's compensation is tied to the stock price and therefore he cares only about the market value of the firm in each period (I motivate this compensation scheme below when describing the model). Before a sale, this market value depends only on the beliefs of the market regarding the value of the assets, while after the sale it simply reflects the amount of cash that was received for the assets.[4]

In each period, the manager is approached, with some probability, by a potential buyer. In order to focus on the asymmetric information between the market and the firm, I simplify the model and assume that the buyer knows the value of the assets. Negotiations result in a price offer. This offer exists for one period only, and if the manager rejects it the offer expires and the buyer invests in a different project. I model the negotiations in reduced form, and examine different possible prices that are a result of different divisions of bargaining power. One extreme case is when the manager makes a take-it-or-leave-it offer to the potential buyer. The other is when the buyer makes such an offer to the manager. The manager can then simply decide whether to sell or not.

A crucial assumption is that offers are not public; therefore, if no sale takes place in a given period, outsiders cannot tell whether an offer was not made or whether it was rejected. As a result, managers with low-value assets are motivated to reject offers, because the price they can get for their assets is lower than the market valuation of these assets if they choose

[4] Section 6 presents an extension of the model where only part of the assets are sold, and market beliefs about the value of the post-sale inventory play an important role.

not to sell.[5] It is shown that in each period there is a threshold value, such that managers who receive an offer and have assets of higher value sell, while managers with assets of lower value refuse to sell. As time passes and no sale is observed, outsiders realize that the firm is likely to have low-value assets, market value decreases, and managers with assets of lower value agree to sell. The unique equilibrium I find here has the following properties:

- The market value of firms that have not been sold and average sale prices are decreasing over time.
- The "threshold type" (the manager who is indifferent between selling and not selling), and the average value of sold assets are decreasing over time. Thus, firms with more valuable assets are sold earlier on average.
- In each period, there is a positive fraction of managers who choose to sell despite the fact that the price they receive is lower than the market value their firm will receive if they don't sell.

The last property arises from the fact that because they expect to get lower prices in the future and are moreover unsure when will they get the next offer, managers are willing to sell today despite the fact that they can get a higher market value if they reject the offer. I show that the properties above hold for a model with any number of periods, including an infinite horizon.[6]

I then analyze how selling behavior is affected by the division of bargaining power. I assume that each manager receives an offer that is between her expected payoff if she does not sell (the minimum she is willing to sell for) and her assets' value (the maximum the buyer is willing to pay). I am able to obtain analytic proofs for the two-period case for any of those prices, and show that:

- When buyers have higher bargaining power (and thus sale prices are lower), sale thresholds are actually lower and thus the firm is sold earlier.

[5]When it is publicly known that the firm has received an offer, then all types sell immediately. This "unraveling" result was first presented by Grossman (1981) and Milgrom (1981).

[6]In the infinite horizon case uniqueness is only for the set of equilibria with threshold strategies.

- When buyers have higher bargaining power, the market value has a higher probability of falling as a result of a sale. In the extreme case where buyers make take-it-or-leave-it offers, market prices always fall after a sale.

When buyers have higher bargaining power, current and future prices are lower. The latter is more pronounced in equilibrium: the manager expects future prices to be low and therefore has a greater incentive to accept offers today. In the extreme case where the buyer makes a take-it-or-leave-it offer, I show that in equilibrium the buyer offers a constant price to the manager that is independent of the assets' value (of course, such a price is offered only to firms with an assets' value that is above that price). In each period, this price is not only below the current market value of the firm, but also below the market value of the firm if it rejects the offer. Nevertheless, managers do accept this offer.

I next proceed to analyze the case where only part of the firm's assets are for sale. In such a case, the information that is disclosed to the market as a result of a sale affects the market's beliefs about the value of the remaining inventory. I show that the impact of inventory is as follows:

- When only part of the assets is for sale thresholds increase, and so on average it takes longer to sell.

The intuition behind this result is that, since high types sell early, a sale always improves the belief of the market about the value of the inventory. As long as the seller has some bargaining power, prices are strictly increasing in value, and thus, since prices are publicly observable, the market can infer the value of the inventory. From the manager's point of view this increases the payoff from not only current but also future sales. I show that the latter effect is stronger, and since the manager can expect a higher payoff if he rejects the offer, sale thresholds increase.

Related Literature An established theoretical literature shows that managers with stock-based compensation may not have the same interests as shareholders (Fishman and Hagerty, 1989; Stein, 1989; Paul, 1992; for a recent survey see Bond, Edmans, and Goldstein, 2012, Section 3). In most of these papers, managers choose a non-profit-maximizing action (e.g., underinvestment in Fishman and Hagerty, 1989, earnings manipulation in Stein, 1989) because the market cannot directly observe the actions of the manager. Paul (1992) shows that even when prices aggregate all information about the assets' value, they do not necessarily serve as a good signal of the action of the manager. Bond, Edmans, and Goldstein (2012) refer to this as the difference between "forecasting price efficiency" and "revelatory price efficiency." In such environments efficiency increases when the actions of the manager are more accurately observed by the market, since this gives the managers less incentives to manipulate.[7] A few papers discuss how stock-based compensation may result in actions that are directly meant to conceal or reveal information from the market. An exception is Benmelech, Kandel, and Veronesi (2010). In their paper it is shown that in order to prevent a sharp price reduction, managers may use a suboptimal investment policy to conceal the fact that the firm's growth opportunities have declined.

Several papers discuss dynamic problems that share some features with mine. Grenadier and Malenko (2011), Morellec and Schürhoff (2011), and Bustamante (2012) all analyze dynamic signaling in a real-options model with asymmetric information, where the exercise timing of an option may signal its value. As in my case, they show that when the decision-maker benefits when outsiders believe that the project's value is higher than it is in reality, higher types exercise earlier. In these papers early exercise is done to signal high value, while in my paper low types postpone a sale to prevent information disclosure, taking advantage of the fact that outsiders do not know whether they have gotten a sale offer or not. Janssen and Roy (2002) and Fuchs and Skrzypacz (2013) analyze dynamic versions of Akerlof's lemon market model, where sellers are more informed than buyers, and care about the price they get and not about the market's response. In the equilibrium of their model bad types sell early, because good types can signal their quality by waiting. The difference in results compared to this paper is due to the different source of adverse selection.

This paper is closely related to the literature, mostly in accounting, that discusses voluntary information disclosure by firms. Dye (1985) was the first to analyze a static model of voluntary disclosure when the manager's payoff depends on the beliefs of the market, and

[7] Fishman and Hagerty (1989) show this may actually act as an incentive for the manager to commit to a higher degree of disclosure.

the market does not know whether the manager has information to disclose. I follow Dye (1985) when I assume that the market does not know whether the firm has received a sale offer. This paper, however, deals with real decisions and therefore consider inventories and divisions of bargaining power. I make the plausible assumption that sale offers are available for a limited time, while dynamic models of disclosure, e.g. Guttman, Kremer, and Skrzypacz (2013), assume that once information is acquired, it can be disclosed at any time.[8]

Finally, Milbradt (2012) and Bond and Leitner (2013) present dynamic models where a firm has an inventory of assets and takes into account how its current trade influences the value of the remaining inventory. I present a similar problem in Section 6, where the market learns about the value of the inventory from a sale, and this has an effect on the manager's payoff. Both papers find that firms may choose not to trade in order not to reveal negative information about the value of the inventory. In my model, the impact of inventory is more subtle because inventory affects the payoff of the manager whether she sells now or whether she chooses to wait. I show that an inventory improves the payoff more in the future and thus gives the manager an incentive to defer.

The rest of the paper is organized as follows. The model is formally introduced in Section 2. Some general properties of an equilibrium with threshold strategies appear in Section 3. Section 4 presents the main results in the simple two-period case. Section 5 shows that the nature of the equilibrium is similar for a longer horizon, whether finite or infinite. Section 6 explores how the presence of inventories affects the results. Proofs that do not appear in the main text are relegated to the Appendix.

2 A Model

There is a single firm who owns an asset with a fundamental value of v. The value is randomly drawn in the beginning of the game and remains constant throughout the game. The initial distribution of v can be represented by a cumulative distribution F_1 with a partial distribution

[8] Acharya, DeMarzo, and Kremer (2011) present a dynamic disclosure model where the focus is on how the timing of disclosure is affected by an exogenous public signal. Einhorn and Ziv (2008) and Beyer and Dye (2012) present dynamic disclosure models where the information of the manager is always disclosed in each period and the focus is on reputational effects that do not appear in my model. In my model, information is disclosed only as a result of a sale.

f_1 that is non-atomic and has full support[9] over a subset of $\mathbb{R}$. Notice that v may or may not be bounded from above or below. In what follows I abuse notation and denote by $\underline{V}$ and $\overline{V}$ the lower and upper values, respectively, where $\underline{V} < \overline{V}$, $\underline{V} \in \mathbb{R} \cap \{-\infty\}$, and $\overline{V} \in \mathbb{R} \cap \{\infty\}$. I also treat the interval of v as open (and therefore use limits), though I allow $v = \underline{V}$ and/or $v = \overline{V}$. Time is discrete and starts in period 1 (Section 5 analyzes both finite and infinite horizons), and stage payoffs are discounted with a discount factor of β.

Periodic Sell Offers In each period, with probability q a potential buyer arrives and negotiates with the manager a price offer to buy the firm's assets (Section 6 presents an extension where only part of the firm's assets are for sale). The buyer is *short-lived*: if there is no deal the buyer leaves and the firm has to wait until it receives an additional offer before it can sell.

Sale Prices In order to focus on the asymmetric information between the market and the firm I assume that prices can be conditioned on the value of the firm's assets. That is, either each potential buyer does a due diligence and therefore knows v at the time he approaches the firm, or this investigation is conducted as part of the negotiations.

The selling price in each period is a result of a bargaining process between the buyer and the seller (the firm's manager). I do not model the bargaining explicitly, but represent it as a reduced form price function $p\left(v, u^{\text{NS}}(v)\right)$, where v is the value of the seller's assets and u^{NS} represents its outside option if she does not sell as described below.

If the manager chooses to sell in period t for a price p, then the value of the firm from that point is simply the cash holdings of the firm, which is p. If the manager chooses not to sell, he expects some payoff that depends on the probability he will get offers in the futures and on the prices of these offers. This payoff is the manager's outside option in the current negotiation, and in this model it arises endogenously in equilibrium. Formally, define $U_t^{\text{NS}}(v)$ as the expected discounted payoff of a type v manager if she does not sell in period t, and

[9] These assumptions on f_1 are for simplicity. Most results hold qualitatively even without them.

behaves optimally from period $t+1$ onwards. Now define u_t^{NS} using the equality

$$u_t^{\text{NS}}(v) \equiv \frac{U_t^{\text{NS}}(v)}{\sum_{\tau=t}^{T} \beta^{\tau-t}} = \frac{1-\beta}{1-\beta^{T-t+1}} U_t^{\text{NS}}(v).$$

u_t^{NS} is periodic payout of an annuity of size U_t^{NS}: if she receives u_t^{NS} periodically from period t to the end of the game, the expected discounted payoff of the manager is U_t^{NS}. Normalized that way, it is clear that the manager sells only if $p\left(v, u^{\text{NS}}(v)\right) \geq u_t^{\text{NS}}(v)$.

An asset of type v is worth v to the buyer. I normalize the outside option of the buyer (its payoff in case he does not buy) to zero. Given the above, it is natural to focus on price functions of the type

$$p\left(v, u_t^{\text{NS}}(v)\right) = (1-\lambda) \cdot u_t^{\text{NS}}(v) + \lambda \cdot v, \tag{1}$$

where $\lambda \in [0,1]$ is the relative bargaining power of the seller (the firm). This formulation is standard and nests many models of bargaining with symmetric information, such as Nash (1950) and Rubinstein (1982): λ may be a result of, say, different discount factors, or conditions of supply and demand. The extreme cases where $\lambda = 1$ and $\lambda = 0$ represent the seller and the buyer making a take-it-or-leave-it offer respectively. In such cases the party that receives the offer is left with no surplus due to a sale. Note that $u_t^{\text{NS}}(v)$ is a function of future prices, and therefore is also a function of λ.

While I analyze a price function where the relative bargaining power λ is independent of v, this assumption can be somewhat relaxed without changing the results, as long as the price function is weakly increasing in v.[10] If, however, λ depends also on u^{NS}, then an equilibrium with threshold strategies of the type I characterize below may not exist.[11]

As part of the definition of equilibrium, I assume that the price function $p(\cdot)$ is efficient; that is, if $u_t^{\text{NS}}(v) \leq v$, then type v sells in equilibrium in period t. The idea is that a price $p' < u_t^{\text{NS}}(v)$ (that results in rejection) cannot be part of an equilibrium because the buyer

[10] If p is decreasing in v then a threshold equilibrium may not exist. Notice, however, that in such a case a firm has an incentive to destroy value, and so weak monotonicity is a plausible assumption for an equilibrium price function.

[11] An example of such a case is a price function where type v has a lot of bargaining power today, and therefore $p\left(v, u_t^{\text{NS}}(v)\right) = v$, but has weak bargaining power next period, and so $p\left(v, u_{t+1}^{\text{NS}}(v)\right) = u_{t+1}^{\text{NS}}(v)$. In contrast, type $v' > v$ has weak bargaining power today ($p\left(v', u_{t+1}^{\text{NS}}(v')\right) = u_{t+1}^{\text{NS}}(v')$) and strong bargaining power next period ($p\left(v', u_t^{\text{NS}}(v')\right) = v'$). While type v has an incentive to sell in period t, type v' has an incentive to wait if q is high enough, thus resulting in non-monotone selling strategy.

has an incentive to offer a higher price $p \in \left[u_t^{\text{NS}}(v), v\right]$ that is profitable to both parties, and there is nothing that prevents the buyer from doing so (recall that the buyer is a short-term player, and so his sole consideration is a specific negotiation).

Market Values The firm is traded at a market value of h. For simplicity I assume risk-neutral pricing, where the market value of the firm equals the expected fundamental given the beliefs of the market. Note that actual market values may differ from the fundamental due to bargaining power. That is, since the assets have some probability to be sold for a price that is lower than the fundamental, this should be embodied in the market value. I simplify the model by not taking the bargaining power into account and assuming stock price only reflects the fundamental. This assumption makes the model simpler and tractable to analyze. In the two period case, taking into account the actual sale prices does not change the qualitative results.[12]

The market is uncertain about the value of the firm only before a sell is made, when it owns an asset with unknown value: after the deal the firm's only asset is cash, and therefore its market value simply equals p. In what follows, I let the public belief about the value of a firm that did *not* sell until period t be distributed according to a CDF $F_t(v)$ with PDF $f_t(v)$.

Manager's Compensation The manager's compensation is an increasing function of the firm's price, and therefore she maximizes in each period the discounted sum of future market values. One can think of at least two interpretations for this payoff, both a result of asymmetric information in other operations of the firm, which I leave unmodeled for brevity. First, shareholders may give the manager a stock-based compensation. This may be either in order to incnetivize the manager to exert costly effort in other operations where the manager's action is unobserved (Harris and Raviv, 1979; Hölmstrom, 1979), or as a result of a majority of the shareholders having short-term incentives and wishing to maximize stock-price. Another

[12] The the possibility of a sale results in lower market value compared to the expected fundamental (because the asset will be sold for less than its value). In longer horizons, this results in an "horizon effect": as the games progress, the horizon is shorter and therefore the probability of a sale and its resulting "discount" is lower. Under my analysis, as will be proven later, prices decrease over time. As long as the horizon effect is not too strong the results do not change. This thing can be assured by putting bounds on the parameters. If, however, the horizon effect is strong enough to result in an increasing price trend, then the equilibrium I describe no longer holds.

interpretation is that the market value serves as a proxy for the manager's unobservable ability (as in Holmström, 1999), again because it is a result of additional actions that he can do in order to increase the value of the firm. Under this interpretation, the manager's payoff is a result of career concerns: he wishes to appear as competent as possible in order to increase her future compensation.

Equilibrium I find a perfect Bayesian equilibrium. Such an equilibrium is composed of a strategy of the seller to sell or not to sell in each period ($\{\mathrm{S}, \mathrm{NS}\}$), of a price p_t that depends (through u_t^{NS}) on the current and future posteriors of the market regarding the asset's value and on the seller's strategy, and of a belief of the market about the value of the firm that hasn't yet sold, F_t. This belief F_t is updated using Bayes law from F_{t-1} given the strategy of the firm in period t, and it fully determines the stock price h_t in that period.

3 Properties of Equilibrium with Threshold Strategies

In what follows, I will prove that in the finite horizon case the unique equilibrium has a threshold-selling strategy; that is, in each period all types above some threshold choose to sell. This section reviews some properties of such an equilibrium, which are used later. Henceforth, I denote the threshold type in period t in general by v_t^*.

Posteriors When threshold strategies are used, then the posterior in period $t+1$ following no-sale is

$$f_{t+1}(v) = \begin{cases} \frac{f_t(v)}{1-q+qF_t(v_t^*)} & v < v_t^* \\ \frac{(1-q)f_t(v)}{1-q+qF_t(v_t^*)} & v \geq v_t^* \end{cases} \text{ and } F_{t+1}(v) = \begin{cases} \frac{F_t(v)}{1-q+qF_t(v_t^*)} & v < v_t^* \\ \frac{qF_t(v_t^*)+(1-q)F_t(v)}{1-q+qF_t(v_t^*)} & v \geq v_t^* \end{cases}. \tag{2}$$

f_t and F_t are functions of all the previous threshold strategies $\{v_\tau^*\}_{\tau=1}^t$, as well as q, but in the sequel I usually omit those variables when the context is clear. Notice that F_t FOSD F_{t+1} ($F_{t+1}(v) > F_t(v)$ for all v) and thus $E_t(v) > E_{t+1}(v)$ (where $E_t(v) \equiv E_{F_t}(v)$).

Market value decreasing over time When threshold strategies are used, the market value of a firm that did not sell in period t is (Jung and Kwon, 1988)

$$h_t(v_t^*, q) \equiv \frac{(1-q) \cdot E_t(v) + q \cdot F_t(v_t^*) \cdot E_t(v \mid v \leq v_t^*)}{1 - q + q \cdot F_t(v_t^*)}, \tag{3}$$

where the subscript t in h_t denotes the fact that F_t is used. Notice that $h_t(v_t^*, q) < E_t(v)$ for all v_t^*. Also, by definition, $h_t(v_t^*, q) = E_{t+1}(v)$, because $h_t(v_t^*, q)$ is exactly the expected value of a firm that hasn't sold in period t. The following lemma is therefore immediate:

Lemma 1. *If the firm uses a threshold strategy, its market value is strictly decreasing over time until a sale, that is,* $h_{t+1}(v_{t+1}^*, q) < h_t(v_t^*, q)$ *for all* v_t^* *and* v_{t+1}^*.

The threshold type receives fair pricing An additional property of the threshold equilibrium is that in equilibrium the threshold type receives $p_t(v_t^*) = v_t^*$, no matter what the division of bargaining power is. This is a result of the efficiency requirement.

Lemma 2. *If the firm uses a threshold strategy* v_t^* *and prices are weakly increasing in* v*, then* $p_t(v_t^*) = v_t^*$.

Proof. Assume $p_t(v_t^*) < v_t^*$. Then, there exists a type $p_t(v_t^*) < v' < v_t^*$. Since $p_\tau(v)$ is weakly increasing by definition (as is obvious from 1) for all periods and specifically for $\tau \geq t+1$, then $u_t^{\text{NS}}(v)$ is also weakly increasing in v. Thus, type v' also agrees to sell for $p_t(v_t^*)$ and such a sale is profitable to the buyer – a contradiction of the fact that type v' does not sell. □

4 Equilibrium in the Two-Period Case

I start by finding the equilibrium for the two-period case. This allows me to develop all intuitions in a relatively simple environment. I solve the model using backward induction, thus solving first the last period and then the first period.

4.1 Second Period / A Static Benchmark

The last period is identical to a static one-shot model.[13] In this period, the manager decides between selling and obtaining $p_2(v)$ and not selling and obtaining h. Thus, type v will sell iff $p_2(v) \geq h$ (in the terminology introduced above, $u_2^{\text{NS}}(v) = h$). Because $p_2(v)$ is weakly increasing in v (equation 1), the selling strategy is a threshold strategy. Thus, given a threshold equilibrium v_2^*, the market value is simply $h_2(v_2^*, q)$ (equation (3)). Given Lemma 2, the threshold is defined using the following indifference condition:

$$v_2^* = h_2(v_2^*, q).$$

Notice that the set of types that sell does not depend at all on the selling prices (but does depend on the distribution, which, as we shall see, does depend in a dynamic model on the distribution of bargaining power).

In what follows, it will be useful to define $v^M(F, q)$ as the solution to the equality

$$v^M(F, q) = h\left(v^M; F, q\right),$$

where F represents the distribution of types. We can interpret v^M as the "myopic" solution, when $\beta = 0$ and the manager only cares about the current period. Given this definition, we can write $v_2^* = v_2^M$. A useful property of v^M, found by Acharya, DeMarzo, and Kremer (2011) and extensively used below, is the fact that the price function $h(v; F, q)$ of (3) has a single minimum in v^M :

Fact ("The Minimum Principle," Acharya, DeMarzo, and Kremer, 2011, Proposition 1)**.** v^M *is unique and satisfies* $v^M(F, q) = \min_v h(v; F, q)$.

Given the minimum principle, the equilibrium threshold in the second period is unique for a given F_2.[14] The following lemma sums up the observations above:

[13] This example is very similar to the voluntary disclosure model of Dye (1985) and Jung and Kwon (1988), the major difference being that in case the manager decides to sell (equivalent to disclosure in Dye, 1985), he do not necessarily receive v.

[14] In any threshold equilibrium, there are many prices for values where there is no sale ($v < v^*$) that are in equilibrium (any price below v_2^M), and so the equilibrium is unique up to prices below the threshold. In what follows I ignore this multiplicity and say that the equilibrium is unique when the set of selling types is unique.

Lemma 3. *In the second (and last) period, a manager sells if and only if $v \geq v_2^M$, and the market value of a firm that does not sell is v_2^M. This result is independent of selling prices.*

The sale price depends on the division of bargaining power, and is

$$p_2(v) \equiv p\left(v, v_2^M\right) = (1-\lambda)\cdot v_2^M + \lambda \cdot v \geq v_2^M. \tag{4}$$

Notice that in the scenario where the buyer has all the bargaining power ($\lambda = 0$), all types receive the same payoff, whether they sell or not.

4.2 First Period

In period 1, the firm cares also about its expected payoff in period 2. The following lemma assures that the equilibrium strategy in period 1 is also a threshold.

Lemma 4. *The optimal strategy is a threshold strategy in period 1.*

Proof. Assume an arbitrary strategy in period 1, $x_1 = \{v \mid v \text{ sells in period 1}\}$, and denote no sale by "NS". Let $F_2(x_1)$ be the updated prior of the market in the beginning of period 2, given that the manager used strategy x_1 in period 1.

The payoff of the manager from selling is $(1+\beta)\cdot p_1$ because the market value of the firm following a sale is simply the cash received by the sale. A seller of type v' sells if and only if $p_1(v') \geq u_1^{\text{NS}}(v')$, and given Equation (1) the condition becomes $v' \geq u_1^{\text{NS}}(v')$. The outside option $u_1^{\text{NS}}(v')$ is calculated by

$$(1+\beta)u_1^{\text{NS}}(v') \quad = \quad E_1\left(v|\text{NS}; x_1\right) + \beta(1-q\lambda)\cdot v_2^M + \beta q\lambda \max\left\{v', v_2^M\right\}. \tag{5}$$

Since $\frac{\partial u_1^{\text{NS}}(v')}{\partial v'} < \lambda = \frac{\partial p_1(v')}{\partial v'}$, if the inequality holds for some v' it also holds for all $v > v'$, and so the equilibrium strategy is a threshold. □

I now prove the following properties of the threshold in the first period, v_1^*.

Proposition 1. *In a two-period model:*

1. *Thresholds are decreasing:* $v_1^* > v_2^*$.

2. *In the first period, the market value of the threshold type after a sale is below the market value of a firm that hasn't sold:* $h_1(v_1^*, q) > v_1^*$.

3. *In the first period, the threshold type is lower than the myopic threshold:* $v_1^* < v_1^M$.

Proof. (1) Assume on the contrary that $v_1^* \leq v_2^* = v_2^M$. Then, type v_2^M strictly prefers to sell in period 1:

$$h_1(v_1^*, q) + \beta \cdot v_2^M \quad < \quad (1+\beta) \cdot p_1\left(v_2^M\right) \leq (1+\beta) \cdot v_2^M.$$

We get $h_1(v_1^*, q) < v_2^M = h_2\left(v_2^M, q\right)$ – a contradiction of Lemma 1.

(2) Given part (1), type v_1^* is indifferent in period 1 but strictly prefers to sell in period 2, and using Lemma 2, v_1^* satisfies

$$(1+\beta) \cdot v_1^* = h_1(v_1^*, q) + \beta(1-q)v_2^M + \beta q \cdot p_2(v_1^*). \tag{6}$$

Since $p_2(v_1^*) \leq v_1^*$ and $v_1^* > v_2^M$ (part (1)), then $h_1(v_1^*, q) > v_1^*$. Part (3) is immediate from (2), given the minimum principle. □

Part (3) of Lemma 1 shows that the dynamic setup leads to more sales. This is because a firm that does not sell expects lower prices in the future. Future prices are more important to the threshold type than current prices because this type receives fair pricing (Lemma 2).

One obvious question is what happens to the average value of sold assets over time, as both the thresholds and the value distribution of a firm that has not yet sold change. The following lemma shows that this average value is decreasing with time.

Lemma 5. *The average value of assets that are sold in each period,* $E_t[v \mid v \geq v_t^*]$, *is decreasing over time.*

Proof. Using the posterior at (2), we can see that $E_t[v \mid v \geq v_t^*] = E_{t+1}[v \mid v \geq v_t^*]$. Since thresholds are decreasing, $E_{t+1}\left[v \mid v \geq v_{t+1}^*\right] < E_{t+1}[v \mid v \geq v_t^*]$. Thus, $E_{t+1}\left[v \mid v \geq v_{t+1}^*\right] < E_t[v \mid v \geq v_t^*]$. □

4.3 Bargaining Power

This section analyzes how the thresholds depend on the division of bargaining power that is captured by the parameter λ. Given Lemma 1 one can substitute the prices of period 2 (equation 4) in condition (6) to get

$$v_1^* = \frac{h_1(v_1^*, q) + \beta(1 - q\lambda)v_2^*}{1 + \beta(1 - q\lambda)}. \tag{7}$$

Notice that $h_1(v_1^*, q) > v_1^* > v_2^*$. This, however, is not enough to conclude that v_1^* is increasing in λ because v_2^* is a function of the distribution in the second period, F_2, and therefore is a function of v_1^*. Moreover, from part (3) of Lemma 1 and the minimum principle we know that $h_1(v_1^*, q) = E_2(v)$ is decreasing in v_1^*, and so the impact of the lower threshold in period 1 on the threshold in period 2 is not immediate. Fortunately, this problem can still be solved for two periods and conclude that there are more sales when the buyer has more bargaining power. As part of the proof I also prove the uniqueness of the equilibrium.

Proposition 2. *In a two-period model,*

1. *The equilibrium thresholds are unique.*
2. $\frac{\partial v_t^*}{\partial \lambda} > 0$ *for* $t \in \{1, 2\}$*. That is, more types sell when the buyer has more bargaining power, and the asset is sold faster.*

The result of Proposition 2 is not intuitive, because when the buyer has more bargaining power prices are lower, and yet assets are sold faster. The intuition for the first period is that when the buyer has more bargaining power the seller expects lower prices in the future and thus have an incentive to sell today. Since more types sell in the first period, the distribution of types in the second period is more skewed and this results in more sales in the second period.

4.3.1 Price Reaction and Bargaining Power

Since the market value in this model is always below the mean type and above the minimal type (Equation 3), and since a sale discloses the value of the firm, then following a sale there can be both price jumps and price falls. A natural question is how the relative probability of jumps to falls is affected by the bargaining power. Moreover, from part (2) of Proposition 1, we know there is always a positive probability that a manager sells despite the fact that in that period she can get a higher market value (and payoff) by not selling. The last result is due to the fact that the manager expects lower prices in the future and so she has an incentive to sell today despite the short-term "loss." The next lemma shows that the probability of a price fall as well as a short-term loss is increasing when the buyer has more bargaining power.

Lemma 6. *When the buyer has more bargaining power:*

1. *The probability that the market value of a firm falls following a sale increases.*
2. *The probability that the manager sells in the first period for a price lower than the market value of a firm that does not sell increases.*

When the buyer has enough bargaining power, both of these probabilities equal one.

One can interpret bargaining power as market conditions. Given this interpretation, one receives an empirical prediction that more price falls will be observed when the market conditions are bad (assuming there is no recovery in sight). More importantly, the second part of the lemma suggests that we will see many firms that sell and have significant lower market value than similar firms (i.e., close competitors) that did not sell. This observation is obtained despite the fact that firms do not have any financial stress that results in "fire sales." This is simply a result of the dynamic consideration of firms when market participants have rational expectations.

5 Longer Horizons

5.1 Finite Horizon (T Periods)

In this section I generalize the analysis above to an arbitrary number of periods, and get the same results. In such a model, periods T and $T-1$ are identical to the two-period model. I can then solve for all other periods using backward induction. The main results of my model are summarized in the following proposition.

Proposition 3. *In a model with T periods:*

1. *The equilibrium involves a unique threshold selling strategy in all periods.*

2. *Thresholds are decreasing: $v_t^* > v_{t+1}^*$ for all $t \in \{1, \ldots, T-1)$.*

3. *In every period but the last, the market value of the threshold type falls after the sale and is below the market value of a firm that does not sell: $v_t^* < h_t(v_t^*, q) < h_{t-1}\left(v_{t-1}^*, q\right)$ for all $t \in \{1, \ldots, T-1\}$.*

Given an optimal strategy with decreasing thresholds, one can write explicitly the outside option of the manager. Assume that $v_{t+1}^* \leq v$ and so the manager would like to sell in period $t+1$ and all subsequent periods if she has an offer. We have seen in the proof to the two-period case (Proposition 1) that $u_T^{\mathrm{NS}}(v) = v_T^M$ and $u_{T-1}^{\mathrm{NS}}(v) = \frac{h_{T-1}\left(v_{T-1}^*, q\right) + \beta\left[(1-q\lambda)\cdot u_T^{\mathrm{NS}} + q\lambda v\right]}{1+\beta}$. This can be generalized, using (1), to give the following representation of an outside option

$$u_t^{\mathrm{NS}}(v) = \frac{\sum_{\tau=t}^{T} \left[\beta(1-q\lambda)\right]^{\tau-t} h_\tau(v_\tau^*, q) + v \sum_{\tau=t}^{T} \beta^{\tau-t}\left(1-(1-q\lambda)^{\tau-t}\right)}{\sum_{\tau=t}^{T} \beta^{\tau-t}} \tag{8}$$

for type $v \geq v_{t+1}^*$ that has not yet sold. Given Lemma 2, the indifference condition for type v_t^* in period t is simply $v_t^* = u_t^{\mathrm{NS}}(v_t^*)$. Substituting (8) and rearranging results in

$$v_t^* \equiv \frac{\sum_{\tau=t}^{T} \left[\beta(1-q\lambda)\right]^{\tau-t} h_\tau(v_\tau^*, q)}{\sum_{\tau=t}^{T} \left[\beta(1-q\lambda)\right]^{\tau-t}}. \tag{9}$$

As in the two-period case, the threshold is an average of the market values that the firm will get in the future, discounted and weighted by the probability the firm arrives that period without an offer. This threshold is lower than the myopic threshold v_t^{M} due to the risk that, if the firm does not sell today, it will be a while before it receives an additional offer and will receive a low market value in the meantime. Numeric simulations show that, as in the two-period case, selling thresholds are increasing in λ in the general finite horizon case, but I have not managed to improve this property analytically.

5.2 Infinite Horizon

I can extend my analysis to an infinite horizon. In this case, I cannot prove that the unique equilibrium has a threshold strategy. Instead, I will simply focus on such an equilibrium, motivated by the results of the finite-horizon case. The analysis in this case is somewhat different, since I cannot use backward induction. Nevertheless, I can still prove that thresholds are decreasing.

Lemma 7. *In the infinite-horizon case, the only equilibrium with threshold strategies involves thresholds that are strictly decreasing over time.*

Given the above lemma, one can write the outside option of a type $v > v_{t+1}^*$ that has not yet sold until period t, as

$$u_t^{\text{NS}}(v) = (1-\beta)\left\{\sum_{\tau=t}^{\infty}\left[\beta(1-q\lambda)\right]^{\tau-t} h_\tau(v_\tau^*, q) + v\left(\frac{1}{1-\beta} - \frac{1}{1-\beta(1-q\lambda)}\right)\right\}.$$

The indifference condition $v_t^* = u_t^{\text{NS}}(v_t^*)$ can thus be rewritten as

$$v_t^* = [1-\beta(1-q\lambda)]\sum_{\tau=t}^{\infty}[\beta(1-q)\lambda]^{\tau-t}\, h_\tau(v_\tau^*, q).$$

This condition is similar to the limit of (9) when $T \to \infty$.

6 Inventories

In this section I consider the case where a firm does not sell all its assets. Specifically, I assume that a sale involves only α of the firm's assets, and $1-\alpha$ of the firm's assets stay on its balance sheet. When a firm sells only part of its assets, the manager cares not only about the actual price received for the assets sold, but also about the effect that this sale has on the market value of the remaining inventory.[15]

By definition of the price function (Equation (1)), the price is strictly increasing in v for all $\lambda \in (0,1]$. In what follows I analyze this case, where the sale price is a perfect signal

[15] I could have assumed that the assets that are being sold and the assets that remain on the balance sheet are not identical but that their values are correlated, and the results would have been qualitatively unchanged.

about the remaining inventory's value.[16] In such a case, the payoff of the seller following a sale is simply

$$\alpha \cdot p(v, u^{\mathrm{NS}}) + (1-\alpha)v = \alpha(1-\lambda) \cdot u^{\mathrm{NS}}(v) + [1-\alpha(1-\lambda)]\, v.$$

Thus, the equilibrium is similar to a model where the seller has more bargaining power (the weight of v is higher). From Proposition 2 we know this means that the equilibrium threshold is higher. This is formally written below.

Corollary 1. *In the two-period inventory case, when $\lambda \in (0,1]$ and sale prices are publicly observed, selling a smaller part of the assets results in higher thresholds: $\frac{\partial v_t^*}{\partial \alpha} < 0$ for $\lambda \in (0,1)$ and $\frac{\partial v_t^*}{\partial \alpha} = 0$ for $\lambda = 1$, where $t \in \{1,2\}$.*

That is, when the inventories are higher than the amount sold, fewer types are willing to sell, and the probability of a sale is lower. My result is similar to that of Milbradt (2012) and Bond and Leitner (2013). In both of these papers, inventories may result in "market freeze," where assets are not sold. The underlying mechanism, however, is different. In those papers, firms forgo profitable transactions because these transactions reduce the value of their inventories. In contrast, here the result arises from the dynamic nature of the problem. An inventory makes a sale more profitable, because the manager profits from the disclosure of its type – she receives $p \le v$ for the assets she sells and v for those that remain. Surprisingly, this results in fewer sales in the first period, because the seller can expect a more profitable trade in the future. In Milbradt (2012), the value of the inventory is updated due to fair value accounting of illiquid ("level 3") assets. In his paper, in contrast to mine, rational expectations do not play a role and therefore market value does not change when there is no transactions. Moreover, since in Milbradt (2012) firms can and always trade when the fundamental increases, there cannot be price jumps but only price falls. In contrast, my paper predicts both price jumps and price drops following a sale, and I tie between bargaining power and the relative probability of price falls. In Bond and Leitner (2013), as in my case, the

[16] When $\lambda = 0$ the price conveys no information, and so the seller's payoff following a sale in period t is $p_t(v) + E_t\left(v \mid v \ge v_t^*\right)$. The results shown below do not hold in this case. In fact, the results are reversed and the introduction of inventories results in lower thresholds. For brevity, and since this is a knife-edge case, I do not analyze this case; but the analysis is available upon request.

value of the assets is a result of rational expectations. In their “lemon market” setup, trade always decreases the value of the remaining assets due to adverse selection, while in my case, a sale may reveal positive information for high types. They predict a price increase (“run-up”) and then a halt in trade (“freeze”), while I predict that trade will be delayed (on average), but assets will be sold eventually.

7 Concluding Remarks

In this paper I analyzed a dynamic model of asset sales. While the model is stylized, I believe some lessons can be learned from it. First, managers in public companies may forgo profitable sales or engage in unprofitable sales due to considerations of information that is revealed through prices. [17] This inefficiency exists despite the fact that the bargaining procedure between the buyer and the seller is efficient. Second, rational expectations prevent a complete market breakdown in a dynamic setup, and therefore assets will be sold eventually. Third, the time-to-sale of an asset depends on the expected future prices of this asset in case it is not sold today. Thus, when the seller benefits more in the future, she has fewer incentives to sell today, and this is true no matter what the current prices are. Thus, sales are delayed when the seller has more bargaining power and there is a larger inventory.

A Proofs

A.1 Proof of Proposition 2

Proof. I prove the Proposition in the following steps:

1. First, define a function $v_2^M(v_1)$ implicitly using the following equality:

$$v_2^M(v_1) = h_2\left(v_2^M(v_1), q; v_1\right) = \frac{(1-q)\cdot h_1(v_1, q) + q\cdot F_2\left(v_2^M(v_1); v_1\right)\cdot E_2\left(v \mid v \leq v_2^M(v_1); v_1\right)}{1-q+q\cdot F_2\left(v_2^M(v_1); v_1\right)},$$

[17] The question whether a sale is profitable or not depends on the value of the asset to the shareholders. If, for example, an asset is worth v to the buyer and $v-\Delta$ to the shareholders (where $\Delta > 0$), then trade is always profitable for the firm, while the opposite is true where $\Delta < 0$. Both cases can be incorporated easily into the model.

where $F_2(\cdot; v_1)$ is the distribution of types in period 2 given v_1 as defined in (2), $E_2(\cdot)$ means the expectation is calculated using F_2, and I use the fact $E_2(v) = h_1(v_1, q)$. Notice that $v_2^M(v_1) \leq h_1(v_1, q)$ for all v_1, and the inequality is strict when $F_2(v_2^M; v_1) \neq 1$ or $F_2(v_2^M; v_1) \neq 0$.

2. Define $\widetilde{v}$ by $v_2^M(\widetilde{v}) = \widetilde{v}$. I now prove that $\widetilde{v}$ exists, is unique and satisfies $\widetilde{v} = \min_{v_1} v_2^M(v_1)$.

   I first prove existence of $\widetilde{v}$. From (2) notice that, for any v, $\lim_{v_1 \to \underline{V}^+} F_2(v) = \lim_{v_1 \to \overline{V}^-} F_2(v) = F_1(v)$ (I abuse notation and allow $\underline{V}$ and $\overline{V}$ to be ∞ if the support is not bounded from below or above). Therefore $\lim_{v_1 \to \overline{V}^-} v_2^M(v_1) = \lim_{v_1 \to \underline{V}^+} v_2^M(v_1) = v_1^M$ so $\lim_{v_1 \to \overline{V}^-} \left[v_2^M(v_1) - v_1\right] < 0$ and $\lim_{v_1 \to \underline{V}^+} \left[v_2^M(v_1) - v_1\right] > 0$. By continuity, there exists $\widetilde{v} \in (\underline{V}, \overline{V})$.

   Next notice that, since $v_2^M(v_1) < h_1(v_1, q)$ for $v_1 \in (\underline{V}, \overline{V})$, then $\tilde{v} < v_1^M$ ($v_2^M(v_1)$ crosses the 45° line to the left of $h_1(v_1, q)$).

   Consider the case where $v_2 = v_1 = \widetilde{v}$. That is, if in period 1 types that receive an offer sell iff $v \geq \tilde{v}$, then the optimal strategy for the manager in period 2 is also to sell iff $v \geq \tilde{v}$. In addition, the overall mean value of types who did not sell in period 2 is also $\tilde{v}$. Now consider the case where the disclosure threshold in period 1 is lower, $v_1^* = v' < \tilde{v}$. Comparing between the two cases, we can see that in the later case there are additional types with an offer, $[v', \tilde{v}]$, who sell in the first period. Since these types are lower than $\tilde{v}$, then, if $v_2^* = \tilde{v}$, the mean type that haven't sold period 2 is higher, that is $h_2(\widetilde{v}, q; v') > h_2(\widetilde{v}, q; \widetilde{v}) = \tilde{v}$. From the minimum principle and the definition of v_2^M in Section 4.1, $v_2^M(v') > \tilde{v}$.

   Now compare the case where $v_2 = v_1 = \widetilde{v}$ to a case where the disclosure threshold in period 1 is higher, $v_1^* = v'' > \tilde{v}$. If $v_2^* = \tilde{v}$ then, following the two periods, some of the firms of type $[\tilde{v}, v'']$ that sell in the former case do not sell in the later case (those who only have an offer in period 1). Thus, the type population who did not sell in both periods has additional types that are higher than $\tilde{v}$. Thus, the mean is higher, $h_2(\widetilde{v}, q; v'') > h_2(\widetilde{v}, q; \widetilde{v}) = \tilde{v}$, and again $v_2^M(v'') > \tilde{v}$.

3. Now define $G(v_1, \alpha) \equiv \alpha h_1(v_1, q) + (1 - \alpha) v_2^M(v_1)$. $G(v_1, \alpha)$ is strictly increasing in α.

One can rewrite (7) as $v_1^* = G\left(v_1^*, \frac{1}{1+\beta(1-q\lambda)}\right)$. Given the minimum principle and step 2 above, $G(v_1, \alpha) > v_1$ for $v_1 \le \tilde{v}$ and $G(v_1, \alpha) < v_1$ for $v_1 > v_1^M$.

4. From Lemma 4 we know that there is a unique v_1^* such that $v_1^* = G(v_1^*, \alpha)$: from step 3 it is evident that $G(v_1, \alpha)$ crosses the 45° line an odd number of times, and if the number is greater than one then disclosure is not monotone, in contrast to the Lemma. Thus, equilibrium is unique (first part of Proposition).

5. Finally, given steps 3 and 4, v_1^* is increasing in α, and therefore is increasing in λ. Since $v_1^* > \tilde{v}$ (for all α), then $v_2^M(v_1)$ is increasing and therefore is also increasing in λ.

□

A.2 Proof of Lemma 6

Proof.

1. The market value of a firm decreases if $p_t(v') < E_2(v)$. Define $\hat{v}_t$ as the type that satisfies $p_t(\hat{v}_t) = E_t(v)$, and notice that $v_t^* < h_t(v_t^*, q) < E_t(v) \le \hat{v}_t$. The probability that the market value falls after a sale is

$$\Pr(fall) \equiv \frac{F_t(\hat{v}_t) - F_t(v_t^*)}{1 - F_t(v_t^*)}.$$

By definition, $p_t(v)$ is increasing in λ (this is also due to the fact that $u_t^{\mathrm{NS}}(v)$ is increasing in λ for $t < T$), so $\hat{v}_t$ is decreasing in λ. Together with the fact that v_t^* is increasing in λ, this is enough to establish that $\Pr(\text{fall})$ is decreasing in λ.

2. From Part (2) of Lemma 1 one knows that in period 1 $h_1(v_1^*, q) > v_1^* = p_1(v_1^*)$ and thus there are always types that sell although they can get a higher market value if they choose not to. Let $\tilde{v}_1$ be the type such that $h_1(v_1^*, q) = p_1(\tilde{v}_1; \lambda)$. Notice that $v_1^* < h_1(v_1^*, q) \le \tilde{v}_1$. The probability that the manager "loses" in the period she sells is

$$\Pr(\text{loss in period } 1) \equiv \frac{F_1(\tilde{v}_1) - F_1(v_1^*)}{1 - F_1(v_1^*)}.$$

From Part (2) of Lemma 1 and the minimum principle $h_1(v_1^*, q)$ is decreasing in v_1^*, and from Proposition 2 v_1^* is increasing in λ. Thus, $h_1(v_1^*, q)$ is decreasing in λ. In contrast $p_1(v;\lambda)$ is increasing in λ. Therefore, $\tilde{v}_1(\lambda)$ is decreasing in λ. This is enough to establish that $\Pr(\text{loss in period 1})$ is also decreasing in λ

To see that both probabilities equals 1 when λ is low, I show this is the case where $\lambda = 0$, that is, the buyer has all bargaining power. First notice that in period 2, $p_2(v;\lambda = 0) = u_2^{\text{NS}}(v) = v_2^M$. This price, which is independent of type, is less than the $E_2(v)$ (the market value in the beginning period 2), so $\Pr(\text{fall}) = 1$ in this case. In period 1, in case $\lambda = 0$, if the manager does not sell in period 1 she receives an type-independent expected payoff of $u_1^{\text{NS}}(v) = h_1(v_1^*, q) + \beta v_2^M$. Thus, in period 1 the price is also type-independent, and equals $p_1^B \equiv u_1^{\text{NS}} = \frac{h_1(v_1^*, q) + \beta v_2^M}{1+\beta}$. This type is offers to any profitable types $v \geq v_1^* \equiv u_1^{\text{NS}}$. The market value of all types that sell, p_1^B, is less than the market value before the sale, $E_1(v)$, so $\Pr(\text{fall}) = 1$. It is also less than the market value of firms that do not sell, $h_1(v_1^*, q)$, and thus $\Pr(\text{loss in period 1}) = 1$. When the value of the firm has an upper bound, i.e., $\overline{V} \neq \infty$, it is easy to show that these probabilities are one also for low values of λ that are above 0.

□

A.3 Proof of Proposition 3

Proof. I first provide a useful property of decreasing thresholds, which I later use in the proof.

Lemma 8. *In a threshold equilibrium, for every period t: (1) iff $v_t^* > v_{t+1}^*$ then $v_t^* < h_t(v_t^*, q)$; (2) iff $v_t^* < v_{t+1}^*$ then $v_t^* > h_t(v_t^*, q)$; (3) iff $v_t^* = v_{t+1}^*$ then $v_t^* = h_t(v_t^*, q)$.*

Proof. I prove only (1), as the proof to the other two parts is similar. Type v_t^* is indifferent in period t, that is:

$$h_t(v_t^*, q) + \left[q \cdot p_{t+1}(v_t^*) + (1-q) u_{t+1}^{\text{NS}}(v_t^*)\right] \left(\sum_{\tau=t+1}^{T} \beta^{\tau-t}\right) = v_t^* + v_t^* \left(\sum_{\tau=t+1}^{T} \beta^{\tau-t}\right).$$

Since type v_t^* strictly prefers to sell in period $t+1$, then $u_{t+1}^{\text{NS}}(v_t^*) < p_{t+1}(v_t^*)$. Since $p_{t+1}(v_t^*) \leq v_t^*$, the equality holds iff $v_t^* < h_t(v_t^*, q)$. □

From the two-period case we know that the Proposition holds for periods T and $T-1$. I prove for periods $t \in \{0, 1, ...T-2\}$ using backward induction. I thus prove the Proposition for an arbitrary period t, while using the fact that the Proposition holds for periods $t+1, ..., T$, and therefore there exist disclosure thresholds $v^*_{t+1} > v^*_{t+2} > ... > v^*_T$.

Part (1) – Threshold Strategies. Assume an arbitrary strategy $x_t(v)$ in period t and let $E(v \mid \text{NS}, x_t)$ be the price of a firm that does not sell in period t. Assume a firm of value v', such that $v^*_{t+k+1} \leq v' < v^*_{t+k}$. Thus, if this firm does not sell in period t then it also does not sell in periods $t+1$ to $t+k$, but does accept sale offers from period $t+k+1$ and onward. Denote by $\tilde{u}^{\text{NS}}_t$ the outside option is this case (the tilde refers to the fact that the strategy in period t is arbitrary, though I assume a threshold equilibrium from period $t+1$ onwards), this outside option is calculated using the following equality:

$$\begin{aligned}
\tilde{u}^{\text{NS}}_t(v') \cdot \sum_{\tau=t}^{T} \beta^{\tau-t} &= E_t(v \mid \text{NS}, x_t) + \sum_{\tau=t+1}^{t+k} \beta^{\tau-t} h_\tau(v^*_\tau, q) \\
&+ \sum_{\tau=t+k+1}^{T} \beta^{\tau-t}(1-q\lambda)^{\tau-t-k} h_\tau(v^*_\tau, q) \\
&+ v \sum_{\tau=t+k+1}^{T} \beta^{\tau-t}\left(1-(1-q\lambda)^{\tau-t-k}\right). \qquad (10)
\end{aligned}$$

This outside option also describes types $v' > v^*_{t+1}$ and $v' < v^*_T$: set $k=0$ for the former and $k = T-t$ for the later; in the former case the first argument in the RHS is an "empty sum" and by convention equals zero, while in the later case the same is true for the second and third arguments.

Type v' sells iff $p_t(v') \geq \tilde{u}^{\text{NS}}_t(v')$, which translates, given (1), to the condition $v' \geq \tilde{u}^{\text{NS}}_t(v')$. Notice

$$\frac{\partial \tilde{u}^{\text{NS}}_t(v)}{\partial v} \cdot \sum_{\tau=t}^{T} \beta^{\tau-t} = \sum_{\tau=t+k+1}^{T} \beta^{\tau-t}\left(1-(1-q\lambda)^{\tau-t-k}\right) < \sum_{\tau=t+k+1}^{T} \beta^{\tau-t}$$

so $\frac{\partial \tilde{u}^{\text{NS}}_t(v)}{\partial v} < 1$ for all k. Since for any type $v > v'$ one can write $v^*_{t+k'+1} \leq v < v^*_{t+k'}$ where $k' \leq k$, this fact is sufficient to conclude that, if v' chooses to sell, than all types $v' < v$ also choose to sell.

Part (2) – Thresholds are decreasing. Assume, in contrast, that $v^*_t \leq v^*_{t+1}$. Then, using exactly the same argument as in the proof of Lemma 8, I get $v_t^* \geq h_t(v_t^*, q)$. However, Lemma 8 and $v_{t+1}^* > v_{t+2}^*$ entails $v_{t+1}^* < h_{t+1}(v_{t+1}^*, q)$ and Lemma 1 entails $h_{t+1}\left(v_{t+1}^*, q\right) < h_t\left(v_t^*, q\right)$ – a contradiction.

Part (3). $v_t^* < h_t\left(v_t^*, q\right)$ is immediate from part (2) and Lemma 8, while $h_t\left(v_t^*, q\right) < h_{t-1}$ is from Lemma 1. □

A.4 Proof of Lemma 7

Proof. I first prove that in equilibrium the threshold strategy must be strictly monotone. To do so, I use the results of Lemma 8 (though Lemma 8 was proven for a finite horizon case, its results can be easily extended to infinite horizon: just change T to ∞).

First consider the case that $v_t^* \leq v_{t+1}^* > v_{t+2}^*$. From Lemmas 1 and 8 one receives $v_t^* \geq h_t(v_t^*, q) > h_{t+1}(v_{t+1}^*, q) > v_{t+1}^*$ – a contradiction. Therefore, if thresholds are weakly increasing at some point, then they cannot decrease from that period and on. Now consider the case where, from period t, threshold are always weakly increasing. In such a case, if type v_t^* does not disclose in period t it also does not disclose in subsequent periods (or indifferent to that), so its indifference condition can be written as

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} h_\tau(v_\tau^*, q) = v_t^* \sum_{\tau=t}^{\infty} \beta^{\tau-t} = \frac{v_t^*}{1-\beta}.$$

Since $h_\tau(\cdot)$ is decreasing in τ (Lemma 1) I can write

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} h_\tau(v_\tau^*, q) < h_t(v_t^*, q) \sum_{\tau=t}^{\infty} \beta^{\tau-t} = \frac{h_t(v_t^*, q)}{1-\beta}.$$

I get $v_t^* < h_t(v_t^*, q)$ – contradiction to Lemma 8. Thus, thresholds must be strictly decreasing. □

References

ACHARYA, V. V., P. DEMARZO, AND I. KREMER (2011): "Endogenous Information Flows and the Clustering of Announcements," *The American Economic Review*, 101(7), 2955–

2979.

Agarwal, R., and J. Kolev (2013): "Strategic Corporate Layoffs," Discussion paper.

Benmelech, E., E. Kandel, and P. Veronesi (2010): "Stock-Based Compensation and CEO (Dis)Incentives," *The Quarterly Journal of Economics*, 125(4), 1769–1820.

Beyer, A., and R. Dye (2012): "Reputation Management and the Disclosure of Earnings Forecasts," *Review of Accounting Studies*, 17(4), 877–912.

Bond, P., A. Edmans, and I. Goldstein (2012): "The Real Effects of Financial Markets," *Annual Review of Financial Economics*, 4(1), 339–360.

Bond, P., and Y. Leitner (2013): "Market Run-Ups, Market Freezes, Inventories, and Leverage," Discussion paper.

Bustamante, M. C. (2012): "The Dynamics of Going Public," *Review of Finance*, 16(2), 577–618.

Cohen, D. A., and P. Zarowin (2010): "Accrual-based and real earnings management activities around seasoned equity offerings," *Journal of Accounting and Economics*, 50(1), 2 – 19.

Dye, R. A. (1985): "Disclosure of Nonproprietary Information," *Journal of Accounting Research*, 23(1), 123–145.

Einhorn, E., and A. Ziv (2008): "Intertemporal Dynamics of Corporate Voluntary Disclosures," *Journal of Accounting Research*, 46(3), 567–589.

Fishman, M. J., and K. M. Hagerty (1989): "Disclosure Decisions by Firms and the Competition for Price Efficiency," *The Journal of Finance*, 44(3), 633–646.

Fuchs, W., and A. Skrzypacz (2013): "Costs and Benefits of Dynamic Trading in a Lemons Market," Discussion paper.

Grenadier, S. R., and A. Malenko (2011): "Real Options Signaling Games with Applications to Corporate Finance," *Review of Financial Studies*, 24(12), 3993–4036.

Grossman, S. J. (1981): "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics*, 24(3), pp. 461–483.

Gunny, K. A. (2010): "The Relation Between Earnings Management Using Real Activities Manipulation and Future Performance: Evidence from Meeting Earnings Benchmarks," *Contemporary Accounting Research*, 27(3), 855–888.

Guttman, I., I. Kremer, and A. Skrzypacz (2013): "Not Only What But also When: A Theory of Dynamic Voluntary Disclosure," Discussion paper.

Harris, M., and A. Raviv (1979): "Optimal incentive contracts with imperfect information," *Journal of Economic Theory*, 20(2), 231 – 259.

Hölmstrom, B. (1979): "Moral Hazard and Observability," *The Bell Journal of Economics*, 10(1), pp. 74–91.

Holmström, B. (1999): "Managerial Incentive Problems: A Dynamic Perspective," *The Review of Economic Studies*, 66(1), 169–182.

Janssen, M. C., and S. Roy (2002): "Dynamic Trading in a Durable Good Market with Asymmetric Information," *International Economic Review*, 43(1), 257–282.

Jung, W.-O., and Y. K. Kwon (1988): "Disclosure When the Market Is Unsure of Information Endowment of Managers," *Journal of Accounting Research*, 26(1), pp. 146–153.

Kyle, A. S. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53(6), pp. 1315–1335.

Milbradt, K. (2012): "Level 3 Assets: Booking Profits and Concealing Losses," *Review of Financial Studies*, 25(1), 55–95.

Milgrom, P. R. (1981): "Good News and Bad News: Representation Theorems and Applications," *The Bell Journal of Economics*, 12(2), pp. 380–391.

Morellec, E., and N. Schürhoff (2011): "Corporate Investment and Financing Under Asymmetric Information," *Journal of Financial Economics*, 99(2), 262 – 288.

Nash, J. F. J. (1950): "The Bargaining Problem," *Econometrica*, 18(2), pp. 155–162.

Paul, J. (1992): "On the Efficiency of Stock-Based Compensation," *Review of Financial Studies*, 5(3), 471–502.

Peek, J., and E. S. Rosengren (2005): "Unnatural Selection: Perverse Incentives and the Misallocation of Credit in Japan," *American Economic Review*, 95(4), 1144–1166.

Roychowdhury, S. (2006): "Earnings management through real activities manipulation," *Journal of Accounting and Economics*, 42(3), 335 – 370.

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50(1), pp. 97–109.

Sekine, T., K. Kobayashi, and Y. Saita (2003): "Forbearance Lending: The Case of Japanese Firms," *Monetary and Economic Studies*, 21(2), 69–92.

Stein, J. C. (1989): "Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior," *The Quarterly Journal of Economics*, 104(4), 655–669.

Zang, A. Y. (2011): "Evidence on the Trade-Off between Real Activities Manipulation and Accrual-Based Earnings Management," *The Accounting Review*, 87(2), 675–703.