ERC Workshop 27-31 January 2019

Abstracts

Luis Montejano

Title: On the real geometric hypothesis of Banach

Abstract:

The following is known as the geometric hypothesis of Banach: let V be an m-dimensional Banach space with unit ball B and suppose all n-dimensional subspaces of V are isometric (all the n-sections of B are affinely equivalent). In 1932, Banach conjectured that under this hypothesis V is isometric to a Hilbert space (the boundary of B is an ellipsoid). Gromov proved in 1967 that the conjecture is true for n=even and Dvoretzky derived the same conclusion under the hypothesis n=infinity. We prove this conjecture for n=5 and 9 and give partial results for an integer n of the form 4k+1.

The ingredients of the proof are classical homotopic theory, irreducible representations of the orthogonal group and convex geometry. Suppose B is an (n+1)-dimensional convex body with the property that all its n-sections through the origin are affinity equivalent to a fixed n-dimensional body K. Using the characteristic map of the tangent vector bundle to the n-sphere, it is possible to prove that if n=even, then K must be a ball and using homotopical properties of the irreducible subgroups of SO(5) and SO(9), we prove that if N=5,9, then K must be a body of revolution. Finally, we prove, using convex geometry and topology that, if this is the case, then there must be a section of B which is an ellipsoid and consequently B must be also an ellipsoid.

Arseniy Akopyan

Title: Any cyclic quadrilateral can be inscribed in any closed convex smooth curve

Abstract:

We prove that any cyclic quadrilateral can be inscribed in any closed convex C1-curve. The smoothness condition is not required if the quadrilateral is a rectangle. Joint work with Sergey Avvakumov.

Gabriel Nivasch

Title: Grid peeling and the affine curve-shortening flow

Abstract:

We observe that, experimentally, grid peeling (the convex-layer decomposition of subsets of the integer grid) seems to behave at the limit like the affine curve-shortening flow. We offer some theoretical arguments to explain this phenomenon. We also derive some rigorous results for the special case of peeling the quarter-infinite grid.

Joint work with David Eppstein and Sariel Har-Peled.

Chaya Keller

Title: Improved lower bounds on the Hadwiger-Debrunner numbers

Abstract:

A family of sets F is said to satisfy the (p,q)-property if among any p sets in F, some q have a non-empty intersection. Hadwiger and Debrunner (1957) conjectured that for any $p > q > d$ there exists a constant $c = c_d(p,q)$, such that any family of compact convex sets in R^d that satisfies the (p,q)-property, can be pierced by at most c points. The classical Helly's Theorem is equivalent to the fact that $c_d(p,p)=1$ $(p > d)$.

In a celebrated result from 1992, Alon and Kleitman proved the conjecture. However, obtaining sharp bounds on the minimal such $c_d(p,q)$, called `the Hadwiger-Debrunner numbers', is still a major open problem in combinatorial geometry.

In this talk we present improved lower bounds on the Hadwiger-Debrunner numbers, using the hypergraph container method.

Joint work with Shakhar Smorodinsky.

Wojciech Samotij

Title: Lower bounds for eps-nets for lines in the plane.

Abstract:

Balogh and Solymosi have recently given a randomised construction of point sets in the plane whose smallest eps-net with respect to the family of lines has size at least $(1/eps)(log(1/eps))^{1/3-o(1)}$. In this talk, I will present their argument, which crucially relies on the hypergraph container lemma. I will also indicate how a new `efficient' version of the container lemma improves the exponent 1/3 in the above lower bound to 1/2.

If time permits, I will say a few words about the proof of this `efficient' container lemma, whose key ingredient has a convex geometric flavour. This is joint work with Jozsef Balogh.

Micha Sharir, Tel Aviv University

Title: Radial isotropic position: Theory, algorithms, and applications

Abstract:

A set X of n vectors on the unit sphere in d dimensions is said to be in isotropic position if the sum of the squares of their projections in any direction is constant, independently of the direction. We say that X can be brought to radial isotropic position if there exists a linear tranformation A, such that if we normalize each of the vectors of AX we get a set in isotropic position. We review the elegant theory that shows that almost any set X can be brought to radial isotropic position, show the strong connection between this notion and Singular Value Decomposition, a basic tool in linear algebra, present iterative algorithms, based on gradient descent, for approximating the desired linear transformation, and discuss combinatorial and algorithmic applications, most notably the recent work of Kane, Lovett and Moran on point location in high-dimensional hyperplane arrangements.

Joint work with Shiri Artstein-Avidan and Haim Kaplan.

Florian Frick (Carnegie Mellon University)

Title: New Applications of Tverberg's Theorem: Fair Splittings by Independent Sets

Abstract:

Tverberg's theorem has found several interesting applications, in discrete geometry and further afield, such as providing bounds on the number of halving planes of a point set or for the chromatic number of uniform hypergraphs. In this talk, I will explain new applications of Tverberg's theorem and focus on the problem of fair splittings by independent sets: Given a graph with its vertices arbitrarily colored, we

wish to split most of the vertex set into independent sets that accurately reflect the correct fraction of vertices of each color.

Csaba D. Toth

Title: Convex Polygons in Cartesian Products

Abstract: We study several problems concerning convex polygons whose vertices lie in a Cartesian product (for short, grid) of two sets of n real numbers. We prove that every such grid contains a convex polygon with $\Omega(\log n)$ vertices and that this bound is tight up to a constant factor. We generalize this result to d dimensions (for a fixed d), and obtain a tight lower bound of $\Omega(\log^{d-1}n)$ for the maximum number of points in convex position in a d-dimensional grid. We also present exponential upper and lower bounds on the maximum number of convex polygons in planar grids.These bounds are tight up to polynomial factors. (Joint work with Jean-Lou De Carufel, Adrian Dumitrescu, Wouter Meulemans, Tim Ophelders, Claire Pennarun, and Sander Verdonschot.)

Shakhar Smorodinsky, BGU

Title: on chi-bounded graphs and hypergraphs arising in geometry

A family of graphs $\F$ is called χ-bounded if there exists a function $$C_{\F}:\mathbb{N}\mapsto \mathbb{N}$$ such that for every graph $G \in \F$

we have $\chi(G) \leq C_{\F}(\omega(G))$ where $\chi(G)$ denotes the chromatic number of G and $\omega(G)$ denotes the maximum size of a clique in G.

It is well known that the family of all graphs is not χ-bounded as there exists triangle free graphs with arbitrary large chromatic number.

An analogous definition of the notion of χ-bounded graphs holds for hypergraphs.

In this talk we survey several results regarding χ-bounded geometrically defined graphs and hypergraphs.

Alan Lew

Title: Collapsibility of Simplicial Complexes of Hypergraphs

Abstract:

Let X be a simplicial complex. Given a simplex $\sigma \in X$ of dimension at most $d - 1$ that is contained in a unique maximal face of X, the operation of removing σ and all the simplices that contain it from X is called an elementary d-collapse. We say that X is d-collapsible if it can be reduced to the void complex by performing a sequence of elementary d-collapses. The notion of d-collapsibility was introduced by Wegner, who proved that the nerve of a family of convex sets in R^d is d-collapsible.

Let H be an r-uniform hypergraph. We show that the simplicial complex whose simplices are the hypergraphs $F \subset H$ with covering number at most p is $(\binom{r+p}{r}-1)$-collapsible. Similarly, the simplicial complex whose simplices are the pairwise intersecting hypergraphs $F \subset H$ is $1/2\binom{2r}{r}$-collapsible.

The proof relies on a general method for d-collapsing simplicial complexes, due essentially to Matoušek and Tancer, and on a combinatorial lemma about skew intersecting families of sets, proved independently by Frankl and Kalai.

Roy Meshulam

Title: Topology and combinatorics of the complex of flags

Abstract:

Let V be an n-dimensional space over a fixed finite field. The complex of flags X(V) is the simplicial complex whose vertices are the non-trivial linear subspaces of V, and whose simplices are ascending chains of subspaces. This complex, also known as the spherical building associated to the linear group GL(V), appears in a number of different mathematical areas, including topology, combinatorics and representation theory. After recalling the classical homological properties of X(V), we will discuss some more recent results including:

1. Minimal weight cocycles in the Lusztig-Dupont homology (joint work with Shira Zerbib)
2. Coding theoretic aspects of X(V) and the existence of homological codes.
2. Coboundary expansion of X(V) and its applications.

Janos Pach

Title: Disjointness graphs of strings

Abstract:
Let $\omega(G)$ and $\chi(G)$ denote the clique number and chromatic number of a graph G, respectively.

The {\em disjointness graph} of a family of curves (continuous arcs in the plane) is the graph whose vertices correspond to the curves and in which two vertices are joined by an edge if and only if the corresponding curves are disjoint. A curve is called {\em x-monotone} if every vertical line intersects it in at most one point. An x-monotone curve is {\em grounded} if its left endpoint lies on the y-axis.

We prove that if G is the disjointness graph of a family of grounded x-monotone curves such that $\omega(G)=k$, then $\chi(G)\leq \binom{k+1}{2}$. If we only require that every curve is x-monotone and intersects the y-axis, then we have $\chi(G)\leq \frac{k+1}{2}\binom{k+2}{3}$. Both of these bounds are best possible. The construction showing the tightness of the last result settles a 25 years old problem: it yields that there exist K_k-free disjointness graphs of x-monotone curves such that any proper coloring of them uses at least $\Omega(k^{4})$ colors. This matches the upper bound up to a constant factor.

Joint work with Istv\'an Tomon.

Nati Linial

Title: On the geometry of graphs

Abstract: My talk is comprised of two parts. I first speak about the local theory of graphs. The main underllying question here is “what can the small-scale statistics of a large graph look like”. The second part of the talk concerns the large-scale metric of graphs and in particular I will discuss the relations between the diameter and the girth of large graphs. I will mention many open problems along the way.

Imre Barany
Title: An Application of the Universality Theorem for Tverberg Partitions

Abstract:
We show that, as a consequence of a remarkable new result of Attila P\'or on universal Tverberg partitions, any large-enough set P of points in $\Re^d$ has a $(d+2)$-sized subset whose Radon point has half-space depth at least $c_d \cdot |P|$, where $c_d \in (0, 1)$ depends only on d. We then give an application of this result to computing weak $\eps$-nets by random sampling. Joint work with Nabil Mustafa.

Sergey Avvakumov

Title: "Fair partition of a convex planar pie"

Abstract:
Nandakumar and Ramana Rao asked in 2008 the following question: Is it possible to cut a convex body in the plane in m convex parts of equal area and equal perimeter? Although this problem also has some algorithmic aspects, we only discuss the existence question.

Nandakumar and Ramana Rao themselves noticed that the case m=2 is solved with a simple continuity argument, and suggested an argument for m=4. In fact, fair partition in the case m is a power of two is a simple modification of M. Gromov's argument from 2003 used in his proof of the waist of the Gaussian measure and the waist of the sphere theorems.

The case m=3 was done by B\'ar\'any, Blagojevi\'c, and Sz\H{u}cs in 2010. The case when m is a power of a prime was done by a joint effort of several people, Aronov, Hubard, Blagojevi\'c, Ziegler, and the speaker. Curiously, the essential topological fact used in this proof was established already in 1988 by Vassiliev in his paper about topological complexity of finding non-multiple roots of a complex polynomial.
The last publication on this problem was in 2014 and since then there were no advances. But in 2018 we have managed to find new ideas that solve the problem completely, for any m.

joint with Arseniy Akopyan and Sergey Avvakumov from IST Austria

Gabor Tardos

Title: On a graph coloring conjecture of Erdos and Hajnal

Abstract:

An old and beautiful conjecture of Erdos and Hajnal states that large chromatic graphs have large girth, large chromatic subgraphs. More precisely, for every m and k there is n=f(m,k) such that every graph with chromatic number n has a subgraph of chromatic number m and girth at least k. Rodl proved in 1977 the girth 4 case, the rest of the conjecture is open. Following Mohar and Wu who did this for Kneser graphs, we prove the conjecture for special classes of graphs, including shift graphs and a family of graphs used by Burling and others as thigh chromatic triangle-free geometric intersection graph constructions. In the proof we find a very low threshold f(n,k) for the shift graphs, but very high thresholds for Burling graphs. We show conjectures on certain 2-player combinatorial games that may actually lead to strong lower bounds on f(m,k).

This is joint work with Bartosz Walczak.

Wojciech Samotij

Title: Lower bounds for eps-nets for lines in the plane.

Abstract:

Balogh and Solymosi have recently given a randomised construction of point sets in the plane whose smallest eps-net with respect to the family of lines has size at least (1/eps) (log(1/eps))^{1/3-o(1)}. In this talk, I will present their argument, which crucially relies on the hypergraph container lemma. I will also indicate how a new `efficient' version of the container lemma improves the exponent 1/3 in the above lower bound to 1/2.

If time permits, I will say a few words about the proof of this `efficient' container lemma, whose key ingredient has a convex geometric flavour. This is joint work with Jozsef Balogh.

Boris Aronov

Title: Constructive Polynomial Partitioning for Algebraic Curves in R^3 with Applications

Abstract:
In 2015, Guth proved that, for any set of k-dimensional varieties in R^d and for any positive integer D, there exists a polynomial of degree at most D whose zero-set divides R^d into open connected "cells," so that only a small fraction of the given varieties intersect each cell. Guth's result generalized an earlier result of Guth and Katz for points.

Guth's proof relies on a variant of the Borsuk-Ulam theorem, and for k>0, it is unknown how to obtain an explicit representation of such a partitioning polynomial and how to construct it efficiently. In particular, it is unknown how to effectively construct such a polynomial for curves (or even lines) in R^3.

We present an efficient algorithmic construction for this setting.
Given a set of n input curves and a positive integer D, we efficiently construct a decomposition of space into O(D^3log^3{D}) open cells, each of which meets at most O(n/D^2) curves from the input. The construction time is O(n^2), where the constant of proportionality depends on D and the

maximum degree of the polynomials defining the input curves. For the case of lines in 3-space we present an improved implementation, whose running time is O(n^{4/3} polylog{n}).

As an application, we revisit the problem of eliminating depth cycles among non-vertical pairwise disjoint triangles in 3-space, recently studied by Aronov~\etal~(2017) and De~Berg~(2017). Our main result is an algorithm that cuts n triangles into O(n^{3/2+eps}) pieces that are depth cycle free, for any eps>0. The algorithm runs in O(n^{3/2+eps}) time, which is nearly worst-case optimal. We also sketch several other applications of our effective partitioning for curves in R^3.

Joint work with Esther Ezra and Josh Zahl ====

Natan Rubin

Title: Planar point sets determine many pairwise crossing segments

Abstract: We show that any set of n points in general position in the plane determines at $n^{1-o(1)}$ pairwise crossing segments.

The best previously known lower bound, $\Omega(\sqrt n)$, was proved more than 25 years ago by Aronov, Erdos, Goddard, Kleitman, Klugerman, Pach, and Schulman.

Our proof is fully constructive, and extends to dense geometric graphs.

Joint work with Janos Pach and Gabor Tardos

Roman Glebov

Title: Perfect Matchings in Random Subgraphs of Regular Bipartite Graphs

Abstract:
Consider the random process in which the edges of a graph G are added one by one in a random order. A classical result states that if G is the complete graph K_{2n} or the complete bipartite graph $K_{n,n}$, then typically a perfect matching appears at the moment at which the last isolated vertex disappears. We extend this result to arbitrary k-regular bipartite graphs G on $2n$ vertices for all $k = \omega \left(\frac{n}{\log^{1/3} n} \right)$.

Surprisingly, this is not the case for smaller values of k. Using a construction due to Goel, Kapralov and Khanna, we show that there exist bipartite k-regular graphs in which the last isolated vertex disappears long before a perfect matching appears.

Joint work with Z. Luria and M. Simkin

Andrey Kupavskii

Title: Crossing Tverberg theorem

Abstract:

The famous Tverberg theorem states that, given a family of (d+1)n points in R^d in general position, one can find n vertex-disjoint d-simplices with vertices in P that all share a common point. We show that we can choose these simplices in such a way that, additionally, their boundaries mutually intersect. In particular, on any 3n points in general position on the plane one can find n mutually crossing triangles.

Joint work with R. Fulek, B. Gartner, P. Valtr and U. Wagner