# Cognitive MIMO Radar


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# Outline

- MIMO radar at a glance
- Cognitive radar - introduction
- Cognitive MIMO radar for beamforming and detection
- Conclusion

# MIMO Radar at a Glance

Data model: $\mathbf{X} = \mathbf{H}(\boldsymbol{\Theta})\mathbf{S} + \mathbf{W}$

$[\mathbf{X}]_{m,n}$ - Rx signal at sensor $m$ and time index $n$

$[\mathbf{S}]_{i,n}$ - Tx signal by element $i$ and time index $n$

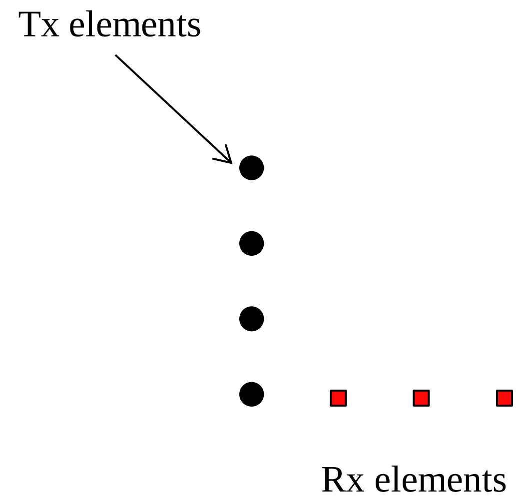
$\mathbf{H}(\boldsymbol{\Theta})$ - Tx-Rx channel matrix

$\boldsymbol{\Theta}$ - Unknown targets' parameters

$$\mathbf{R}_S = \frac{1}{N}\mathbf{S}\mathbf{S}^H$$

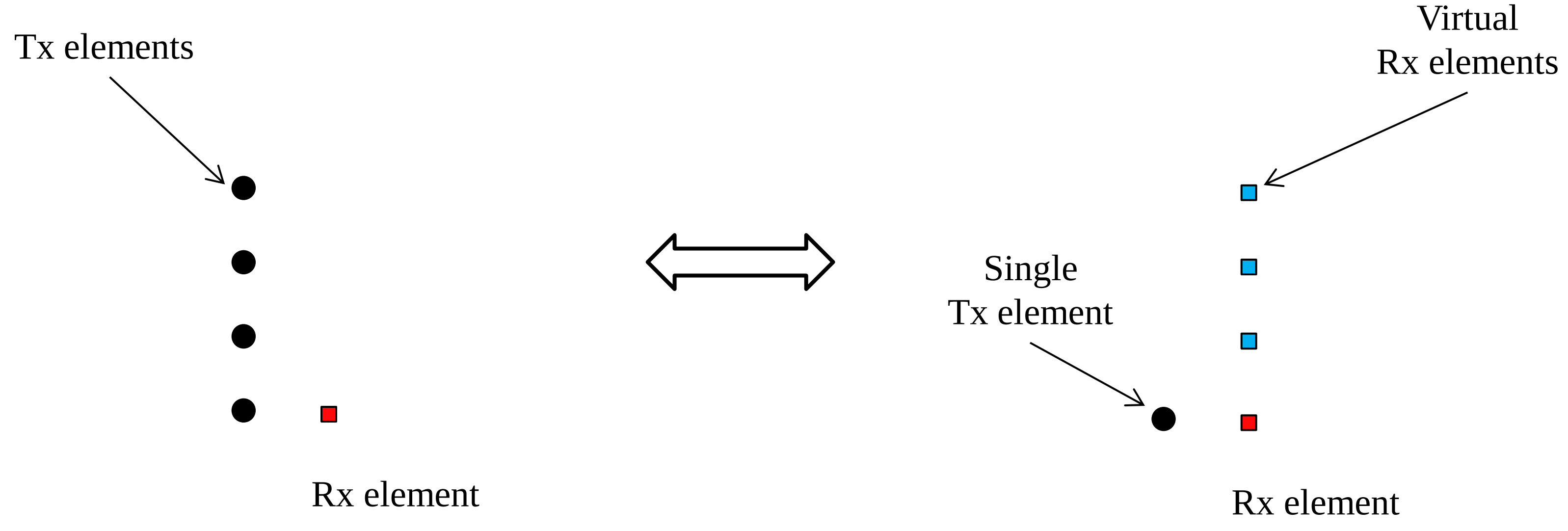
MIMO - $\mathbf{R}_s = c\mathbf{I}$

Phased array - rank($\mathbf{R}_s$)=1



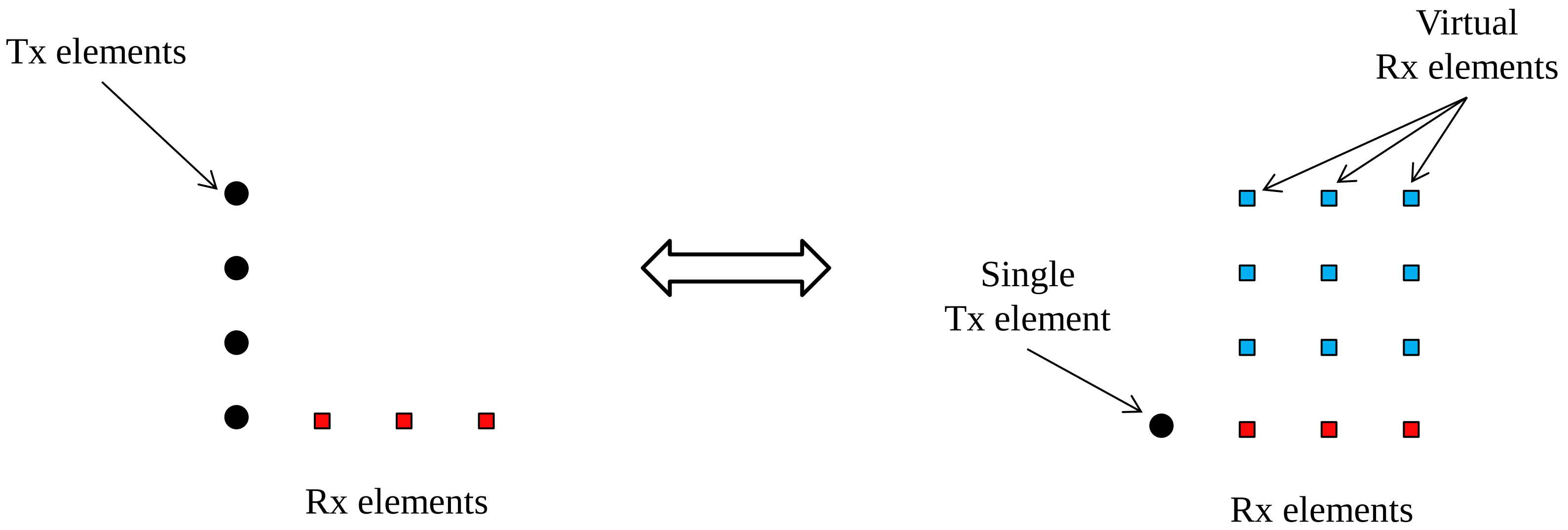
# MIMO Radar at a Glance

Orthogonal Tx signals can be decomposed at the receiver, allowing adaptive beamforming of the Tx signals → Virtual receiving elements:



# MIMO Radar at a Glance

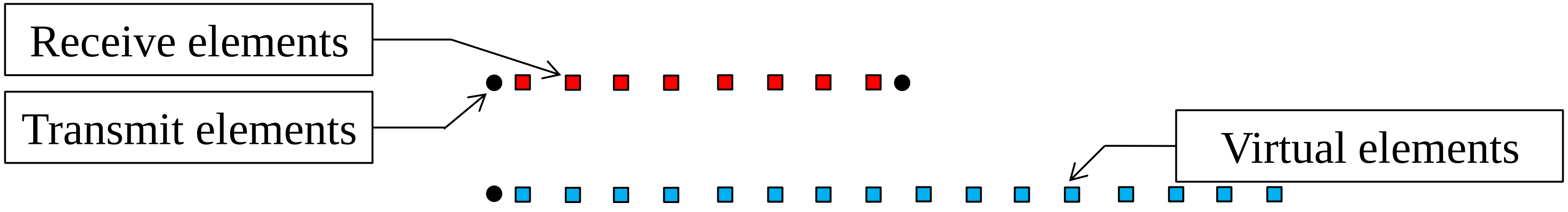
Virtual receiving elements: Orthogonal Tx signals can be decomposed at the receiver, allowing adaptive beamforming of the Tx signals.

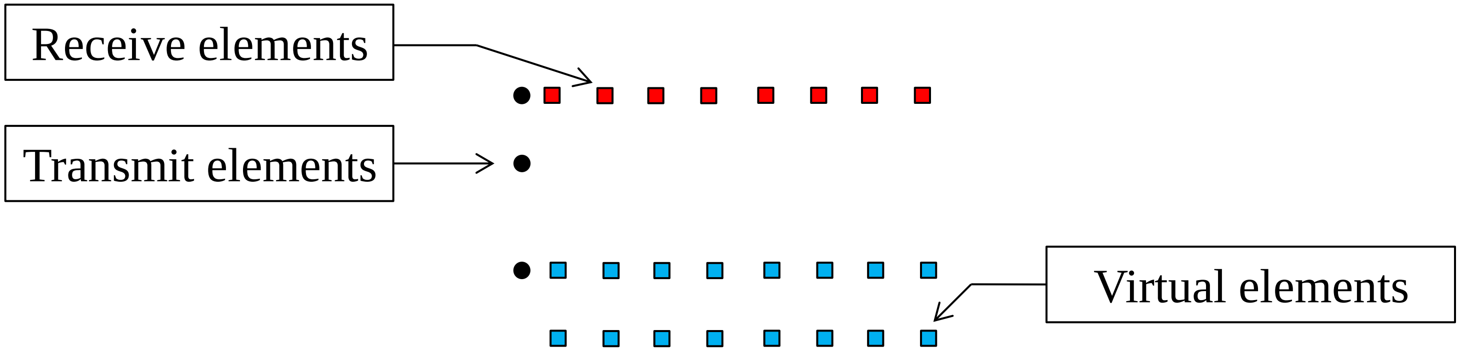



***Colocated (mono-static) MIMO radar: Bekkerman-Tabrikian 2004***
***Distributed (multi-static) MIMO radar: Fishler et al. 2004***

# MIMO Radar Properties

Array aperture extension:



# MIMO Radar Properties

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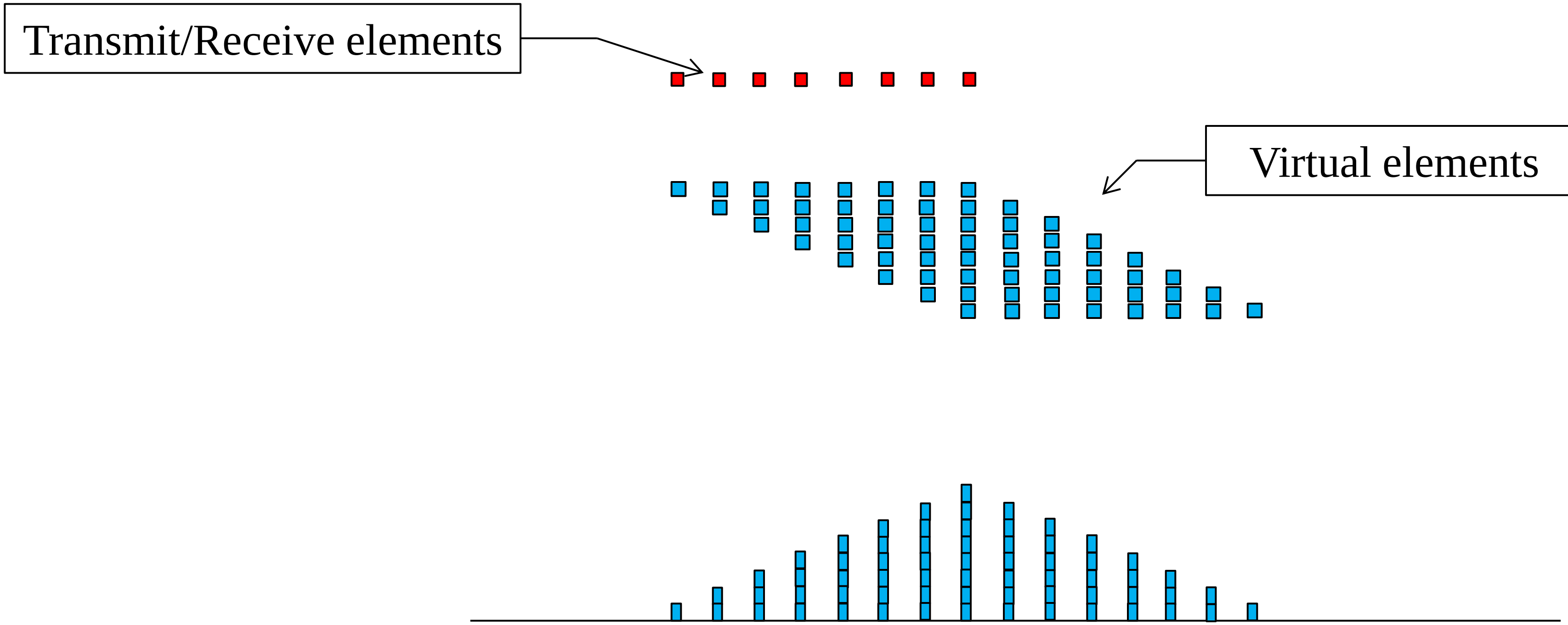


# MIMO Radar Advantages

- More degrees of freedom due to the ***virtual sensors***:
  - Higher angular resolution.
  - Higher number of targets/clutter in a given range-Doppler cell, which can be detected and localized.
  - Lower sidelobes by virtual spatial windowing.
  - Digital beamforming of the Tx beams in addition to the Rx beams, and therefore avoid beam shape loss in cases that the target is not in the center of the beam.
- Decrease the spatial power density of the Tx signal – spatial spread spectrum (SSS) which is critical for low probability of intercept radars (LPIR).

# MIMO Radar Disadvantage

- Implementation
- Gain loss (omni-directional transmission)
  - Not a real problem in **search mode**: omni-directional coverage allows large time-on-target (requires quasi-stationarity or track-before-detect).
  - A real problem in **track/acquisition** modes:

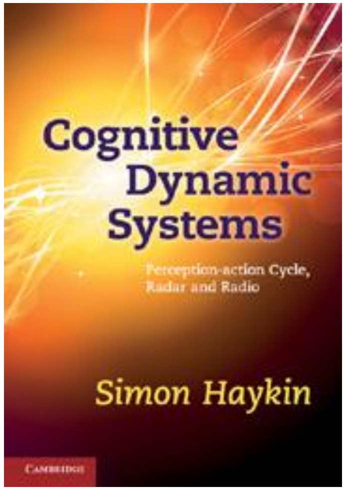
    If the target direction is known with a given degree of accuracy, then MIMO radar “wastes” its energy towards undesired directions.

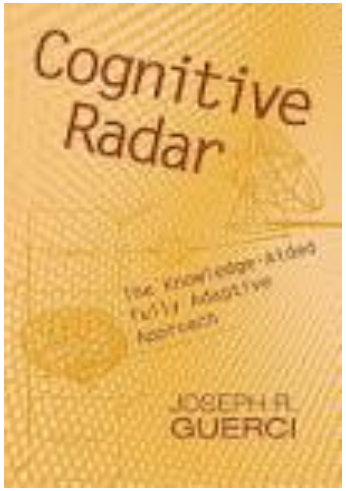
**Solution: Cognitive MIMO Radar**

# Cognitive Radar

## Proposed by Simon Haykin 2006.

A cognitive radar employs ***adaptive Tx-Rx*** based on ***history observation*** and ***environmental information***.




# Cognitive Radar

## Why the term cognitive is used?

## NIH definition:

"*Cognition***:** conscious mental activity that informs a person about his or her environment. Cognitive actions include perceiving, thinking, reasoning, judging, problem solving and remembering."

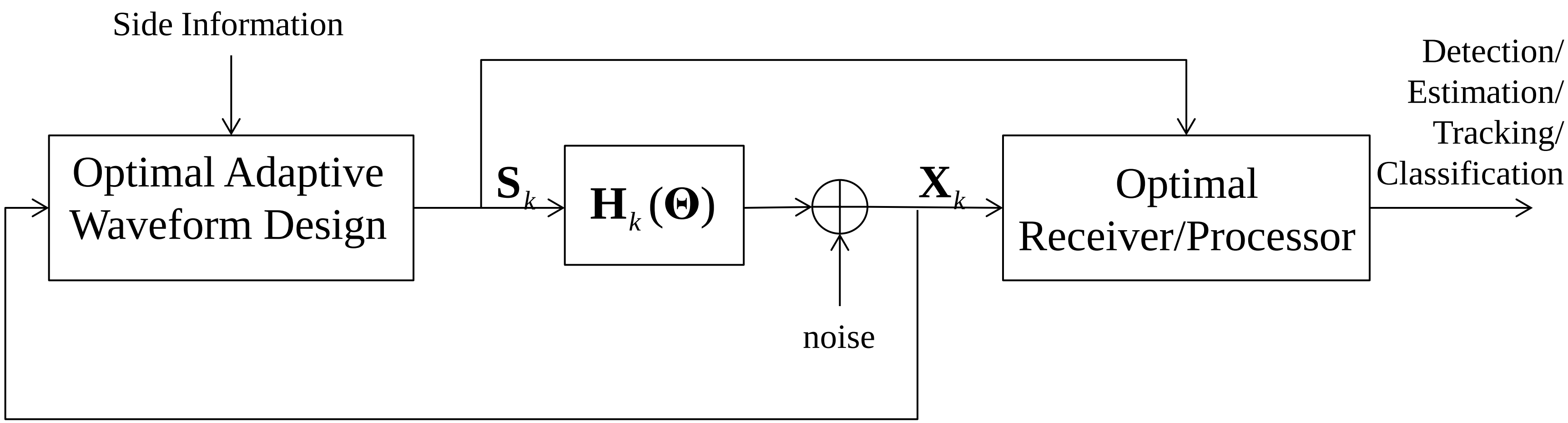
| Biological Cognitive Properties versus Cognitive Radars | |
|---|---|
| **Cognitive Property** | **Cognitive Radar Equivalent** |
| Perceiving | Sensing |
| Thinking, Reasoning, Judging, Problem Solving | Expert Systems, Adaptive Algorithms, and Computation |
| Remembering | Memory, Environmental Database |

*[Guerci 2011]*

# Cognitive MIMO Radar

Data model at the $k^{\text{th}}$ step: $\mathbf{X}_k = \mathbf{H}_k(\mathbf{\Theta})\mathbf{S}_k + \mathbf{W}_k$

$\mathbf{H}_k(\mathbf{\Theta})$ - MIMO channel matrix, $\mathbf{\Theta}$ - Target parameters




***Optimal processor:*** Detect/localize/track/classify the target(s) based on available measurements, $\mathbf{X}^{(k)} = [\mathbf{X}_1, \ldots, \mathbf{X}_k]$.

***Optimal adaptive waveform design:*** Design the transmit signal at the $k^{\text{th}}$ pulse, $\mathbf{S}_k$, based on the measurements during the previous pulses, $\mathbf{X}^{(k-1)} = [\mathbf{X}_1, \ldots, \mathbf{X}_{k-1}]$ to optimize a given criterion.

# Cognitive MIMO Radar

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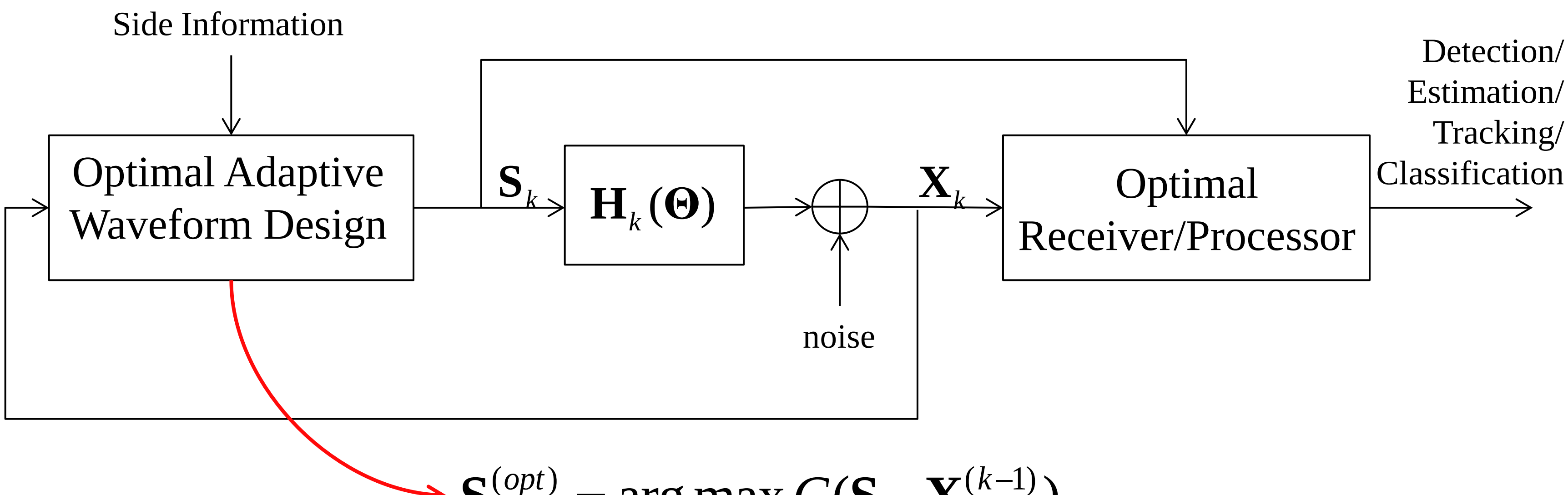



$$\mathbf{S}_k^{(opt)} = \arg\max_{\mathbf{S}_k} C(\mathbf{S}_k, \mathbf{X}^{(k-1)})$$

s.t. Tx power constraint

$C(\mathbf{S}_k, \mathbf{X}^{(k-1)})$ - Criterion for perofrmance optimization

# Cognitive Beamforming

Criterion for estimation accuracy: performance bound on the mean-squared-error (MSE):

- Bayesian Cramér-Rao bound (BCRB): Simple, but not tight.
- Bobrovski-Zakai, Reuven-Messer, or Weiss-Weinstein bounds: High computational complexity, but tighter.

It can be shown that with Gaussian noise, the bounds depend on

the Tx auto-correlation matrix: $\mathbf{R}_{\mathbf{s}_k} = \frac{1}{N}\mathbf{S}_k\mathbf{S}_k^H$

Power constraint: $\left\|\mathbf{S}_k\right\|_F^2 = \mathrm{tr}\left(\mathbf{R}_{\mathbf{s}_k}\right) \leq P$

or $\left[\mathbf{R}_{\mathbf{s}_k}\right]_{n,n} = P / N_T, \quad n = 1, \ldots, N_T$

# Cognitive Beamforming

For single unknown parameter, $\theta$, with total Tx power constraint, and zero-mean Gaussian noise with cov. $\mathbf{R}_\mathbf{v}$:

$$\mathbf{R}_{\mathbf{s}_k}^{(opt)} = P\mathbf{u}_k\mathbf{u}_k^H$$

$\mathbf{u}_k$ - eigenvector corresponding to the maximum eigenvalue of

$$\mathbf{\Gamma}_k(\mathbf{X}^{(k-1)}) = \mathrm{E}\left(\dot{\mathbf{H}}_k^H(\theta)\mathbf{R}_\mathbf{v}^{-1}\dot{\mathbf{H}}_k(\theta)\middle|\mathbf{X}^{(k-1)}\right)$$

**Vector parameter case, $\mathbf{\Theta} \in \mathbb{R}^Q$ - weighted BCRB:**

Convex optimization problems, and thus can be solved efficiently (Boyd and Vandenberghe (2004)).

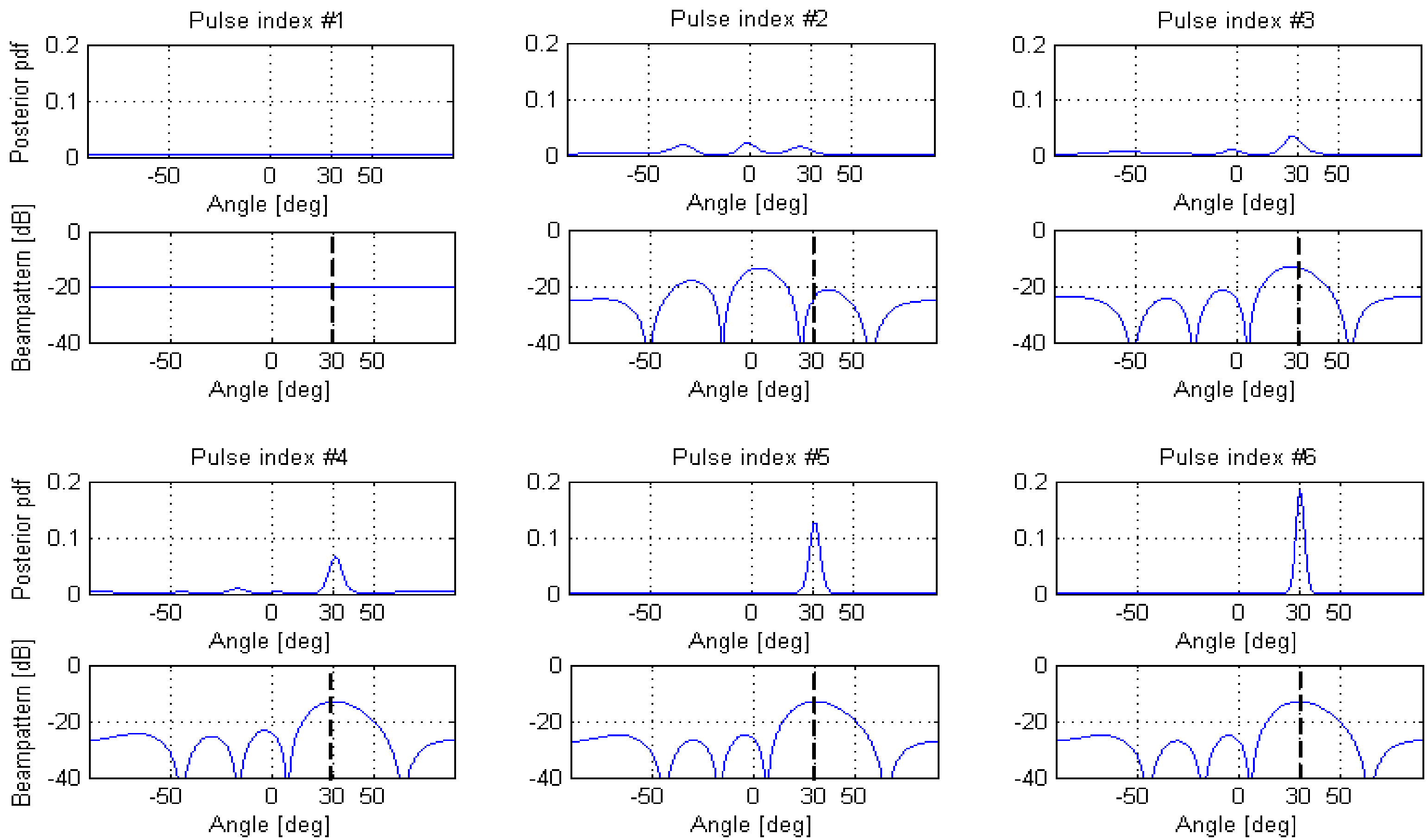
# Example – Cognitive Beamforming

**Scenario** :

- Uniform linear array of transceivers $N_R = N_T = 7$ elements with $\lambda/2$ inter-element spacing.
- AWGN with covariance $\mathbf{R} = \sigma^2 \mathbf{I}_{N_R}$.
- ASNR $= |\alpha|^2 NPN_R / \sigma^2 = -6$dB.

# Example – Cognitive Beamforming

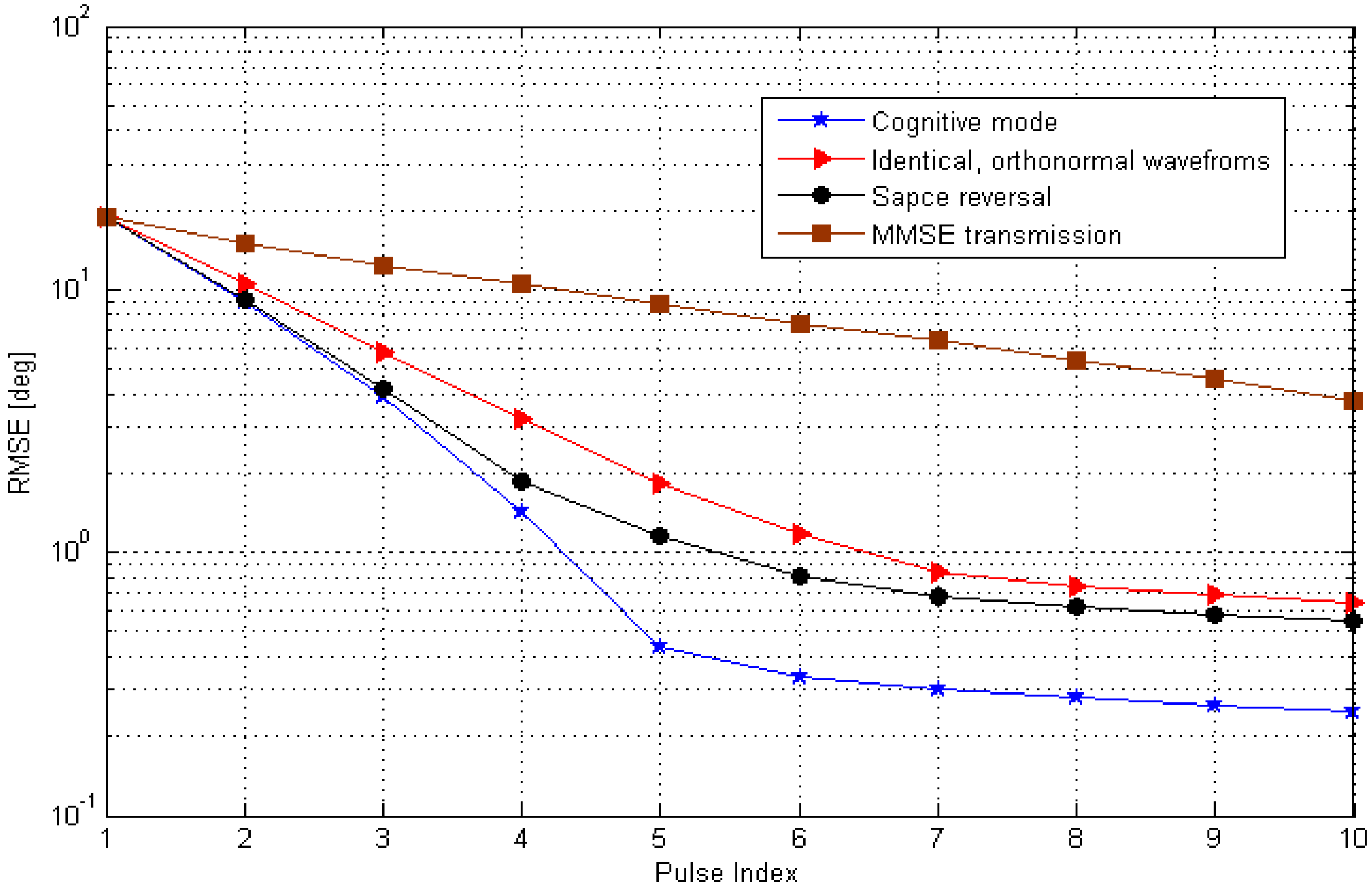
Posterior pdf's versus transmit beampatterns $P_k(\varphi) = \mathbf{a}_T^T(\varphi)\mathbf{R}_{\mathbf{s}_k}^{*}\mathbf{a}_T(\varphi)$.




Auto-focusing effect: Automatic beamforming before detection/estimation.

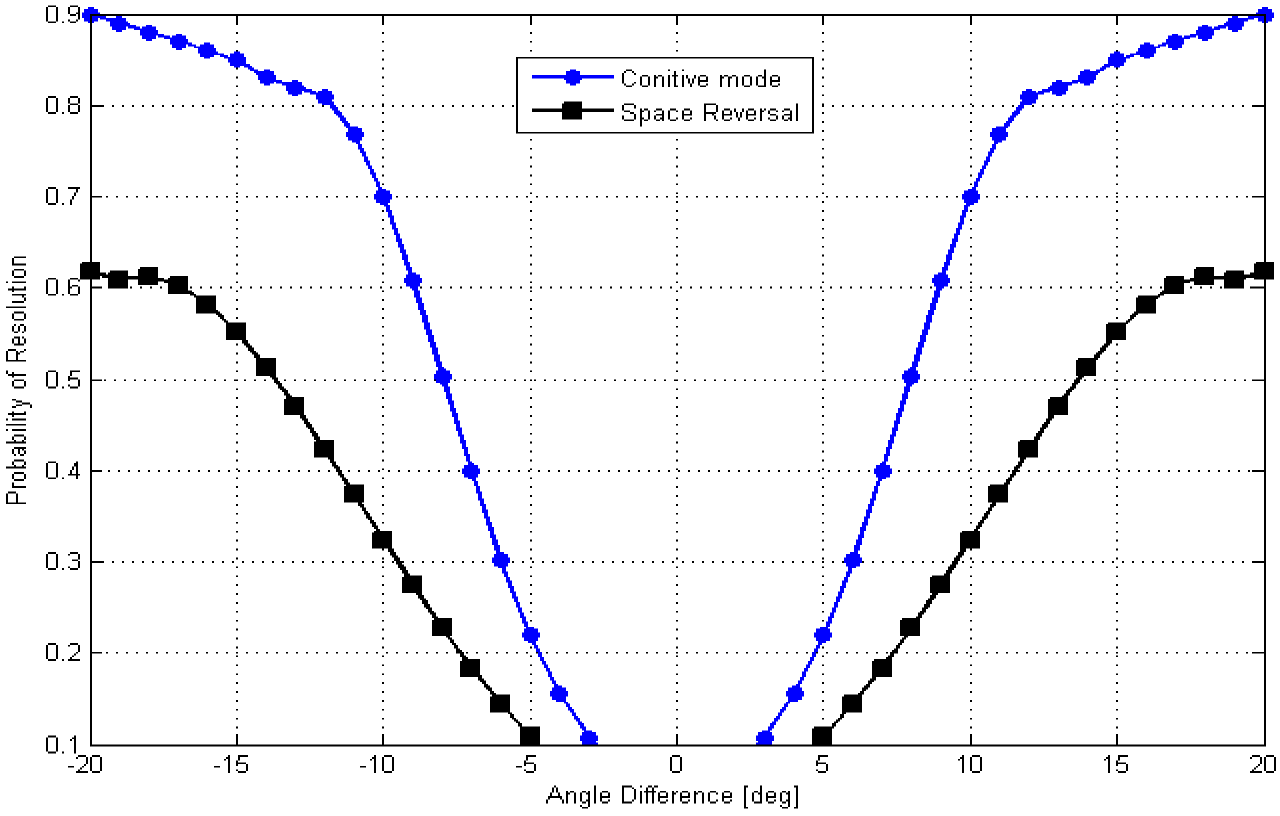
# Example – Cognitive Beamforming

Single target – direction estimation accuracy, 7 transceivers



# Example – Cognitive Beamforming

Probability of resolution compared to space-reversal method.
Two targets – SNR=-2 dB, $k$=10, 7 transceivers.

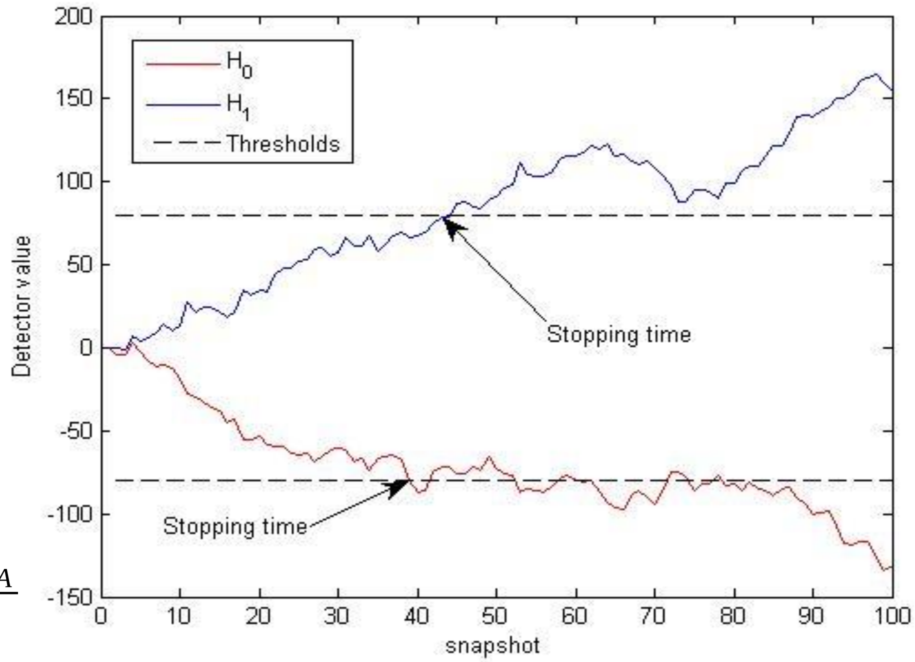


# Cognitive Detection

Sequential Hypothesis Testing (SHT):

$$\begin{cases} H_1: & \mathbf{x}_{k,l} = \mathbf{H}_{k,l}(\boldsymbol{\Theta})\mathbf{s}_{k,l} + \mathbf{w}_{k,l} \\ H_0: & \mathbf{x}_{k,l} = \mathbf{w}_{k,l} \end{cases}, \quad k = 1, 2, \ldots, \quad l = 1, \ldots, L,$$

Goal: Minimize Average Sample Number (ASN) to achieve given error probabilities: $1 - P_D,\ P_{FA}$ .




$$\text{Decide } H_1 \text{ if: } \log \frac{f_{\mathbf{X}^{(k)}}(\mathbf{X}^{(k)} \mid H_1)}{f_{\mathbf{X}^{(k)}}(\mathbf{X}^{(k)} \mid H_0)} > \frac{P_D}{1 - P_D}$$

$$\text{Decide } H_0 \text{ if: } \log \frac{f_{\mathbf{X}^{(k)}}(\mathbf{X}^{(k)} \mid H_1)}{f_{\mathbf{X}^{(k)}}(\mathbf{X}^{(k)} \mid H_0)} < -\frac{1 - P_{FA}}{P_{FA}}$$

# Cognitive Detection

Two hypotheses:

$$ASN \geq \underbrace{\max\left\{\frac{-\log(1-P_D)}{KLD_k(H_1 \| H_0)}, \frac{-\log P_{FA}}{KLD_k(H_0 \| H_1)}\right\}}_{C\left(\mathbf{s}_k, \mathbf{X}^{(k-1)}\right)}.$$

Optimal signal design:

$$\mathbf{S}_{k,opt} = \arg\min_{\mathbf{s}_k} \max\left\{\frac{-\log(1-P_D)}{KLD_k(H_1 \| H_0)}, \frac{-\log P_{FA}}{KLD_k(H_0 \| H_1)}\right\}.$$

$KLD_k(H_m \| H_n)$ - conditional Kullback-Leibler Divergence given $\mathbf{X}^{(k-1)}$.

# Cognitive Detection

Criterion:

$$\mathbf{S}_{k,opt} = \arg\min_{\mathbf{s}_k} \sum_{l=1}^{L} \mathbf{s}_{k,l}^H \mathrm{E}\left( \mathbf{H}_{k,l}^H(\mathbf{\Theta}) \mathbf{R}_{\mathbf{v}}^{-1} \mathbf{H}_{k,l}(\mathbf{\Theta}) \middle| \mathbf{X}^{(k-1)} \right) \mathbf{s}_{k,l}$$

$$\text{s.t. } \sum_{l=1}^{L} \left\| \mathbf{s}_{k,l} \right\|^2 \le P$$

$$\mathbf{H}_{k,l}(\mathbf{\Theta}) = \alpha e^{-j\omega_D T k} \mathbf{a}_R(\theta) \mathbf{a}_T^T(\theta) e^{-j\frac{2\pi}{T} l\tau}, \quad k = 1, 2, \ldots, \; l = 1, \ldots L,$$

$$\mathbf{R}_{\mathbf{s}_k,opt} = P \mathbf{u}_{\max} \mathbf{u}_{\max}^H$$

$\mathbf{u}_{\max}$ - Eigenvector corresponding to the maximal eigenvalue of the matrix $\mathrm{E}\left( \mathbf{a}_T^*(\theta) \mathbf{a}_T^T(\theta) \middle| \mathbf{X}^{(k-1)} \right)$.

# Example - Cognitive Detection

4Tx, 16 Rx

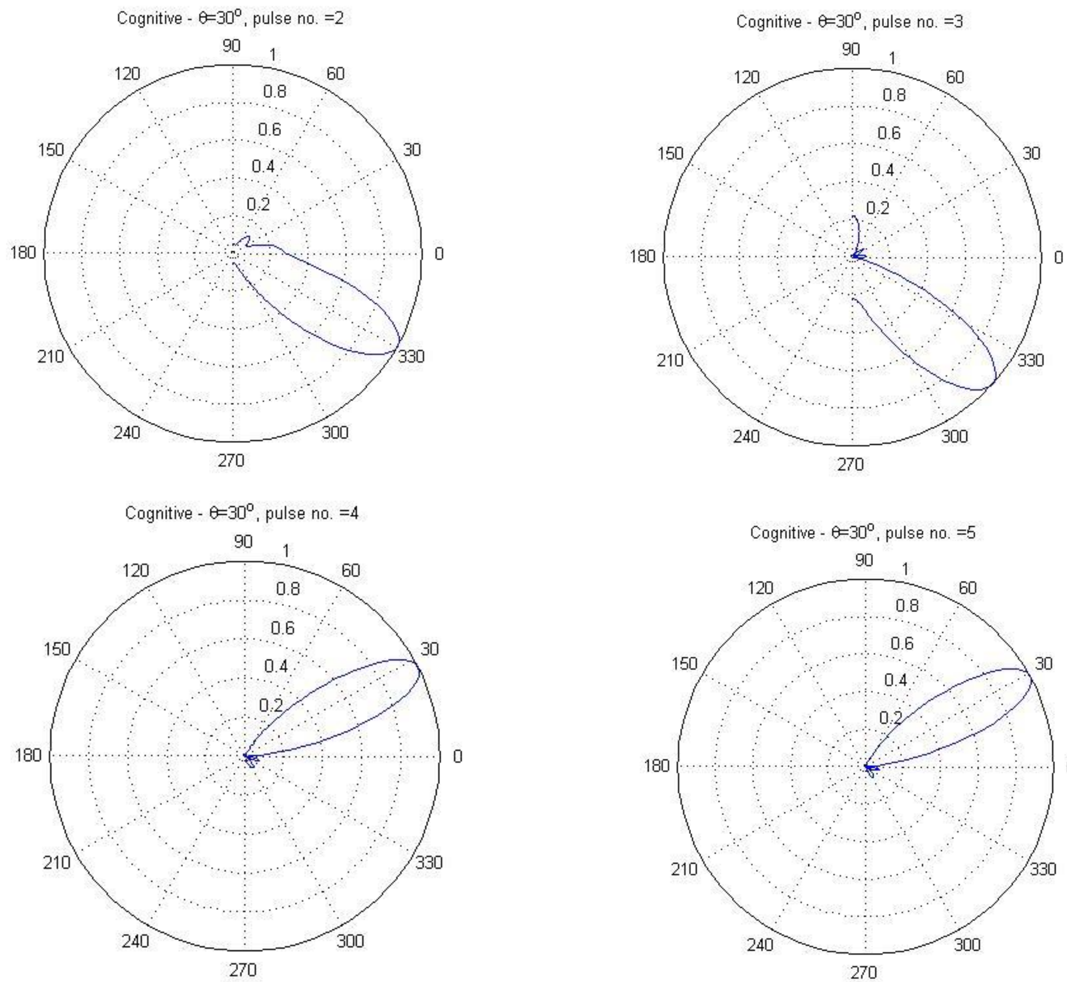
$\lambda/2$ inter-element spacing

$NF = 7dB$

$RCS = 1m^2$

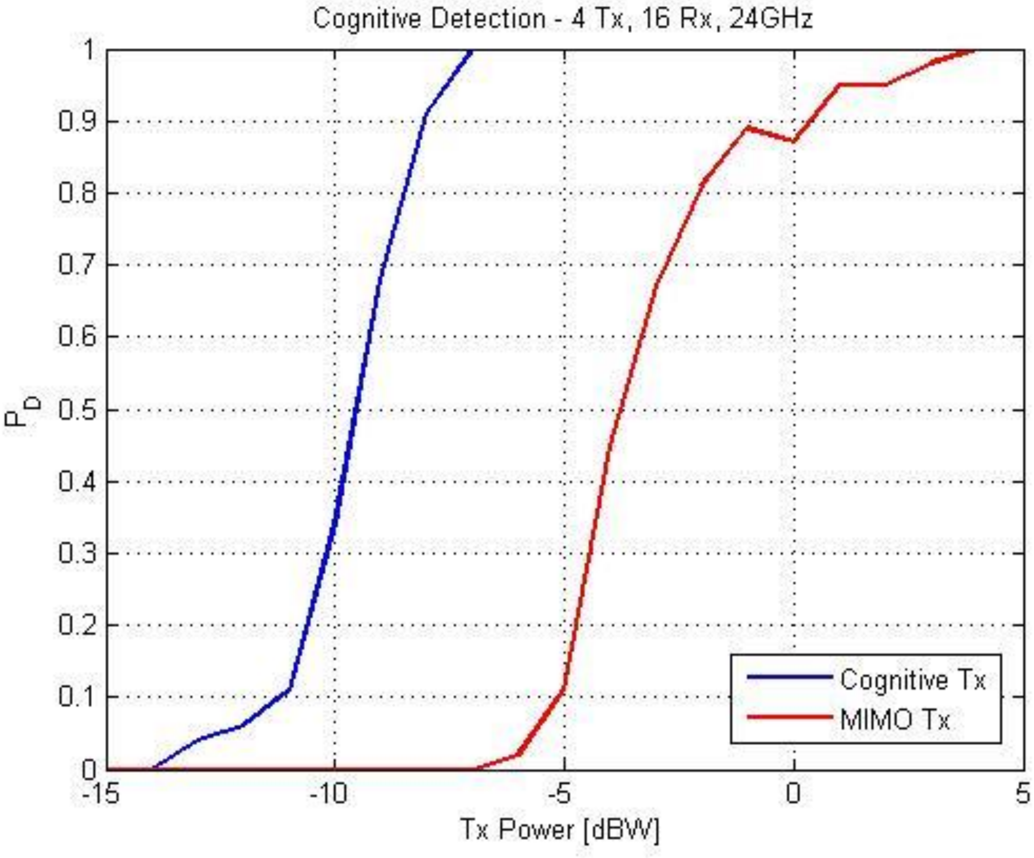
Range=50m

Azimuth=$30^{\circ}$



# Example - Cognitive Detection

$$4\text{Tx},\ 16\ \text{Rx},\ \text{NF}=7\text{dB},\ \text{RCS}=1\text{m}^2,\ \text{range=50m, azimuth=30}^\circ$$



# Conclusions and Future Research

- MIMO radar offers great advantages but needs to be used with care.
- In cognitive MIMO radar, Tx signal auto-correlation matrix is adaptively optimized. The optimized signal is not necessarily orthogonal (MIMO) or fully correlated (phased array).
- Two new cognitive Tx beamforming approaches were presented to optimize: localization accuracy and detection performance

  This approach provides an automatic focusing array: beamforming before detectionqestimation.
- Future research:

  Considering other criteria, such as probability of resolution, or target classification performance.

# Thank you!