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Cloaking –The Road to Realization

by

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Radar Symposium

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Building 98, Auditorium 001
Ben-Gurion University of the Negev

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Outline

- ***Introduction***
- ***Transformation Optics***
- ***Laplace's Equation- Transformation***
- ***Realization***
- ***Summary***

Introduction

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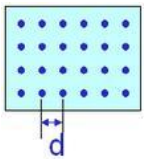
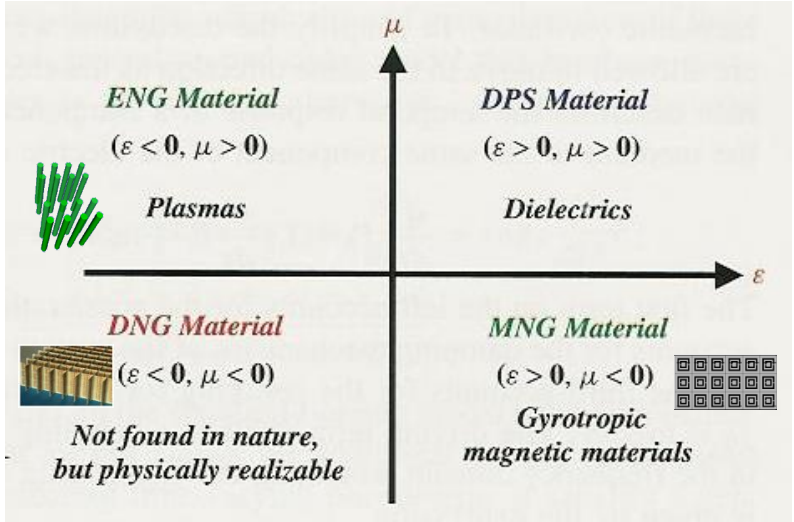
Metamaterials

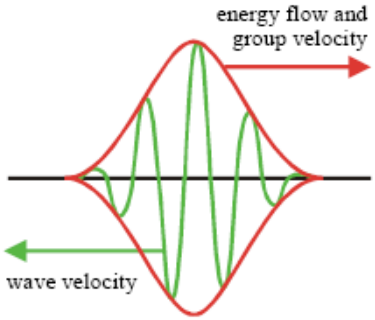


The prefix “meta” comes from Greek and means “after” or “beyond”. These are engineered composites (usually periodic structures) that exhibit superior properties that are not found in nature and not observed in the constituent materials.

Material classification

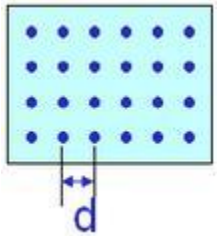


In double negative (DNG) materials the phase velocity is negative, although the group velocity stays positive. Vectors k, E and H will form a left hand set.



V.G.Veselago : ‘The Electrodynamics of Substances with Simultaneously Negative Values of P and M’, Sov. Phys. Usp., 1968, 10, (4), pp. 509–514

Introduction



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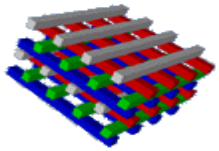
History of Metamaterials

Frequency Selective Surfaces (FSS) and Electromagnetic Band Gap (EBG) materials (d ~ λ)

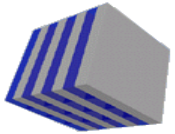
- ***Element Distances are on the order of half a wavelength or more (Periodic medium concepts)***
- ***Electromagnetic Band Gap materials (EBG), Artificial Magnetic Conductors (AMC), High Impedance Surfaces (HIS)***

Metamaterials (d << λ)

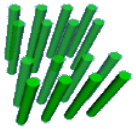
- ***Elements and distances between them are much smaller than a wavelength (simultaneous effective negative permittivity and permeability)***
- ***Have several names including left-handed materials, backward-wave materials, negative index of refraction (NIR) materials, etc.***



Woodpile



Stacked dielectric layer



Dielectric rods

Printed elements

Split Ring Resonators (SRR)

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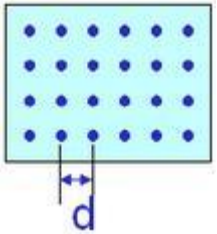


DNG Structures

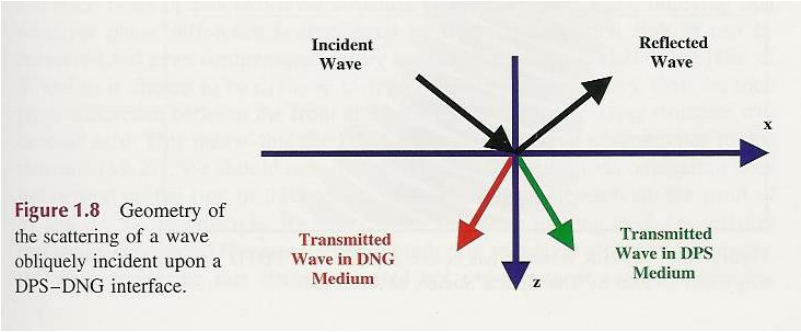
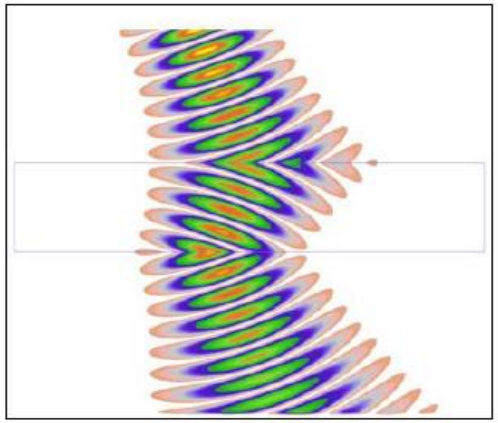
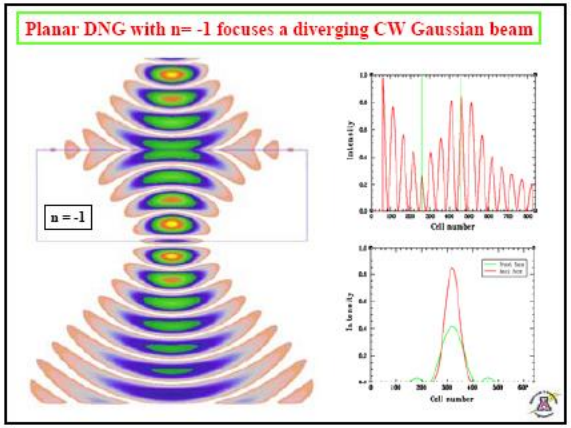


Figure 1.8 Geometry of the scattering of a wave obliquely incident upon a DPS–DNG interface.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow Snell's\ law$$





ε_2=-1 ; μ_2=-1 (no reflection)

J.B. Pendry, "Negative Refraction Makes a Perfect Lens," Phys. Rev. Lett., vol.85, no.18, pp.3966-3969, Oct. 2000.

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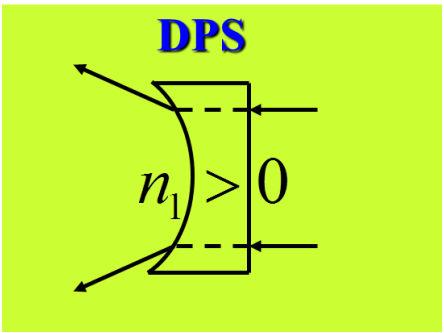
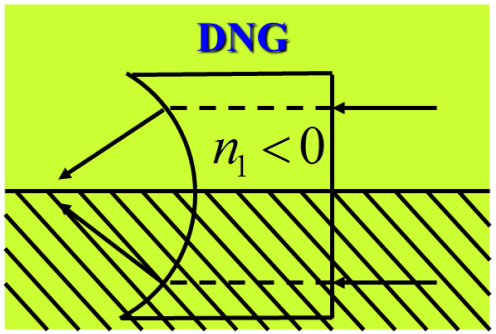
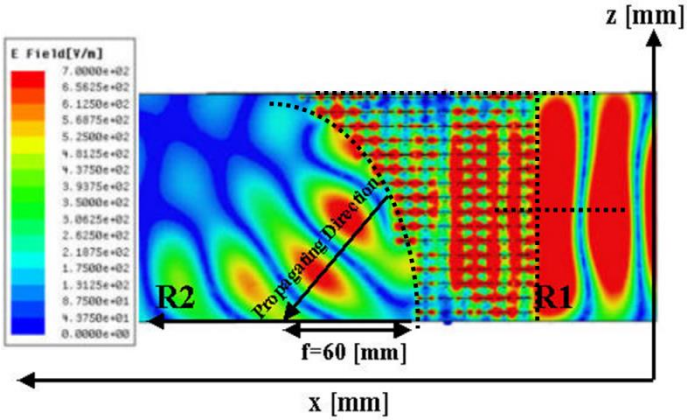
DNG Lens







Zero index flat Lens

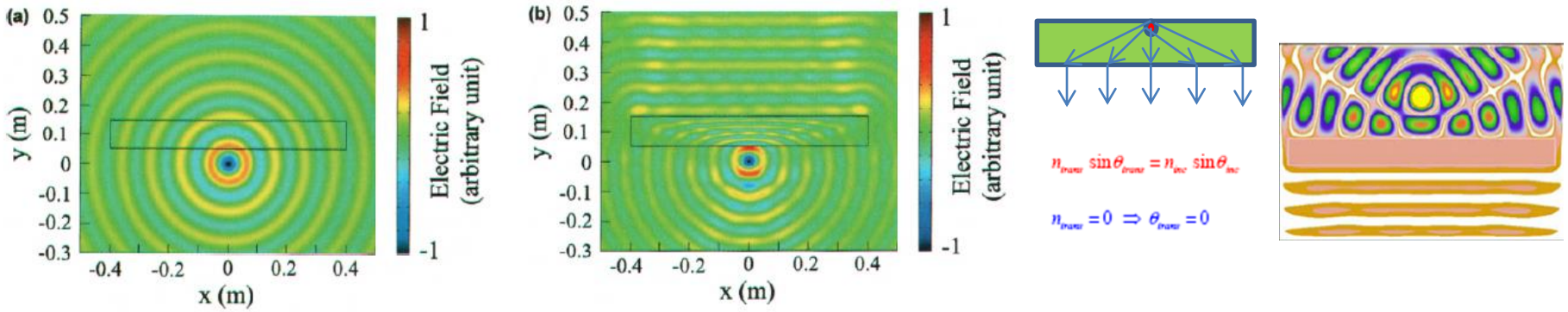


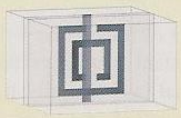
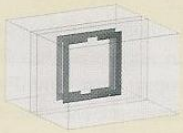
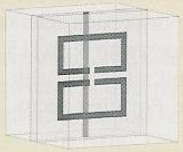
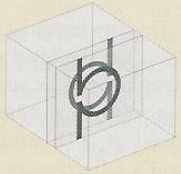
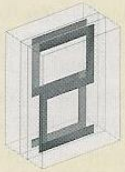
Figure 18. Snapshots of the total electric field for a line source radiating at the coordinate origin: (a) for the line source radiating in free space, (b) for the line source radiating in the presence of the far-zone-focusing flat lens.

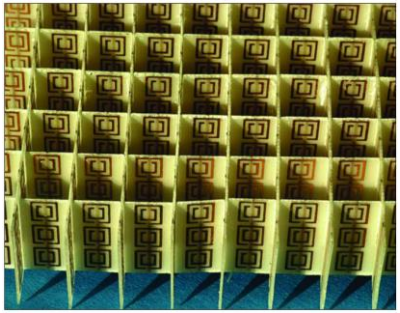
R.A. Shelby, D.r. Smith and S. Schultz, "Experimental Verification of a Negative Index of Refraction," Science, vol. 292, pp.77-79, April 2001.

Introduction

Metamaterial Structures

TABLE 4.1 Overview of Geometries and Features of Main Rings Known to Date

Nomenclature	Geometry	Features
Edge-coupled SRR		• Presents an asymmetry which seems detrimental to good transmission in waveguides (seen in simulations) • Contact issue of the rods
Broadside SRR		• Similar to the previous ring but cancels bi-anisotropy • Contact issue of the rods
Axially symmetric SRR		• Numerical simulations show a very good transmission and a good field symmetry • Contact issue of the rods
Omega SRR		• Single structure combining a ring and a rod • Two rings back to back cancel bi-anisotropy • Contact issue of the rods
S ring		• Single structure producing a permittivity and a permeability effect • No rod issue, which is a very significant advantage in waveguide measurements • Wide bandwidth



Lattice spacing: 6mm~λ/5

Double negative band: 10.2 …10.8 GHz
2D construction! Polarized waves.

R. A. Shelby, D. R. Smith, S. Schultz, Science 292, 77 (2001)

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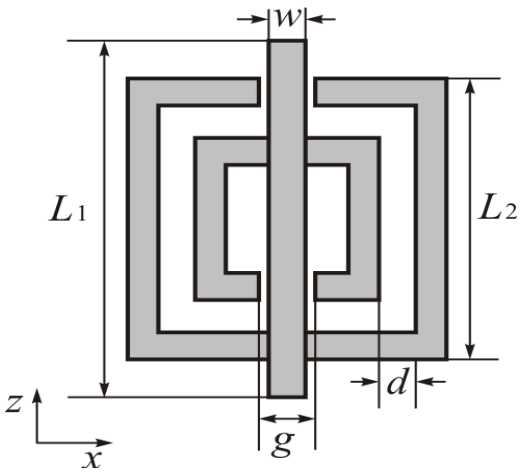
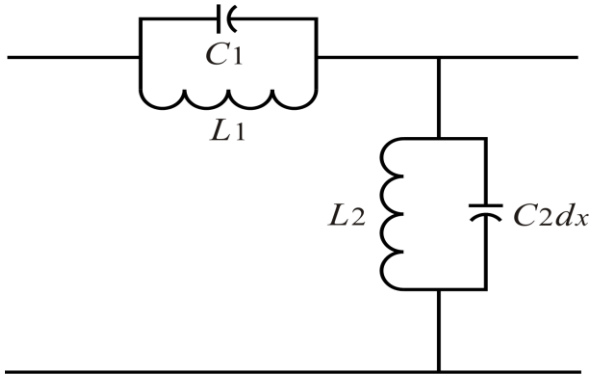
Split Ring Resonator (SRR) Element



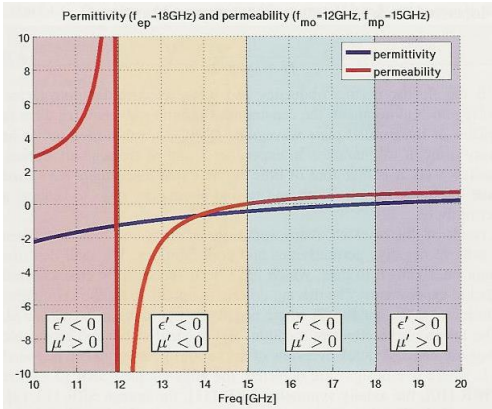


$$\mu_{eq} = \frac{L_1}{d_x}\left[\frac{1}{1-\omega^2/\omega_1^2}\right]$$

$$\varepsilon_{eq} = C_2\left(1-\frac{\omega_2^2}{\omega^2}\right)$$

$$\omega_1 = 1/\sqrt{L_1 C_1}$$

$$\omega_2 = 1/\sqrt{L_2(C_2 d_x)}$$



ω_1 ω_2

J.B. Pendry, D.Schurig and D.R. Smith, "Controlling Electromagnetic Fields," Science, 312, June 2006, pp. 1780-1782.

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Optical Invisibility Cloak

H.G.Wells, “The Invisible Man”, 1897.

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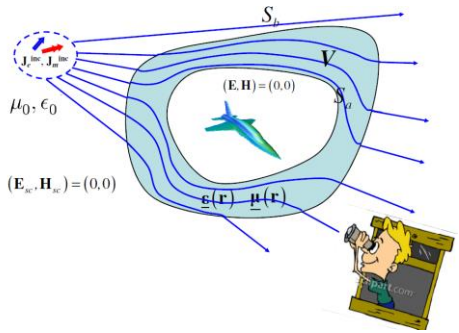
RF Cloak based on tensor type metamaterial



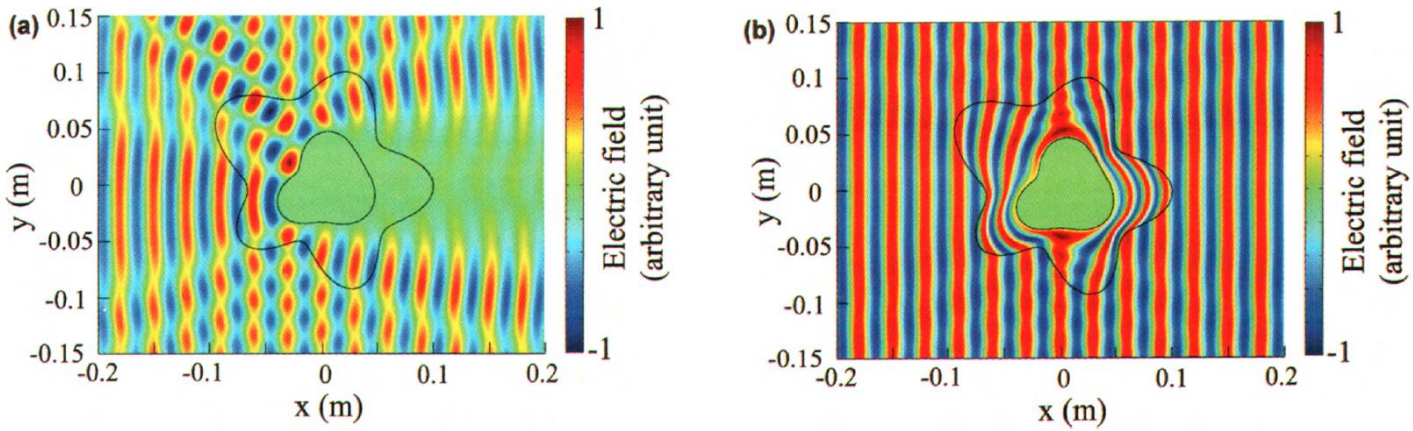


Figure 7. Snapshots of the total $\hat{z}$-directed electric field for a two-dimensional annular cloak having an arbitrary cross section: (a) the perfectly conducting cylinder exposed to the incoming TE-mode plane-wave illumination propagating in the $+\hat{x}$ direction, (b) the same perfectly conducting cylinder enclosed in an annular cloak.

No unique solution to the problem

Transformation Optics

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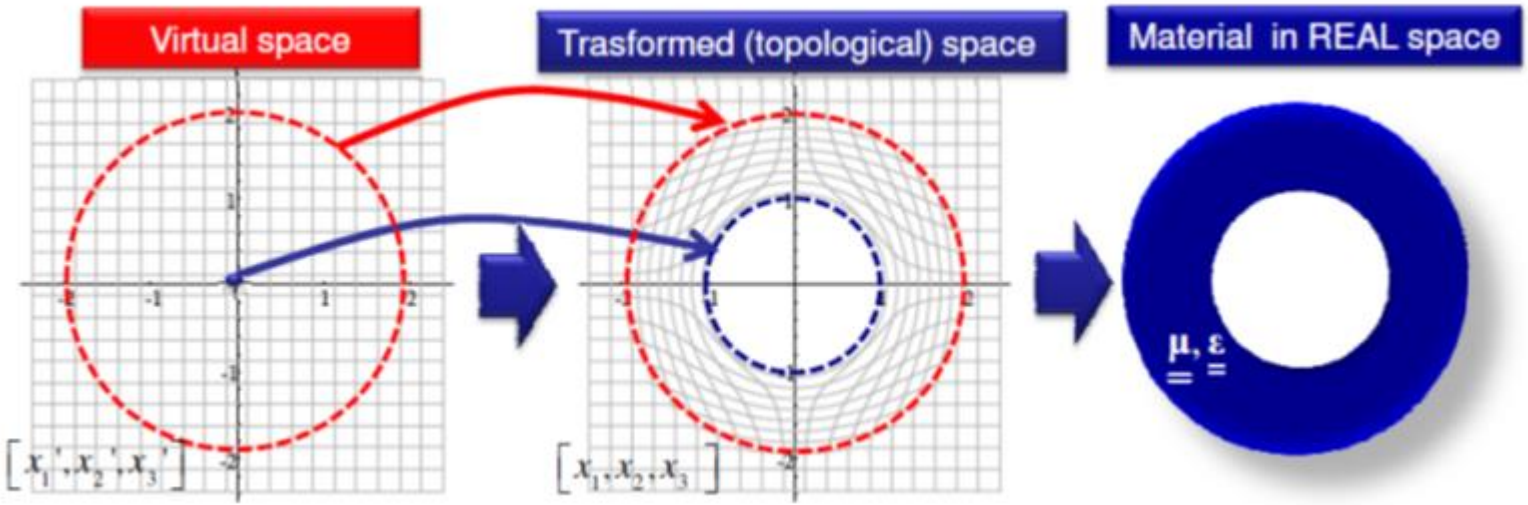

$$\nabla \times \underline{E} = -j\omega \underline{\underline{\mu}}\, \underline{H}$$

Maxwell eqs. are invariant

$$\nabla \times \underline{H} = j\omega \underline{\underline{\varepsilon}}\, \underline{E}$$

$$\left.\begin{aligned} x &= x(x', y', z') \\ y &= y(x', y', z') \\ z &= z(x', y', z') \end{aligned}\right\} \Rightarrow \mathbf{A} = \begin{bmatrix} \partial x/\partial x' & \partial x/\partial y' & \partial x/\partial z' \\ \partial y/\partial x' & \partial y/\partial y' & \partial y/\partial z' \\ \partial z/\partial x' & \partial z/\partial y' & \partial z/\partial z' \end{bmatrix} \quad \textit{Jacobian}$$

$$\underline{\underline{\mu}} = \frac{\mathbf{A}\mu'\mathbf{A}^T}{\det \mathbf{A}} \quad ; \quad \underline{\underline{\varepsilon}} = \frac{\mathbf{A}\varepsilon'\mathbf{A}^T}{\det \mathbf{A}}$$

Transformation Optics

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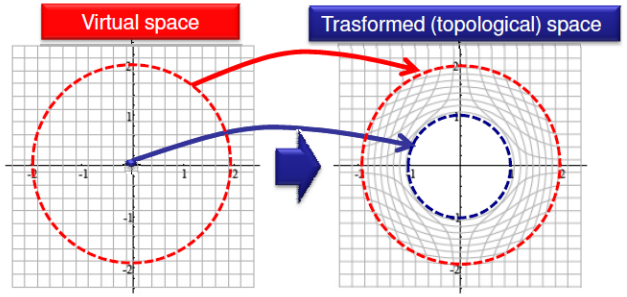
Cylindrical 2D scatterer

$$r = \frac{b-a}{b} r' + a$$
$$\phi = \phi'$$
$$z = z'$$

$$\mathbf{A} = \begin{bmatrix} (b-a)/b & 0 & 0 \\ 0 & r/r' & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ *Jacobian in cylindrical coordinates*

$$\bar{\bar{\varepsilon}} = \bar{\bar{\mu}} = \begin{bmatrix} \frac{r-a}{r} & 0 & 0 \\ 0 & \frac{r}{r-a} & 0 \\ 0 & 0 & \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \end{bmatrix}$$ *in cylindrical coordinates*

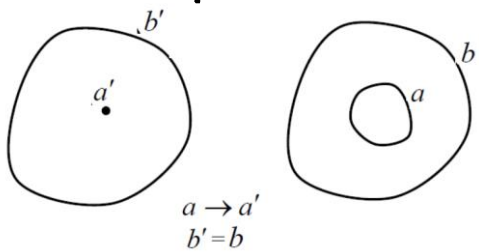


$$\bar{\bar{\varepsilon}} = \bar{\bar{\mu}} = \begin{bmatrix} \frac{r-a}{r}\cos^2\phi + \frac{r}{r-a}\sin^2\phi & \left(\frac{r-a}{r} - \frac{r}{r-a}\right)\sin\phi\cos\phi & 0 \\ \left(\frac{r-a}{r} - \frac{r}{r-a}\right)\sin\phi\cos\phi & \frac{r-a}{r}\sin^2\phi + \frac{r}{r-a}\cos^2\phi & 0 \\ 0 & 0 & \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \end{bmatrix}$$ *in cartesian coordinates*

Laplace's Equation-Transformation

The Laplace equation transformation fits very well arbitrarily scattering shape, because its solution is numerical. The method is used to compute the cloak material parameters.

Space transformation from a flat space x' to a distorted space x (x'), yields the permittivity $\bar{\bar{\varepsilon}}$ and permeability $\bar{\bar{\mu}}$



$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)U_i = 0\,,\; i=1,2,3$$

$$U_1(x,y,z)\big|_a = 0\big|_{a'} \quad ; \quad U_1(x,y,z)\big|_b = x'\big|_{b'}$$

$$U_2(x,y,z)\big|_a = 0\big|_{a'} \quad ; \quad U_2(x,y,z)\big|_b = y'\big|_{b'}$$

$$U_3(x,y,z)\big|_a = 0\big|_{a'} \quad ; \quad U_3(x,y,z)\big|_b = z'\big|_{b'}$$

$$\overline{\overline{\mu}} = \frac{\mathbf{A}\mu'\mathbf{A}^T}{\det \mathbf{A}} \quad ; \quad \overline{\overline{\varepsilon}} = \frac{\mathbf{A}\varepsilon'\mathbf{A}^T}{\det \mathbf{A}} \quad ; \quad \mathbf{A} = \begin{bmatrix} \partial U_1/\partial x & \partial U_1/\partial y & \partial U_1/\partial z \\ \partial U_2/\partial x & \partial U_2/\partial y & \partial U_2/\partial z \\ \partial U_3/\partial x & \partial U_3/\partial y & \partial U_3/\partial z \end{bmatrix}$$

Laplace's Equation-Transformation

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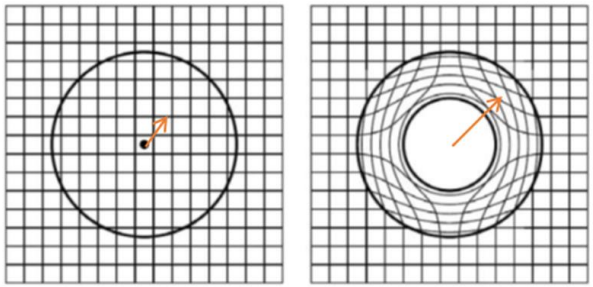
Example: *2D circular cylinder*

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}\right]U_i=0 \quad ; \quad U_1=r',U_2=\phi'=\phi,U_3=z'=z$$

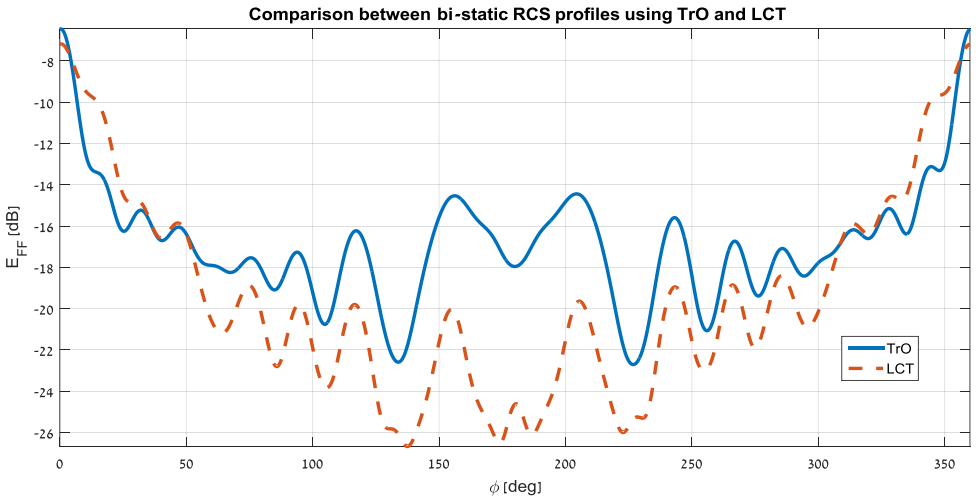
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial r'}{\partial r}\right)=0 \quad ; r'(a)=0\,,\,r'(b)=b$$

$$r'=b\log_{b/a}\left(\frac{r}{a}\right)\Rightarrow r=a\left(\frac{b}{a}\right)^{\frac{r'}{b}} \qquad r=a\left(\frac{b}{a}\right)^{\frac{r'}{b}}\,,\,\phi=\phi'\,,\,z=z'$$



(r',ϕ',z') $\qquad$ (r,ϕ,z)

$$\overline{\overline{\varepsilon}}=\overline{\overline{\mu}}=\begin{pmatrix} \ln\left(\frac{r}{a}\right) & 0 & 0 \\ 0 & \frac{1}{\ln\left(\frac{r}{a}\right)} & 0 \\ 0 & 0 & \frac{b^2\ln\left(\frac{r}{a}\right)}{r^2\ln^2\left(\frac{b}{a}\right)} \end{pmatrix}$$



Realization

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Challenges in realization:

- Requires anisotropic materials.
- The required electrical parameters (ε, μ) vary on a very wide range ($0 \rightarrow \infty$) in different regions of the cloaking.
- Implementation requires metamaterials which are: narrow band, dispersive, lossy, sensitive to polarization and incident angle.

Simplifications:

- Consider single polarization TE or TM.
- Consider normal incidence.
- Use quantization instead of continuous media with cell size smaller than 0.1λ.

Realization

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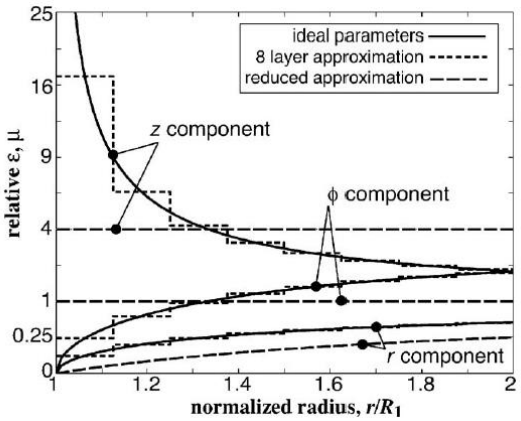
Quantization



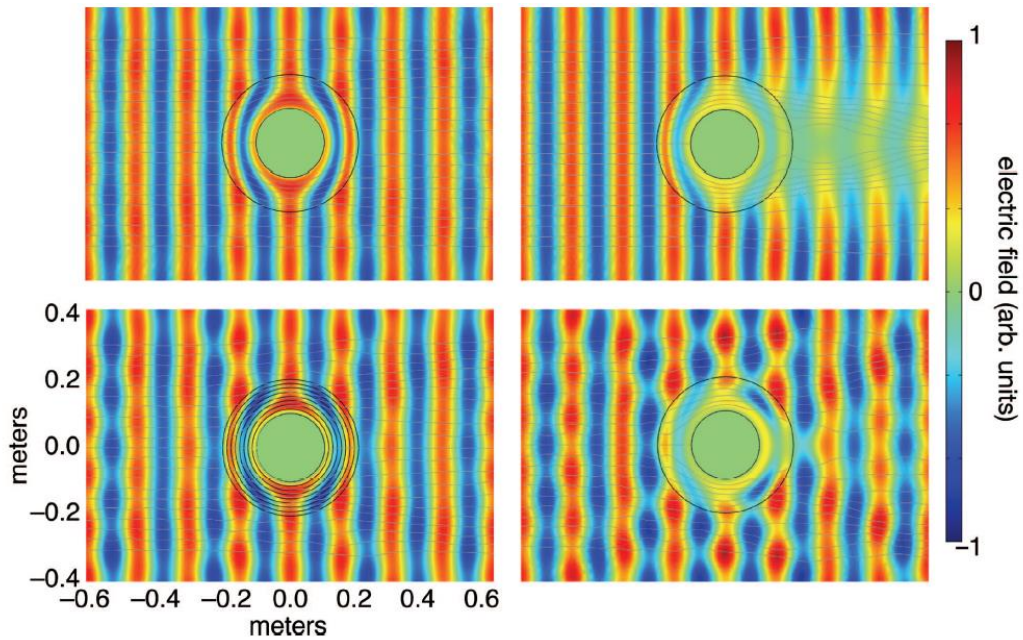


FIG. 3. (Color) The resulting electric-field distribution in the vicinity of the cloaked object. Power-flow lines (in gray) show the smooth deviation of electromagnetic power around the cloaked PEC shell. In all cases power flow is from left to right. Upper left (case 1): Ideal parameters. Upper right (case 2): Ideal parameters with a loss tangent of 0.1. Lower left (case 3): eight-layer stepwise approximation of the ideal parameters. Lower left (case 4): Reduced material parameters.

Cummer etal, "Full Wave Simulations of EM Cloaking Structures", Physical Review E, vol.74, 2006

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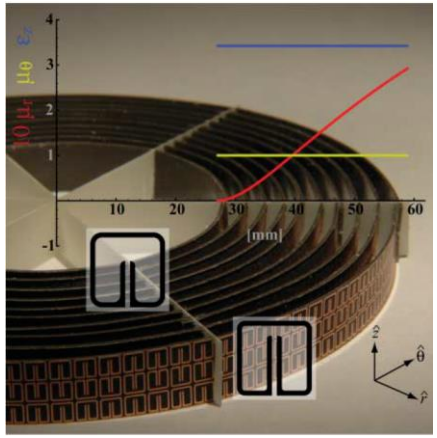


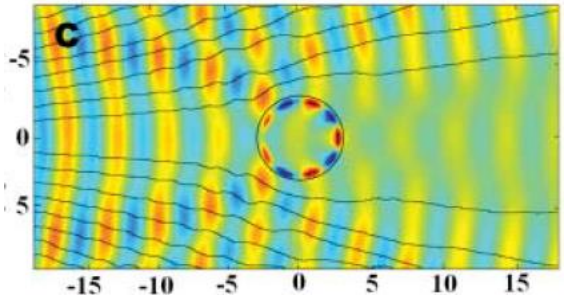
For TM polarization and normal incidence
only $\mu_{rr}^{cloak}, \mu_{\phi\phi}^{cloak}, \varepsilon_{zz}^{cloak}$ *are relevant parameters*

TrO Cloaking

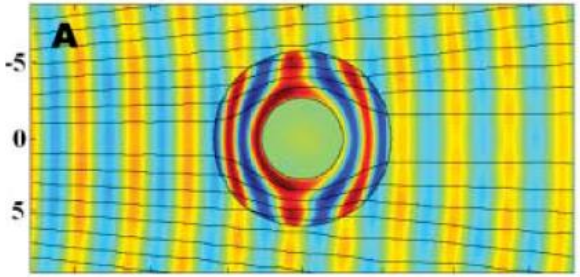
$$\mu_{rr}^{cloak} = \left(\frac{r-a}{r}\right)^2 \quad ; 0 < \mu_{rr}^{cloak} < \left(\frac{b-a}{b}\right)^2$$

$$\mu_{\phi\phi}^{cloak} = 1$$

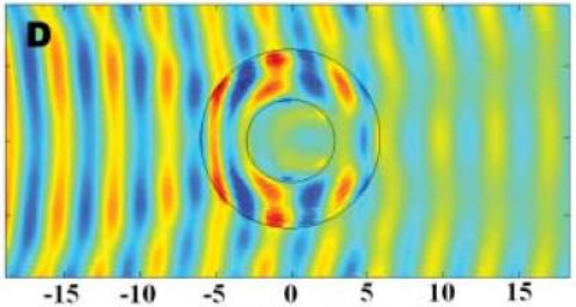
$$\varepsilon_{zz}^{cloak} = \left(\frac{b}{b-a}\right)^2$$



Metal Scatterer



Analytical Cloaking

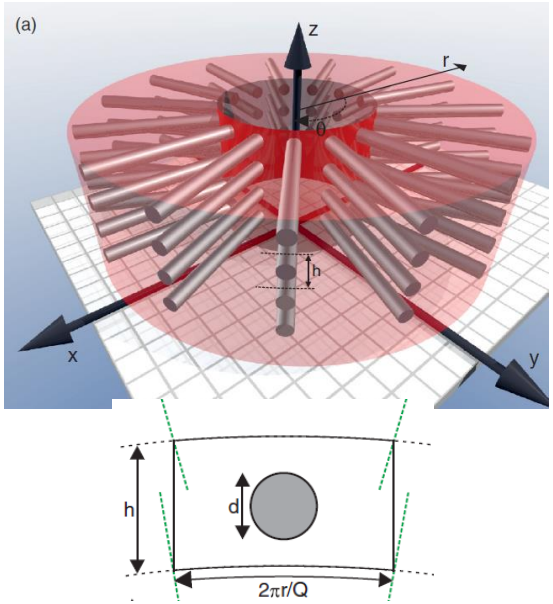


Physical Cloaking

TM *polarization-f*=8.5GHz, *a*=27.1mm (0.76λ), *b*=58.9mm (1.67λ), h=10mm

Schurig etal, "Metamaterial Electromagnetic Cloak at Microwaves Frequencies", Science, vol.314, 2006

Realization



For TE polarization and normal incidence only $\varepsilon_{rr}^{cloak}, \varepsilon_{\phi\phi}^{cloak}, \mu_{zz}^{cloak}$ *are relevant parameters*

TrO Cloaking

$$\varepsilon_{rr}^{cloak} = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right)^2 \quad ; 0 < \varepsilon_{rr}^{cloak} < 1$$

$$\varepsilon_{\phi\phi}^{cloak} = \left(\frac{b}{b-a}\right)^2$$

$$\mu_{zz}^{cloak} = 1$$

$$s = \frac{1}{1 - \frac{\varepsilon_m(\omega)}{\varepsilon_h}}$$

$\varepsilon_m(\omega)$ – *permittiviy of metal*

$\varepsilon_h = 1$ – *permittivity environment*

$$L_r = \left(\frac{d}{b-a}\right)^2 \ln\left(\frac{b-a}{d}\right) \quad ; \quad \textit{depolarization factor}$$

$$f = \frac{d^2 Q}{8rh} \quad ; \quad \textit{filling factor (metal)}$$

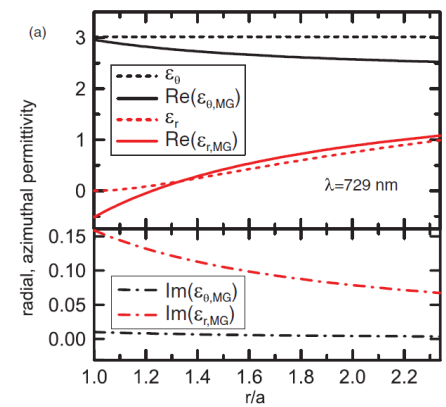
Q – *# of cylinders over* $\phi = 2\pi$ *rotation*

Maxwell – Garnet Model

$$\varepsilon_{rr}^{MG} = \left(1 - \frac{f}{s - L_r(1-f)}\right)$$

$$\varepsilon_{\phi\phi}^{MG} = \left(1 + \frac{2f}{(f-1)(L_r-1) - 2s}\right)$$

$$\mu_{zz}^{MG} = 1$$



***TE** polarization- a*=0.2m (1.33λ), *b*=0.4m (2.67λ)

Rottler etal, "Route towards cylindrical cloaking at visible frequencies using an optimization algorithm", Physical Review B, vol.86, 2012

Realization

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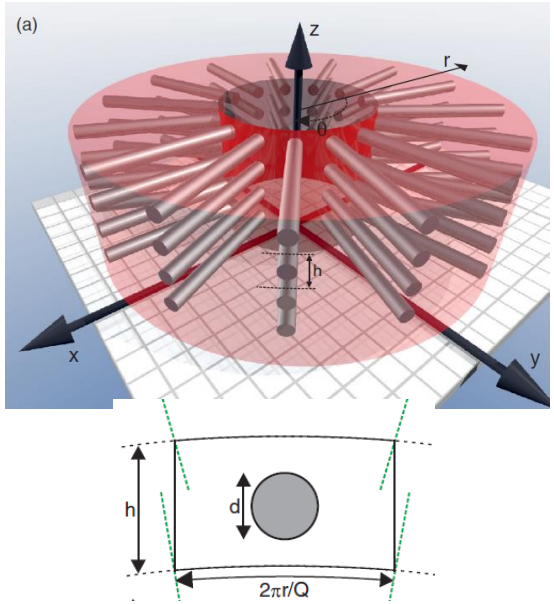

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$$\varepsilon_{\phi\phi}^{cloak} = \left(\frac{b}{b-a}\right)^2$$

$$\mu_{zz}^{cloak} = 1$$

Maxwell – Garnet Model

$$\varepsilon_{rr}^{MG} = \left(1 - \frac{f}{s - L_r(1-f)}\right)$$

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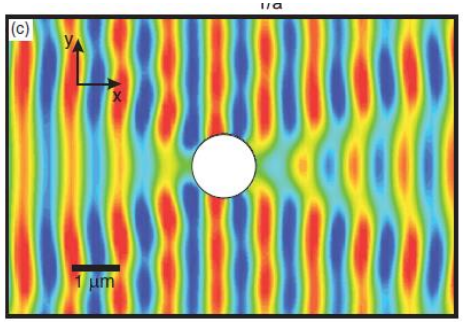
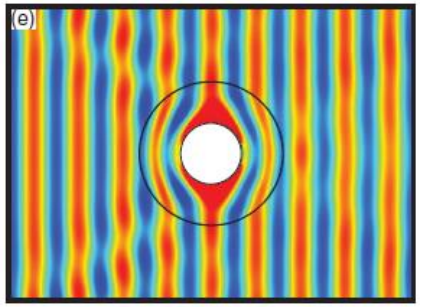
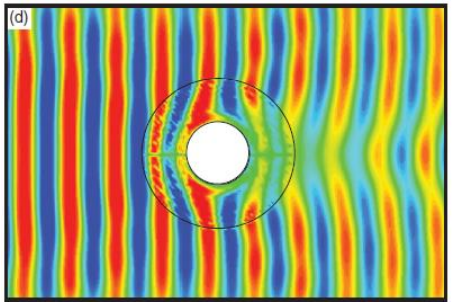
$$\mu_{zz}^{MG} = 1$$



Metal Scatterer



Analytical Cloaking



Physical Cloaking

***TE** polarization- a=0.2m (1.33λ), b=0.4m (2.67λ)*

Rottler etal, "Route towards cylindrical cloaking at visible frequencies using an optimization algorithm", Physical Review B, vol.86, 2012

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Summary

- It has been demonstrated that cloaking is feasible for both canonical and arbitrary shape targets .

- Invisible cloak in RF is possible, but further research effort is needed (theoretical and experimental) to comply with polarization and frequency bandwidth requirements.

- Metamaterials technology constitute a promising road towards implementation.