Cloaking –The Road to Realization

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Outline

- Introduction
- Transformation Optics
- Laplace’s Equation - Transformation
- Realization
- Summary
**Introduction**

**Metamaterials**

The prefix “meta” comes from Greek and means “after” or “beyond”. These are engineered composites (usually periodic structures) that exhibit superior properties that are not found in nature and not observed in the constituent materials.

**Material classification**

In double negative (DNG) materials the phase velocity is negative, although the group velocity stays positive. Vectors $k$, $E$ and $H$ will form a left hand set.

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History of Metamaterials

Frequency Selective Surfaces (FSS) and Electromagnetic Band Gap (EBG) materials ($d \sim \lambda$)

- Element Distances are on the order of half a wavelength or more (Periodic medium concepts)
- Electromagnetic Band Gap materials (EBG), Artificial Magnetic Conductors (AMC), High Impedance Surfaces (HIS)

Metamaterials ($d << \lambda$)

- Elements and distances between them are much smaller than a wavelength (simultaneous effective negative permittivity and permeability)
- Have several names including left-handed materials, backward-wave materials, negative index of refraction (NIR) materials, etc.

Introduction
Introduction

DNG Structures

$n_1 \sin \theta_1 = n_2 \sin \theta_2 \Rightarrow \text{Snell's law}$

Figure 1.8 Geometry of the scattering of a wave obliquely incident upon a DPS–DNG interface.

$\varepsilon_2 = -1 \ ; \ \mu_2 = -1$ (no reflection)

**Introduction**

**DNG Lens**

**Zero index flat Lens**

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Introduction

**Metamaterial Structures**

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Geometry</th>
<th>Features</th>
</tr>
</thead>
</table>
| Edge-coupled SRR   | ![Image](image1.png) | - Presents an asymmetry which seems detrimental to good transmission in waveguides (seen in simulations)  
                      - Contact issue of the rods                                             |
| Broadside SRR      | ![Image](image2.png) | - Similar to the previous ring but cancels bi-anisotropy  
                      - Contact issue of the rods                                             |
| Axially symmetric SRR | ![Image](image3.png) | - Numerical simulations show a very good transmission and a good field symmetry  
                      - Contact issue of the rods                                             |
| Omega SRR          | ![Image](image4.png) | - Single structure combining a ring and a rod  
                      - Two rings back to back cancel bi-anisotropy  
                      - Contact issue of the rods                                             |
| S ring              | ![Image](image5.png) | - Single structure producing a permittivity and a permeability effect  
                      - No rod issue, which is a very significant advantage in waveguide measurements  
                      - Wide bandwidth                                                        |

**Lattice spacing:** 6mm~\(\lambda/5\)

**Double negative band:** 10.2...10.8 GHz

2D construction! Polarized waves.

Introduction

Split Ring Resonator (SRR) Element

\[ \mu_{eq} = \frac{L_1}{d_x} \left[ \frac{1}{1 - \omega^2/\omega_1^2} \right] \]

\[ \varepsilon_{eq} = C_2 \left( 1 - \frac{\omega_2^2}{\omega^2} \right) \]

\[ \omega_1 = \frac{1}{\sqrt{L_1 C_1}} \]

\[ \omega_2 = \frac{1}{\sqrt{L_2 (C_2 d_x)}} \]

Introduction

Optical Invisibility Cloak

Introduction

RF Cloak based on tensor type metamaterial

No unique solution to the problem

Figure 7. Snapshots of the total $\hat{z}$-directed electric field for a two-dimensional annular cloak having an arbitrary cross section: (a) the perfectly conducting cylinder exposed to the incoming TE-mode plane-wave illumination propagating in the $+\hat{x}$ direction, (b) the same perfectly conducting cylinder enclosed in an annular cloak.
Transformation Optics

\[ \nabla \times \mathbf{E} = -j \omega \mu \mathbf{H} \]
\[ \nabla \times \mathbf{H} = j \omega \varepsilon \mathbf{E} \]

Maxwell eqs. are invariant

\[ x = x(x', y', z') \] \[ y = y(x', y', z') \] \[ z = z(x', y', z') \] \[ \Rightarrow \mathbf{A} = \begin{bmatrix} \partial x / \partial x' & \partial x / \partial y' & \partial x / \partial z' \\ \partial y / \partial x' & \partial y / \partial y' & \partial y / \partial z' \\ \partial z / \partial x' & \partial z / \partial y' & \partial z / \partial z' \end{bmatrix} \] Jacobian

\[ \mu = \frac{\mathbf{A} \mu' \mathbf{A}^T}{\det \mathbf{A}} \]; \[ \varepsilon = \frac{\mathbf{A} \varepsilon' \mathbf{A}^T}{\det \mathbf{A}} \]
**Transformation Optics**

**Cylindrical 2D scatterer**

\[
\begin{align*}
  r &= \frac{b-a}{b} r' + a \\
  \phi &= \phi' \\
  z &= z'
\end{align*}
\]

\[
A = \begin{bmatrix}
  \frac{(b-a)}{b} & 0 & 0 \\
  0 & r/r' & 0 \\
  0 & 0 & 1
\end{bmatrix}
\] Jacobian in cylindrical coordinates

\[
= \varepsilon = \mu = \begin{bmatrix}
  \frac{r-a}{r} & 0 & 0 \\
  0 & \frac{r}{r-a} & 0 \\
  0 & 0 & \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}
\end{bmatrix}
\] in cylindrical coordinates

\[
= \varepsilon = \mu = \begin{bmatrix}
  \frac{r-a}{r} \cos^2 \phi + \frac{r}{r-a} \sin^2 \phi & \left(\frac{r-a}{r} - \frac{r}{r-a}\right) \sin \phi \cos \phi & 0 \\
  \left(\frac{r-a}{r} - \frac{r}{r-a}\right) \sin \phi \cos \phi & \frac{r-a}{r} \sin^2 \phi + \frac{r}{r-a} \cos^2 \phi & 0 \\
  0 & 0 & \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}
\end{bmatrix}
\] in cartesian coordinates
Laplace’s Equation-Transformation

The Laplace equation transformation fits very well arbitrarily scattering shape, because its solution is numerical. The method is used to compute the cloak material parameters. Space transformation from a flat space \( x' \) to a distorted space \( x (x') \), yields the permittivity \( \tilde{\varepsilon} \) and permeability \( \tilde{\mu} \)

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) U_i = 0, \quad i = 1, 2, 3
\]

\[
U_1(x, y, z) \big|_a = 0 \big|_{a'}; \quad U_1(x, y, z) \big|_b = x' \big|_b;
\]

\[
U_2(x, y, z) \big|_a = 0 \big|_{a'}; \quad U_2(x, y, z) \big|_b = y' \big|_b;
\]

\[
U_3(x, y, z) \big|_a = 0 \big|_{a'}; \quad U_3(x, y, z) \big|_b = z' \big|_b;
\]

\[
\mu = \frac{A \mu' A^T}{\det A} \quad ; \quad \varepsilon = \frac{A \varepsilon' A^T}{\det A} \quad ; \quad A = \begin{bmatrix}
\partial U_1 / \partial x & \partial U_1 / \partial y & \partial U_1 / \partial z \\
\partial U_2 / \partial x & \partial U_2 / \partial y & \partial U_2 / \partial z \\
\partial U_3 / \partial x & \partial U_3 / \partial y & \partial U_3 / \partial z
\end{bmatrix}
\]
Laplace’s Equation-Transformation

Example: 2D circular cylinder

\[
\begin{bmatrix}
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \end{bmatrix} U_i = 0 \quad ; \quad U_1 = r', U_2 = \phi' = \phi, U_3 = z' = z
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial r'}{\partial r} \right) = 0 \quad ; \quad r'(a) = 0, \quad r'(b) = b
\]

\[
r' = b \log_{b/a} \left( \frac{r}{a} \right) \Rightarrow r = a \left( \frac{b}{a} \right)^{r'}
\]

\[
(r, \phi, z) \quad \Rightarrow \quad (r', \phi', z')
\]

\[
\begin{pmatrix}
\ln \left( \frac{r}{a} \right) & 0 & 0 \\
0 & 1 & 0 \\
\ln \left( \frac{r}{a} \right) & 0 & \frac{b^2 \ln \left( \frac{r}{a} \right)}{r^2 \ln^2 \left( \frac{b}{a} \right)}
\end{pmatrix}
\]

Comparison between bi-static RCS profiles using TrO and LCT
Realization

Challenges in realization:
- Requires anisotropic materials.
- The required electrical parameters ($\varepsilon$, $\mu$) vary on a very wide range ($0\rightarrow\infty$) in different regions of the cloaking.
- Implementation requires metamaterials which are: narrow band, dispersive, lossy, sensitive to polarization and incident angle.

Simplifications:
- Consider single polarization TE or TM.
- Consider normal incidence.
- Use quantization instead of continuous media with cell size smaller than $0.1\lambda$. 
Realization

Quantization

FIG. 3. (Color) The resulting electric-field distribution in the vicinity of the cloaked object. Power-flow lines (in gray) show the smooth deviation of electromagnetic power around the cloaked PEC shell. In all cases power flow is from left to right. Upper left (case 1): Ideal parameters. Upper right (case 2): Ideal parameters with a loss tangent of 0.1. Lower left (case 3): eight-layer stepwise approximation of the ideal parameters. Lower left (case 4): Reduced material parameters.

**Realization**

For TM polarization and normal incidence only $\mu_{rr}^{\text{cloak}}, \mu_{\phi\phi}^{\text{cloak}}, \varepsilon_{zz}^{\text{cloak}}$ are relevant parameters.

**TrO Cloaking**

\[
\mu_{rr}^{\text{cloak}} = \left( \frac{r - a}{r} \right)^2 ; 0 < \mu_{rr}^{\text{cloak}} < \left( \frac{b - a}{b} \right)^2
\]

$\mu_{\phi\phi}^{\text{cloak}} = 1$

$\varepsilon_{zz}^{\text{cloak}} = \left( \frac{b}{b - a} \right)^2$

**Metal Scatterer**

**Analytical Cloaking**

**Physical Cloaking**

**TM polarization** $f = 8.5\text{GHz}$, $a = 27.1\text{mm} \; (0.76\lambda)$, $b = 58.9\text{mm} \; (1.67\lambda)$, $h = 10\text{mm}$

Realization

For TE polarization and normal incidence only $\varepsilon_{rr}^{\text{cloak}}$, $\varepsilon_{\phi\phi}^{\text{cloak}}$, $\mu_{zz}^{\text{cloak}}$ are relevant parameters

**TrO Cloaking**

$$
\varepsilon_{rr}^{\text{cloak}} = \left( \frac{b}{b-a} \right)^2 \left( \frac{r-a}{r} \right)^2 ; 0 < \varepsilon_{rr}^{\text{cloak}} < 1
$$

$$
\varepsilon_{\phi\phi}^{\text{cloak}} = \left( \frac{b}{b-a} \right)^2
$$

$$
\mu_{zz}^{\text{cloak}} = 1
$$

**Maxwell – Garnet Model**

$$
\varepsilon_{rr}^{\text{MG}} = \left( 1 - \frac{f}{s - L_r(1-f)} \right)
$$

$$
\varepsilon_{\phi\phi}^{\text{MG}} = \left( 1 + \frac{2f}{(f-1)(L_r-1)-2s} \right)
$$

$$
\mu_{zz}^{\text{MG}} = 1
$$

**TE polarization** - $a=0.2m$ (1.33$\lambda$), $b=0.4m$ (2.67$\lambda$)

Realization

TrO Cloaking

\[ \varepsilon_{rr}^{\text{cloak}} = \left( \frac{b}{b-a} \right)^2 \left( \frac{r-a}{r} \right)^2 ; \quad 0 < \varepsilon_{rr}^{\text{cloak}} < 1 \]

\[ \varepsilon_{\phi\phi}^{\text{cloak}} = \left( \frac{b}{b-a} \right)^2 \]

\[ \mu_{zz}^{\text{cloak}} = 1 \]

Maxwell – Garnet Model

\[ \varepsilon_{rr}^{\text{MG}} = \left( 1 - \frac{f}{s - r(1 - f)} \right) \]

\[ \varepsilon_{\phi\phi}^{\text{MG}} = \left( 1 + \frac{2f}{(f-1)(L_r - 1) - 2s} \right) \]

\[ \mu_{zz}^{\text{MG}} = 1 \]

TE polarization - \( a = 0.2 \text{m} (1.33\lambda) \), \( b = 0.4 \text{m} (2.67\lambda) \)

Summary

- It has been demonstrated that cloaking is feasible for both canonical and arbitrary shape targets.

- Invisible cloak in RF is possible, but further research effort is needed (theoretical and experimental) to comply with polarization and frequency bandwidth requirements.

- Metamaterials technology constitute a promising road towards implementation.