Heterogeneous Track-to-Track Fusion

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Motivation and Difficulties of Heterogeneous T2TF

Motivation: There is need to fuse tracks from *active* and *passive* sensors.

Compared with homogeneous track-to-track fusion (T2TF) that assumes the same system model for different local trackers, the heterogeneous case poses two major difficulties:

- The model heterogeneity problem: fuse tracks from different state spaces (related by a certain nonlinear transformation).
- The estimation errors' dependence problem: recognized as the "common process noise effect", which is quantified by the crosscovariance matrix.

Consider the following state-space models

- at sensor i

$$\mathbf{x}^i(k+1) = f^i[\mathbf{x}^i(k)] + \mathbf{v}^i(k) \quad (1)$$

$$\mathbf{z}^i(k) = h^i[\mathbf{x}^i(k)] + \mathbf{w}^i(k) \quad (2)$$

- at sensor j

$$\mathbf{x}^j(k+1) = f^j[\mathbf{x}^j(k)] + \mathbf{v}^j(k) \quad (3)$$

$$\mathbf{z}^j(k) = h^j[\mathbf{x}^j(k)] + \mathbf{w}^j(k) \quad (4)$$

where

- $\mathbf{x}^i$ and $\mathbf{x}^j$ are in different state spaces (with unequal dimensions).
- $f^*(\cdot)$ and $h^*(\cdot)$ are nonlinear in general
- $\mathbf{v}^*(\cdot)$ denote the process noises
- $\mathbf{w}^*(\cdot)$ denote measurement noises.

Note that the two heterogeneous trackers are assumed synchronized and the time index k for sampling time t_k will be omitted if there is no ambiguity.

Let $\mathbf{x}^i$ be the larger dimension state (e.g., full Cartesian position and velocity in 2-dimensional space for tracking with an active sensor)

$$\mathbf{x}^i = [\ x \quad \dot{x} \quad y \quad \dot{y}\ ]' \tag{5}$$

and $\mathbf{x}^j$ be the smaller dimension state (e.g., angular position and velocity for tracking with a passive sensor)

$$\mathbf{x}^j = [\ \theta \quad \dot{\theta}\ ]' \tag{6}$$

These state vectors have the nonlinear relationship

$$\mathbf{x}^j \triangleq g(\mathbf{x}^i) \tag{7}$$

From sensor i one has

- the track $\hat{\mathbf{x}}^i$
- the covariance matrix P^i.

From sensor j one has

- the track $\hat{\mathbf{x}}^j$
- the covariance matrix P^j.

The problem is how to carry out the fusion of the track $\hat{\mathbf{x}}^i$ with P^i and the track $\hat{\mathbf{x}}^j$ with P^j to achieve

- improved estimation performance over single sensor track quality.
- comparable estimation performance to the track quality of centralized measurement tracker/fuser (CTF).

The LMMSE fused estimate of $\mathbf{x} = \mathbf{x}^i$ with “observation" $\mathbf{z} = \hat{\mathbf{x}}^j$ (using the fundamental equations of LMMSE) is

$$\hat{\mathbf{x}}^i_{\text{LMMSE}} = \hat{\mathbf{x}}^i + P_{\mathbf{xz}} P_{\mathbf{zz}}^{-1} \left[\hat{\mathbf{x}}^j - g\left(\hat{\mathbf{x}}^i\right) \right] \tag{8}$$

with the corresponding fused covariance matrix

$$P^i_{\text{LMMSE}} = P^i - P_{\mathbf{xz}} P_{\mathbf{zz}}^{-1} P'_{\mathbf{xz}} \tag{9}$$

where

$$\begin{aligned} P_{\mathbf{xz}} &\triangleq E\left[\left(\mathbf{x}^i - \hat{\mathbf{x}}^i \right) \left(\hat{\mathbf{x}}^j - g(\hat{\mathbf{x}}^i) \right)' \right] \\ &\approx P^i (G^i)' - P^{ij} \end{aligned} \tag{10}$$

$$\begin{aligned} P_{\mathbf{zz}} &\triangleq E\left[\left(\hat{\mathbf{x}}^j - g(\hat{\mathbf{x}}^i) \right) \left(\hat{\mathbf{x}}^j - g(\hat{\mathbf{x}}^i) \right)' \right] \\ &\approx P^j - G^i P^{ij} - P^{ji} (G^i)' + G^i P^i (G^i)' \end{aligned} \tag{11}$$

with G^i the Jacobian of $g(\mathbf{x}^i)$

$$G^i \triangleq \left[\nabla_{\mathbf{x}^i} g(\mathbf{x}^i)' \right]'_{\mathbf{x}^i = \hat{\mathbf{x}}^i} \tag{12}$$

and P^{ij} the crosscovariance matrix.

Under the Gaussian assumption, the heterogeneous T2TF problem can be solved by minimizing the negative log-likelihood function

$$\begin{aligned} L &= -\ln p(\hat{\mathbf{x}}^i, \hat{\mathbf{x}}^j | \mathbf{x}^i) \\ &\propto \left(\begin{bmatrix} \hat{\mathbf{x}}^i \\ \hat{\mathbf{x}}^j \end{bmatrix} - \begin{bmatrix} \mathbf{x}^i \\ \mathbf{x}^j \end{bmatrix} \right)' \begin{bmatrix} P^i & P^{ij} \\ P^{ji} & P^j \end{bmatrix}^{-1} \left(\begin{bmatrix} \hat{\mathbf{x}}^i \\ \hat{\mathbf{x}}^j \end{bmatrix} - \begin{bmatrix} \mathbf{x}^i \\ \mathbf{x}^j \end{bmatrix} \right) \end{aligned} \quad (13)$$

Then, with $\mathbf{x}^j = g(\mathbf{x}^i)$, the ML fused estimate is the solution of

$$\nabla_{\mathbf{x}^i} L = 0 \quad (14)$$

Because of the nonlinearity of the function $g(\mathbf{x}^i)$, we solve (14) by numerical search.

The fusion result is denoted as $\hat{\mathbf{x}}^i_{ML}$ with the corresponding covariance matrix

$$P^i_{\text{ML}} = \left(\begin{bmatrix} I & G^i \end{bmatrix} \begin{bmatrix} P^i & P^{ij} \\ P^{ji} & P^j \end{bmatrix}^{-1} \begin{bmatrix} I \\ G^i \end{bmatrix} \right)^{-1} \quad (15)$$

where G^i is defined in (12) and I is the identity matrix (4×4 in our case).

- The measurements
  - an active sensor located at $(x_a,\ y_a)$ with measurements
    range: $r = \sqrt{(x - x_a)^2 + (y - y_a)^2} + w_r$
    azimuth angle: $\theta_a = \tan^{-1}\left(\frac{y - y_a}{x - x_a}\right) + w_a$
  - a passive sensor located at $(x_p,\ y_p)$ with measurements
    only azimuth angle: $\theta_p = \tan^{-1}\left(\frac{y - y_p}{x - x_p}\right) + w_p$

  where w_r, w_a and w_p are assumed to be mutually independent zero mean white Gaussian noises with standard deviations (SD) σ_r, σ_a and σ_p, respectively.
- The ground truth

  A target moving with a constant speed of 250 m/s with initial state in Cartesian coordinates (with position in m)

$$\mathbf{x}(0) = \begin{bmatrix} x(0) & \dot{x}(0) & y(0) & \dot{y}(0) \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 20000 & 250 \end{bmatrix}' \quad (16)$$

  At $k = 10$ ($t = 100$ s) it starts a left turn of 2°/s for 30 s, then continues straight until $k = 20$, at which time it turns right with 1°/s for 50 s, then left with 1°/s for 90 s, then right with 1°/s for 50 s, then continues straight until 50 s.

A Typical Scenario – overview

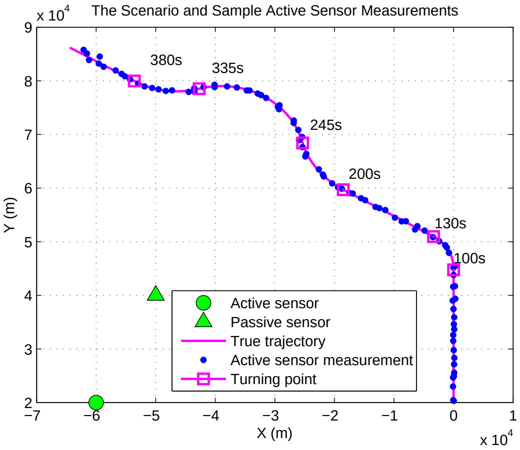

Figure 1: The scenario, with the target true speed 250 m/s, the active sensor located at $\left(-6 \times 10^4,\ 2 \times 10^4\right)$ m with sampling interval $T_a = 5$ s and the passive sensor located at $\left(-5 \times 10^4,\ 4 \times 10^4\right)$ m with sampling interval $T_p = 1$ s.

The active sensor IMM estimator has two modes

mode 1 linear nearly constant acceleration (NCA) model: implemented as discretized continuous white noise acceleration (CWNA) model .

mode 2 nonlinear nearly coordinate turn (NCT) model: implemented as discretized continuous coordinate turn (CCT) model [Morelande&Gordon, ICASSP 2005].

The (target state-dependent) process noise covariance matrix of the NCT model is (details in [MG2005])

$$Q_a^i[\mathbf{x}(k)] = \begin{bmatrix} \frac{T_a^3}{3}\frac{\dot{x}^2(k)}{\dot{x}^2(k)+\dot{y}^2(k)}q_v & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \frac{T_a^3}{3}\frac{\dot{y}^2(k)}{\dot{x}^2(k)+\dot{y}^2(k)}q_v & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & Tq_\Omega \end{bmatrix} \tag{17}$$

where q_a and q_Ω are the power spectral densities (PSDs). Note that the process noise induced RMS change in the velocity and in the turn rate over sampling interval T_a are

$$d_v \triangleq \frac{\sqrt{q_v T_a}}{T_a} \qquad d_\Omega \triangleq \frac{\sqrt{q_\Omega T_a}}{T_a} \tag{18}$$

whose physical dimensions are linear acceleration and turn acceleration, respectively. The CTF uses the same IMM design (CTF IMM for short) as the active sensor IMM.

For the passive sensor, in the scenario considered, the target maneuvering index is very small and the target maneuvers are nearly unobservable by the passive sensor. Consequently, a linear KF (rather than IMM estimator) is used [KB2003].

The motion model used is the discretized continuous Wiener process acceleration (CWPA) model (with angle, angle rate and angle acceleration). The covariance matrix of the process noise is

$$Q_p^j(k) = \begin{bmatrix} \frac{T_p^5}{20} & \frac{T_p^4}{8} & \frac{T_p^3}{6} \\ \frac{T_p^4}{8} & \frac{T_p^3}{3} & \frac{T_p^2}{2} \\ \frac{T_p^3}{6} & \frac{T_p^2}{2} & T_p \end{bmatrix} q_p \tag{19}$$

where q_p is the process noise PSD. The process noise induced RMS change in the angular acceleration over T_p are

$$d_p \triangleq \frac{\sqrt{q_p T_p}}{T_p} \tag{20}$$

whose physical dimension is the angular jerk (derivative of acceleration).

Note that d_p with d_v and d_Ω as in (18) are the design values used to select the process noise PSDs for the local trackers.

- The measurement noises: the active sensor $\sigma_r = 20\,$m and $\sigma_a = 5\,$mrad; the passive sensor $\sigma_p = 1\,$mrad.

  An unbiased measurement conversion from polar coordinates to Cartesian coordinates is done for the active sensor measurements for filtering.

- The process noise intensities settings
  - Active sensor:

	d_a (m/s^2)	d_Ω (mrad/s^2)
Mode 1 (NCA)	0.2	N/A
Mode 2 (NCT)	1	2

  - Passive sensor: $d_p = 0.04$ mrad/s^3.
- The IMM transition probability matrix is

$$\pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \tag{21}$$

  with initial mode probability vector $[\,0.9,\ 0.1\,]$.
- The estimate $\hat{\mathbf{x}}^i(k)$ from the active sensor IMM with the corresponding covariance matrix $P^i(k)$ and the estimate $\hat{\mathbf{x}}^j(k)$ from the passive KF with the corresponding covariance matrix $P^j(k)$ are used for the heterogeneous T2TF.
- The fusion performance is compared with the corresponding single active sensor IMM track and the CTF IMM track.

In view of the fact that there is no known way to evaluate the crosscovariance of the estimation errors in the case of heterogeneous trackers, a Monte Carlo (MC) investigation of these errors' crosscorrelations is carried out.

The *sample crosscorrelation coefficient* between the lth component of $\mathbf{x}^i$ and the hth component of $\mathbf{x}^j$ in M MC runs at a particular point in time is

$$\hat{\rho}^{M}_{\mathbf{x}^i_l \mathbf{x}^j_h} \triangleq \frac{\sum_{m=1}^{M}(\hat{\mathbf{x}}^i_{l,m} - \mathbf{x}^i_l)(\hat{\mathbf{x}}^j_{h,m} - \mathbf{x}^i_h)}{\sqrt{\left[\sum_{m=1}^{M}(\hat{\mathbf{x}}^i_{l,m} - \mathbf{x}^i_l)^2\right]\left[\sum_{m=1}^{M}(\hat{\mathbf{x}}^j_{h,m} - \mathbf{x}^j_h)^2\right]}} \tag{22}$$

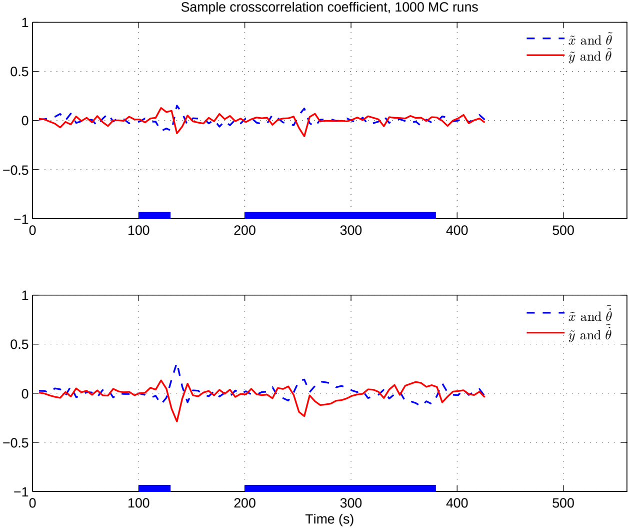

Figure 2: The sample crosscorrelation for $\tilde{x}$ and $\tilde{y}$ with $\tilde{\theta}$ and $\tilde{\dot{\theta}}$. (Some are positive and some are negative)

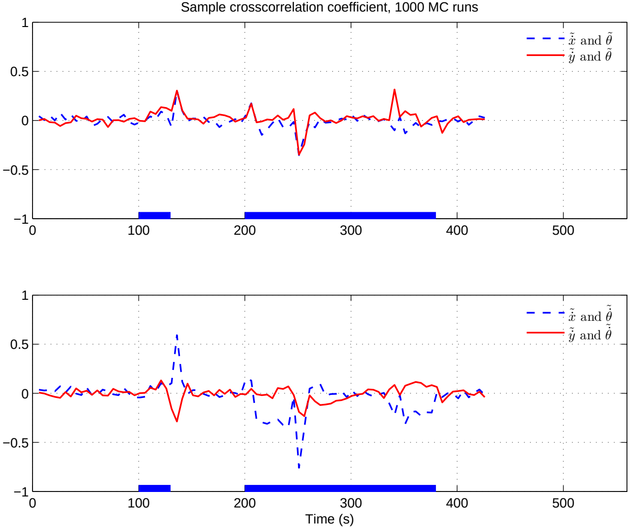

Figure 3: The sample crosscorrelation for $\tilde{\dot{x}}$ and $\tilde{\dot{y}}$ with $\tilde{\theta}$ and $\tilde{\dot{\theta}}$.
(Some are positive and some are negative)

The Sample Crosscorrelation – negligible crosscorrelation

It can be seen from the MC simulations that

- Some of the crosscorrelations are positive and some are negative.
- The crosscorrelations depend on the relative geometry of the two sensors and the target, as well as the target maneuvers.
- For the nonlinear case, neglecting the crosscorrelations makes the fusion sometimes optimistic and sometimes pessimistic, but the effect is small.

This supports the approach of ignoring the dependency between the tracks from different local sensors. Thus, since the maneuvers are unknown and scenario dependent, we pursue the heterogeneous T2TF without considering the crosscorrelation between the estimation errors.

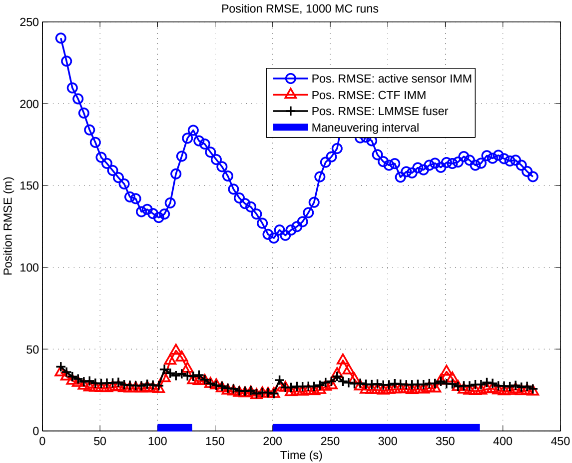

Figure 4: The position RMSE for LMMSE fuser.
(Heterogeneouse T2TF is superior to CTF IMM during model switching)
(ML fuser has practical the same performance as LMMSE fuser)

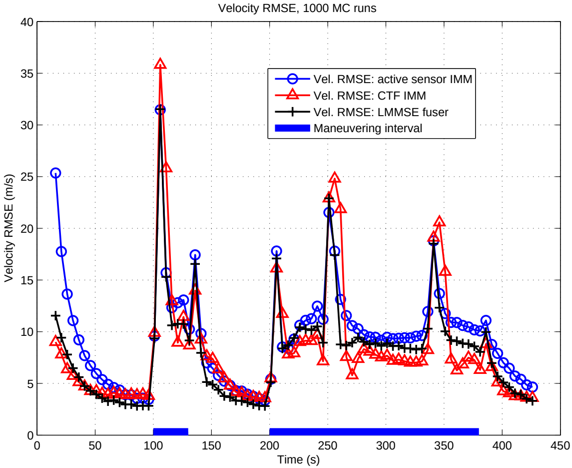

Figure 5: The velocity RMSE for LMMSE fuser.

(Heterogeneouse T2TF is superior to CTF IMM during model switching)

(ML fuser has practical the same performance as LMMSE fuser)

Simulation Results – maneuvering mode probability (NCT)

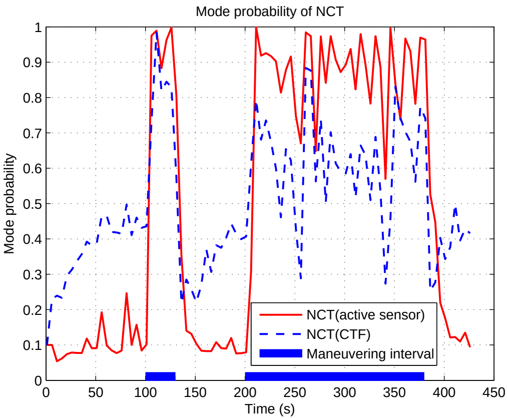

Figure 6: Maneuvering mode probability (NCT) in the active sensor IMM and CTF IMM. (Active sensor IMM is superior to CTF IMM!)

Conclusions

- The LMMSE and the ML approaches for heterogenous T2TF can effectively achieve improved performance over the single sensor track quality and comparable performance to the CTF track.
- The estimation errors' crosscorrelation has been examined by MC simulations. The crosscorrelation of the estimation errors from heterogeneous local sensors is too complicated to capture.
- The use of the passive measurements in the CTF IMM “clouds" the maneuvers – it is preferable to have an active sensor IMM (which does detect the maneuvers) and a passive sensor KF (since the passive sensor is almost “blind" to the maneuvers) and fuse the outputs of these two local trackers.
- The freedom available to each local sensor to flexibly design a more suitable local estimator allows the heterogeneous T2TF approach to achieve a better estimation performance than the CTF IMM in the scenario considered.
- The LMMSE T2TF has practically the same performance as the ML T2TF and can be considered as an effective and efficient alternative for the numerical search required by the ML approach.