Complementary pair radar pulse waveform - revisited

פולסי מכ״ם משלימים – בחינה מחדש

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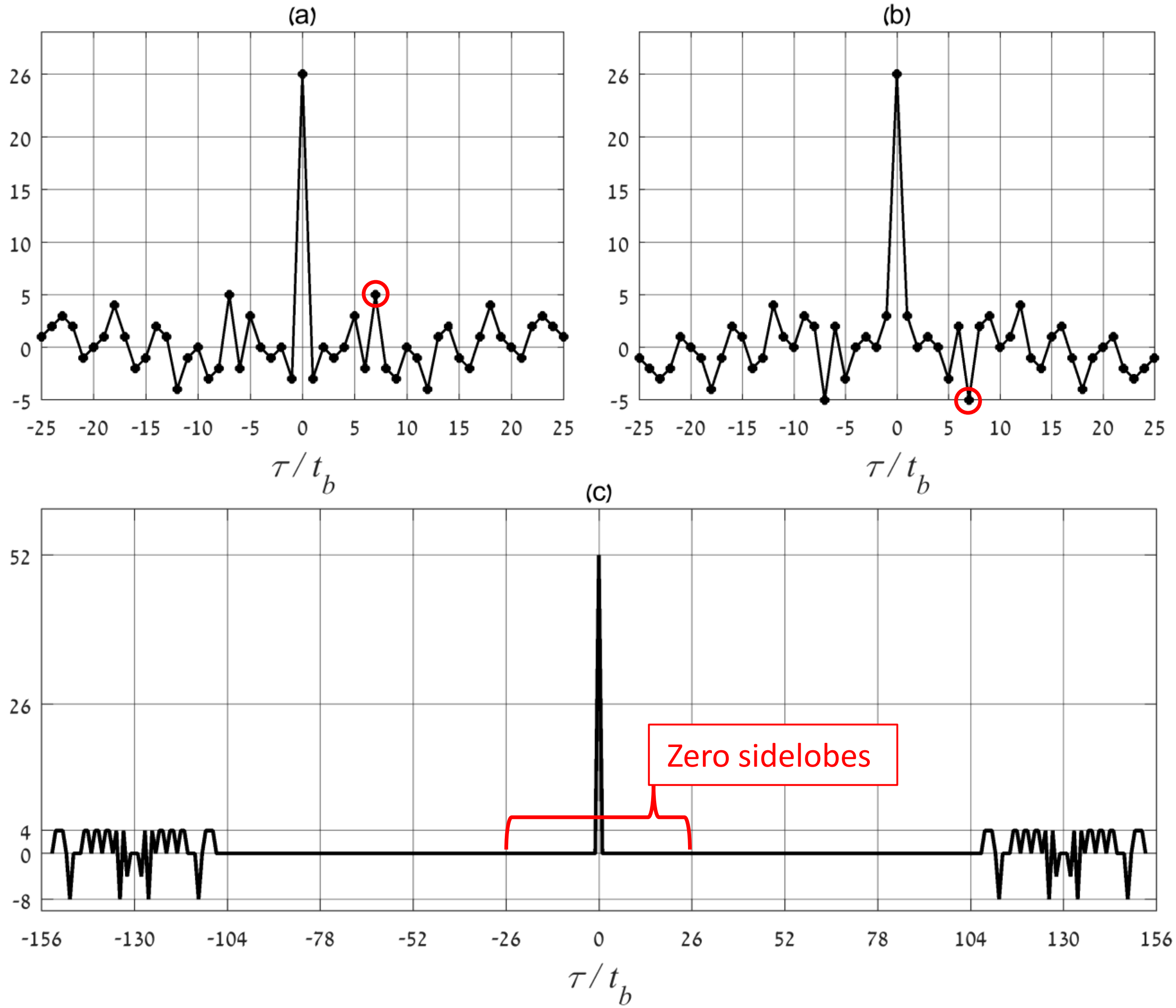
Radar pulse waveform – wish list

- **Delay response**:
  - Narrow (wide bandwidth, pulse compression)
  - Low range sidelobes (mismatched filter, entails SNR loss)
  - Zero range sidelobes (is it possible ?)
- **Doppler response**:
  - Narrow (long **C**oherent **P**rocessing **I**nterval)
  - Low Doppler sidelobes (inter-pulse weight window on receive, = SNR loss)
- **High energy**
  - High energy pulse <u>but</u> low peak power (pulse compression)
- **Constant amplitude**
- **Spectral efficiency** (Allows coexistence of many radars, allows low sampling rate)
  - Uniform spectrum (LFM, multicarrier)
  - Fast decaying spectral sidelobes (FSK rather than PSK)
- **Waveform separability** (motivated by MIMO or multistatic systems)
  - Many separable waveforms of the same family (coding rather than LFM)
- **Low Probability of Intercept – LPI**
  - Low detectability (low peak power, unconventional pulse compression)
  - Rapidly switchable waveforms (code family with many members)

Complementary pair waveform

- **Delay response**:
  - √⇒ Narrow (wide bandwidth, pulse compression)
  - Low range sidelobes (mismatched filter, entails SNR loss)
  - √⇒ Zero range sidelobes (is it possible ?)
- **Doppler response**:
  - √⇒ Narrow (long **C**oherent **P**rocessing **I**nterval)
  - √⇒ Low Doppler sidelobes (inter-pulse weight window on receive, = SNR loss)
- **High energy**
  - √⇒ High energy pulse <u>but</u> low peak power (pulse compression)
- **Constant amplitude** ⇐ **almost**
- **Spectral efficiency** (Allows coexistence of many radars, allows low sampling rate)
  - Uniform spectrum (LFM, multicarrier)
  - √⇒ Fast decaying spectral sidelobes (FSK rather than PSK)
- **Waveform separability** (motivated by MIMO or multistatic systems)
  - √⇒ Many separable waveforms of the same family (coding rather than LFM)
- **Low Probability of Intercept – LPI**
  - √⇒ Low detectability (low peak power, unconventional pulse compression)
  - √⇒ Rapidly switchable waveforms (code family with many members)

Coherent pulse train constructed from complementary pairs



⇐ Periodic correlation

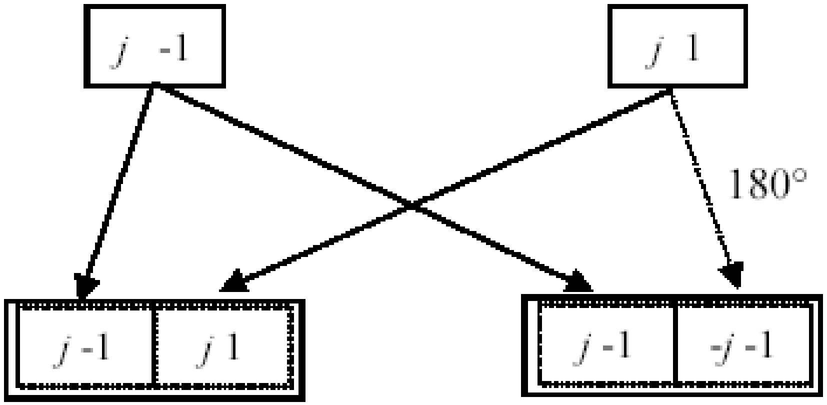
Some kernels of known poly phase complementary sets

S	L	Phase sequence
2	2	$[0\ 0]$, $[0\ \pi]$
2	10	$[0\ 0\ \pi\ \pi\ \pi\ \pi\ \pi\ 0\ \pi\ \pi]$, $[0\ 0\ \pi\ 0\ \pi\ 0\ \pi\ \pi\ 0\ 0]$
2	26	$[0\ 0\ 0\ \pi\ \pi\ 0\ 0\ 0\ \pi\ 0\ \pi\ \pi\ 0\ \pi\ 0\ \pi\ 0\ \pi\ \pi\ 0\ 0\ \pi\ 0\ 0\ 0\ 0]$, $[0\ 0\ 0\ 0\ \pi\ 0\ 0\ \pi\ \pi\ 0\ \pi\ 0\ 0\ 0\ 0\ 0\ \pi\ 0\ \pi\ \pi\ \pi\ 0\ 0\ \pi\ \pi\ \pi]$
2	3	$[0\ 0\ \pi]$, $[0\ \pi/2\ 0]$
2	4	$[0\ 3\pi/2\ \ 0\ \pi/2]$, $[0\ \pi/2\ 0\ 3\pi/2]$
3	3	$[0\ \pi\ \pi]$, $[0\ 2\pi/3\ 7\pi/3]$, $[0\ \pi/3\ 5\pi/3]$
3	2	$[0\ \ \ 0]$, $[0\ \ 2\pi/3]$, $[0\ \ 4\pi/3]$
2	5	$[\pi\ \ 0\ \pi\ \ \pi/2\ \ \ \pi/2]$, $[\pi/2\ \ \pi\ \ -\pi/2\ \ -\pi/2\ \ \pi]$

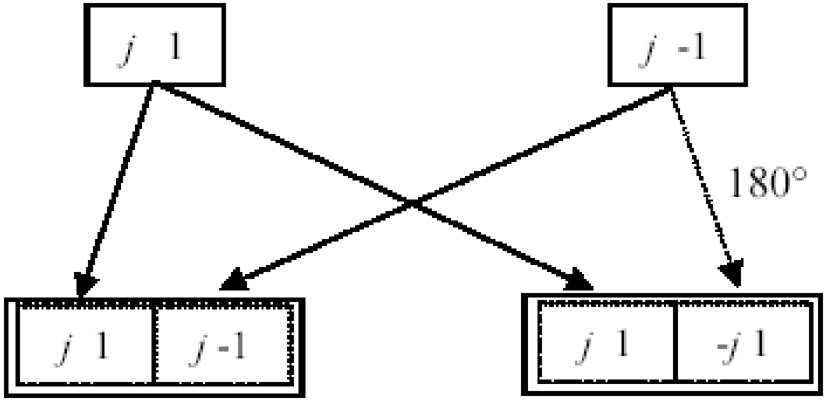
Complementary binary code pairs are known only for code length N of the form: $N = 2^a 10^b 26^c$, where $a, b,$ and c are non-negative integers. For $N \le 100$, only those CC pairs of length $N = 1,2,4,8,10,16,20,32,40,52,64,80,100$ were found.

Generating a complementary pair of 4-element sequences

using a complementary pair of 2-element sequences



switch roles



JOURNAL OF COMBINATORIAL THEORY (A) 16, 313–333 (1974)

Hadamard Matrices, Baumert-Hall Units, Four-Symbol Sequences, Pulse Compression, and Surface Wave Encodings

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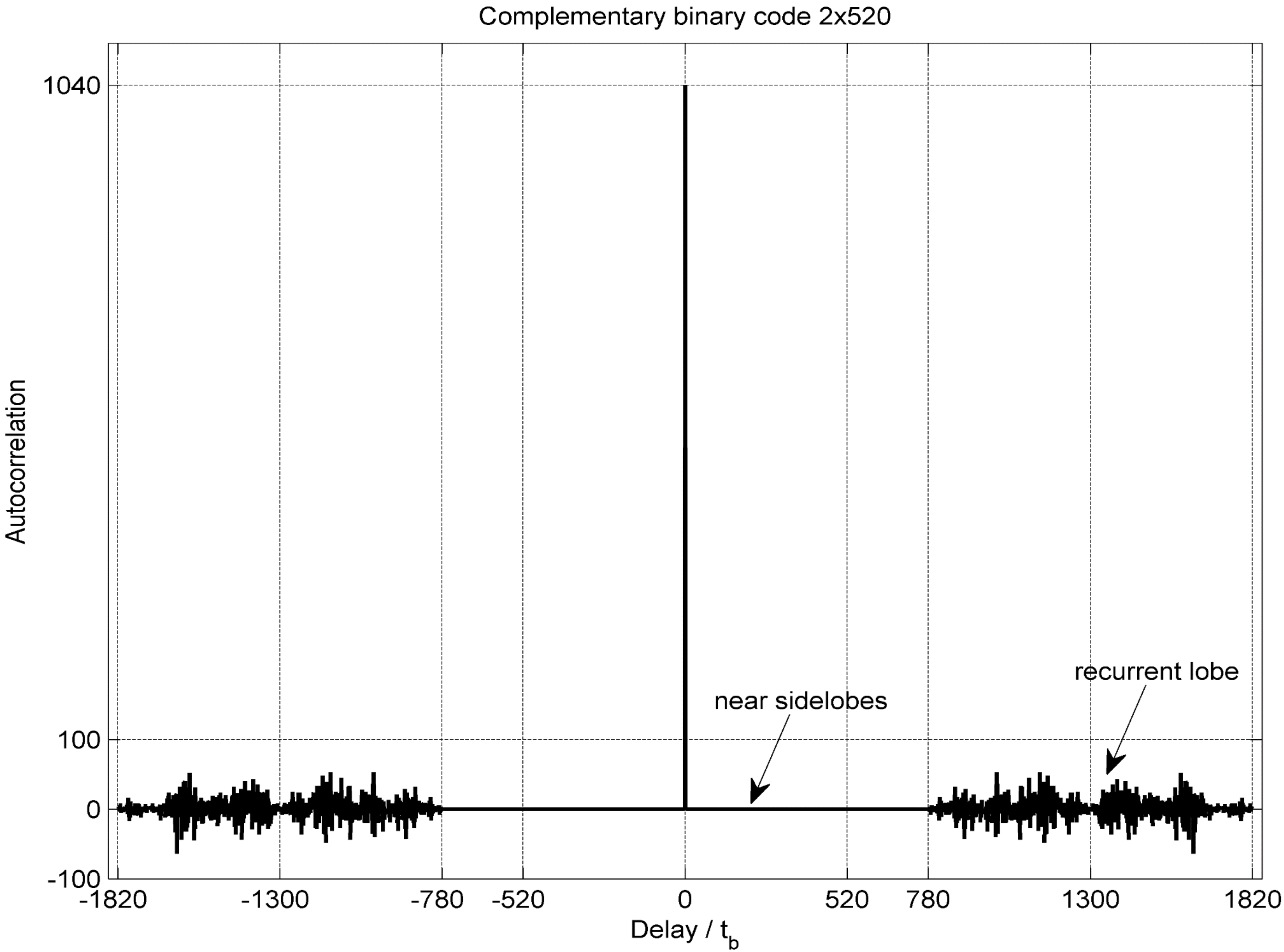
If a Williamson matrix of order $4w$ exists and a special type of design, a set of Baumert-Hall units of order $4t$, exists, then there exists a Hadamard matrix of order $4tw$. A number of special Baumert-Hall sets of units, including an infinite class, are constructed here; these give the densest known classes of Hadamard matrices. The constructions relate to various topics such as pulse compression and image encodings.

Constructing a pair of binary complementary sequences, with sequence length 2MN, from two pairs of binary complementary sequences with sequences lengths of M and N.

```
a= -1+2*[0 0 0 1 1 0 0 0 1 0 1 1 0 1 0 1 0 1 1 0 0 1 0 0 0 0];
b= -1+2*[0 0 0 0 1 0 0 1 1 0 1 0 0 0 0 0 1 0 1 1 1 0 0 1 1 1];
c= -1+2*[1 1 0 1 0 1 0 0 1 1];
d= -1+2*[1 1 0 1 1 1 1 1 0 0];

ac=kron(c,a); % Kronecker product
bd=kron(d,b);
sig_left=[ac bd]; % concutanation

df=fliplr(d); % flipping left-right
ncf=fliplr(-c); % negation and flipping
adf=kron(df,a); % Kronecker product
bncf=kron(ncf,b);
sig_right=[adf bncf]; % concutanation

mac=length(a)*length(c);
sr=round(mac*3);
space2=zeros(1,sr);

ss_all=[sig_left  space2 sig_right];
ss_all_cor=xcorr(ss_all);

peak_pos=0.5*(length(ss_all_cor)+1);
xscale=0:length(ss_all_cor)-1;
xscale=xscale-peak_pos +1;

xtick_locations=[-(2*length(sig_left)+sr)  -(length(sig_left)+sr) -sr  -length(sig_left) ...
  0 length(sig_left) sr (length(sig_left)+sr) (2*length(sig_left)+sr)] ;

figure(1), clf
plot(xscale, ss_all_cor,'k', 'linewidth',1.5)
xlabel(' Delay / t_b ')
ylabel(' Autocorrelation ')
title(' Complementary binary code 2x520 ')
axis([xtick_locations(1)-30  xtick_locations(end)+30 -100  1100])
set( gca, 'YTick',[-100 0 100 1040], 'YGrid','on', 'XTick',[xtick_locations], 'XGrid','on')
```


Complementary binary code 2x520
1040
100
0
-100
Autocorrelation
-1820
-1300
-780
-520
0
520
780
1300
1820
Delay / t_b
near sidelobes
recurrent lobe

A quadriphase signal *s(t)* can be generated from a binary sequence B_n of length M, by a binary-to-quadriphase transformation (t_b is the duration of a code element):

$$s(t) = \sum_{m=0}^{M-1} j^n B_n p(t - mt_b)$$

$$p(t) = \begin{cases} \cos\left(\frac{\pi}{2}\frac{t}{t_b}\right) & |t| \le t_p \\ 0 & |t| > t_p \end{cases}$$

binary ⇒ quadri-phase transformation

$$B = [0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ 1],\quad M = 10$$

$$q_1 = 0$$

xor

$$q_m = \left[q_{m-1} + \tfrac{1}{2} - \left(B_m \oplus B_{m-1} \right) \right]_{\mathrm{mod}\,2},\quad m = 2, 3, \ldots, M$$

$$\therefore q = \left[0 \quad \tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 0 \quad \tfrac{-1}{2} \quad -1 \quad \tfrac{-1}{2} \right]$$

$$d = \left[\exp(j\pi q) \quad 0 \right]$$

Creating N samples per bit (e.g., N = 4)

$$N = 4$$

N-1

$$a = [1\ 0\ 0\ 0]$$

$$f = \mathtt{kron}(d, a)$$

N

$$W = \cos\left(\frac{\pi}{2N} [-4\ \ -3\ \ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4] \right)$$

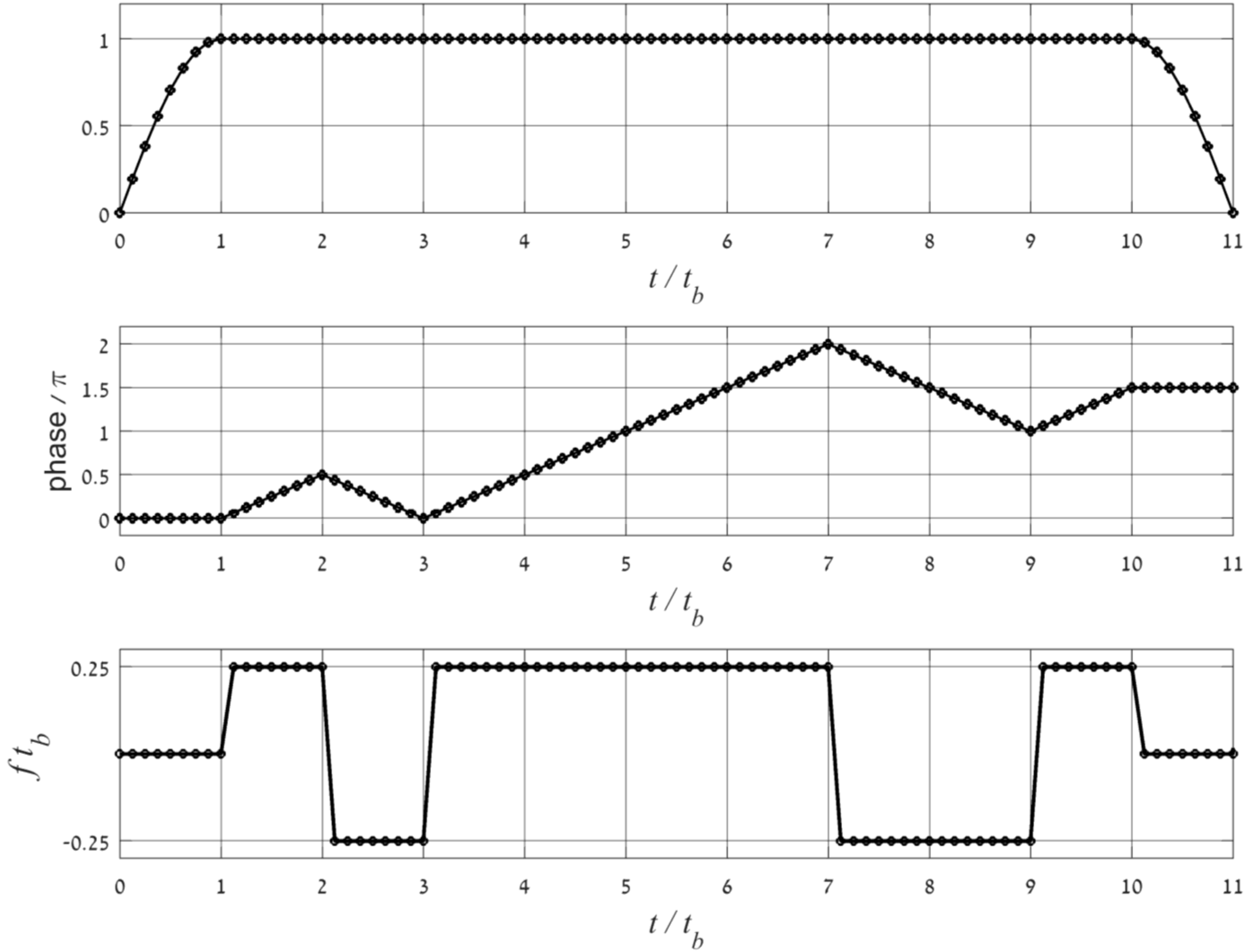
$$s = [\mathtt{filter}\left(W, 1, f \right)\ \ 0]$$

$\mathtt{kron}$, $\mathtt{filter}$ = MATLAB expressions

Achieving spectral efficiency

by using binary => quadri-phase transformation

Quadri-phase coded waveform = BFSK coded waveform



Converted 10 element complementary code

Taylor, J. W., and Blinchikoff, H. J. "Quadriphase code: a radar pulse compression signal with unique characteristics" *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 24, 2, (1988), 156-170.

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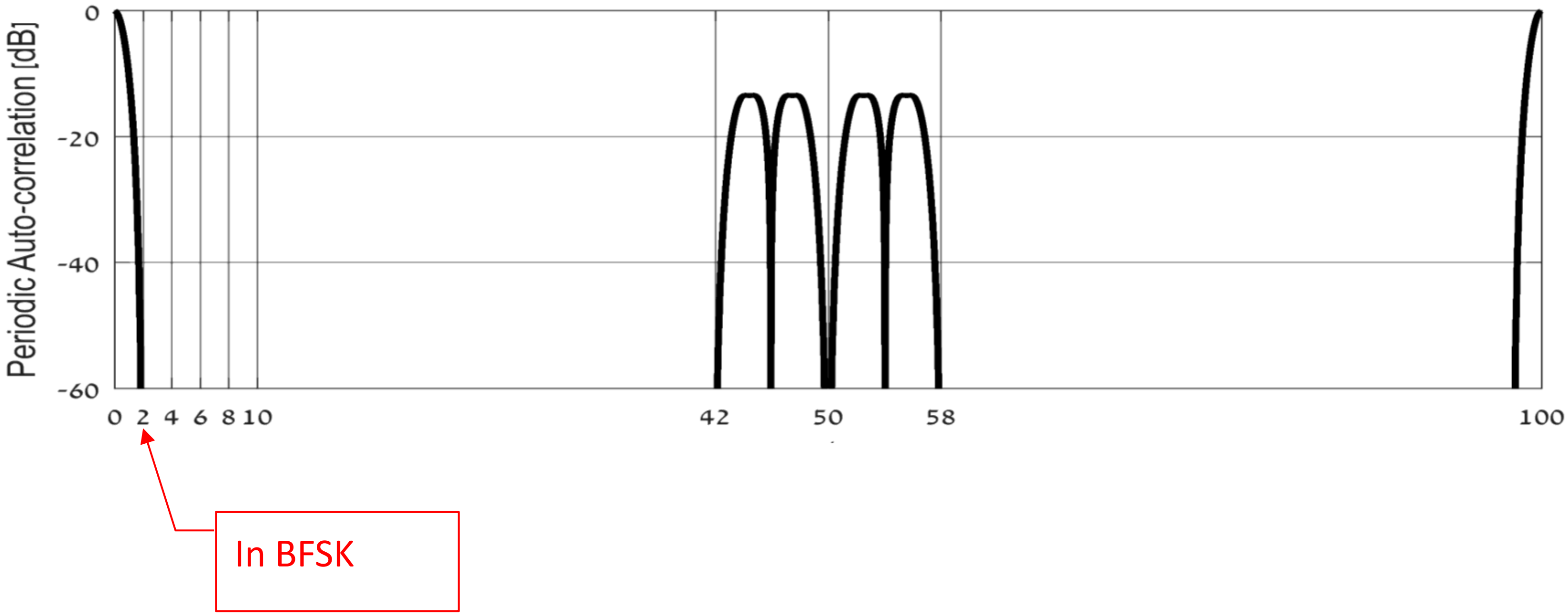

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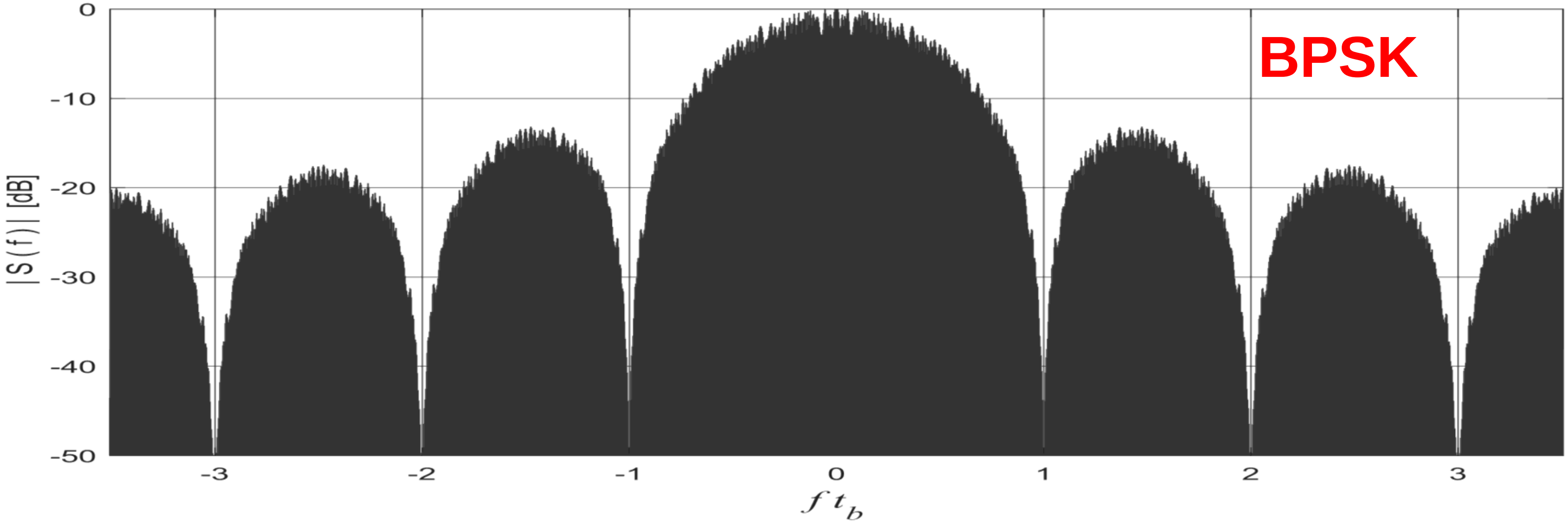

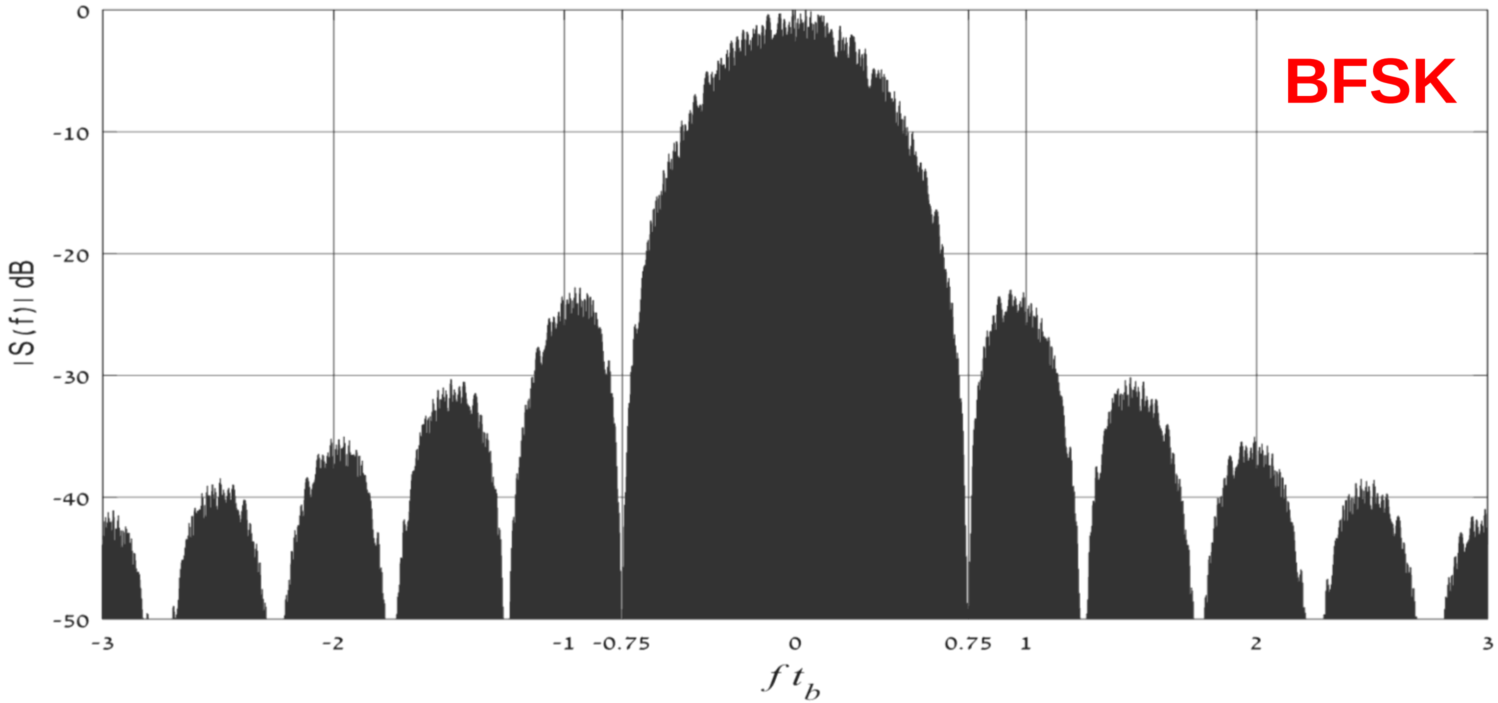

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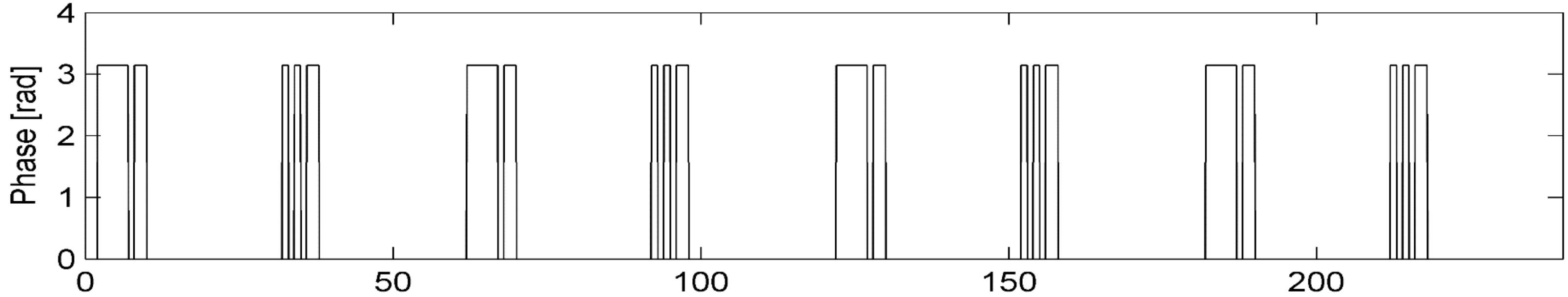
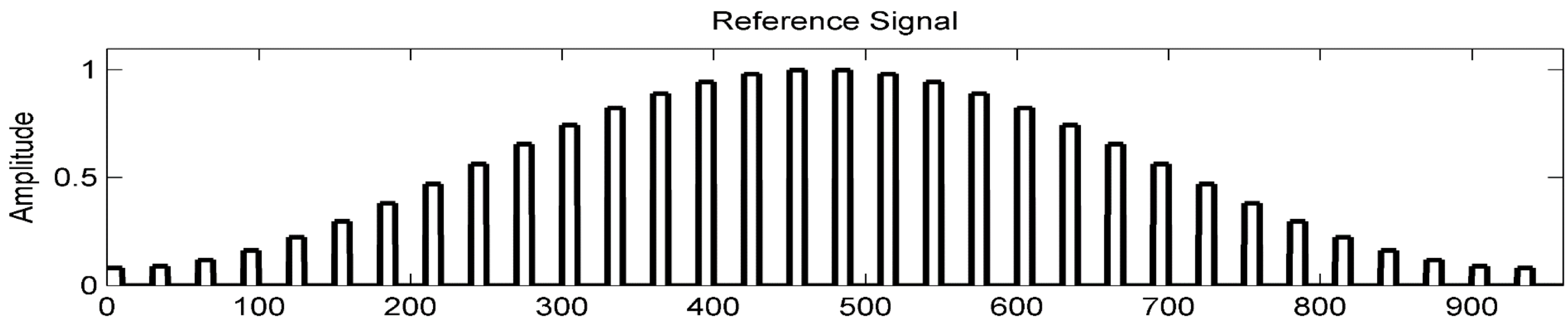
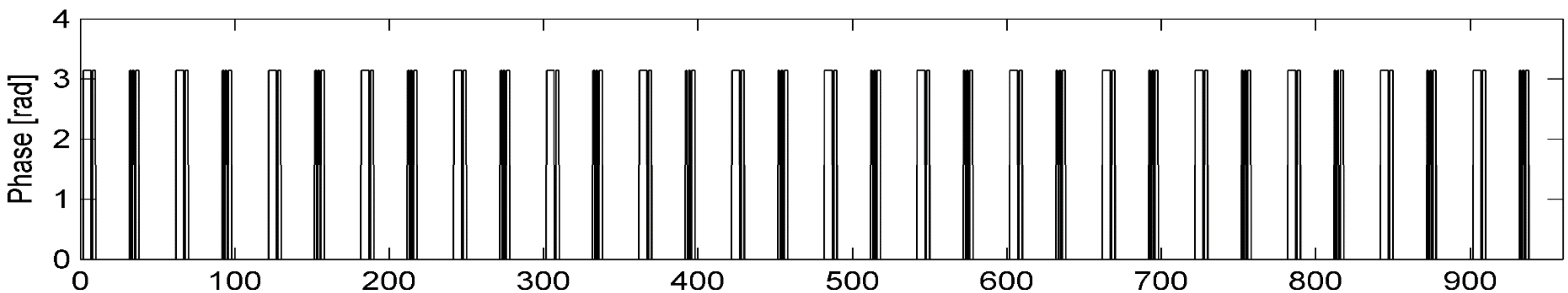
Reducing Doppler sidelobes by transmitting a train of complementary pairs with amplitude-weighted reference

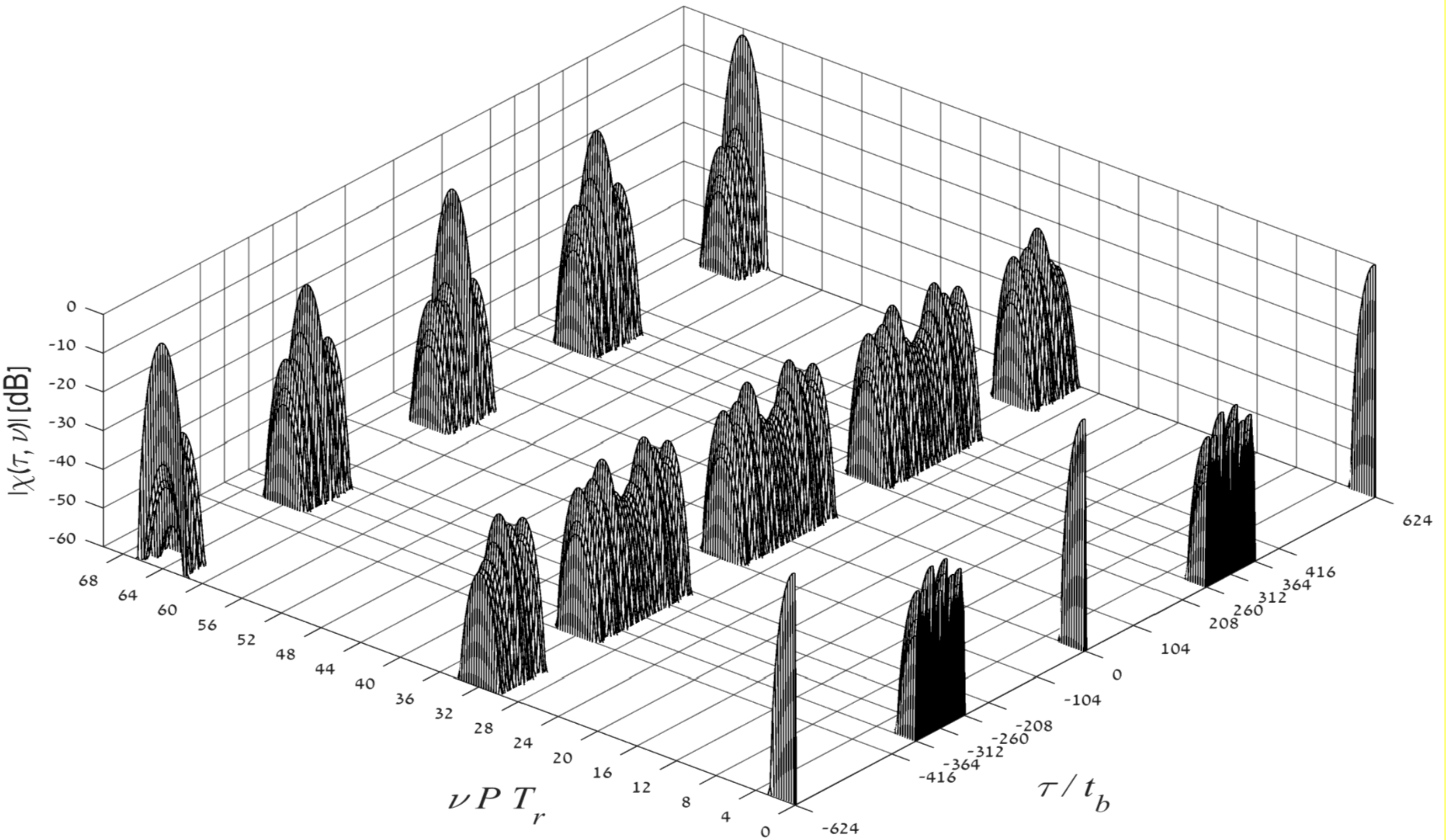
```
x0=[1 1 b b b b b 1 b b];   x1=[1 1 b 1 b 1 b b 1 1];   b=-1
```





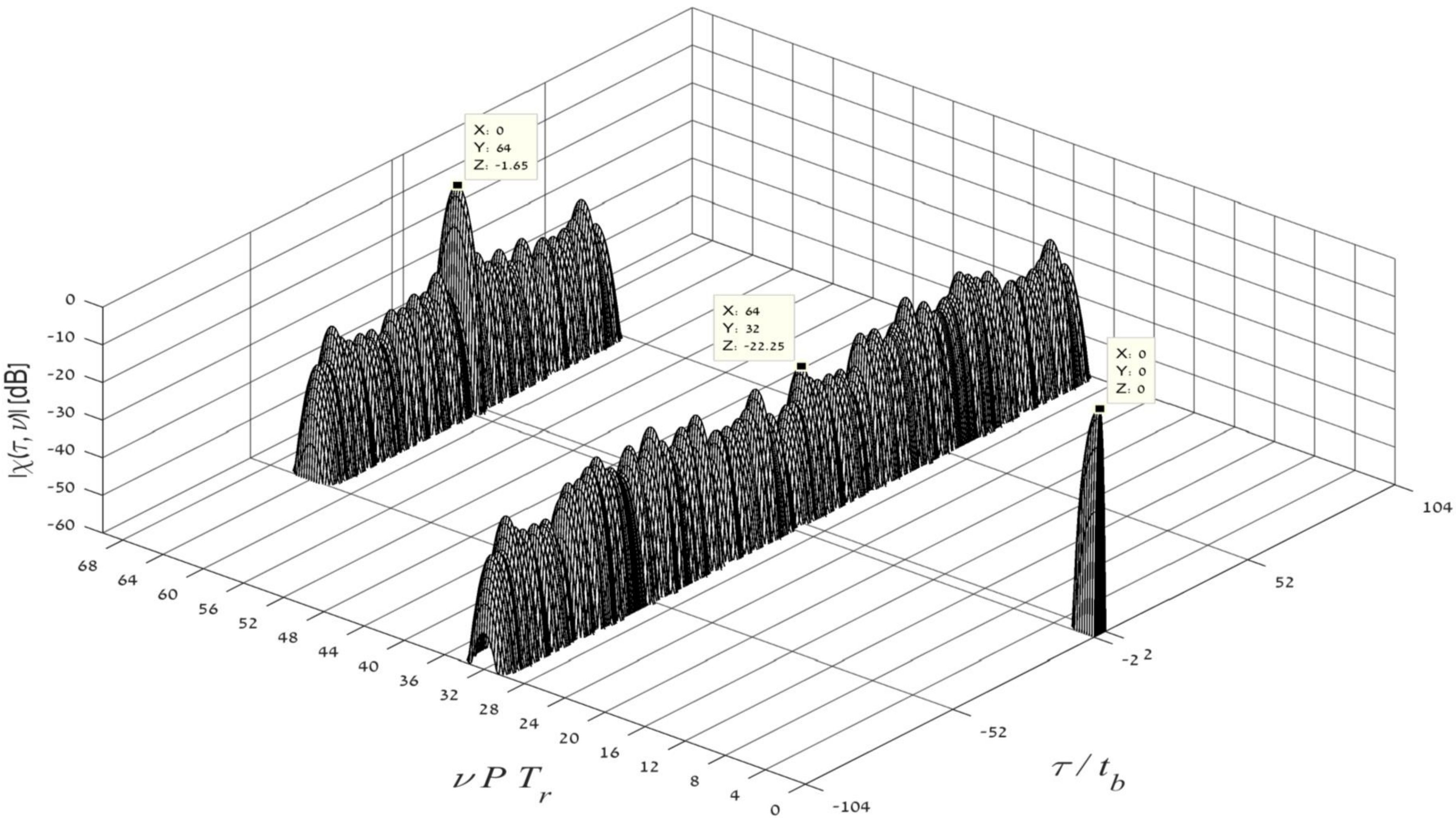


Delay-Doppler response
Coherent train of 32 complementary pairs (=64 pulses)
104 BFSK coded elements in each pulse, duty cycle=30%



Delay-Doppler response, zoom on one pulse

Coherent train of 32 complementary pairs (=64 pulses) 104 BFSK coded elements



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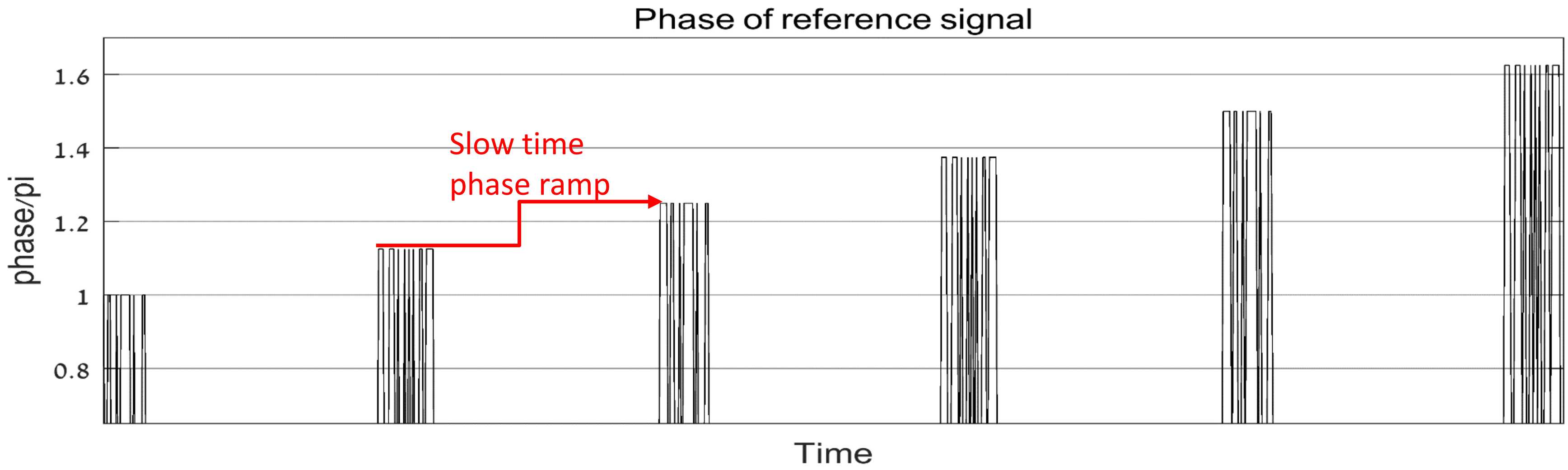
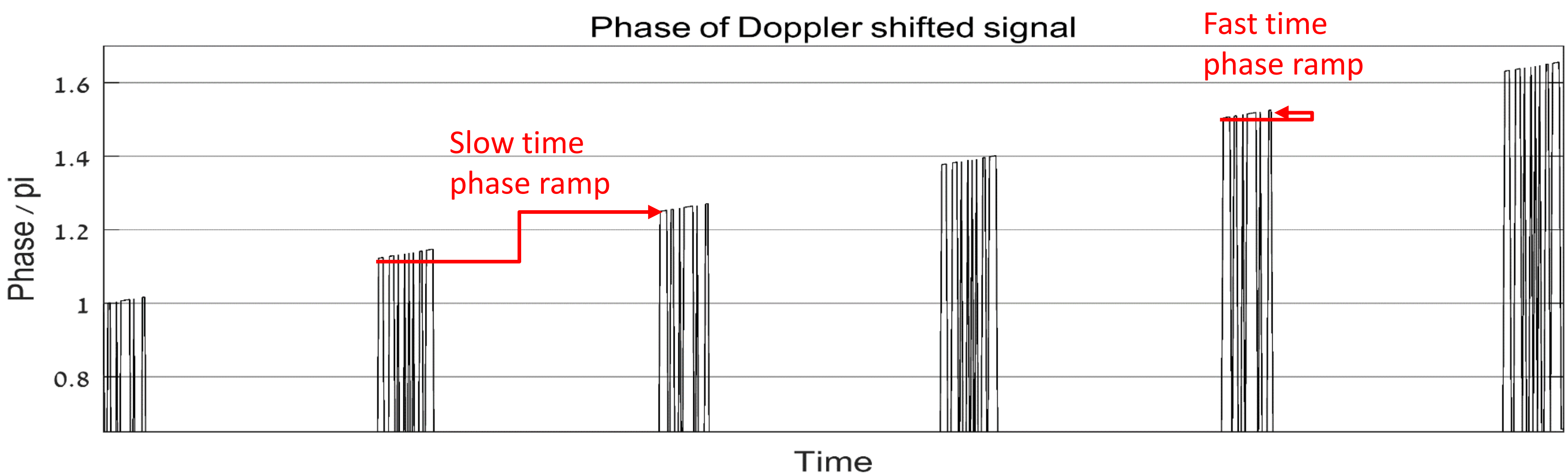
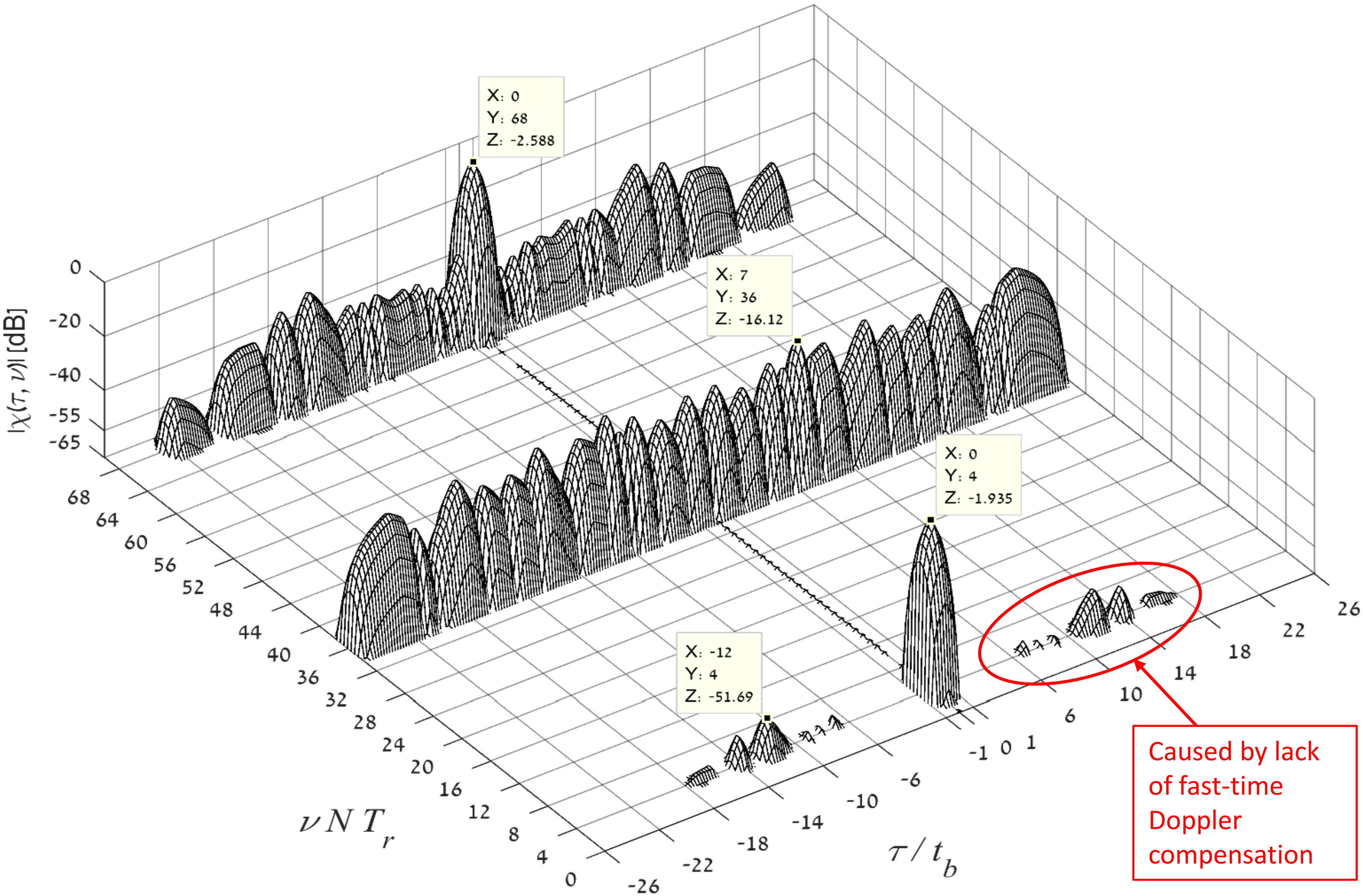
Range sidelobe's sensitivity to Doppler





Complementary pair, 26 elements/pulse, 64 pulses, `chebwin(64,62)` inter-pulse weight window, BPSK implementation, Delay-Doppler response of the 4'th FFT output.



Why is the sensitivity to Doppler of a complementary pair so emphasized?

In a conventional pulse compression, with matched or mismatched filter, a sidelobe level of -50 dB is considered good. Hence SL of -51dB caused by lack of fast-time Doppler compensation is not felt.

In complementary pair the normal SL level is zero (= $-\infty$ dB), hence SL increase to -51dB is clearly felt.

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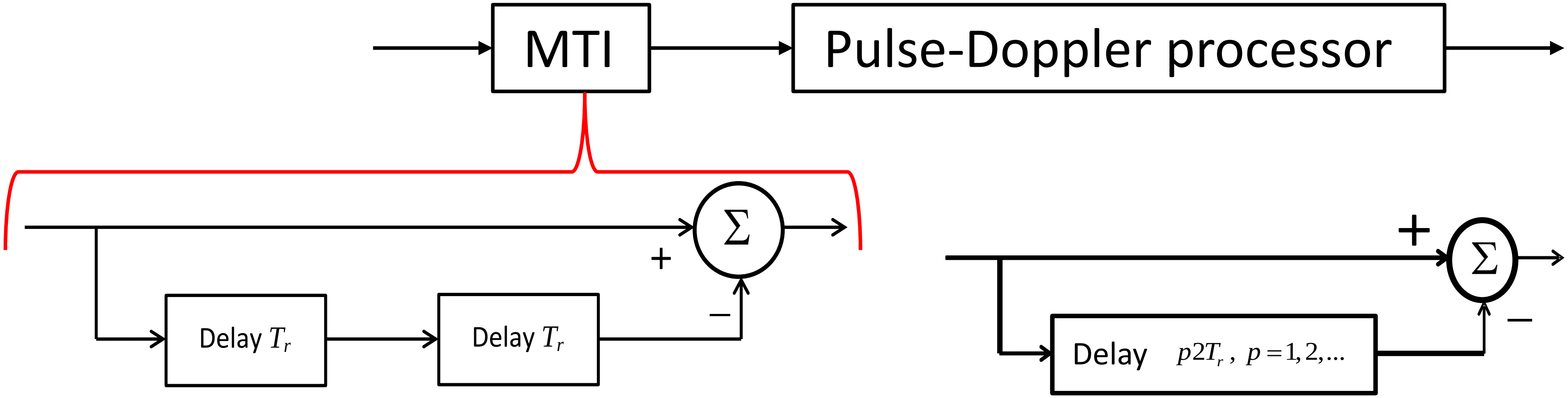
Observing slow moving targets in stationary clutter

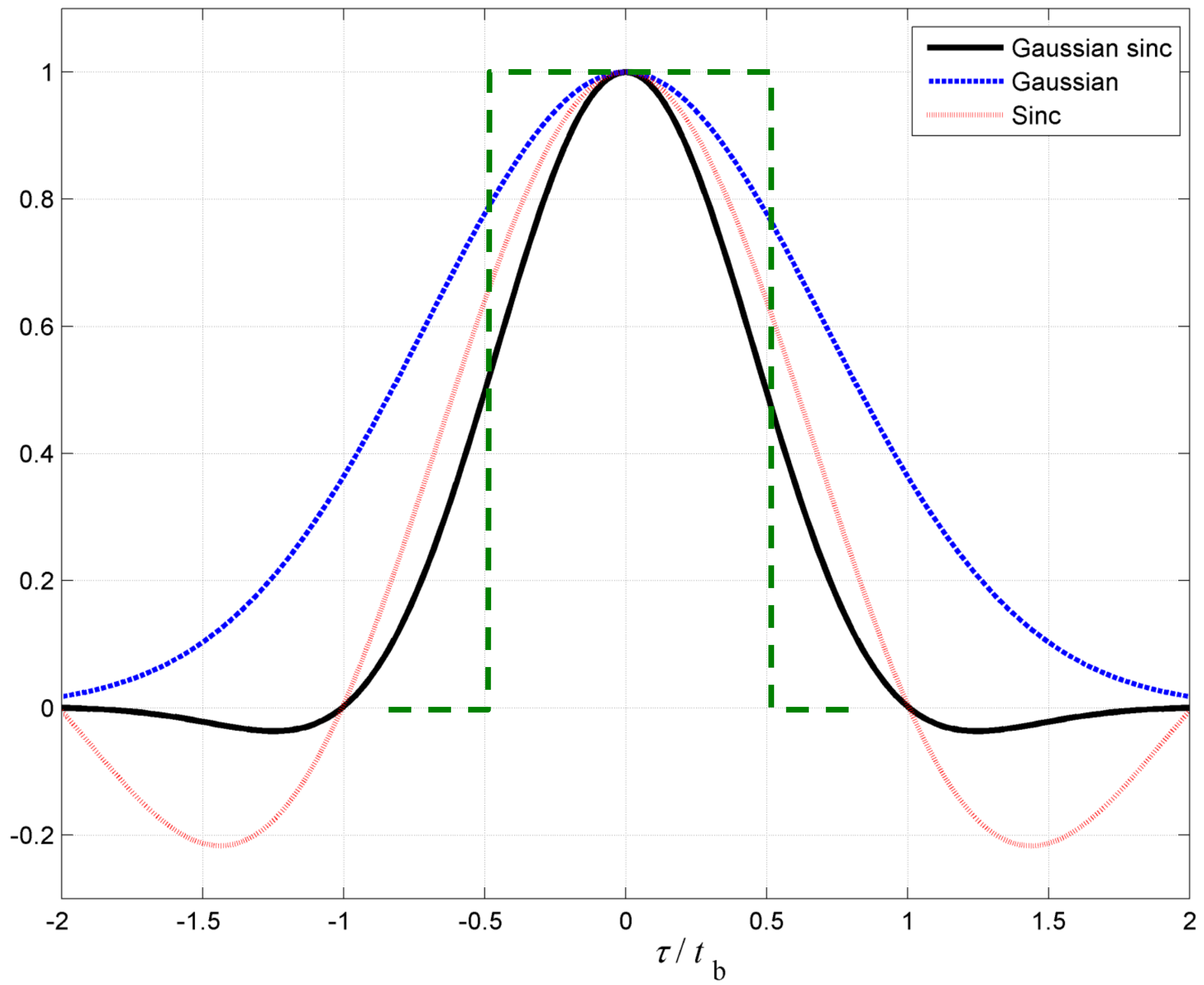
Combining **Pulse-Doppler** and **MTI** processing



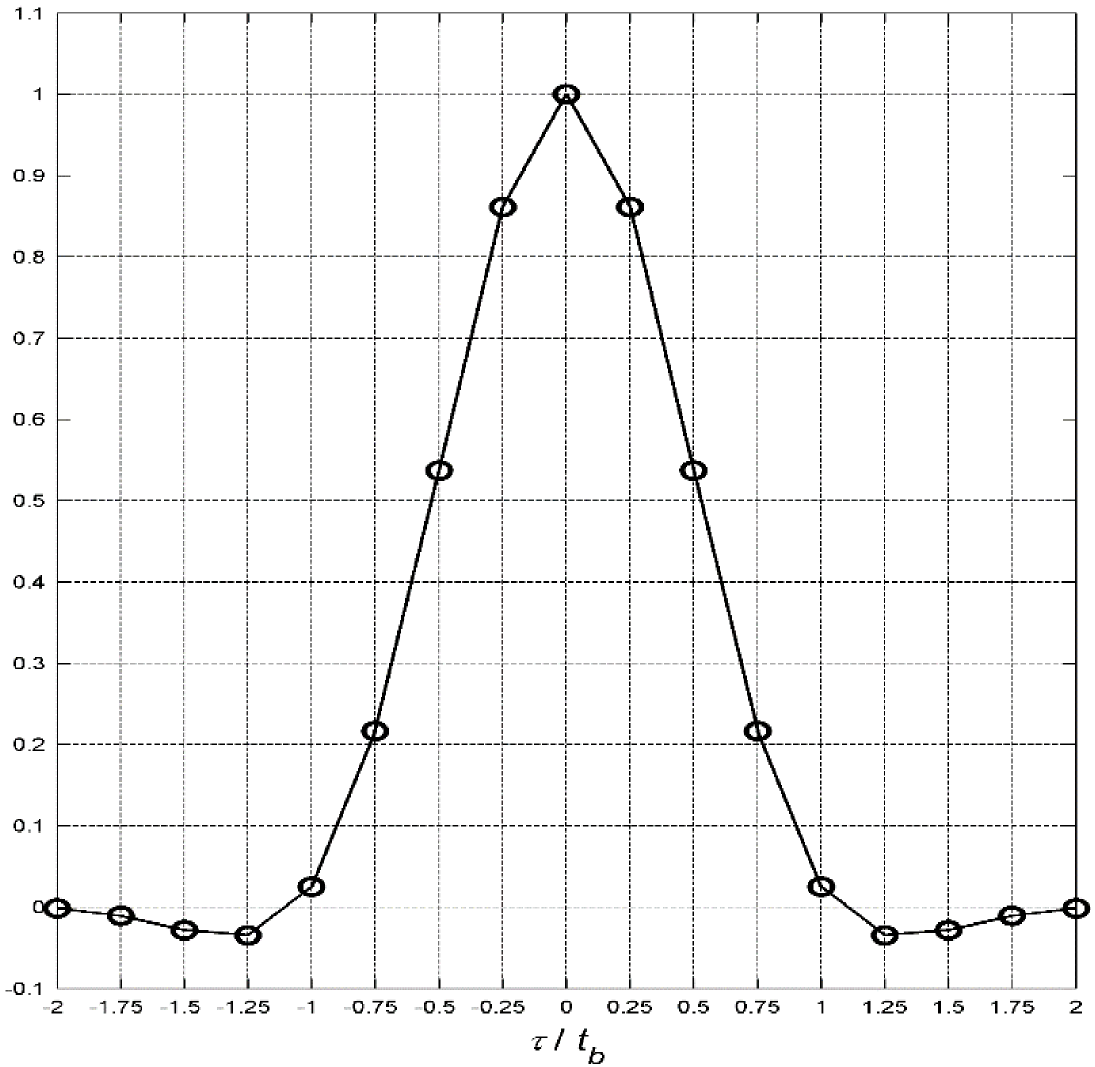
When using **complementary pulse-pair** radar waveform

More general 2-pulse canceller

Another frequency efficient implementation: **G**aussian **W**indowed **S**inc



Chen, R., and Cantrell, B., "Highly bandlimited radar signals", 2002 IEEE Radar Conf., Long Beach CA, April 2002, pp. 220-226.



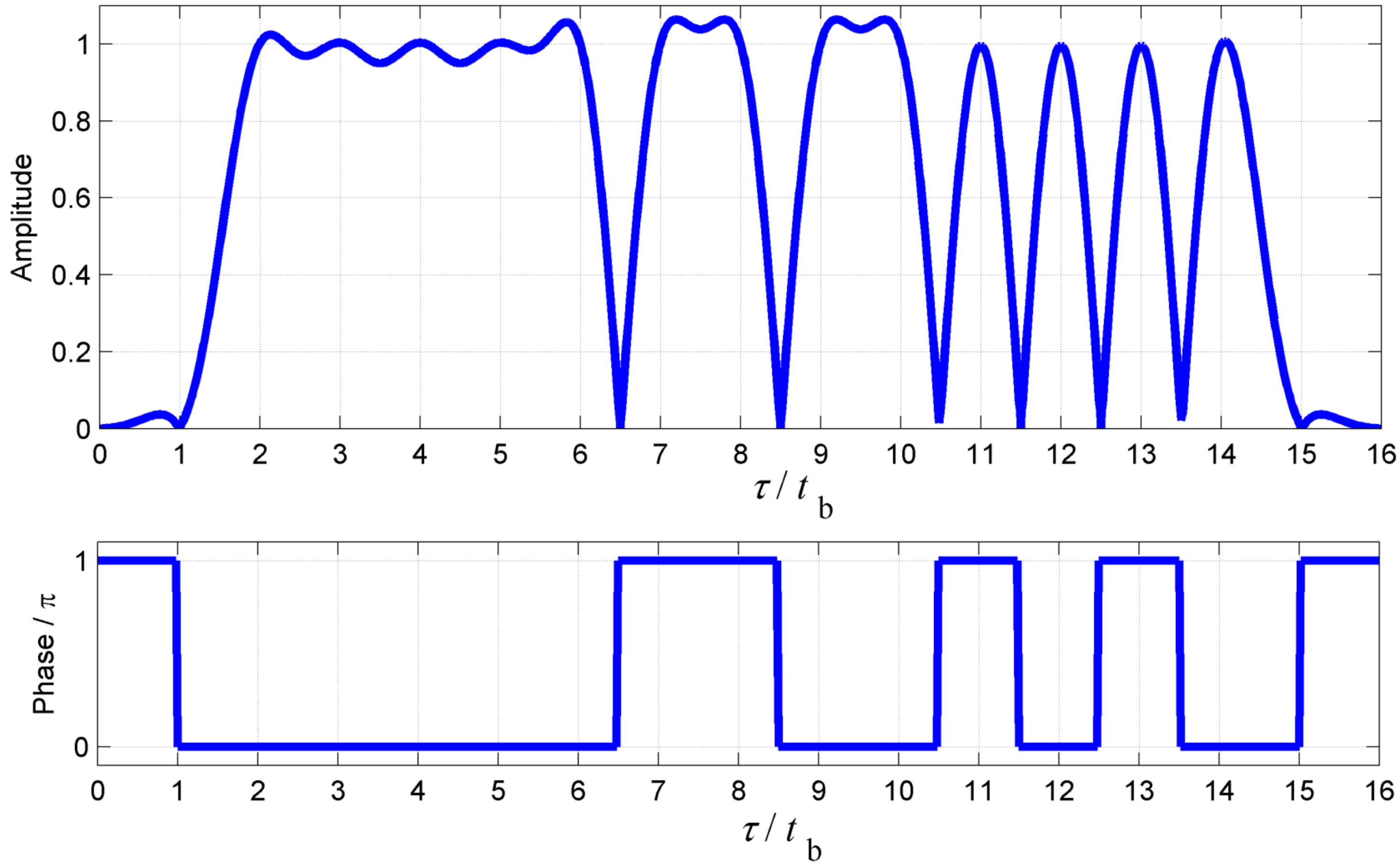
$M = 4$

M is the number of samples per code element (bit).

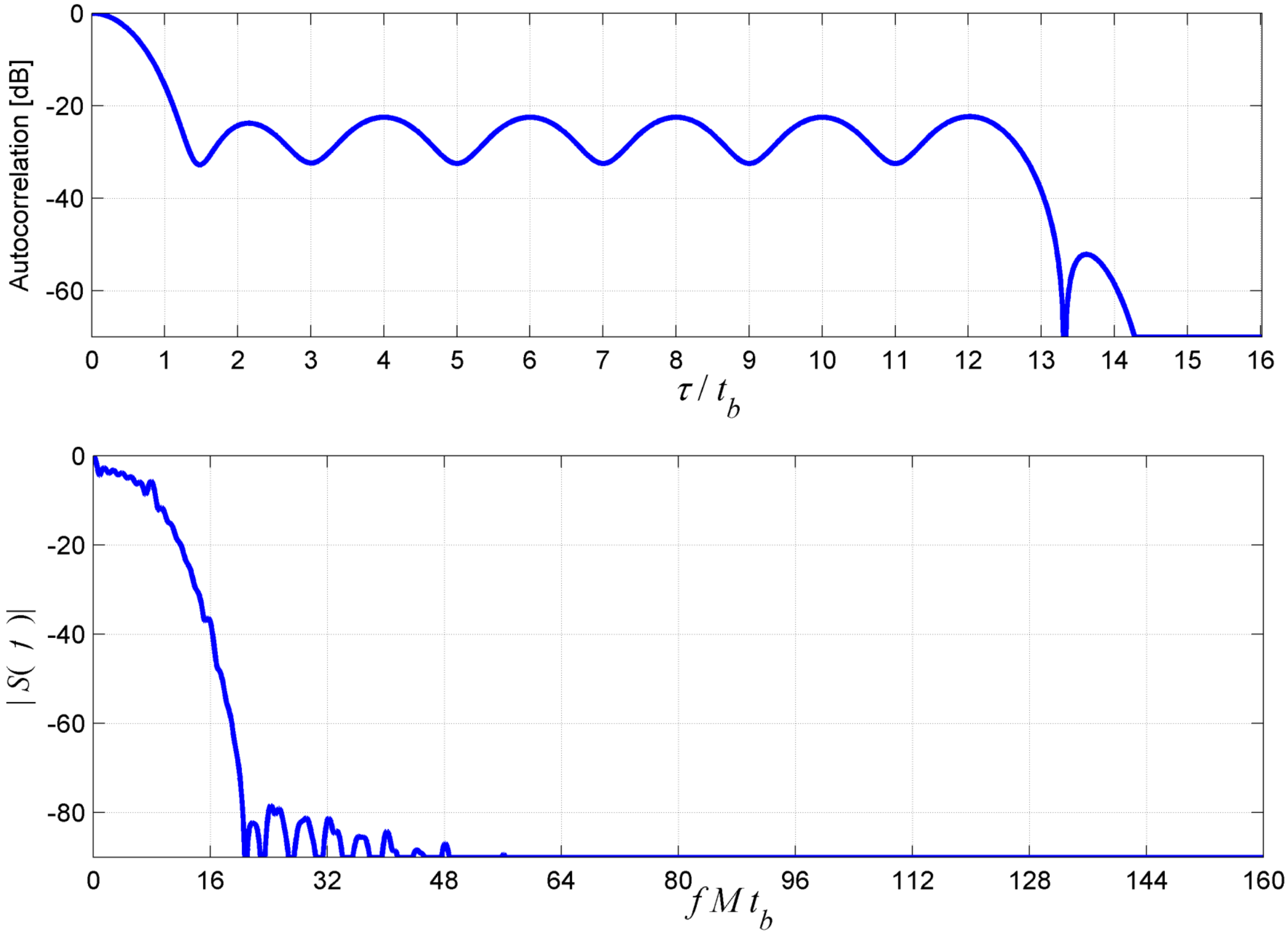
σ is a parameter chosen as 0.7.

$$GWS_m = \exp\left[-\frac{1}{2}\left(\frac{4m}{\sigma(4M+1)}\right)^2\right]\frac{\sin\alpha_m}{\alpha_m},\ \alpha_m = \frac{4\pi m}{4M+1},\ m = -2M, -2M+1, \ldots, 2M$$

Barker 13 – using Gaussian windowed sinc



Barker 13 – using Gaussian windowed sinc



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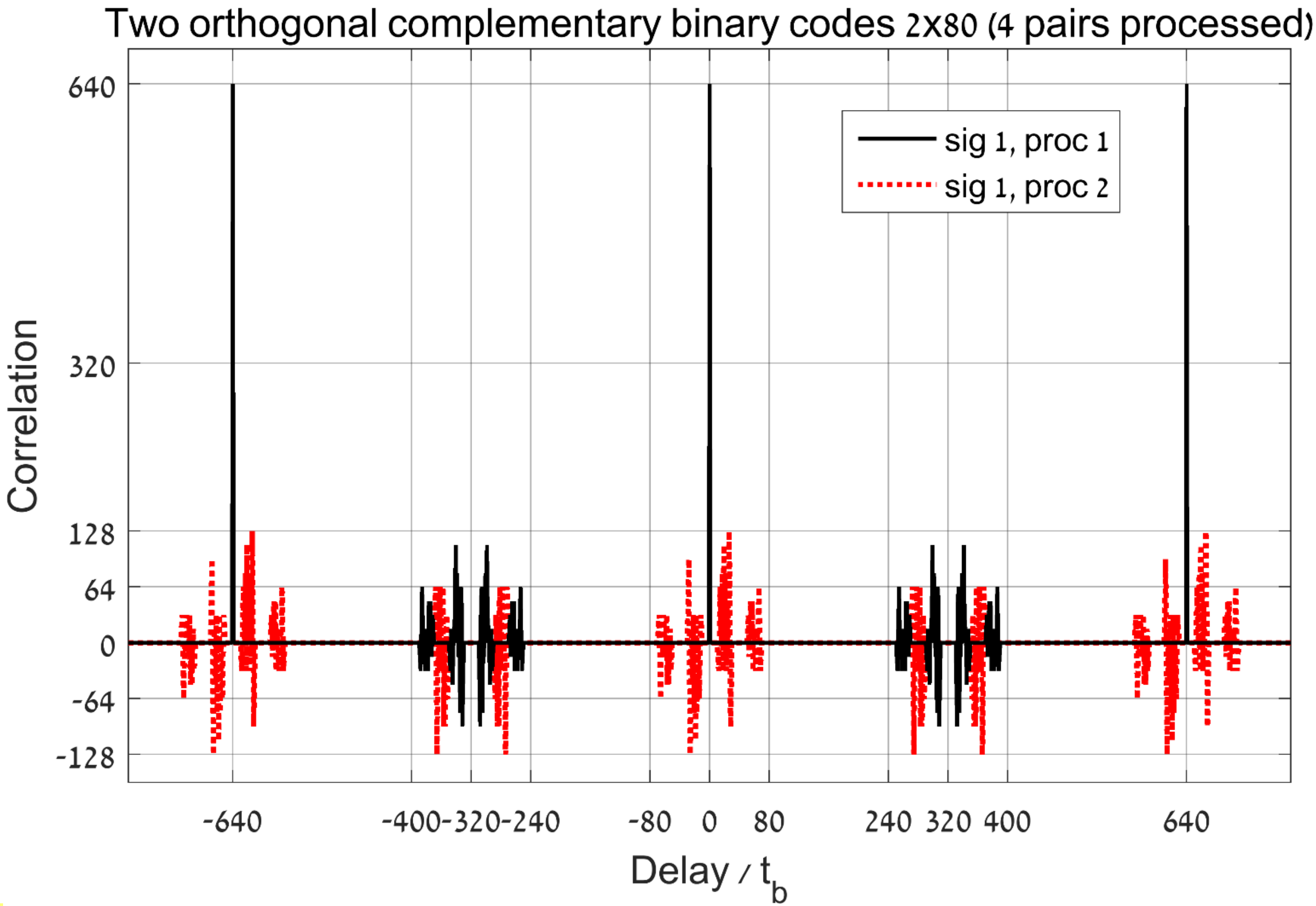
Gaussian Window Sinc (GWS) implementation (4 samples/bit)

104 element complementary pair, **Real signal**

Two (and more) binary complementary pairs can be found that are orthogonal.
Namely, no correlation peak when one signal is processed by a filter matched to the second signal.



Feature Article:

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Complementary Pair Radar Waveforms—Evaluating and Mitigating Some Drawbacks

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INTRODUCTION

Complementary pulse pair is a radar waveform that achieves the ultimate range sidelobe reduction–zero sidelobe. It is an early and simple embodiment of radar waveform diversity (WD), presently a popular topic. However, the use of complementary pulse waveforms is not widely spread because of several drawbacks. The main problem is the sensitivity to Doppler shift. Usually the two complementary coded pulses are separated in time. Doppler shift causes a phase ramp as function of time. That ramp causes two problems: (a) the two pulses in a pair are centered on different average phases; (b) there is a phase ramp during each pulse. Problem (a) also known as slow-time mismatch, is handled by the pulse-to-pulse conventional Doppler processing, which provides slow-time phase compensation. Problem (b) requires fast-time compensation, not provided by a simple linear Doppler processor. It causes loss of the ideal delay-sidelobe cancellation resulting in near range-sidelobes. Those near sidelobes increase with longer codes and with higher Doppler shifts. At the same time a complementary pulse pair also causes a difficulty at low Doppler shifts.

pulses in the CPI, while maintaining the pulse repetition interval (PRI); (b) range resolution (e.g., by pulse compression); (c) Doppler sidelobe reduction (e.g., by improved weighting windows); (d) range sidelobe reduction (e.g., by using mismatched filters).

Since complementary pairs are phase-coded they suffer from high- and slow-decaying spectral sidelobes. There are several measures [4, sec. 6.8] to improve spectral efficiency of phase-coded waveforms. When applied to complementary pairs they raise the question of how well the zero range-sidelobes property is preserved.

This article considers the above issues, suggests mitigating measures, and evaluates performances. The specific complementary Golay binary pair in this demonstration is the longest ($L = 26$ element) known binary sequence pair [5] that is not constructed from shorter sequences. The phases of the pair are given by

$$\phi_1 = \pi[00011000101101010110010000]$$
$$\phi_2 = \pi[00001001101000001011100111]$$

The autocorrelation functions (ACF) of each coded pulse by pair also causes a difficulty at low Doppler shifts.

Moving target indication (MTI) is a radar processing approach designed to help stationary pulse-Doppler radars to separate weak reflections of moving targets from strong returns of stationary clutter. This task becomes more difficult at low Doppler (slow targets). A classical MTI processor is constructed from a pulse canceller followed by discrete Fourier transform (DFT). A pulse canceller subtracts returns from consecutive pulses, assuming stationary clutter returns are identical and will cancel out. This concept fails if consecutive pulses are differently coded.

A very early version of MTI was used in the FPS-18 radar [1]. The receiver included a 3-pulse canceller followed by 8-pulse DFT. The interpulse weighting was a raised cosine. Special measures were added to circumvent the excessive attenuation of returns from very low-Doppler targets, caused by the three-pulse canceller.

Progress in devices and signal processing [2] allows considerable improvements in: (a) Doppler resolution (e.g., by increasing the coherent processing interval (CPI) by increasing the number of

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Review handled by D. O'Hagan.
0885/8985/17/$26.00

The autocorrelation functions (ACF) of each coded pulse by itself are shown in subplots (a) and (b) of Figure 1. Note the equal magnitudes but opposite polarities at each delay, except at the origin. That fact is responsible for the sidelobes cancellation when the sidelobes of the two correlations are added. Such addition happens when a train of repeated complementary pulse pairs $\{s_1\ s_2\ s_1\ s_2\ s_1\ s_2\ s_1\ s_2\ \ldots\}$ is cross-correlated with at least one reference pair. The resulting periodic cross-correlation, with a reference containing one complementary pulse pair $\{s_1\ s_2\}$, is shown in subplot (c) of Figure 1. Selected duty cycle of $d = 0.2$ resulted in a PRI five times longer than the pulse duration, namely

$$T_r = t_p/d = Lt_b/d = 26t_b/0.2 = 130t_b$$

where t_b is the duration of a code element (bit), L is the code length, t_p is the pulse duration, and T_r is the PRI.

The main property of a complementary pair is demonstrated in Figure 1(c) by the zero near-sidelobes at $1 \le |\tau/t_b| \le L = 26$. When the delay equals the PRI, signal and reference pulses overlap again but now the overlapping pulses are not matched. Signal pulse 1 is aligned with reference pulse 2 and signal pulse 2 overlaps reference pulse 1. This results in the recurrent delay lobes at the delay spans

$$(T_r - t_p)/t_b = (130 - 26) < |\tau/t_b| < (130 + 26) = (T_r + t_p)/t_b$$

Conclusions

Try it !