Cognitive/Adaptive Spatial Sampling

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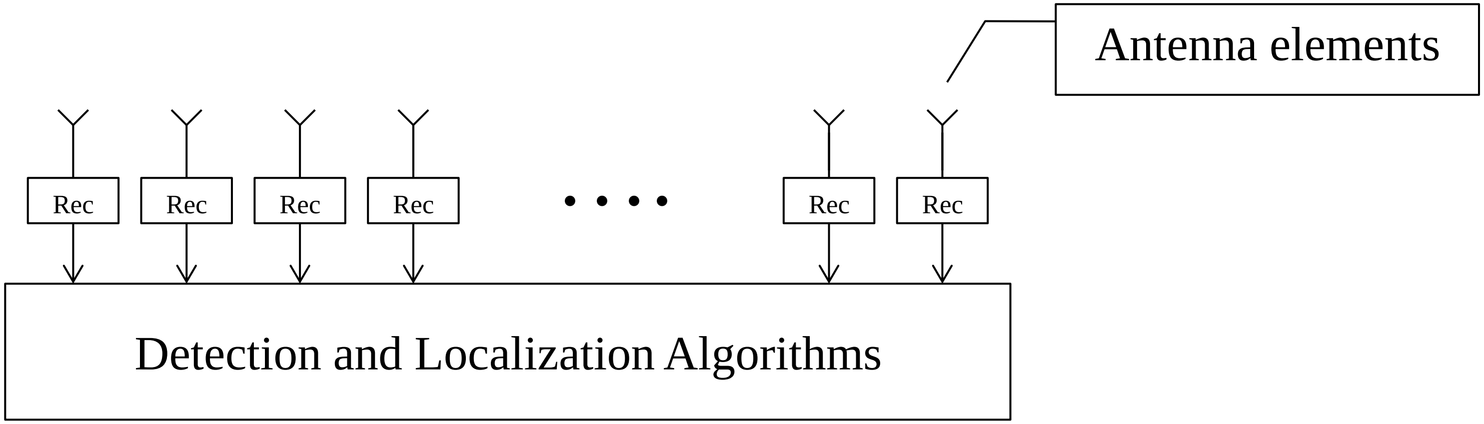
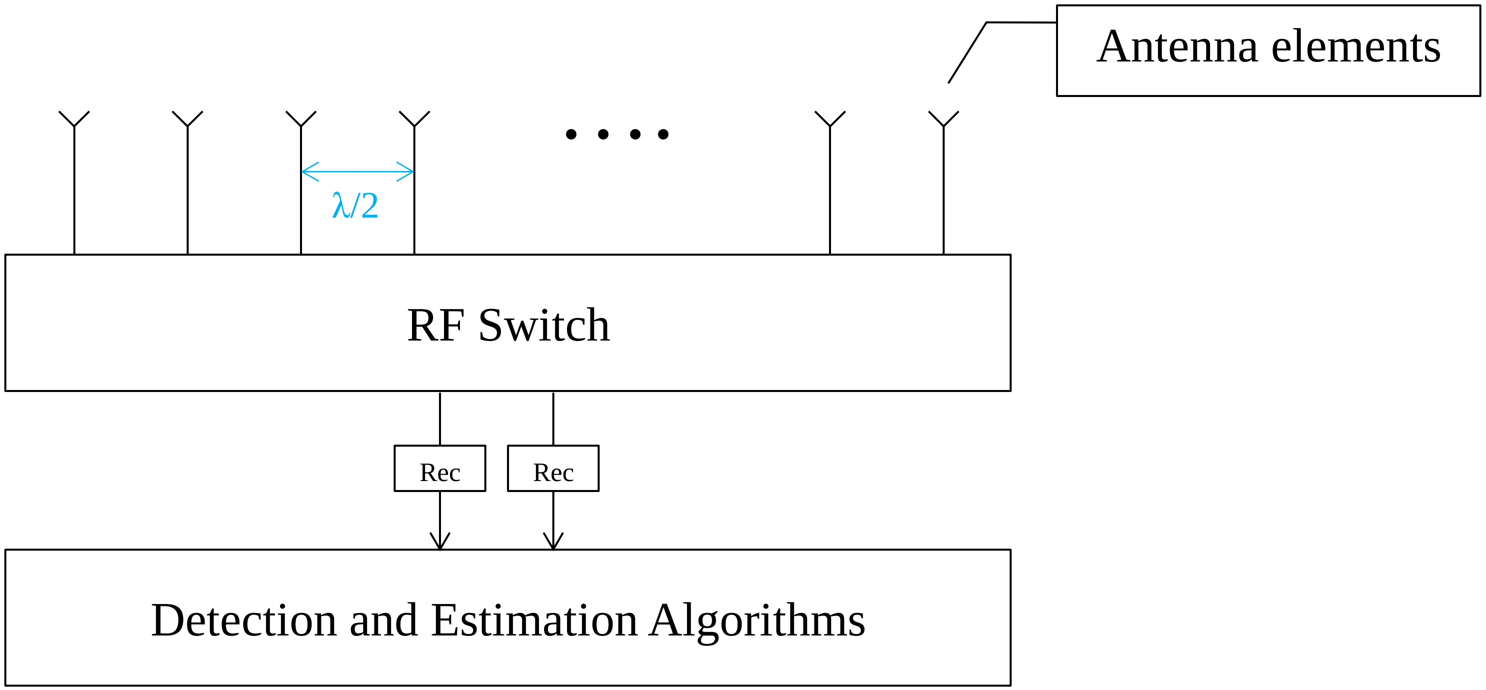
Introduction



- Expensive receiving channels
- Allows full flexibility in data processing
- Good accuracy without ambiguity in direction-of-arrival (DOA)

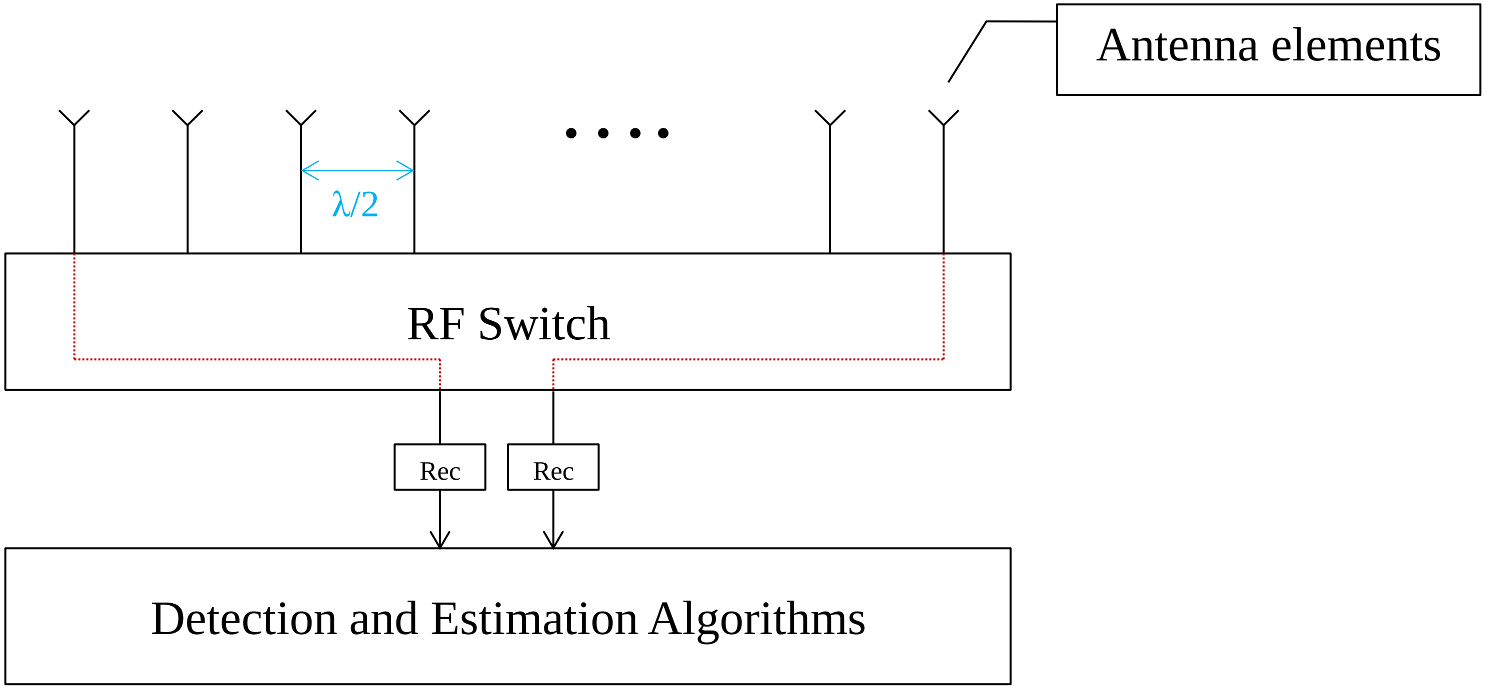
Introduction



- Low cost

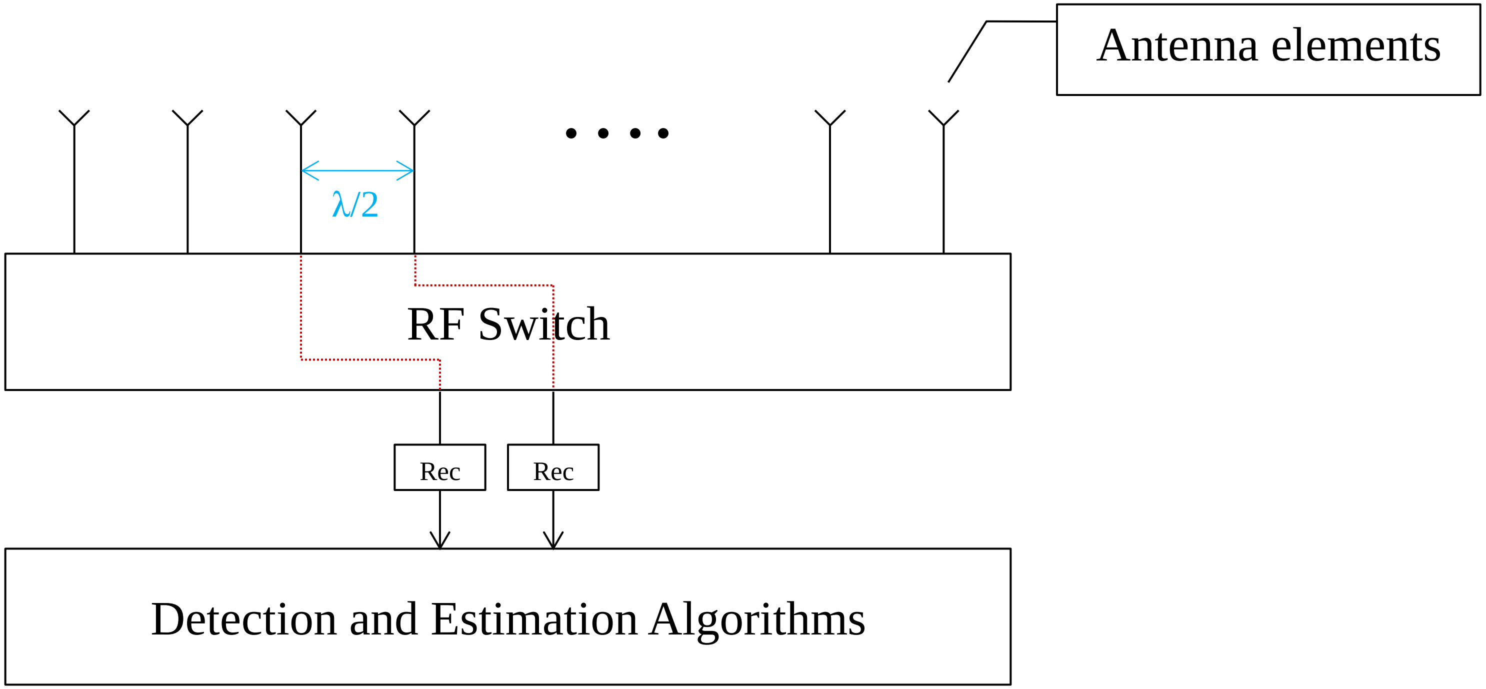
What is the optimal switch configuration?

Introduction



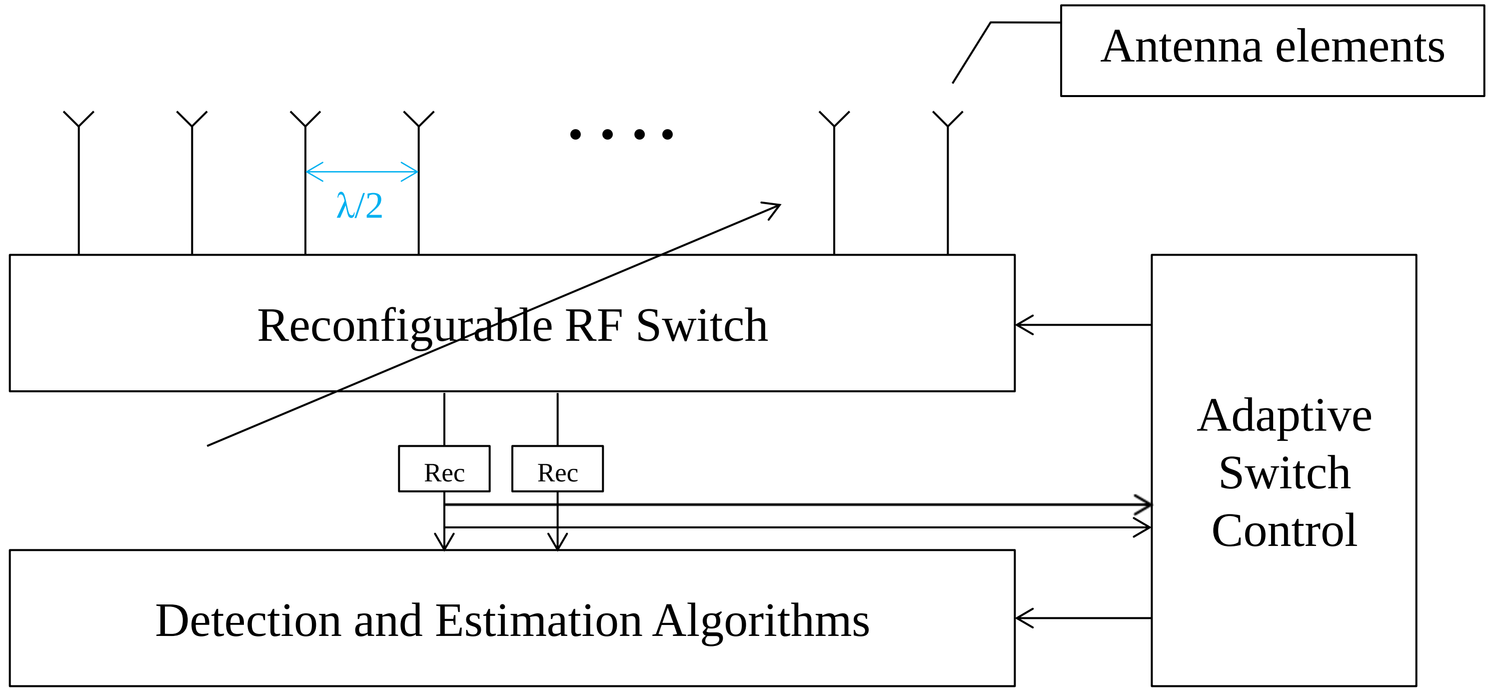
- Low cost
- Large aperture - high angular resolution/accuracy
- Ambiguity in DOA

Introduction



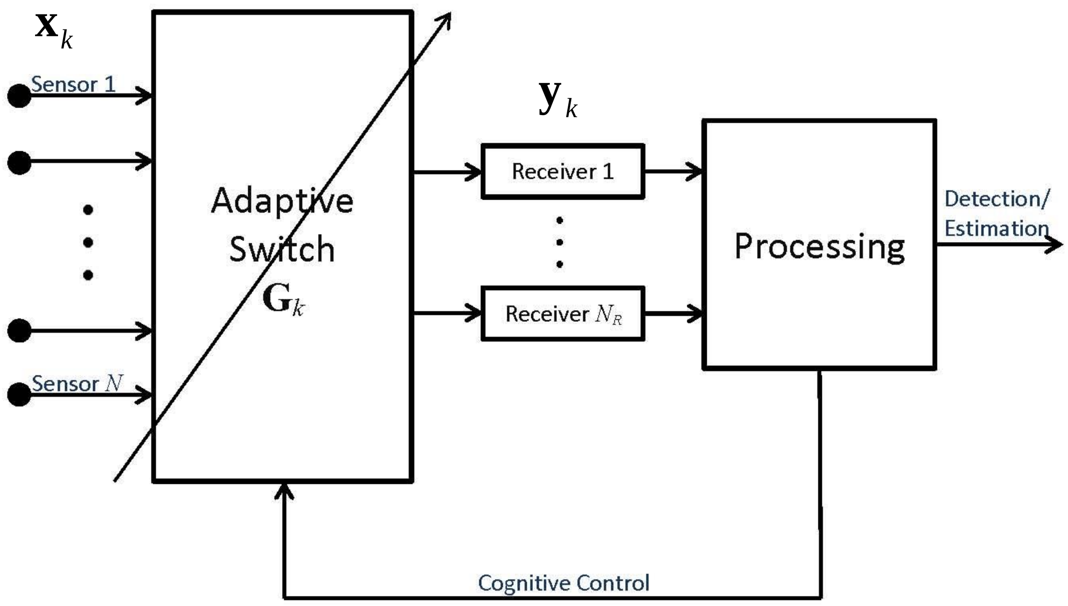
- Low cost
- Small aperture - low angular resolution/accuracy
- No DOA ambiguity

Introduction



Goal: Optimal adaptive switch control based on history observations.

Problem Statement



$$\mathbf{x}_k = \mathbf{a}(\theta)s_k + \mathbf{v}_k, \qquad k = 1, 2, \ldots$$

$$\mathbf{y}_k = \mathbf{G}_k\left(\mathbf{a}(\theta)s_k + \mathbf{v}_k\right),$$

$$[\mathbf{G}_k]_{ij} \in \{0,1\} \text{ and } \mathbf{G}_k \mathbf{1}_N = \mathbf{1}_{N_R}$$

$$\mathbf{G}_k = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & 1 & \cdots & 0 \end{bmatrix}$$

Problem statement: $\underset{\mathbf{G}_k}{\text{optimize}}\, C(\mathbf{G}_k, \mathbf{X}^{(k-1)})$

$\mathbf{X}^{(k-1)} = [\mathbf{x}_1, \ldots, \mathbf{x}_{k-1}]$ - history measurements

$C(\cdot,\cdot)$ - Chosen criterion - e.g. conditional lower bound on the estimation accuracy.

Problem Statement - Signal Models

Time - invariant signal :

$$\mathbf{y}_k = \mathbf{G}_k\left(\mathbf{a}(\theta)s + \mathbf{v}_k\right),\ \ k = 1, 2, \ldots$$

$$\mathbf{y}_k \mid s, \theta \sim CN(\mathbf{G}_k \mathbf{a}(\theta)s, \sigma^2 \mathbf{I}_{N_R})$$

Passive radar: (Observation time $\times$ Bandwidth) $\ll$ 1

Active radar: considering given Doppler cell with non-fluctuating target

Time - varying signal :

$$\mathbf{y}_k = \mathbf{G}_k\left(\mathbf{a}(\theta)s_k + \mathbf{v}_k\right),\ \ k = 1, 2, \ldots$$

$$\mathbf{y}_k \mid \sigma_s^2, \theta \overset{i.i.d.}{\sim} CN(\mathbf{0}, \sigma_s^2 \mathbf{G}_k \mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{G}_k^H, \sigma^2 \mathbf{I}_{N_R})$$

Passive radar: non-negligible (Observation time $\times$ Bandwidth)

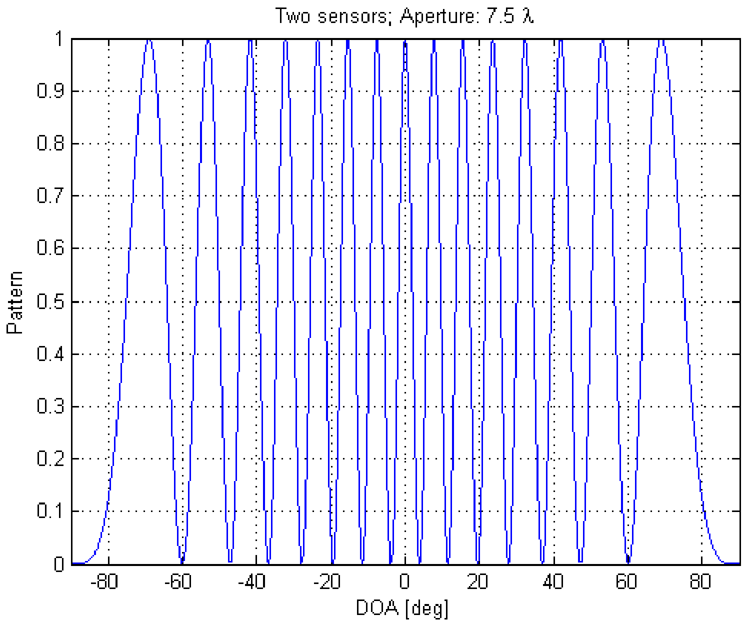
Active radar: moving/fluctuating targets

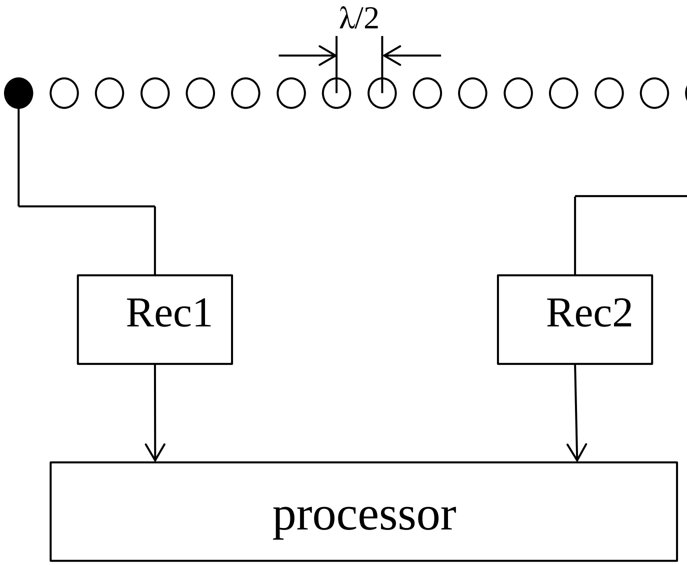
Criterion for Antenna Selection

Bayesian Cramér-Rao bound (BCRB) is a popular tool for performance analysis.

Advantage: Simplicity
Disadvantage: Ignores "large errors" due to ambiguity.

Example: single source $\mathbf{x}_k = \mathbf{a}(\theta)s + \mathbf{v}_k, \quad \mathbf{v}_k \sim CN(\mathbf{0}, \sigma^2 \mathbf{I})$





Cognitive Antenna Selection (CASE)

Proposed approach: Using the Bobrovsky-Zakai bound (BZB) as a criterion.

Advantage: Takes into account "large errors" due to grating lobes and ambiguity.
Disadvantage: Complexity

Optimization criterion:

$$\mathbf{G}_k^{opt}\left(\mathbf{X}^{(k-1)}\right) = \arg\min_{\mathbf{G}_k} BZB_{\varphi}\left(\mathbf{G}_k, \mathbf{X}^{(k-1)}\right)$$

The optimization is performed using an iterative greedy approach:
At each iteration, the optimization is performed w.r.t. one switch (one row of the matrix $\mathbf{G}_k$) while the other rows are fixed. This process is repeated for all the rows of $\mathbf{G}_k$.

$$\mathbf{G}_k = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & 1 & \cdots & 0 \end{bmatrix}$$

Cognitive Antenna Selection (CASE)

Sequential BZB:

$$BZB_\theta\left(\mathbf{G}_k, \mathbf{X}^{(k-1)}\right) = \sup_{\mathbf{h}} \frac{h_\theta^2}{B\left(\mathbf{G}_k, \mathbf{X}^{(k-1)}, \mathbf{h}\right) - 1} \qquad \boldsymbol{\psi} = [\theta, s]^T, \quad \mathbf{h} = [h_\theta, h_s]^T$$

$$B\left(\mathbf{G}_k, \mathbf{X}^{(k-1)}, \mathbf{h}\right) = \mathrm{E}\left[Z_k^{(k-1)} \cdot W^{(k-1)} \middle| \mathbf{X}^{(k-1)} \right]$$

$$Z_k^{(k-1)} = \underbrace{\mathrm{E}\left(\frac{f^2(\mathbf{x}_k | \boldsymbol{\psi} + \mathbf{h}, \mathbf{X}^{(k-1)})}{f^2(\mathbf{x}_k | \boldsymbol{\psi}, \mathbf{X}^{(k-1)})} \middle| \mathbf{X}^{(k-1)} \right)}_{\text{Current Info}} \quad W^{(k-1)} = \underbrace{\frac{f^2\left(\boldsymbol{\psi} + \mathbf{h} \middle| \mathbf{X}^{(k-1)}\right)}{f^2\left(\boldsymbol{\psi} \middle| \mathbf{X}^{(k-1)}\right)}}_{\text{History Info}}$$

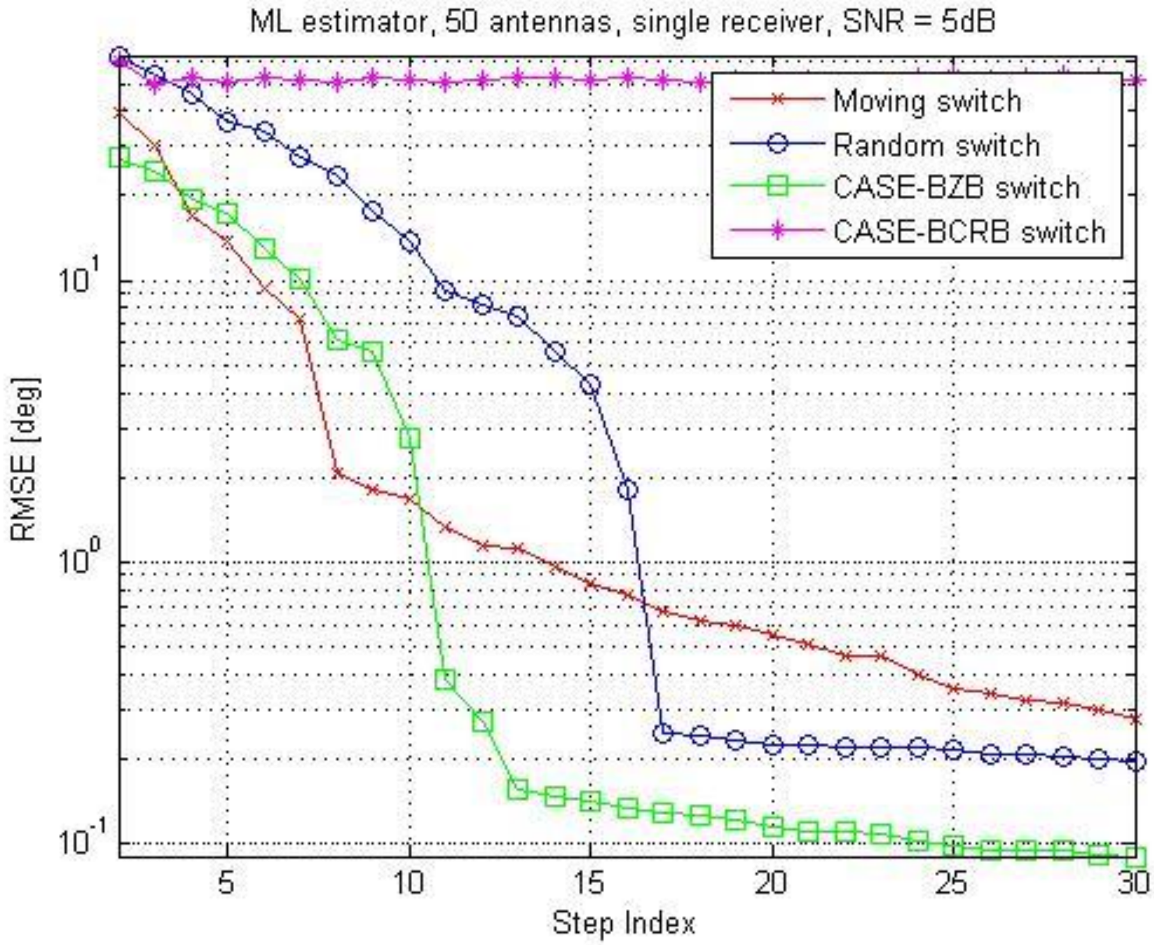
In our problem (time-invariant signals):

$$Z_k^{(k-1)} = \exp\left(\frac{2}{\sigma^2} \left\| \alpha \mathbf{G}_k \mathbf{a}(\theta) - (s + h_s) \mathbf{G}_k \mathbf{a}(\theta + h_\theta) \right\|^2 \right)$$

$$W^{(k-1)} = \exp\left(\frac{2}{\sigma^2} \sum_{m=1}^{k-1} \left[-\left\| \mathbf{x}_m - (s + h_s) \mathbf{G}_k \mathbf{a}(\theta + h_\theta) \right\|^2 + \left\| \mathbf{x}_m - s \mathbf{G}_k \mathbf{a}(\theta) \right\| \right]^2 \right)$$

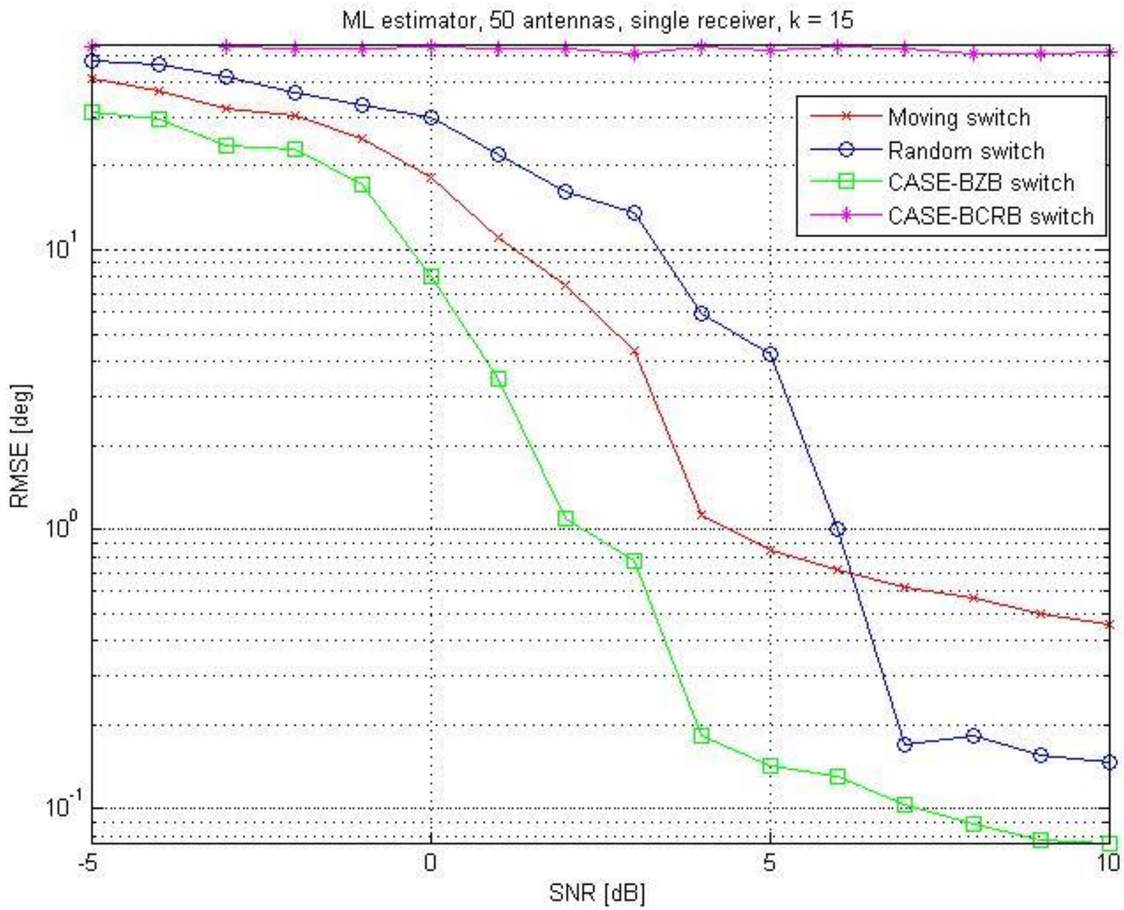
Results - Single Target

Time-invariant signal
1 Tx, 1 Rx
50 antennas
Azimuth~U(-80^0,80^0)
SNR=5dB



Results - Single Target

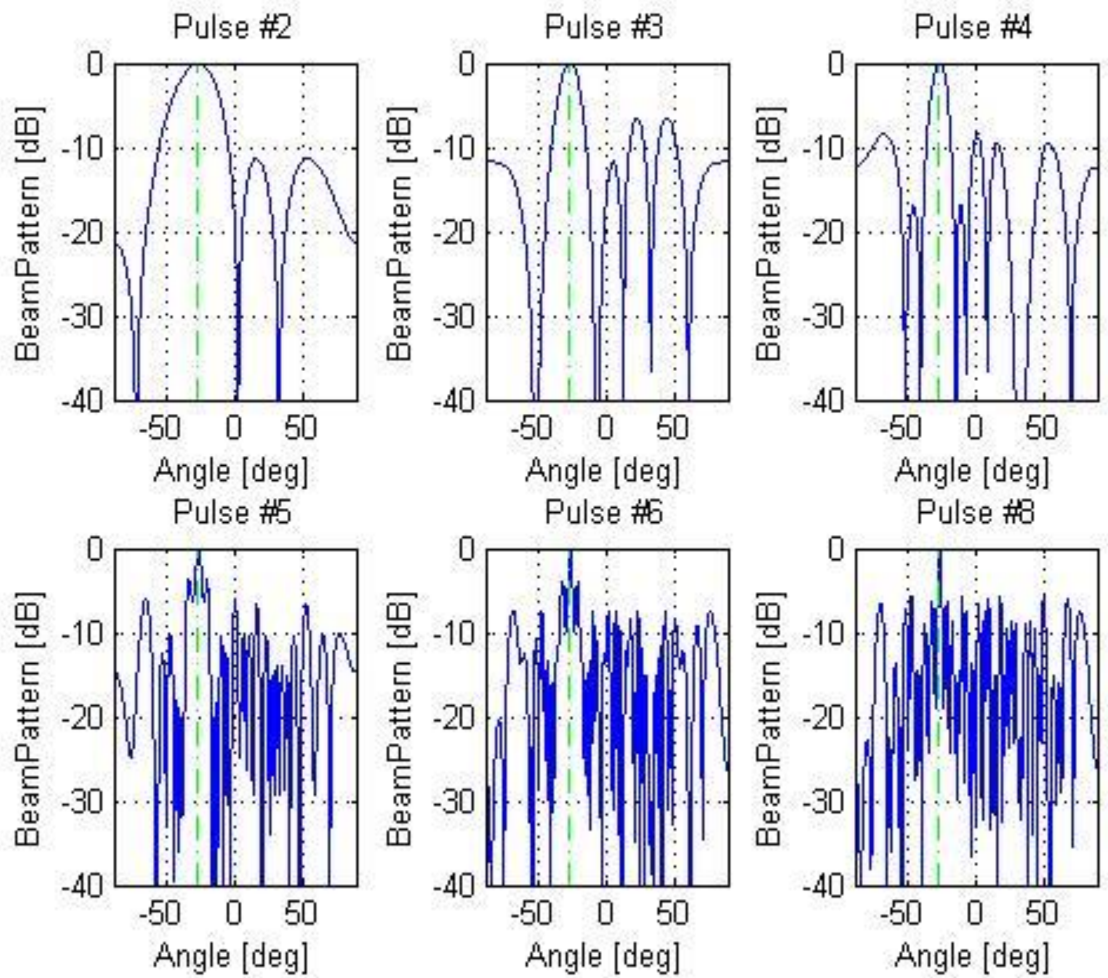
1 Tx, 1 Rx,
50 antennas
Azimuth~U(-80^0,80^0)
k=15 pulses



Results - Single Target

Beampattern

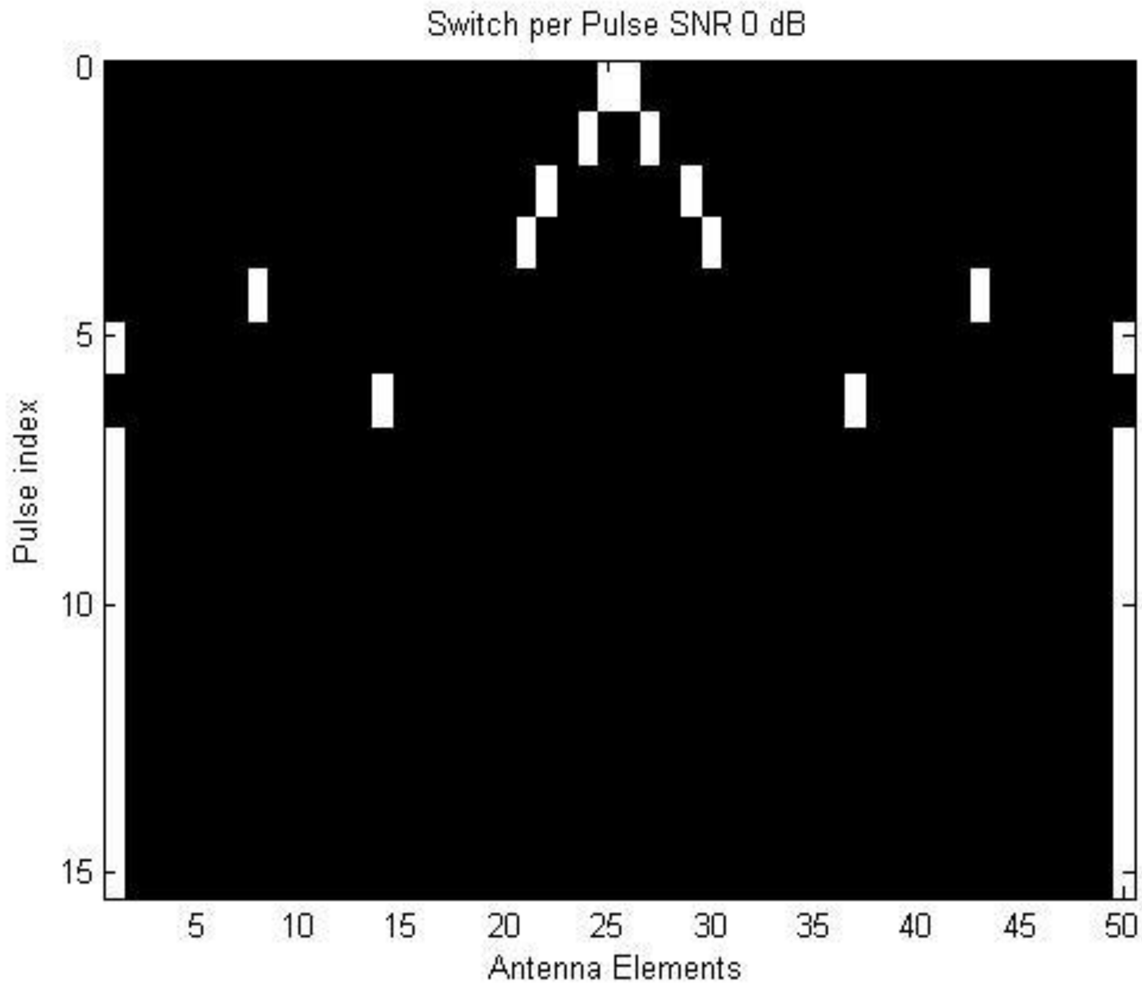
1 Tx, 2 Rx
50 antennas
Azimuth~U(-80^0,80^0)
SNR=0dB



Results - Single Target

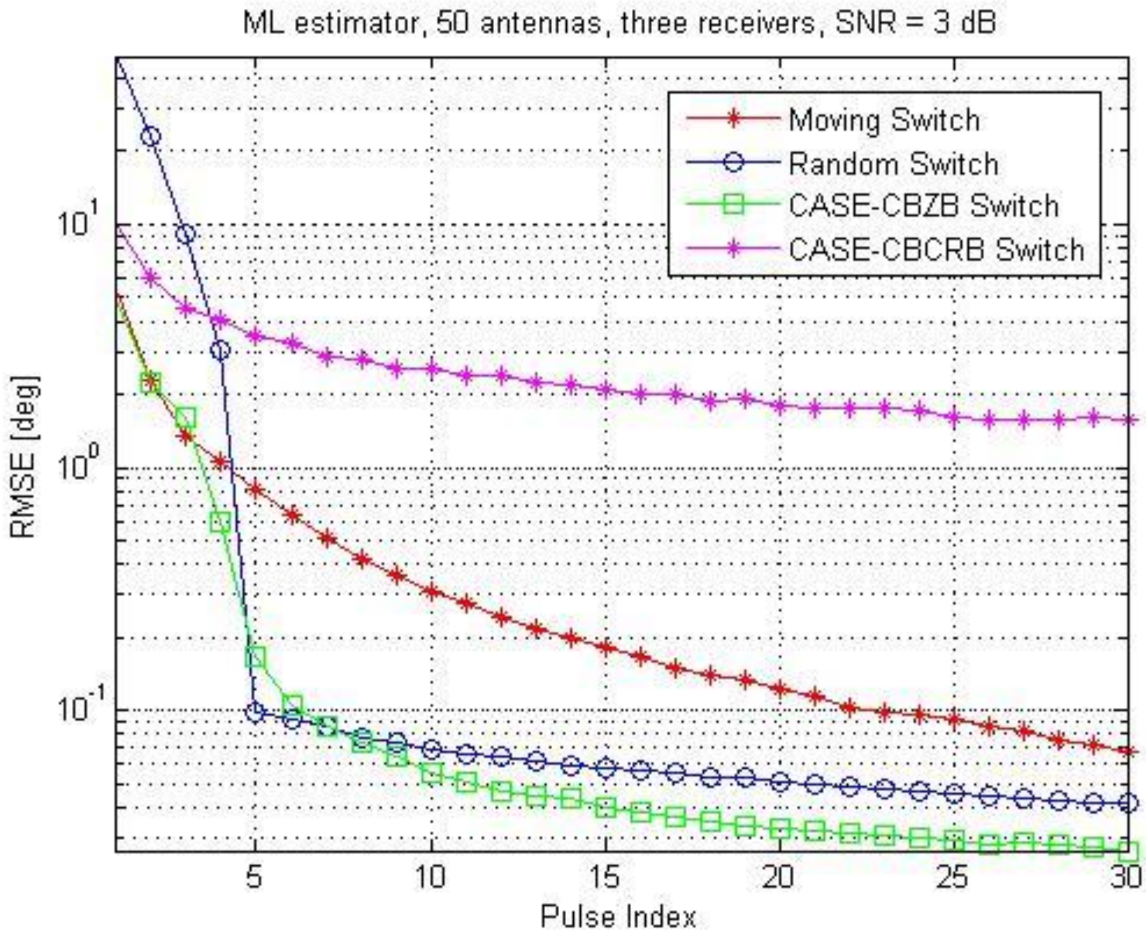
Switching

1 Tx, 2 Rx
50 antennas
Azimuth~U(-80^0,80^0)
SNR=0dB



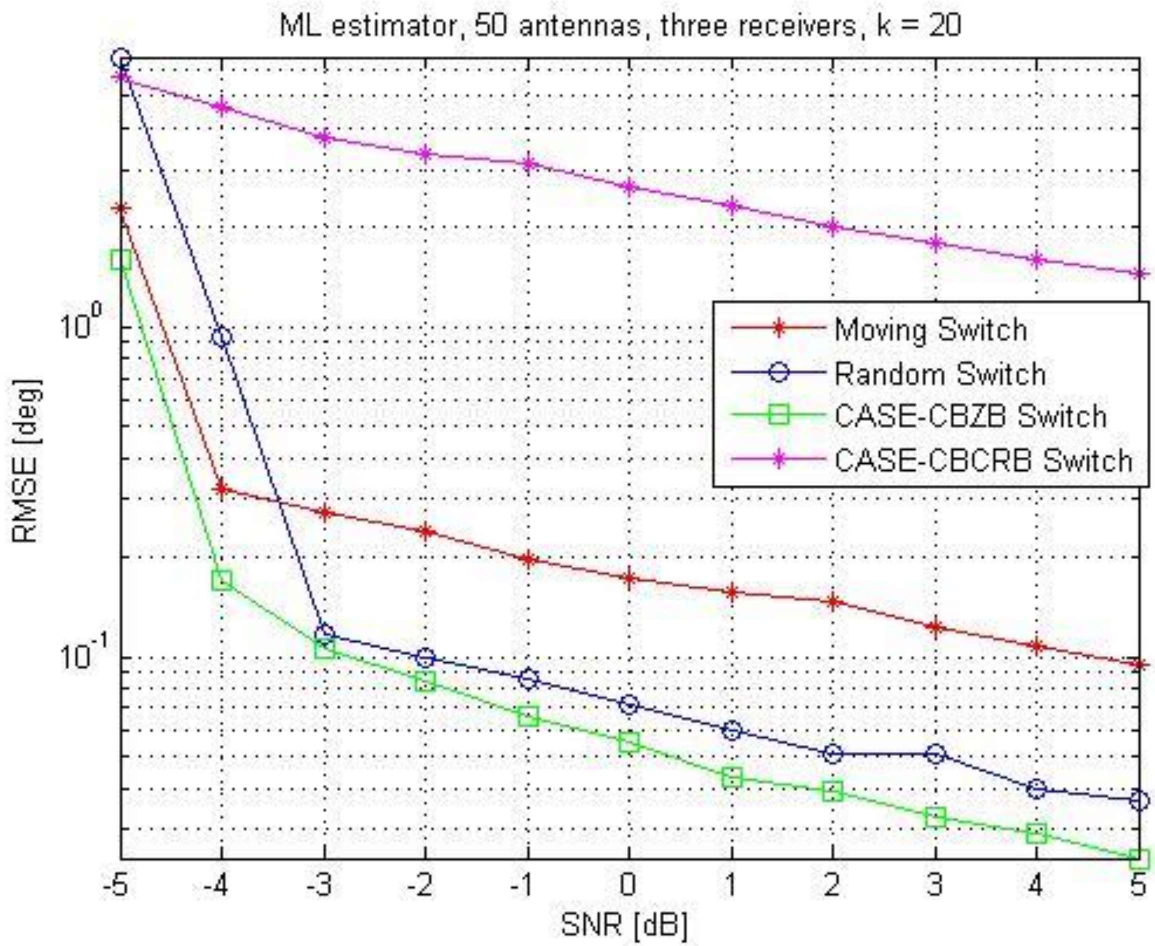
Results – Two Targets

Time-varying signal
1 Tx, 3 Rx
50 antennas
Two targets:
Azimuth~U(-80^0,80^0)
SNR=3dB
20 snapshots per pulse



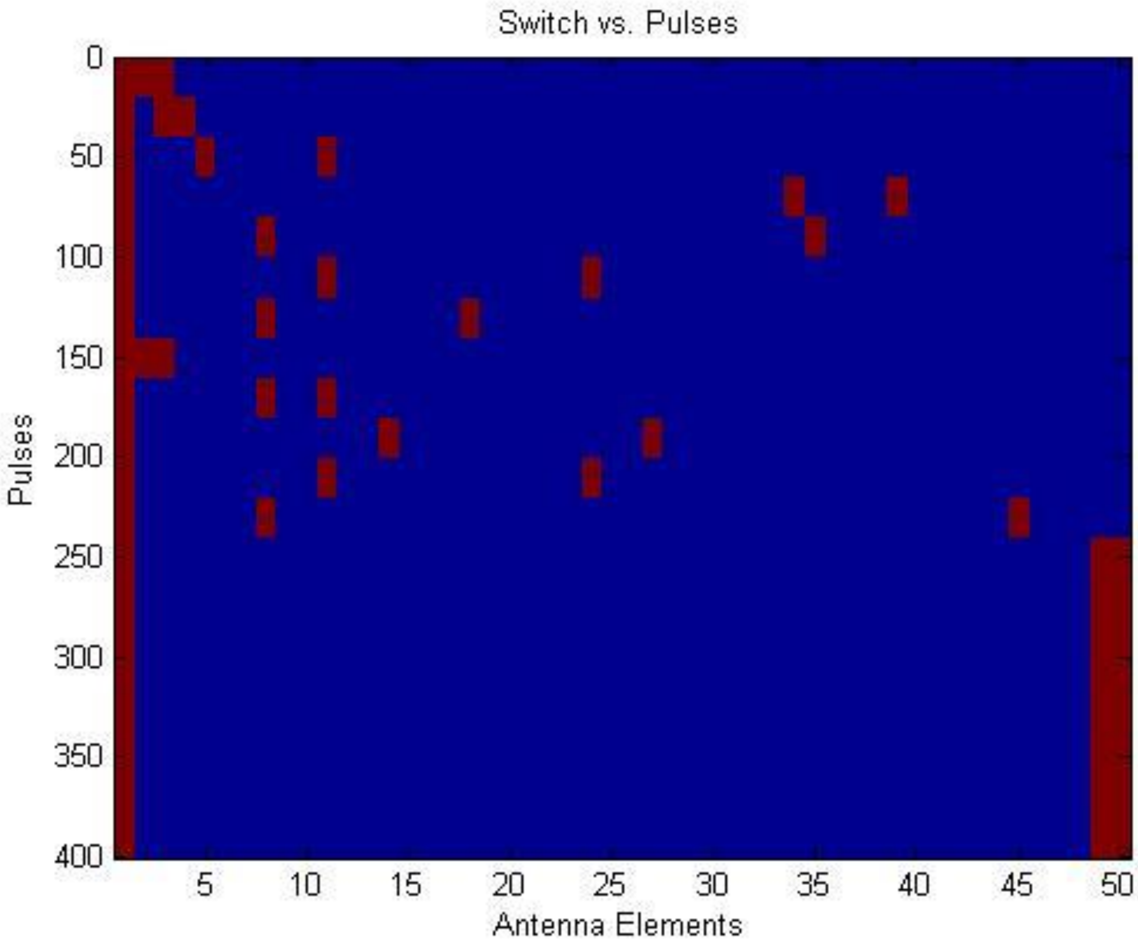
Results – Two Targets

Time-varying signal
1 Tx, 3 Rx
50 antennas
Two targets:
Azimuth~U(-80^0,80^0)
k=20 pulses
20 snapshots per pulse



Results – Two Targets

Time-varying signal
1 Tx, 3 Rx
50 antennas
Two targets:
Azimuth~U(-80^0,80^0)
k=20 pulses
20 snapshots per pulse



Conclusion and Future Research

- A new adaptive antenna selection approach was presented based on minimizing performance bounds for DOA estimation.
- The BCRB was found to be inappropriate as a criterion, since it takes into account only “local errors”. Therefore, the BZB was chosen as a criterion.
- Using small number of receiving channels, this approach allows to exploit the benefit of large array aperture, while avoiding ambiguity in DOA.
- The proposed technique was tested in the presence of one or two sources, and exhibited very good performance compared to other tested methods.
- Future/current research:
  - Considering hardware beamformer (phased array) instead of switch.
  - Taking into account Doppler information: needs considering joint DOA-Doppler ambiguity function (Doppler and DOA are not decoupled anymore).
  - Taking range information with multiple DOA/range distributed targets.

Thank you!