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Alternating para/diamagnetic domains in a p-wave superconductor

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Abstract. – The mesoscopic one-dimensional magnetic texture caused by small Zeeman coupling in a p-wave superconductor in the magnetic field is found. The field orientation is opposite on the two sides of the texture and the magnitude can be even smaller than H_{c1}. The texture is not built from various singular topological defects, like vortices, as previously found in the simplest three-component Ginzburg-Landau model of superconductors.

Introduction. – The order parameter in many classes of superconductors including organics of the type of Bechgaard salts [1], unconventional [2] and multiband superconductors [3] like heavy fermions or SrRuO, generally has several components. It describes coexisting condensates of Cooper pairs. The symmetry of the order parameter in unconventional superconductors is related to the crystallographic symmetry group of the material, the structure of the Fermi surface and the nature of the pairing mechanism. Although the number of charged fields and their transformation properties under rotations are different in any case, the common feature of theories describing these diverse systems remains the $U(1)$ local gauge invariance. In systems of this kind, in the presence of an external magnetic field the major role is played by various topological defects. It is well known that, while in the simplest case, that of the one-component order parameter, the Abrikosov vortices (AV) are the only kind of topological defects, in the multicomponent case other types of defects exist.

Machida [4] argued that in the p-wave superconductors with weak spin-orbit coupling the triplet pairing function can be represented by a three-dimensional complex vector field. The simplest Ginzburg-Landau model describing heavy fermion UPt_3, even subjected to an external magnetic field B *under condition of negligible Zeeman coupling*, has an approximate $SO(3)$ symmetry making it more amenable (and interesting) to mathematical analysis. This model was subsequently studied in more detail and various topologically distinct "non-Abrikosov" defects with sophisticated structure were found: coreless magnetic skyrmions [5,6], field-carrying

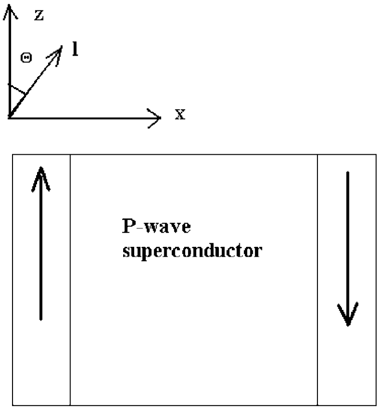


Fig. 1 – The sample subjected to an opposite-directed external magnetic field.

topological textures [7], knot solitons [3] and the "vector vortices" (VV) [8, 9], which have a complex core more typical of the superfluid ^{3}He rather than of a superconductor.

The Zeeman paramagnetic term responsible for direct interaction between the Cooper pair spin and the external magnetic field although usually small, might considerably affect topological solitons. It has not been taken into account in theoretical considerations with an exception for the case of the skyrmions. In the latter case, a sufficiently strong paramagnetic effect led to skyrmion's instability [6], but no qualitatively new effects. In the present communication we consider a new mesoscopic one-dimensional magnetic texture caused by small Zeeman coupling in the three-component p-wave superconductor. It is not built from the singular topological defects and can be realized as a p-wave superconducting plate subjected to an external magnetic field of the configuration shown in fig. 1. The external-magnetic-field configuration can be created, for example, by currents flowing on the left of the slab in direction y and on the right of the slab in the opposite direction $-y$. Note that these currents flow outside the superconducting slab.

Basic equations. – We start with the Ginzburg-Landau free-energy density describing a system with a three-component complex vector order parameter ψ_i:

$$\mathcal{F} = F_{\mathrm{pot}} + F_{\mathrm{grad}} + F_{\mathrm{Zeeman}} + \frac{1}{8\pi} B_j^2, \tag{1}$$

where

$$F_{\mathrm{pot}} = -\alpha\psi_i\psi_i^* + \frac{\beta_1}{2}\left(\psi_i\psi_i^*\right)^2 + \frac{\beta_2}{2}|\psi_i\psi_i|^2, \tag{2}$$

$$F_{\mathrm{grad}} = \frac{\hbar^2}{2m^*}\left(D_j\psi_i\right)\left(D_j\psi_i\right)^*, \tag{3}$$

$$F_{\mathrm{Zeeman}} = -\mu\vec{S}\cdot\vec{B}. \tag{4}$$

Here $D_j \equiv (\partial_j - i2eA_j/\hbar c)$ are covariant derivatives, $B_j \equiv (\nabla\times\vec{A})_j$ is the magnetic induction and $\mu = ge\hbar/(m^*c)$ is determined by the dimensionless gyromagnetic ratio g. The average spin of the Cooper pair at a specific point in space is given by

$$S_i = \psi_j^*(\vec{r})\left(-i\varepsilon_{ijk}\right)\psi_k(\vec{r}). \tag{5}$$

Writing the order parameter ψ in the form

$$\vec{\psi} = f(\vec{n}\cos\varphi + i\vec{m}\sin\varphi), \tag{6}$$

one can easily obtain two different uniform ground-state solutions:

$$\mathrm{I}: \ \beta_2 > 0, \qquad \vec{\psi} = f\frac{\vec{n}+i\vec{m}}{\sqrt{2}}, \qquad \vec{n}\perp\vec{m}, \qquad \varphi = \frac{\pi}{4}, \qquad f^2 = \frac{\alpha}{\beta_1}; \tag{7}$$

$$\mathrm{II}: \ \beta_2 < 0, \qquad \vec{\psi} = fe^{i\varphi}\vec{n}, \qquad \vec{n} = \pm\vec{m}, \qquad f^2 = \frac{\alpha}{\beta_1+\beta_2}. \tag{8}$$

Here the "length" $f > 0$, $0 \le \varphi \le \pi/2$, $\vec{n}$ and $\vec{m}$ are unit vectors. The spin density is $\vec{S} = f^2\sin 2\varphi\vec{l}$, where $\vec{l}$ completes the "triad": $\vec{l} \equiv \vec{n}\times\vec{m}$. While it vanishes for type II, $S = f^2$ for the type-I ground state.

In the London approximation the length f of the order parameter $\vec{\psi}$ is fixed. For type I substituting the order parameter $\vec{\psi}$ from eq. (7) into eqs. (1)-(4), we obtain the London free-energy density in dimensionless form as

$$F_{\mathrm{L}} = \frac{1}{2}\left(\partial_j\vec{l}\right)^2 + (a_i)^2 + b_i^2 - gl_ib_i. \tag{9}$$

For convenience, we express all the physical quantities in dimensionless units as follows:

$$\begin{aligned} x &\equiv \lambda\widetilde{x}, \qquad F \equiv \frac{\alpha^2}{\beta_1\kappa^2}\widetilde{F}, \qquad f^2 \equiv \frac{\alpha}{\beta_1}\widetilde{f}^2, \\ \vec{A} &\equiv \frac{\Phi_0}{2\pi\lambda}\vec{a}, \qquad \vec{B} \equiv \frac{\Phi_0}{2\pi\lambda^2}\vec{b}, \end{aligned} \tag{10}$$

where the magnetic penetration depth $\lambda \equiv c/e\sqrt{\beta_1 m^*/(\pi\alpha)}$, the coherence length $\xi \equiv \hbar/\sqrt{2m^*\alpha}$, the flux quantum $\Phi_0 \equiv hc/2e$, and the Ginzburg-Landau parameter $\kappa \equiv \lambda/\xi$. The tildes will be omitted hereafter.

Periodic para/diamagnetic structure. – Let us consider the case where a superconducting sample of size L in the x-direction, and infinite in the y-direction (see fig. 1) is subjected to an external magnetic field h directed along the z-axis at $x = 0$, while at $x = L$ it is directed oppositely ($H_{\mathrm{ext}}(L) = -h$). We also assume that the vector $\vec{m}$ is constant in the y-direction. In this case we obtain from the London energy (9),

$$F_{\mathrm{L}} = \int_0^L \left[\frac{1}{2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2 + a^2 + \left(\frac{\mathrm{d}a}{\mathrm{d}x}\right)^2 - g\cos\theta\frac{\mathrm{d}a}{\mathrm{d}x}\right]\mathrm{d}x. \tag{11}$$

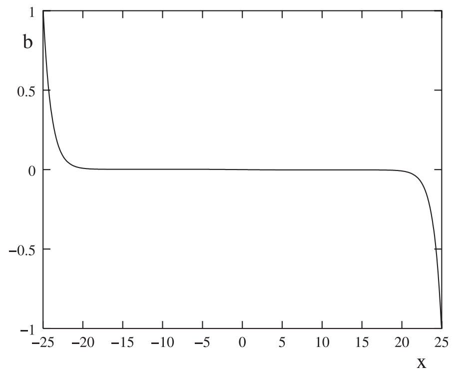
Here θ is the angle between the vector $\vec{l}$ and the magnetic field, and a is the y-component of the vector potential. Variation of the London free energy provides two equations describing the spatial distribution of both the magnetic induction and the spin direction in the sample:

$$\begin{aligned} \frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} &= g\sin\theta\frac{\mathrm{d}a}{\mathrm{d}x}, \\ \frac{\mathrm{d}^2 a}{\mathrm{d}x^2} - a &= -\frac{g}{2}\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}x}. \end{aligned} \tag{12}$$

This set of equations should be supplemented by the boundary conditions as follows:

$$\theta(0) = 0, \qquad \theta(L) = \pi, \qquad \left.\frac{\mathrm{d}a}{\mathrm{d}x}\right|_{x=0} = h, \qquad \left.\frac{\mathrm{d}a}{\mathrm{d}x}\right|_{x=L} = -h. \tag{13}$$

We solved these equations both numerically and approximately analytically for different values of the dimensionless parameter g.



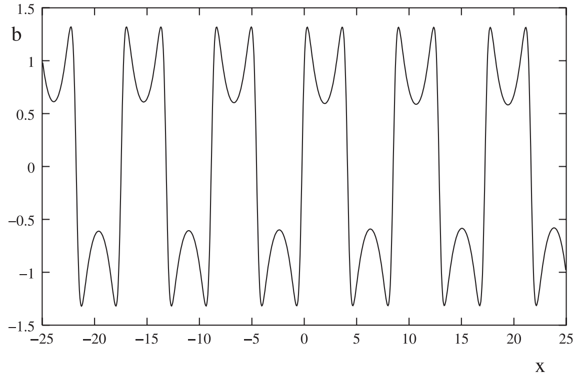


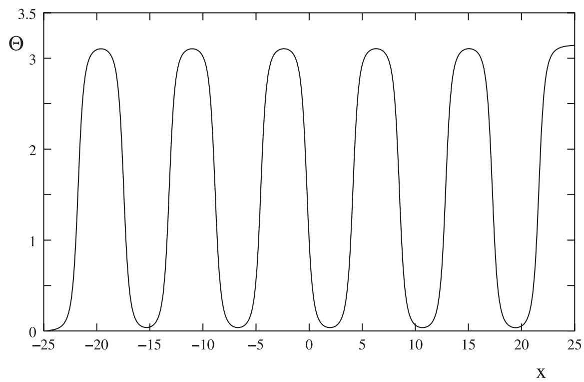
Fig. 2

Fig. 3

Fig. 2 – Spatial distribution of the magnetic induction across the sample ($g = 1$).

Fig. 3 – Magnetic-induction structure across the sample ($g = 5$).

Numerical results. – The set of equations (12), (13) has been solved numerically using the Crank-Nickolson method. The numerical results for the angle θ and the magnetic induction $b = \mathrm{d}a/\mathrm{d}x$ for different values of g are presented in figs. 2-4. One can conclude that for sufficiently small g ($g = 1$) there is just the usual Meissner effect, *i.e.* the magnetic induction decreases exponentially inside the layer, while if g exceeds some critical value, $g_\mathrm{c} \approx 2.68$, alternating domains with opposite magnetic induction appear. It should be noted that the maximal magnitude of the magnetic induction inside the domains is generally different from the values of the external magnetic field. The domain size strongly depends on the paramagnetic parameter g and does not depend of the external-magnetic-field strength.



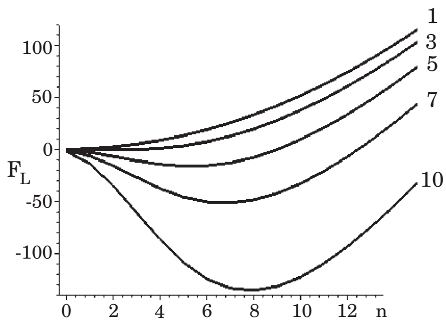


Fig. 4

Fig. 5

Fig. 4 – The angle between the spin direction and the external magnetic field, as a function of x ($g = 5$).

Fig. 5 – The free energy as a function of n (here n is the number of domains), for $g = 1$, 3, 5, 7, 10 (the number beside each line shows the value of g).

Variational solution. – Motivated by the numerical results we have chosen a trial function for the angle θ and the effective magnetic induction b inside the sample, in the form

$$\begin{aligned}
\theta &= \pi \sin^2\left(\frac{\pi}{d}x\right),\\
b &= b_0, \quad \text{for } md \le x < md + \frac{d}{4} \quad \text{or} \quad md + \frac{3d}{4} < x < (m+1)d,\\
b &= -b_0, \quad \text{for } md + \frac{d}{4} < x < md + \frac{3d}{4},
\end{aligned} \tag{14}$$

where m is an integer, while the amplitude of the magnetic induction b_0 inside the domain and the domains size d are considered as variational parameters. Substituting these functions into the free-energy functional (eq. (11)) and taking into account the boundary conditions eq. (13), one obtains the following London energy of the sample:

$$f = \frac{\pi^4(2n+1)^2}{16L} + Lb_0^2 + \frac{2L}{3}b_0^2\left(\frac{L}{2(2n+1)}\right)^2 - gLb_0\frac{R_1}{\pi}, \tag{15}$$

where

$$n = \frac{2L-d}{2d}$$

and $R_1 \approx 2.36$. Here n is the number of magnetic domains inside the sample. Minimizing the free energy (15) with respect to b_0, one obtains

$$\begin{aligned}
\frac{\partial f}{\partial b_0} &= \left(2L + \frac{4L}{3}\left(\frac{L}{2(2n+1)}\right)^2\right)b_0 - Lg\frac{1}{\pi}R_1 = 0,\\
b_0 &= \frac{gR_1}{2\pi\left[1+\frac{2}{3}(L/2(2n+1))^2\right]}.
\end{aligned} \tag{16}$$

Combining eqs. (15) and (16), we obtain the London free energy as a function of the variational parameter n:

$$f(n) = \frac{\pi^4(2n+1)^2}{16L} - L\left(\frac{gR_1}{2\pi}\right)^2\frac{1}{1+\frac{2}{3}(L/2(2n+1))^2}. \tag{17}$$

The energy $f(n)$ plotted in fig. 5 for various g's as a function of n has a minimum at $n = 0$ for $g < g_c$, while for $g > g_c$ the energy reaches its minimum at $n \neq 0$.

The characteristic sizes of the alternating domains are estimated as (see fig. 6): $d \simeq 10\lambda$ for the paramagnetic parameter varying in the range $g \risingdotseq 3$–10. The dimensionless amplitude of the magnetic induction as a function of g is presented in fig. 7. The induction inside domains $B_0 = (\Phi_0/2\pi\lambda^2)b_0$ is usually smaller than the first critical magnetic field $H_{c1} = (\Phi_0/2\pi\lambda^2)\log(\kappa)$ at which Abrikosov vortices enter the sample. The texture therefore has little to do with Abrikosov vortices, as well as with other single-soliton structures. We believe therefore that the solution is a stable nontopological soliton. From figs. 6 and 7 one can see that there is a good agreement between the numerical and the trial function solutions for the domain size, but there is a small difference for the internal magnetic-induction magnitude. The difference probably arises due to the fact that the magnetic induction found numerically has a structure inside the domain, while the trial function is uniform.

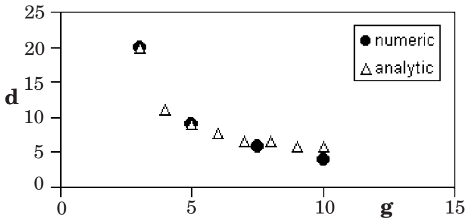


Fig. 6

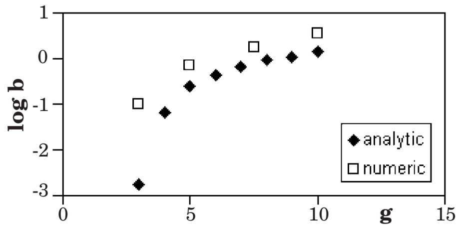


Fig. 7

Fig. 6 – The domain size d (in units of λ) as a function of the gyromagnetic ratio g (numeric and trial function results).

Fig. 7 – The magnetic induction inside the domain b_0 *vs.* the gyromagnetic ratio g (numeric and trial function results).

Summary and discussion. – The p-wave superconducting layer subjected to a weak external magnetic field ($h < H_{c1}$) with opposite field directions on the layer edges has a nontrivial vortex free magnetic structure, consisting of alternating macroscopic para and diamagnetic domains. Both the magnetic induction inside the domain and the domain size are completely defined by the paramagnetic parameter g, and not by the strength of the external magnetic field. It should be noted that the alternating magnetic domains inside the layer appear only above the critical paramagnetic parameter value $g_c \approx 2.68$. Both the characteristic size of the domain d and the internal domain magnetic induction b_0 are only slightly affected by the sample size and almost completely determined by the paramagnetic parameter.

Let us compare and contrast the 1D structure with several similar 1D structures in other p-wave pairing systems. In the A-phase of rotating liquid-^{3}He the vortex sheet configuration was found theoretically [10] and observed experimentally [11]. There is a well-known analogy between the rotating superfluid and a superconductor in the external magnetic field with the rotation pseudovector Ω in the superfluid playing the role of the magnetic field in a superconductor. However, one cannot push the analogy too far. The order parameter in the superfluid (within the London-type approximation in the A-phase) contains two vector fields $\boldsymbol{d}$ and $\boldsymbol{l}$, differently from one vector $\boldsymbol{l}$ in the superconductor (within similar approximation).

The configuration we considered corresponds in the superfluid to the rotation of the two sides of the vessel in opposite directions and is hardly realizable experimentally. On the contrary, in superconductors there is no experimental obstacle to realize such a configuration. A superconductor analog of the vortex sheet would have the magnetic field oriented in the same direction on opposite sides of the junction. However, we note that in the rotating ^{3}He system due to the coupling between vectors $\boldsymbol{d}$ and $\boldsymbol{l}$ there are two degenerate ground-state configurations with either parallel or antiparallel directions of $\boldsymbol{d}$ and $\boldsymbol{l}$. The vortex sheet configuration interpolates between the two states. In p-wave superconductors there is no analog of such a coupling or degeneracy and therefore the configuration is topologically trivial.

Kopnin and Burlachkov (KB) [7] found a spin texture in p-wave superconductors subjected to a homogeneous external magnetic field. It is caused by the anisotropy arising from the spin-orbit interaction rather than from the Zeeman coupling essential in our manuscript. Zeeman coupling has been neglected in [7]. It is clear that the alternating domains are essentially different from those considered in [7]. For example, the magnetic field of the KB domains does not change its direction (see fig. 1 of [7]), while in our case the alternating magnetic domains are the essential issue.

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