Hysteresis and Ferromagnetism

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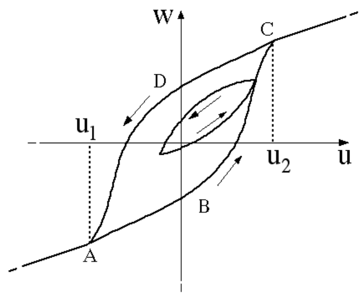

Figure 1: continuous hysteresis

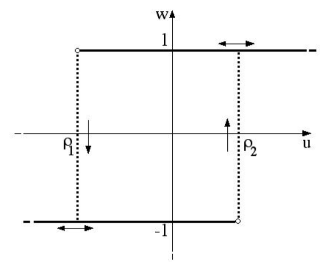

Figure 2: discontinuous hysteresis

This talk is expected to address an audience mainly of physicists, and will illustrate the tenets of hysteresis modeling.

Hysteresis. Hysteresis occurs in several phenomena in physics and other disciplines: typical examples include plasticity, ferromagnetism, ferroelectricity, superconductivity, friction, and so on. Mathematically hysteresis is defined as *rate-independent memory;* in reality this is often combined with rate-dependent effects.

Classical models of hysteresis in magnetism and elasto-plasticity were proposed in the last century by a number of scientists: Duhem, Prandtl, Ishlinskiĭ, Preisach and others.

The Preisach model [Pr] is especially relevant: it is based the idea of representing a continuous hysteresis loop and its interior subloops (see Figure 1), by superposing a weighted population of rectangular hysteresis loops with different thresholds (see Figure 2). This model and its vector extension have been successfully applied to ferromagnetism, ferroelectricity, porous media filtration and other phenomena [Ma].

In the 1970s the Russian school of Krasnosel'skiĭ, Pokrovskiĭ [KrPo] and others introduced and investigated the notion of *hysteresis operator* between spaces of time dependent functions; this is characterized by *rate-independent* memory. Further studies on partial differential equations containing those operators followed in the 1980s [Vi1]. Details may be found e.g. in the monograph [Vi1] and the survey [Vi3]. A different approach to hysteresis is addressed in the recent monograph [MiRo].

Micromagnetism. The classical model of micromagnetism of Landau and Lifshitz [LaLi] is an issue apart. This represents relaxation and precession, and is thus rate-dependent; it becomes rate-independent as the relaxation parameter a vanishes:

$$a\frac{\partial \vec{M}}{\partial t} = \lambda_1 \vec{M} \times \vec{H}^e - \lambda_2 \vec{M} \times (\vec{M} \times \vec{H}^e); \tag{1}$$

here the *effective magnetic field* $\vec{H}^e$ is defined as $\vec{H}^e := \vec{H} - \varphi'(\vec{M}) + \nabla^2 \vec{M}$, see e.g. [BeMaSe]. An equivalent formulation is known as the *Gilbert equation.*

A rate-independent amendment of this model [Vi2] accounts for dry friction in domain-wall displacement, due to magnetic inclusions.

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