Introduction to rare events 001.2.4062

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3 credits

Rare events (large deviations) in natural systems, despite being unusual, can have dramatic or even catastrophic consequences. Examples include earthquakes, heatwaves, floods, pandemic outbreaks, stock market crashes, population extinction, etc.
The goal of this course is to teach students from different backgrounds how to apply state-of-the-art theoretical and numerical tools to predict the likelihood of rare events in stochastic (probabilistic) models, with many examples. The students will also understand the mechanisms that are most likely to produce rare events of different kinds.

Prerequisite: Basic probability theory

Syllabus:

Lectures/Exercises/Credit: 3 weekly lecture hours, 3 credit points.

- Review of selected topics in linear algebra and differential equations.
- Probability theory and stochastic processes
- Optimization problems, calculus of variations
- Large deviations of sums of i.i.d. random variables (Cramér's theorem)
- Large deviations in Markov chains
- Weak-noise theory for Brownian motion
- Monte-Carlo simulations of stochastic systems, the Metropolis algorithm
- Introduction to importance sampling methods for simulating rare events

Bibliography:

- S. N. Majumdar and G. Schehr, *Large deviations*, ICTS Newsletter 2017 (Volume 3, Issue 2); arxiv 1711:07571.
- H. Touchette, *The large deviation approach to statistical mechanics*, Phys. Rep. **478**, 1 (2009).
- A. Dembo and O. Zeitouni, *Large Deviations Techniques and Applications*, 2nd ed. (Springer, New York, 1998).
- F. den Hollander, *Large Deviations, Fields Institute Monographs*, vol. 14 (AMS, Providence, Rhode Island, 2000).
- B. Meerson and N. R. Smith, *Geometrical optics of constrained Brownian motion: three short stories*, J. Phys. A: Math. Theor. 52, 415001 (2019)
- A. K. Hartmann, *Sampling rare events: Statistics of local sequence alignments*, Phys. Rev. E **65**, 056102 (2002).

Grade: Exercises + Exam/final project