



Technical Note

The validity of dynamic block displacement prediction using DDA

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1. Introduction

In 1965, Newmark [1] published his classic Rankine lecture on “effects of earthquakes on dams and embankments”. In his paper, Newmark presented solutions for displacement of a mass along circular or planar sliding surface under earthquake loading. Newmark made the assumption that the mass moves as a single rigid body with resistance mobilized along the sliding surface. Newmark further considered only a single pulse of magnitude A_g lasting for a time interval t_0 , arguing that introduction of a sinusoidal pulse would complicate the expressions unnecessarily. Thus, the so-called “Newmark method” provided an estimate for the amount of mass displacement to be expected under ground acceleration of constant magnitude and given duration. Newmark admitted that his approach would generally overestimate the actual displacement because it ignores the earthquake pulse in the opposite direction.

Goodman and Seed [2] studied experimentally the shear resistance of sand to cyclic loading and suggested an expression for shear strength degradation as a function of displacement. They used numerical integration to find the velocity and displacement of a block on an incline subjected to a sinusoidal acceleration function of the form

$$a_t = A \sin(\omega t + \theta) \quad (1)$$

in which θ is the phase angle required to satisfy the initial condition $a = a_y$ at the instant sliding begins ($t = 0$), where a_y is defined as the yield acceleration. Goodman and Seed showed that for frictional sliding only, where cohesion along the sliding surface is zero, the down slope, horizontal, yields acceleration for a block resting on a plane with inclination α and friction angle ϕ is given by

$$a_y = \tan(\phi_{eq} - \alpha)g, \quad (2)$$

where ϕ_{eq} is a displacement dependent friction angle, which for all practical purposes in rock mechanics could be replaced by ϕ . Similarly, it can be shown that for up slope sliding the horizontal yield acceleration is given by

$$a_y = \tan(\alpha + \phi)g. \quad (3)$$

In Fig. 1 horizontal yield acceleration in units of g is plotted as a function of friction angle and inclination of the sliding surface. It is apparent that up slope motions require significantly higher accelerations, and therefore Newmark’s treatment of downhill motions only, seems justified.

When real earthquake ground motions are considered, neither the originally proposed Newmark’s method nor the Goodman and Seed method are adequate because the accelerogram will not follow a simple sinusoidal function with constant amplitude and period of oscillations. In the realistic case, application of Newmark’s method requires double integration of the acceleration record for every $a_t > a_y$ (e.g. [3]), and indeed some computer codes have been developed in BASIC [4], FORTRAN [5], and C [6] to enable a “Newmark’s type” analysis.

While application of Newmark’s type analysis requires significant time and computing resources, it should be remembered that the solution is only valid for a single, rigid, block undergoing sliding deformation. A real rock mass typically consists of several rigid blocks, which interact with one another during shaking at contact points or along common planes. Furthermore, different blocks in the rock mass may exhibit different failure modes as a response to the same input motion. Therefore, a Newmark’s type analysis will not be valid for the more general case of a blocky rock mass.

In order to study realistic rock slope behavior during ground shaking it would be desirable to check the validity of existing numerical methods that solve for the displacements in a block system under the application of dynamic load. In this paper, we test the validity of discontinuous deformation analysis (DDA) [7,8] by

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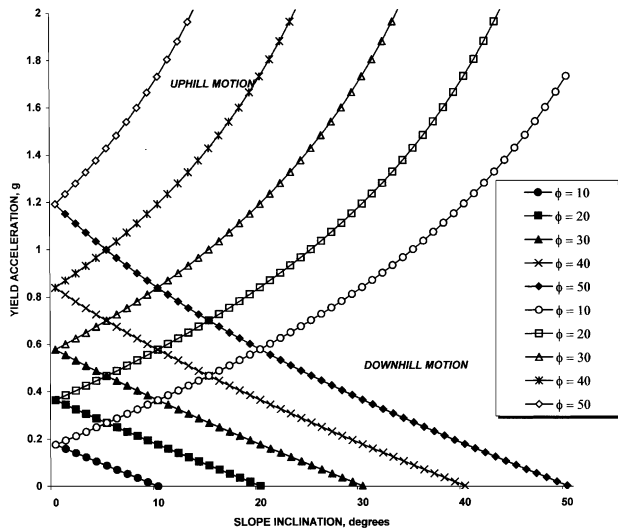


Fig. 1. Horizontal yield accelerations for downhill and uphill motion of a block on an inclined plane.

comparing its solution for dynamic block displacement with analytical solutions. This by no means indicates that we believe that the analytical solution produces true block behaviour under dynamic load; it is, however, a traditional method to address accuracy and shortcomings of numerical tools. Ultimately, a comparison with a physical model such as shaking table output would be desired.

We perform analytical integration of sinusoidal functions of increasing complexity to find the velocity and displacement function with time. We then compare the analytical results with the numerical solution of DDA for the displacement of a single block which rests on an inclined plane with a given friction angle and is subjected to the same acceleration function as solved analytically. Naturally, the analytical validation of the numerical method can only be performed for a single block. In order to validate dynamic displacements, determined by DDA for a system of blocks, comparison with physical models on a shaking table would be necessary. However, validation of DDA for a single block experiencing relatively high dynamic loads and undergoing very large displacements would be the first necessary step in this broader task.

2. The numerical discontinuous deformation analysis

DDA was developed by Shi [7,8] and its validity has been tested and confirmed using simple problems for which analytical or semi-analytical solutions exist [9–13]. In all previously published validations, the block or block system were subjected to gravitational loading only. In this paper we attempt to validate DDA for dynamic loading as well, namely by introducing time dependent acceleration.

The DDA method incorporates dynamics, kinematics, and elastic deformability of the rock, and models actual displacements of individual blocks in the rock mass using a time-step marching scheme. The formulation is based on minimization of potential energy and uses a “penalty” method to prevent penetration of blocks. Numerical penalties in the form of stiff springs are applied at the contacts to prevent either penetration or tension between blocks. Tension or penetration at the contacts will result in expansion or contraction of these springs, a process which requires energy; the minimum energy solution is therefore one with no tension or penetration. When the system converges to an equilibrium state, however, there are inevitable penetration energies at each contact, which balance the contact forces. Thus the energy of the penetration (the deformation of the springs) can be used to calculate the normal and shear contact forces.

Shear displacement along boundaries is modeled in DDA using the Coulomb–Mohr failure criterion. The fixed boundaries are implemented using the same penalty method formulation: stiff springs are applied at the fixed points. Since displacement of the fixed points requires great energy, the minimum energy solution will not permit fixed point displacement. The blocks are simply deformable: stresses and strains within a block are constant across the whole region of the block. This feature requires a minimum number of blocks in the mesh in order to accurately calculate stress and strain distribution throughout the medium. A sub-blocking algorithm has been introduced to DDA [14] and recently water pressures have been implemented as well [15].

In this research a new C/PC version of DDA, recently developed by Gen-Hua Shi [16], is used. In this new version, earthquake acceleration can be input directly in every time step. A necessary condition for direct input of earthquake acceleration is that the numerical computation has no artificial damping because damping may reduce the earthquake dynamic energy and the damage may be underestimated. In DDA, the solution of the equilibrium equations is performed without damping [16]. There are three possible ways to input recorded earthquake waves: time dependent acceleration, time dependent displacement, and time dependent velocity. The time dependent acceleration input is in fact a multi-block Newmark type analysis. DDA in its two dimensional formulation can take as input time dependent displacements, and it may be possible to input time dependent velocities as well. However, it is a common practice in earthquake engineering to use accelerograms as they are readily available for different fault systems and earthquake events around the world. In this research, therefore we input time dependent accelerations directly and compare DDA displacements with the analytical results.

3. A block on an incline subjected to gravitational load only

This is a classic case of dynamics which was studied by MacLaughlin [10] using an updated version of DDA for Windows™ environment [17]. MacLaughlin’s validation test is repeated here for calibration purposes only, using the new DDA C/PC code [16]. For a single block resting on a plane inclined at an angle α with friction along the interface ϕ , and subjected to gravitational acceleration g , the analytical solution for displacement s as a function of time t is given by

$$s_t = \frac{1}{2}at^2 = \frac{1}{2}(g \sin \alpha - g \cos \alpha \tan \phi)t^2. \tag{4}$$

A comparison between the analytical solution in Eq. 4 and DDA solution is shown in Fig. 2. The inclination of the modeled plane is 22.6° , this will be the inclination we will use throughout the validation study in this paper. Four friction angle values are studied, $\phi = 5^\circ, 10^\circ, 15^\circ, 20^\circ$ and the accumulated displacements are calculated up to 2 s. With 5° friction angle the block displacement during the studied time period is very large with respect to the case where the friction angle approaches the value of the inclination angle (the $\phi = 20^\circ$ case, see Fig. 3). This trend will remain the same for smaller and smaller time intervals, down to the actual time step size used in the numerical model which is typically in the range of 0.001–0.1 s. While this has no effect on the accuracy of the analytical solution, the numerical solution is very sensitive to these differences.

Application of the forward modeling code of DDA (DF-code) requires as input the assumed maximum displacement ratio (referred to herein as g_2) where $(g_2)(H/2)$ is the assumed maximum displacement per time step and where H is the height of the analysis domain measured in the vertical direction. The actual displacement computed by DDA within a time step is

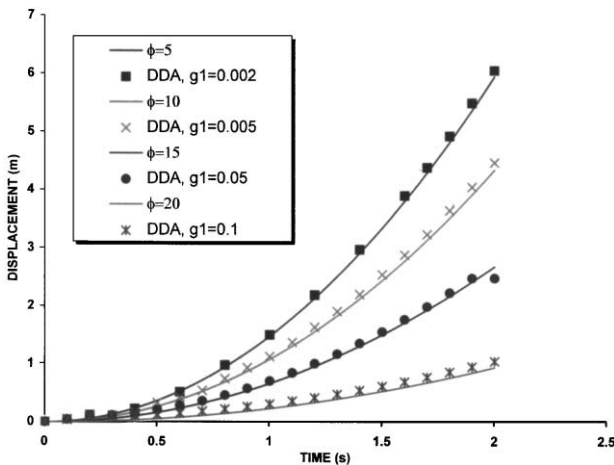


Fig. 2. Block displacement on an incline—gravitational loading only. Comparison between analytical solution (solid line) and DDA results.

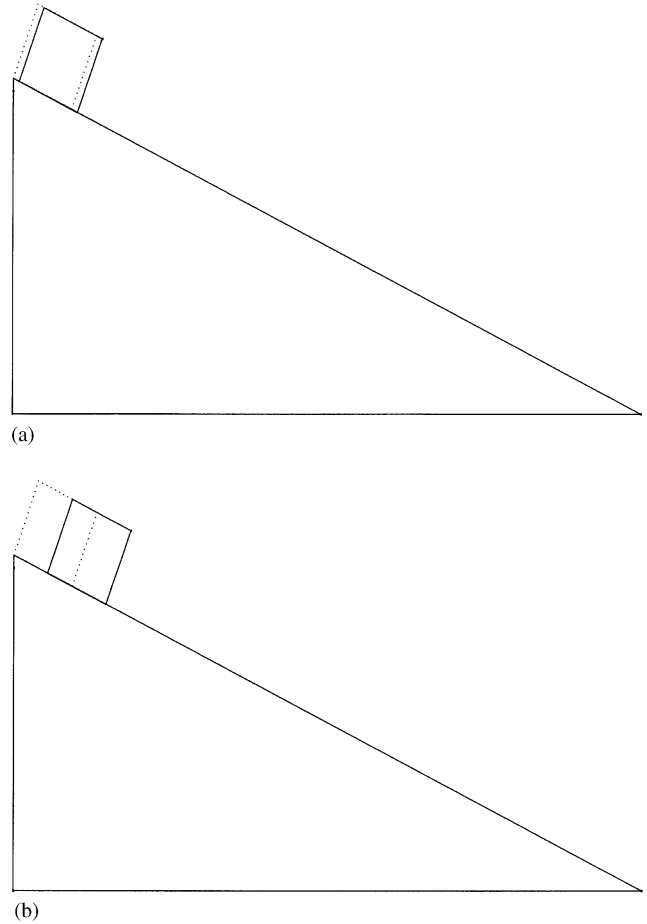


Fig. 3. The single block on an incline model used in this study. Dashed line—initial position of block. Plane inclination 22.6° . Gravitational loading only (a) friction angle 20° , $g_1 = 0.1$, $g_2 = 0.01$, total time 2 s (b) friction angle 5° , $g_1 = 0.1$, $g_2 = 0.01$, total time 2 s.

limited to the assumed maximum displacement which is defined by the user. This ensures that the assumption of infinitesimal displacement is nearly satisfied and that convergence of the open close iterations [7,8] is achieved. Similarly, the maximum displacement per time step is limited by the upper limit of time interval used in each time step. This quantity, referred to herein as g_1 , is also user defined.

As in the case of g_2 the chosen time interval size g_1 should also conform with the assumption of infinitesimal displacement per time step.

A good way to verify the suitability of the numerical parameters g_2 and g_1 is to check the ratio between the actual, or “real”, displacement per time step as computed by DDA (referred to herein as R), and the assumed maximum displacement per time step (referred to herein as A , where $A = (g_2 H)/2$). At the end of an analysis the obtained ratio R/A should be close to 1 if great accuracy is sought. The obtained failure modes in the multi block case are much less sensitive to the user

defined parameters g_1 and g_2 , and their optimization for a failure mode analysis should not be necessary.

In the apparently simple case of a single block on an incline the value of g_1 had to be changed for each case to achieve optimal accuracy (see Fig. 2) while g_2 was kept at a constant value of 0.01 corresponding to a maximum displacement of 25 cm per time step. It can be seen in Fig. 2 that for a rapidly sliding block ($\phi = 5^\circ$) g_1 was set to 0.002 s whereas for a slowly sliding block ($\phi = 20^\circ$) g_1 was increased to as much as 0.1 s in order to get an accurate solution. This sensitivity of the numerical code is inevitable, as was explained above, however, this simple case illustrates the care and responsibility which must be exercised by the user in any attempt to perform dynamic calculations using DDA.

4. A block on an incline subjected to dynamic load

4.1. A simple dynamic function

The simplest form of a sinusoidal function would have the form

$$a_t = A \sin t, \quad (5)$$

where a_t is time dependent acceleration and A is amplitude of oscillations. In order to find the velocity and displacement generated by this acceleration function the acceleration has to be integrated twice over a range from θ to t , where θ is the time at which $a_t = a_y$. The corresponding analytical solutions for the velocity v_t and displacement s_t are given by Eqs. 6 and 7 below

$$v_t = \int_{\theta}^t a_t dt = A(-\cos t + \cos \theta), \quad (6)$$

$$s_t = A \left[\int_{\theta}^t (-\cos t) dt + \cos \theta(t - \theta) \right] \\ = A[-\sin t + \sin \theta + \cos \theta(t - \theta)] \quad (7)$$

The acceleration, velocity, and displacement function derived in Eqs. (5)–(7) are plotted in Fig. 4a for an amplitude $A = 9.81 \text{ m/s}^2 = 1 \text{ g}$. Since the angular frequency ω in this case is 1, a complete cycle lasts 2π seconds. Considering a plane which is inclined 22.6° and has a friction angle of 30° , the yield acceleration would be 0.1299 g according to Eq. (2) above, and the corresponding time interval θ until which no motion should commence would be $t_0 = 0.13 \text{ s}$, since in the simple case of Eq. (5) $t_0 = \sin^{-1}(a_y/A)$. A comparison between the analytical solution, DDA and the Goodman and Seed [2] solution is shown in Fig. 4b. The input acceleration record for DDA was the horizontal East–West component only, where the analyzed section (as in Fig. 3) is an East–West cross section. The other acceleration components (N–S, up–down) were set to zero. The acceleration data points were input to DDA at

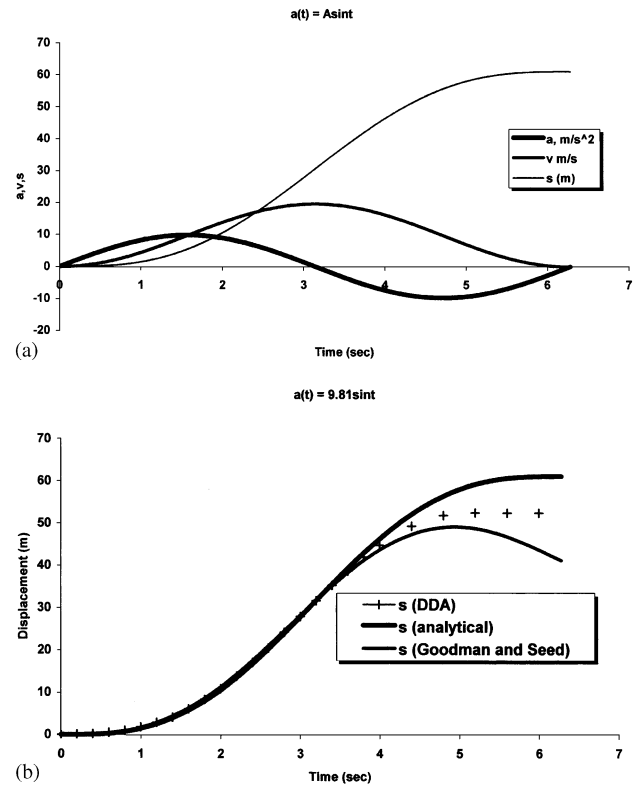


Fig. 4. The function $a(t) = A \sin t$. (a) analytical solution for acceleration (heavy line), velocity (dashed line), displacement (dotted line). (b) Comparison between analytical solution for displacement, DDA solution, and Goodman and Seed solution.

a rate of 200 Hz, namely the input acceleration was updated every 0.005 s. These conventions were maintained in all DDA calculations.

The agreement between DDA, Goodman and Seed solution and the analytical solution is remarkable for a distance of sliding of 40 m. While this is not surprising with respect to Goodman and Seed solution who used numerical integration, it is indeed an excellent validation of DDA performance under relatively high dynamic load (up to 1 g) and for a very large displacement. At the end of the cycle DDA solution is about 10% lower than the analytical solution and this could probably be improved by optimizing the numerical parameters g_1 and g_2 ; it was not attempted in this run.

4.2. A typical harmonic function $a_t = A \sin \omega t$

The function $a_t = A \sin \omega t$ allows us to model different ground motion frequencies using the parameter ω . The analytical solutions for the velocity and displacement are given in Eqs (8) and (9) below

$$v_t = \int_{\theta}^t A \sin \omega t dt = \frac{A}{\omega} [-\cos \omega t + \cos \omega \theta], \quad (8)$$

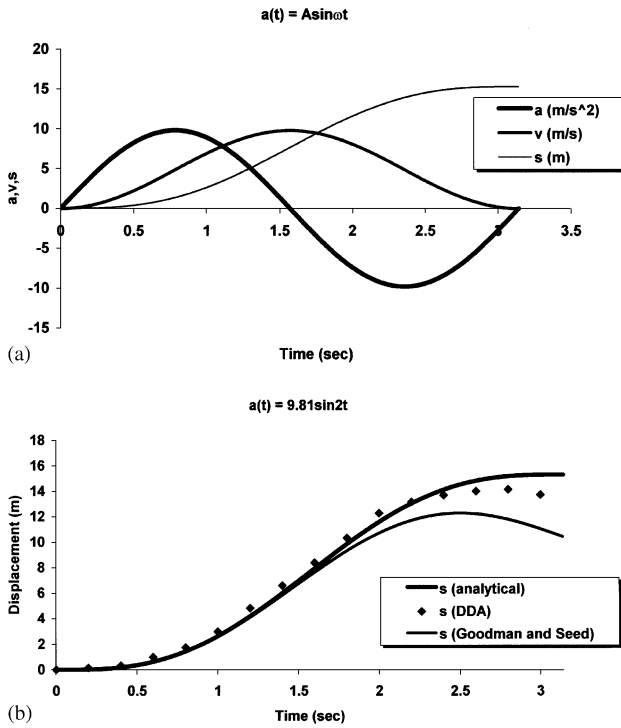


Fig. 5. The function $a(t) = A \sin t$. (a) analytical solution for acceleration (a), velocity (v), and displacement (s). (b) Comparison between analytical solution for displacement, DDA solution, and Goodman and Seed solution.

$$s_t = \frac{A}{\omega} \left[\frac{-\sin \omega t}{\omega} + \frac{\sin \omega \theta}{\omega} + \cos \omega \theta (t - \theta) \right]. \quad (9)$$

The shape of the acceleration, velocity, and displacement functions for $A = 1 g = 9.81 \text{ m/s}^2$, and $\omega = 2$, is shown in Fig. 5a. Since $\omega = 2$, a complete cycle lasts π seconds only. The yield acceleration remains the same because the inclination and friction angles of the plane have been maintained, however, it is attained after 0.065 s only. A comparison between the analytical solution, DDA, and the Goodman and Seed solution is shown in Fig. 5b. Indeed the DDA solution tracks the analytical solution remarkably well up to the end of the cycle, while Goodman and Seed's numerical integration deviates slightly towards the end of the cycle.

4.3. A sum of two sines

A real earthquake will produce a train of oscillations composed of sine components, which may be represented using a Fourier series. Because of the assumed initial conditions (at time $t = 0$ the acceleration is zero) there will be no cosine terms in the series, and the simplest case would be the sum of two sine functions

$$a_t = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t. \quad (10)$$

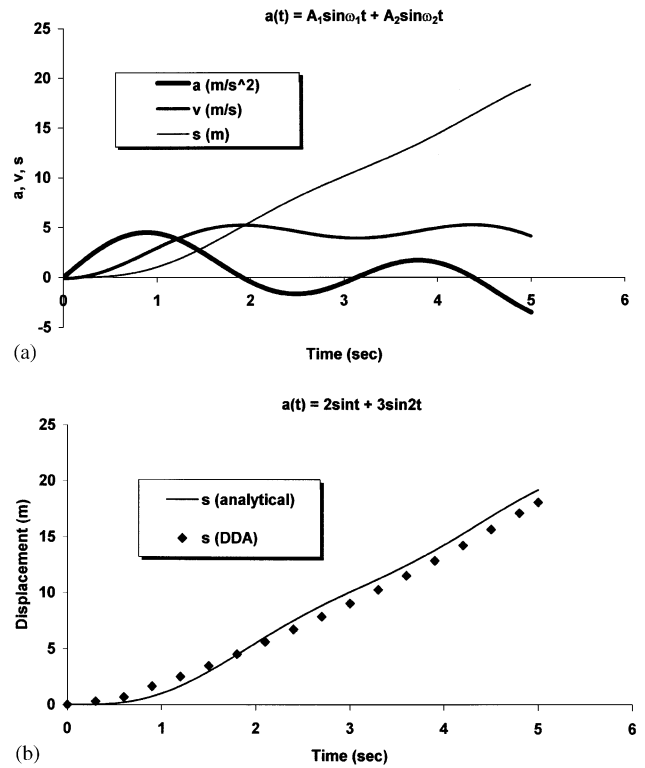


Fig. 6. The function $a_t = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$. (a) analytical solution for acceleration (a), velocity (v), and displacement (s). (b) Comparison between analytical and DDA solution for displacement.

The velocity and displacement corresponding to this function are given by Eqs. (11) and (12) below

$$v_t = \int_{\theta}^t (a_t) dt = a_1 \left(\frac{-\cos \omega_1 t + \cos \omega_1 \theta}{\omega_1} \right) + a_2 \left(\frac{-\cos \omega_2 t - \cos \omega_2 \theta}{\omega_2} \right), \quad (11)$$

$$s_t = a_1 \left[\frac{-\sin \omega_1 t}{\omega_1^2} + \frac{\sin \omega_1 \theta}{\omega_1^2} \right] + a_2 \left[\frac{-\sin \omega_2 t}{\omega_2^2} + \frac{\sin \omega_2 \theta}{\omega_2^2} \right] + \frac{a_1 \cos \omega_1 \theta}{\omega_1} (t - \theta) + \frac{a_2 \cos \omega_2 \theta}{\omega_2} (t - \theta). \quad (12)$$

This case cannot be solved using the Goodman and Seed procedure which is restricted to harmonic oscillations as in Section 4.2. In Fig. 6a the two sine functions and their velocity and displacement functions are plotted for the following constants: $A_1 = 2$; $\omega_1 = 1$; $A_2 = 3$; $\omega_2 = 2$. As in the previous cases the yield acceleration remains the same but it is attained here after 0.17 s of shaking. The corresponding numerical solution for this dynamic case is plotted in Fig. 6b together with the analytical solution and a remarkably good fit is obtained. The numerical parameters g_1 and g_2 did not need to be optimized and the selected values here are $g_1 = g_2 = 0.1$.

4.4. A sum of three sines

It can be shown that the influence of higher order terms in a series of sine functions is negligible and therefore we will not proceed our validation effort beyond the sum of three sine functions. We hence study the function

$$a_t = a_1 \sin \omega_1 t + a_2 \sin \omega_2 t + a_3 \sin \omega_3 t \quad (13)$$

$$v_t = \int_{\theta}^t a_t dt = \left(-\frac{a_1}{\omega_1} \cos \omega_1 t + \frac{a_1}{\omega_1} \cos \omega_1 \theta \right) + \left(-\frac{a_2}{\omega_2} \cos \omega_2 t + \frac{a_2}{\omega_2} \cos \omega_2 \theta \right) + \left(-\frac{a_3}{\omega_3} \cos \omega_3 t + \frac{a_3}{\omega_3} \cos \omega_3 \theta \right), \quad (14)$$

$$s_t = -\frac{a_1}{\omega_1^2} [\sin \omega_1 t - \sin \omega_1 \theta] + \frac{a_1}{\omega_1} \cos \omega_1 \theta (t - \theta) - \frac{a_2}{\omega_2^2} [\sin \omega_2 t - \sin \omega_2 \theta] + \frac{a_2}{\omega_2} \cos \omega_2 \theta (t - \theta) - \frac{a_3}{\omega_3^2} [\sin \omega_3 t - \sin \omega_3 \theta] + \frac{a_3}{\omega_3} \cos \omega_3 \theta (t - \theta) \quad (15)$$

The sum of three sines function is plotted in Fig. 7a with the corresponding velocities and displacements for the following parameters: $A_1 = \omega_1 = 1$; $A_2 = \omega_2 = 2$; $A_3 = \omega_3 = 3$. The yield acceleration is attained in this case after 0.093 s. The acceleration function under the studied parameters seems to decay rather rapidly and after 4.5 s the acceleration is about a third of the maximum acceleration ($5.35 \text{ m/s}^2 = 0.54 \text{ g}$) which is attained after 0.6 s.

A comparison between the analytical solution and DDA is shown in Fig. 7b. Two sets of DDA output data are plotted, one for maximum time step size $g_1 = 0.1 \text{ s}$ and the other for $g_1 = 0.01 \text{ s}$; the assumed maximum displacement per time step is maintained constant at $g_2 = 0.1$ (since $H = 50 \text{ m}$ the assumed maximum displacement per time step “A” is 2.5 m). Clearly the numerical result is very sensitive to the user defined maximum time step size g_1 , and/or to the user defined assumed maximum displacement per time step g_2 . A better R/A ratio (see Section 3 above) was reached with $g_1 = 0.01$ and $g_2 = 0.1$ (the obtained R/A ratio was =0.54) and indeed the agreement between the analytical and numerical solutions is much better. The general behavior of the time dependent displacement function is matched by the DDA results and the accuracy is within 15%. The accuracy could be further improved by optimizing g_1 and g_2 in order to get an R/A ratio closer to 1.0.

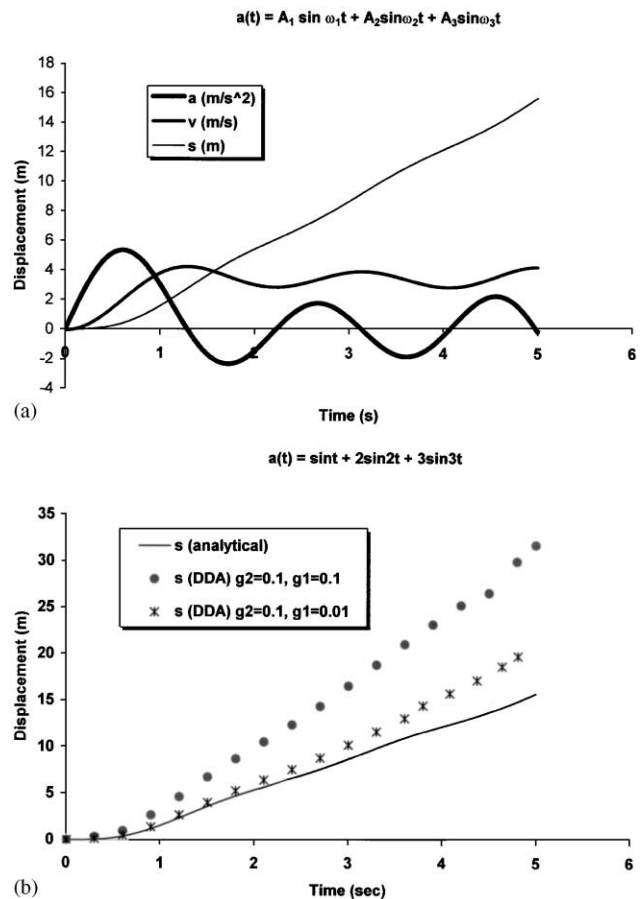


Fig. 7. The function $a_t = a_1 \sin \omega_1 t + a_2 \sin \omega_2 t + a_3 \sin \omega_3 t$. (a) analytical solution for acceleration (a), velocity (v), and displacement (s). (b) Comparison between analytical and DDA solution for displacement.

5. Discussion and conclusions

5.1. Limitation. of the classical Newmark's method

Dynamic block displacement along a surface, for which the shear resistance is provided by friction only, has been estimated for many years by various applications of an approach proposed by Newmark in 1965 [1] and Goodman and Seed [2]. The so-called “Newmark’s method” is a straight forward application of principles of classical mechanics, primarily statics and dynamics, hence its widespread popularity among scientists and engineers. The advantage of the method is its ability to estimate the net displacement a mass would experience as a response to earthquake induced ground accelerations for a given duration of shaking. Provided that sound double integration algorithms are available, realistic and characteristic accelerograms for a given location may be used to estimate displacement. There are, however, two significant restrictions: (A) the procedure is applicable for a single, continuous mass of rock or soil, and (B) the failure mode is pre-assumed

and is restricted to sliding. These limitations are not necessarily a problem in soil mechanics applications. Soils have negligible tensile strength and typically form intact masses with little or no fractures. Therefore, an unstable soil mass could be expected to undergo dynamic displacements as a single, continuous body. Furthermore, “active” failure planes in soil masses typically have an inclination angle from several degrees and up to 60° . Soils masses resting on such planes indeed tend to deform by sliding only. In rock masses, however, vertical or sub-vertical joints are widespread along with very steeply dipping shears, bedding, or foliation planes. The mere existence of discontinuities in rock typically results in a blocky structure; therefore any realistic dynamic displacement computation for a rock mass must consider inter block reactions during shaking. Furthermore, the steeply inclined planes may yield failure modes other than sliding, notably toppling or block slumping [18], which cannot be modeled using Newmark’s type analysis.

5.2. DDA modeling

DDA has been proven to be very successful in computing accurate block displacements in very simple multi block configurations subjected to static loading using analytical solutions [9,10,13,19] or using field case histories [11,12]. Similarly, a multitude of different failure modes have been accurately predicted by DDA for a large number of blocks including sliding or slumping on planar or curved surfaces [20], backward rotation, forward rotation or toppling [21], and block slumping [13]. It is therefore natural to extend DDA validation for real dynamic computations in a blocky structure. This paper presents the first necessary step of analytical validation via the case of a single block on an inclined plane with a given friction angle. Validation of dynamic displacements in the multi block case must be performed using physical models on a shaking table. Several conclusions regarding dynamic DDA modeling emerge from this study:

- Dynamic DDA solutions are extremely sensitive to numerical parameters which must be entered by the user, namely the assumed maximum displacement per time step (g_2) and the maximum time step size (g_1). If relatively large displacements per time step are anticipated due to either high load or low shear resistance, the maximum time step size should be reduced for a given value of g_2 . This numerical sensitivity is illustrated in the dynamic solutions which are plotted in Fig. 2 for the case of gravitational loading only.
- In order to optimize the numerical parameters the ratio between the assumed maximum displacement per time step (A) and the actual calculated displace-

ment during a time step (R) must be checked at the end of the analysis and its value should be as close as possible to 1.0.

- The initial g_2 value was recommended by Shi [22] to be in the range of 0.001–0.01. In this dynamic study we find that the initial g_2 value could be as high as 0.1. The optimal g_2 value also depends, however, on the selected value for g_1 .
- The maximum time step size g_1 in this research was found to be the most sensitive parameter for dynamic calculations. Optimal g_1 values spanned two orders of magnitude from $g_1 = 0.001$ for cases of rapid sliding to $g_1 = 0.1$ for cases of slow sliding. When an inappropriate g_1 value is used, the results of the dynamic calculations may be erroneous.
- It has been shown (Fig. 7b) that the error due to inappropriate g_1 input value is on the conservative side, as greater displacement is calculated with increasing time step length. However, it is not recommended to rely on that observation in any attempted analysis. Rather, the input parameters g_1 and g_2 should be optimized as discussed above.

5.3. Analytical vs. DDA solutions

It is common to believe that the analytical solution correctly represents the physical reality. This is not necessarily so. The problem which is analyzed here involves friction, a complicated physical phenomenon not yet fully understood in rock mechanics context. In DDA, sliding is initiated when Coulomb failure law is reached along the contact between the block and the plane, which is provided by springs [7,8]. When the normal component of the contact force is compressive and the shear component is large enough to cause sliding, a stiff spring normal to the contact is applied to allow sliding to take place along the contact. A sliding force is hence applied against the sliding direction which is defined by the normal force of the previous iteration. However, when the normal component of the contact force is compressive but the shear component is less than required by Coulomb’s law the contact point is locked by normal and shear springs, and sliding is not allowed. Under dynamic loading, the block experiences stop and go motions, and the block displacement at the end of each time step will also be influenced by the energy of the normal contact spring [23]. The analytical solution, however, does not involve spring energies at all and the calculated displacements are derived directly from the integration of acceleration and velocity from $t_0 = \theta$ to t . The analytical solution therefore yields a smooth and continuous displacement function whereas the DDA solution goes through a series of open close iterations [7,8], the final product of which results in net displacement at the end of a time step. We have no way of telling at present which approach is closer to the physical

reality. It is demonstrated in this paper that the agreement between the spring analog in DDA and the analytical solution can be quite good. However, deviations from absolute agreement between the numerical and analytical solutions should not be surprising, and may even be expected, because of the different rationale behind the two methods of analysis.

5.4. Recommended future research

Future research should be directed towards understanding the compatibility between numerical and analytical solutions. The first task should be a comparison between both DDA and the analytical solution, with results from shaking table experiments performed under the exact same modeled conditions (plane inclination and friction, block stiffness, induced wave function). Such a comparison will allow critical review of the validity of the analytical solution, and will provide constraints for the validity of DDA. Once this basic issue is resolved, multi block tests in experimental settings which can be modeled easily using DDA, should be studied.

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