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SOPHISTICATED VOTING UNDER THE SEQUENTIAL VOTING BY VETO[1]

ABSTRACT. The research reported here was the first empirical examination of strategic voting under the Sequential Voting by Veto (SVV) voting procedure, proposed by Mueller (1978). According to this procedure, a sequence of n voters must select s out of $s+m$ alternatives ($m \geq n \geq 2$; $s > 0$). Hence, the number of alternatives exceeds the number of participants by one (n+1). When the ith voter casts her vote, she vetoes the alternative against which a veto has not yet been cast, and the s remaining non-vetoed alternatives are elected. The SVV procedure invokes the minority principle, and it has advantages over all majoritarian procedures; this makes SVV a very desirable means for relatively small groups to make collective decisions. Felsenthal and Machover (1992) pointed out three models of voting under SVV: *sincere*, *optimal*, and *canonical*. The current research investigated, through laboratory experiments, which cognitive model better accounts for the voters' observed behavior and the likelihood of obtaining the optimal outcome as a function of the size of n (when $s = 1$). The findings suggest that while voters under SVV use all three models, their choice is conditioned by group size. In the small groups ($n = 3$), the canonical mode was a better predictor than the sincere model. In the larger groups ($n = 5$), the sincere model was a better predictor than the canonical model. There is also evidence of players' learning during the experiment.

KEY WORDS: Minority principle, Strategic voting, Veto, Voters' behavior

1. INTRODUCTION

The social choice literature has long been interested in whether voters in non-cooperative sequential voting games, under various sequential majoritarian voting procedures, behave sophisticatedly or sincerely. A second area of interest in this regard has been order-of-voting effects (Black, 1958; Farquharson, 1969; Kramer, 1972; McKelvey and Niemi, 1978; Plott and Levine, 1978; Wilson, 1986; Wilson and Pearson, 1987; Herzberg and Wilson, 1988; and many more). A sincere voter opts for the proposal she prefers most, in

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line with her own order of preference, regardless of the other voters' selections and orders of preference. A sophisticated voter takes into account the other voters' selections, prior to casting her vote, adapting her strategy accordingly.

Farquharson (1969) was the first to propose a model attempting to trace the sophisticated voter's calculations. Many others followed suit, constructing various models for various voting procedures. McKelvey and Niemi (1978) extended the theory to binary procedures; Mueller proposed an algorithm for the Sequential Voting by Veto; and Niemi and Frank (1985) did so for the plurality procedure. In practice, voters evidently employ sophisticated voting strategies. Cases in point are the United States Senate's Education and Housing Committees (Riker, 1965); the United States Congress (Enelow and Koehler, 1980); and the Israeli 1981 parliamentary elections (Felsenthal and Brichta, 1985). The best way to test the extent of sophisticated behavior is by conducting experimental studies. However, the findings of the relatively few controlled experiments conducted seem to be contradictory (Plott and Levine, 1978; Wilson, 1986; Wilson and Pearson, 1987; Herzberg and Wilson, 1988; Rapoport et al., 1991). Plott and Levine (1978) found that individuals consistently used myopic voting strategies.[2] Wilson and Pearson (1987) reported that, "the bulk of voters are sincere myopic voters" although "there is some evidence that participants act with limited foresight" (p. 270). Wilson (1986) reported that sophisticated voting was obtainable under non-cooperative conditions. Rapoport, Felsenthal, and Maoz (1991) found strong evidence of strategic voting when voters operated under Approval Voting and Plurality Voting procedures, whereas Herzberg and Wilson (1988) reported that empirically sophisticated strategies tend to be infrequently selected by voters under binary agendas voting, even though the theoretical studies suggest that extensive sophisticated voting should occur. Similarly, theoretical studies regarding the Sequential Voting by Veto (SVV) procedure predict a sophisticated behavior (Mueller, 1978; Moulin, 1983; Felsenthal and Machover, 1992,) despite the fact that under SVV all voters have an incentive to state their proposals sincerely and to rank the other voters' proposals sincerely (Mueller, 1978).

The literature has long pointed to the possibility that under the SVV as well as under various majoritarian procedures, the sincere selected outcome is different from the sophisticated, and that a sophisticated vote for a less preferred alternative may well prove more beneficial to a voter than a sincere vote for her most preferred alternative (Felsenthal, 1990). Consequently, this study examined whether empirical evidence provides any support for these theoretical results when voters operate under SVV procedure – a relatively new and theoretically very desirable, non-majoritarian and sequential voting procedure which has not yet been implemented nor subjected to controlled experimentation. Specifically, the purpose of this research is to investigate whether voters operating under SVV opt to act sincerely or sophisticatedly, and if so, which cognitive processes do voters go through in deciding how to cast their vote. These questions were investigated by conducting laboratory experiments, whose results were expected to shed light on some of the likely consequences of actually implementing SVV.

2. THE VOTING PROCEDURE SVV

The SVV was proposed by Mueller (1978) as a method of dividing some goods among a group. It consists of two stages: (1) Each of the n persons makes a division proposal. The proposal to leave the status quo in place and have no one obtain any portion of the goods also exists. Thus, the number of proposals exceeds the number of participants by one $(n + 1)$; (2) Each of these persons, in a randomly pre-determined sequence, vetoes one of the $n+1$ alternatives against which a veto has not yet been cast. The single proposal that is left un-vetoed wins and is implemented. To illustrate, suppose there are three voters, A, B, and C, who have to select, by using SVV, one out of four alternatives, w, x, y, and z. Let us suppose further that voter A casts her veto first, B casts second, and C casts last. Each voter in turn chooses to cast a veto against a single still un-vetoed alternative. Suppose voter A rules out alternative w by vetoing it. This leaves B with the choice of casting a veto against x, y, or z. Suppose B chooses to rule out x. This leaves C with the choice of two alternatives only: y or z. If C vetoes z, y wins, and vice versa. Generally, a group of n voters selects a number of alternatives (s) out of the number of competing alternatives available $(m+s)$. According to this procedure, the number of alternatives available must exceed the number of voters ($s > 0$; $m \geq n \geq 2$) because the ith voter casts, in her turn, a veto against a certain alternative (k_i) ($\Sigma k_i = m$; $m > k_i \geq 1$) and the s remaining alternatives, left un-vetoed, therefore win.

The SVV procedure invokes the minority principle. It grants every participating voter – whether in the majority or the minority – an equal right to defend herself against her least preferred proposal, by casting a vote against that proposal. In contrast, voting procedures that invoke the majority principle produce the proposal preferred by the majority of voters, even if a large minority (49% of the voters) considers that proposal to be the worst one available.

Mueller argued that the SVV procedure has essentially three advantages. First, it produces a unique winning proposal, regardless of the voters' proposals, the order of voting, and the occurrence of a cyclical order of social preference.[3] Second, if voters act rationally, the selected proposal must be Pareto-optimal[4] and must also constitute no voter's least preferred proposal. Third, the procedure is fair because although the winning proposal may depend on both the type of proposals made and also on the order of voting, each voter has an equal chance to be located at any particular place in this order. Consequently, if the order in which the voters are to cast their veto-votes is made known only after the proposals have been stated, all voters have an incentive to state their proposals sincerely, as well as to rank the other voters' proposals sincerely.

The first and the second advantages are over all extant and proposed majoritarian procedures. The last-mentioned advantage is especially noteworthy because only two other known non-dictatorial voting procedures exist in which voters have no incentive to misrepresent their preferences among the competing alternatives. The first is the procedure in which all the voters must unanimously support the s selected alternatives. The second is the procedure whereby the s selected alternatives are picked at random from among the s top preferences of each voter (Pattanaik, 1978). However, if no unanimity is reached under the first procedure, the result is either a paralyzing vacuum where no alternative is selected, or an indefinite maintenance of the status quo (which may be Pareto-inferior). The second procedure, in contrast, albeit both Pareto-optimal and fair – in the sense that it provides every voter with an equal chance of getting her top s preferences selected – may nevertheless result in selecting the bottom preference(s) of an absolute majority of voters!

These three advantages make SVV a desirable and viable voting procedure for making collective decisions by relatively small groups (e.g., determining the size of a budget and its allocation, prioritizing among goals and strategies of action, and selecting candidates for executive positions, etc.). SVV is also suitable for decision makers in committees or subcommittees in legislature or parliament when transforming the voters' message as expressed in elections into public policy. Nevertheless, despite its theoretical advantages SVV has rarely been implemented. It is probable that the number of competing alternatives in small groups will exceed the number of voters (by one), while in large groups, voters usually outnumber the competing alternatives. SVV cannot be implemented in general elections until a theoretical solution to this problem is found.

Mueller presented an algorithm for determining the winning proposal, given the voting order. He showed that when $n = 2$, the voter who moves first has an advantage, because the second voter will be forced to veto the status quo, provided the first voter's proposal allots her some positive portion of the goods. However, when $n \geq 3$ the voter who moves first may no longer have an advantage. Although Mueller's algorithm is correct, his proof of this fact suffers some minor lacunae and relies directly on an incompletely explicated notion of rationality. Moulin (1981, 1983: 138–140) extended Mueller's idea to any situation in which n fully informed voters have to select one out of $n + 1$ alternatives. Moreover, Moulin also provided a rigorous proof of the correctness of Mueller's algorithm and observed, that this result easily extends to the seemingly more general case where there are $m + 1$ alternatives ($m \geq n$), and each voter is allowed to veto a number of alternatives. However, both Mueller and Moulin addressed themselves only to situations in which a single alternative must be selected. Felsenthal and Machover (1992) extended and generalized the Mueller–Moulin result to a situation in which n voters must select s out of $m + s$ alternatives ($s > 0$; $m \geq n \geq 2$).

3. OPTIMAL AND CANONICAL SEQUENCES

First, one should be aware of the possibility that several (sophisticated) voting patterns – all leading to the selection of the same (unique and optimal) outcome – may exist. Example 1 clarifies this possibility: Suppose there are three voters, 1, 2, and 3, who, through SVV, have to select one out of four alternatives, *a, b, c*, or *d.* Suppose further that the voters' (linear) preference orderings among the alternatives are as follows: voter 1 – $a>b>c>d$; voter 2 – $b>c>a>d$; voter 3 – $d>a>b>c$. Finally, suppose that each voter can cast a veto vote against one of the alternatives and that the sequence in which the three voters are to cast their veto is such that voter 1 is first, voter 2 is second, and 3 is last. Consequently, in this example $n = 3$; $m = 4$; $s = k_i = 1$.

Due to the fact that voter 1 can veto any one of the four alternatives, voter 2 can veto any one of the three alternatives left after voter 1 has cast her veto, and voter 3 can veto any one of the two alternatives left after voters 1 and 2 have cast their veto votes, there are altogether 24 ($m!$) possible sequences in which the three voters may decide to cast their veto. However, since all voters are assumed to behave rationally, i.e., to seek an outcome that is as high as possible in their preference ordering, no voter is expected to veto her most preferred surviving alternative. Consequently, in the above example, voter 1 will contemplate only three (rather than four) voting strategies (to veto *b, c*, or *d*), voter 2 will contemplate only two (rather than three) voting strategies; that is, of the three alternatives left after voter 1 has voted, she contemplates whether to veto her least or her second least preferred alternative. Voter 3, however, is left with only one rational strategy, namely to veto her least preferred alternative out of the two left after voters 1 and 2 have voted. So instead of 24 possible sequences, we are now left with only six [$(m - 1)!$] initially undominated sequences. These six sequences and their resulting outcome are listed below.

Sequence #	*Voter 1 vetoes*	*Voter 2 vetoes*	*Voter 3 vetoes*	*Outcome*
1	*b*	*a*	*c*	*d*
2	*b*	*d*	*c*	*a*
3	*c*	*a*	*b*	*d*
4	*c*	*d*	*b*	*a*
5	*d*	*c*	*b*	*a*
6	*d*	*a*	*c*	*b*

Given that all voters are aware of all other voters' preference orderings, voter 1 knows that sequences #1, 3, 5 are dominated from voter 2's point of view and hence are not feasible. Sequence #1 is not feasible because voter 1 knows that if she vetoes *b,* then voter 2 will surely veto *d* rather than *a*. This is so because voter 3 is expected to veto *c* regardless of whether voter 2 vetoes *a* or *d*, and voter 2 prefers, of course, that *a* rather than *d* will be selected. For a similar reason sequence #3 is also not feasible, because if voter 1 vetoes *c,* then voter 2 will surely veto *d,* because voter 3 is expected to veto *b*, and voter 2 prefers that *a* rather than *d* will be selected. Finally, sequence #5 is also not feasible because if voter 1 were to veto *d,* then voter 2 would surely veto *a,* thereby forcing voter 3 to veto *c* and make *b* the final outcome. Since two of the remaining three sequences (#2, 4) result in the selection of *a,* whereas the third (#6) results in the selection of *b,* and since voter 1 prefers *a* as the winner rather than *b,* he is not expected to veto *d* and seems to be indifferent as to whether to veto *b* or *c*.

However, due to the sequential character of SVV procedure, we must bear in mind that the voters' decisions in the above example depends clearly and essentially on the assumption that voters announce their preferences before the voting order is chosen, and from this point on, they cannot revise their preference ordering until the election is over. Otherwise, this example becomes much more complicated and has many more voting sequences based on the different ways in which each voter could change her preferences after any given voter has cast her veto.

Using the terminology of Felsenthal and Machover (1992), there are therefore two *optimal sequences* in the above example, both resulting in the selection of *a:* sequence #2 – voters 1, 2 and 3 veto *b*, *d*, and *c* respectively; or sequence #4—voters 1, 2 and 3 veto *c*, *d*, and *b* respectively. A decision tree best illustrates this example:

Voter 1:	$\bar{b}$		$\bar{c}$		$\bar{d}$	
Voter 2:	$\bar{a}$	$\bar{d}$	$\bar{a}$	$\bar{d}$	$\bar{c}$	$\bar{a}$
Voter 3:	$\bar{c}$	$\bar{c}$	$\bar{b}$	$\bar{b}$	$\bar{b}$	$\bar{c}$
Outcome:	*d*	**a*	*d*	**a*	*a*	***b*

Figure 1. A Decision Tree

Here, the sign (—) above the letters denotes casting a veto against that specific alternative; (∗∗) denotes the result obtained through a sincere sequence; and (∗) denotes the (optimal) result obtained through a sophisticated sequence.

However, as this rather simple example demonstrates, the task of identifying all optimal sequences may become quite demanding and time-consuming as m increases. Consequently, Felsenthal and Machover (1992) argued that voters operating under SVV are unlikely to attempt to identify all optimal sequences, but instead are likely to identify only a singly *canonical sequence.* Before describing the canonical sequence, note that the existence of any optimal sequence depends crucially on the assumption that the voters' orders of preference among the m alternatives do not admit indifferences[5]. For this reason one must assume that the voters' preferences are linear (i.e., strictly ordered).

Consider now a general case where $m = n + 1$ and $s = 1$. The *canonical sequence* for this case $\langle y_1, y_2, \ldots y_n \rangle$ is obtained by taking y_n to be the alternative least preferred by the last voter V_n; then y_{n-1} is taken as the alternative which the penultimate voter V_{n-1} least prefers among the remaining alternatives; and so on, going backwards. Finally, y_1 is taken as the alternative that the first voter V_1 least prefers of the $s + 1$ remaining alternatives. Thus, in terms of Example 1, sequence #2 is a canonical (and optimal) sequence, whereas sequence #4 is merely an optimal sequence.

The idea behind the canonical sequence results from the following rational consideration. Since y_n is the last voter's least preferred alternative, none of the $n - 1$ preceding voters need bother to veto this alternative, because they all know that no matter what they do, the last voter is bound to veto it if it survives until it is her turn to vote. As far as they are concerned, this alternative is as good as dead and can be ignored. If they all think along these reasonable lines, then y_n indeed survives until the turn of the last voter arrives, and then she has to veto it. A similar reasoning is now applied to the alternative that the penultimate voter least prefers among the remaining $m - 1$ alternatives, leading to the conclusion that none of the previous $n - 2$ voters will bother to veto it, and therefore the penultimate voter will indeed veto it. And so on, backwards, until we reach the first voter. As far as she is concerned, $n - 1$ alternatives are as good as dead and may be ignored; so she will consider only the $s + 1$ remaining alternatives and will veto whichever of these she prefers least.

The definition of the canonical sequence follows a negative rationale: each voter decides not to bother to veto an alternative that she knows will have to be vetoed by a subsequent voter if it survives until this subsequent voter must vote. In contrast, the rationale behind the definition of optimal sequences is a positive one: each voter V_i vetoes any alternative that leaves the following voters with an $(n - 1)$-ary scheme such that, if they too follow the same reasoning, the set of alternatives that will finally be selected is the best possible one from V_i's point of view. Since both approaches seem reasonable, it is natural to conjecture that they should lead to the same end result. Felsenthal and Machover (1992) proved that this is indeed the case.

Consequently, this experimental research focuses on investigating these theoretical arguments about sophisticated voting under SVV. Hence, the question is whether the voters operating under SVV will indeed tend to employ a sophisticated strategy, as voters to a large extent do under different majoritarian procedures. In view of the above discussion, one may expect a monotonical decrease in the proportion of voters using a sophisticated strategy as the number of voters (n) increases. That is, the voters will tend to use sophisticated strategies when n is small and to use sincere action when n is large, since enlarging n makes the calculation of the strategic voting more complicated. The theoretical logic underlying Felsenthal and Machover's study supports this hypothesis, as does the empirical examination of strategic voting in binary choice settings, which found that as the size of the agenda increased (from four elements to eight), the voters were less likely to use sophisticated voting, since the calculation of the strategic voting became complex (Herzberg and Wilson, 1988). In addition, as Felsenthal and Machover argued, canonical thought is less complicated than that required by the optimal model; hence the strategic voters are expected to prefer canonical voting as n increases.

Furthermore, given the voters' preference orderings among m alternatives, it is obvious that, regardless of whether voters vote sincerely or sophisticatedly, the outcome under SVV may depend on the order in which each voter is to cast her veto. Thus, in example 1 above, if the order in which the three voters were to cast their veto votes were changed, such that voter 3 moves first, voter 2 moves second, and voter 1 moves last, the outcome according to the canonical sequence would be the selection of b. However, if all voters were to vote sincerely, a would be selected. In general, it can be easily verified that, if all (fully informed) voters were to adopt the canonical sequence, then each voter would obtain a better outcome if she did not move last than if she did. Conversely, if all voters were to vote sincerely, then each of them could obtain a better outcome by not moving first than by moving first. Hence, given $m > n > 2$, the voters' preference orderings, and that the canonical and sincere sequences result in different outcomes, we may verify once again the empirical occurrence of this theoretical assumption that the voter who moves first under SVV will usually obtain a more preferred outcome than if she does not move first. A comprehensive computer simulation is the best way to pursue this inquiry, but that process requires a formidable allocation of resources. Thus, this study examined the effect of voting order only partially. Specifically, each participant in a vote was granted equal opportunity to occupy a position that generated the best result in an optimal sequence. One may possibly extrapolate from this to the general effect of voting order.

Verifying these three issues regarding SVV – testing the extent of *s* voting; determining which of the sincere, optimal, or canonical models best describes voters' decisions; and inquiring as to whether group size as well as voting order influence strategic voting – will enhance the comparative study of sophisticated voting procedures and should help determine whether the theoretical attention paid to the advantages of SVV is justified and indeed borne out in reality.

4. METHOD

In order to verify the said hypotheses, a series of laboratory experiments were conducted. In each of the experiments a group of (n) voters were asked to select, under SVV, one candidate out of $(n + 1)$ candidates. Each voter, in turn, cast a veto against one of the (surviving) candidates.

Eighty undergraduate students from the University of Haifa volunteered to participate in the experiment. Each one participated in only one of the 20 groups, playing a number of voting games under SVV. The subjects were seated well apart in a spacious room, so as to render inter-communication impossible, and were given paper and pencil. At the beginning of each game each subject was instructed that he or she was about to participate in voting games under SVV. Every group played a number of games, each of which was iterated several times. The games depicted a voting situation in which the group was bound to select any one of the competing alternatives, so each participant sequentially – when his or her turn came – could veto a single (surviving) candidate, and the candidate who ultimately survived last was elected. Each subject was given complete information, in tabular form, about: (1) the series of voting games under SVV to be played by the group; (2) the payoff for each voter associated with the election of each of the candidates (that prompted the voters' artificial preference orderings); and (3) the order in which the voters were to cast their veto votes in each iteration of each game. In addition, each voter was instructed to cast each of his or her available veto votes so that the candidate ultimately elected would maximize his or her monetary gain.

Finally, each subject in his or her turn was asked to loudly announce his or her vetoed alternative so that all of the group's members were updated as to which candidates had already been vetoed by the preceding voters. The group could then move on to hear the subsequent voter's decision, and so forth until the last voter cast her veto.

In order to trace the strategies adopted by the voters, each game met two conditions: it had both an optimal and a canonical path, and the outcome obtained through a sincere sequence differed from the outcome obtained through a sophisticated (canonical and optimal) sequence.

The total of 80 subjects were divided into 20 experimental groups as follows: in order to examine the effect of group size (n) on voters' behavior, the subjects were divided into ten groups each with $n = 3$, and ten other groups each with $n = 5$. Each participant took part in one group only.

To control for the effect of the order in which a voter was to cast his or her veto vote, each subject was granted an equal opportunity to be placed in any one of the n voting placements. Accordingly, each game was iterated several times, with all variables held constant except the sequence in which the voters were to cast their veto votes. Each one of the ten groups with $n = 3$ played a series of nine different SVV voting games, each of which iterated four times,[6] for a total of 36 trials. Similarly, each one of the ten groups with $n = 5$ played a series of six different SVV voting games, each of which iterated six times, for a total of 36 trials.[7]

In order to create an artificial preference ordering, for any given game in a group with $n = 3$, rewards were offered as follows: if the winning alternative ranked last in one's preference orderings, one was rewarded with NIS 0.0 (New Israel Shekel valued at 4.50 to \$1.00). One was rewarded with NIS 1.0, NIS 1.5, and NIS 2.0 respectively if the winning alternative ranked third, second, or first in one's preference orderings. Similarly, subjects in the ten groups with n=5 were rewarded thus: if the winning alternative ranked last fifth, fourth, third, second, or first in one's preference orderings, one was rewarded with NIS 0.0, NIS 1.0, NIS 1.5, NIS 2.0, NIS 2.5, and NIS 3.0 respectively. Throughout the game, the subject was fully informed of the matrix of rewards of all the players and could earn between NIS 65 and NIS 85, depending on both their personal and group voting patterns.

TABLE I

Voting outcomes by group size.

	Sophisticated outcome	Sincere sequence	Other outcomes	Row total
3-person group	47%	24%	29%	100%
	(506)	(265)	(309)	(1080)
5-person group	19%	45%	36%	100%
	(333)	(805)	(662)	(1800)

5. RESULTS

Seeking to establish whether the players adopted a sincere or sophisticated strategy, I calculated the number of decisions made by each individual player. A player whose veto casting matched the prediction of the sincere model was regarded as making a sincere decision. Similarly, a veto casting in line with the sophisticated models (canonical or optimal) was counted as reflecting a sophisticated strategy. To be sure, in some cases the prediction of the sincere model and the prediction of one of the sophisticated models is one and the same; under these circumstances, it is not feasible empirically to distinguish between the models and categorize the decision. These special cases were counted and grouped together with the 'other' category.

The experimental results show participants chose fully sophisticated strategies 29.1% of the time. However, breaking down these results by group size (Table I) blurred these findings and even supported the above conjectures. In triplet games 47% of the decisions were found to denote sophisticated behavior, and only 42% of them corresponded to the sincere vote. Conversely, 19% of the votes in quintet games corresponded to the strategic vote, but 45% denoted the sincere vote. Clearly, the strategic voting decreases as the group size grows larger. The canonical model better described triplet-game voting strategy, while the sincere model best described quintet-game voting strategy. These findings fit well with our expectations.

However, a competitive test of the models cannot rely solely on an inspection of frequency distributions of voting strategies, because the models are dissimilar in several dimensions. Hildebrand et al. (1977) developed seven essential theoretical dimensions for evaluating the predictive power of models. They proposed a measure for bivariate data, which satisfies these criteria, denoted del (∇). The measure del may be interpreted as the proportion of reduction in error attained by a given prediction, given the knowledge of each observation's location on the independent variable, over that expected when the prediction is randomly applied according to the marginal distribution of the independent variable (Rapoport et al., 1991: 218). This measure proved instrumental in comparing models by pointing out which model better corresponded to the observed behavior of participants (Felsenthal, 1990; Rapoport et al., 1991). For a *kXn* voting strategies by voter table of cross classifications, ∇ is calculated by[8]:

$$\nabla = 1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{k} w_{ij} p_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{k} w_{ij} p_{i\bullet} p_{j\bullet}}$$

where $i = 1 \ldots n$; $j = 1 \ldots k$; i represents the categories of the dependent variable, actual voting; j represents the categories of the independent variable, voting as predicted by the model; W_{ij} the model's prediction errors. When $\mathrm{W}_{ij} = 1$, the model predicted an error, while when $\mathrm{W}_{ij} = 0$, no prediction error occurred. p_{ij} is the probability of cases in the cell at the intersection of the (i) line and the (j) column. $p_{j\bullet}$ and $p_{i\bullet}$ are the matrix margin probabilities (Hildebrand et al., 1977: Ch. 6).

Tables II and III display the calculated del values of the predictions of each of the three models with regard to the ten groups of triplet games and the ten groups of quintet games. Inspection of Tables II and III shows that in the triplet-game, del values ascribed a superior predictive capacity to the sincere model in 47% of the 36 rounds and a superior predictive capacity to the canonical sequence in 44% of the cases. The optimal model held sway in only 8% of the cases. Conversely, in quintet games the sincere model proved a superior predictive tool in 69% of the cases, while the canonical model held sway in only 31% of the cases. As was expected, the voters were more likely to use a sophisticated strategy when the group size was small. Moreover, the sophisticated voters preferred the canonical sequence in both three-person and five-person games.

Del values offer further interesting information about individuals' decision-making. Discernible was an (incomplete) ascent in the use of the canonical strategy and an (incomplete) descent in the use of the sincere strategy, both across games and in between any given game's rounds (see Tables II and III). These trends were stronger in the triplet games and weaker in the quintet games, even though these trends gained momentum toward the last games in both group sets (in divergent degrees). On the other hand, the utilization of the optimal sequence seemed quite haphazard.

These interesting trends may hint at a learning process across games. If a learning process occurred, the conclusions as to voting behavior should be drawn on the basis of the rounds performed after the subject's learning process was complete, and the results had become stable. Inspection of Tables 2 and 3 suggests iteration effects on the voters' behavior, that is, voters may learn sophisticated behavior while exercising their vote during games. To test for this iteration effect, the data was subjected to a one-way ANOVA. For that purpose the games' rounds were split into three equal stages, each containing 12 sequential rounds, and the proportion of subjects using each strategy (sincere, optimal, or canonical) was calculated.

The first 12 rounds differed significantly from the last 12 rounds in both triplet and quintet games ($p < 0.05$) regarding the canonical strategy. In fact, the learning process of using the canonical behavior had evidently taken place, compellingly so in the triplet games, and more moderately so in the quintet games. Most players acted sincerely at the outset of games and then altered course and shifted to a canonical strategy. Further examination of the data shows that canonical voting is used more frequently during the practice rounds of the games in both triplet and quintet games (Pearson's $r = 0.402$, $p = 0.015$ and $r = 0.459$, $p = 0.05$ respectively). However, in line with the theoretical hypothesis, using the same analysis for the optimal sequence results insignificantly.

The third theoretical expectation, that of voting order effect, was tested as well. Using sequential procedure, participants may also

TABLE II

Del values, triplet games.

Game number	Sincere sequence	Optimal sequence	Canonical sequence
1.1	0.409	0.077	0.308
1.2	0.031	0.273	0.227
1.3	0.539	0.194	0.015
1.4	0.162	0.032	0.516
		−(0.113)	
2.1	0.354	0.060	0.283
		−(0.120)	
2.2	0.373	0.169	0.123
2.3	0.718	−0.210	0.238
2.4	0.409	−0.108	0.216
3.1	0.446	0.364	−0.091
3.2	0.308	−0.045	0.409
		−(0.136)	
3.3	0.328	0.216	0.169
3.4	0.318	0.031	0.216
4.1	0.484	0.181	0.090
4.2	0.250	−0.333	0.762
		−(0.048)	
4.3	0.641	−0.210	0.372
4.4	0.409	−0.108	0.492
5.1	0.446	0.031	0.354
5.2	0.193	0.202	0.390
5.3	−0.059	−0.182	0.545
		(0.000)	
5.4	0.272	0.250	0.250
6.1	0.014	0.237	0.285
6.2	0.193	−0.173	0.531
		−(0.126)	
6.3	0.238	0.190	0.285
6.4	0.446	0.045	0.409
7.1	0.409	−0.045	0.364
7.2	0.400	0.090	0.180
7.3	0.328	−0.267	0.672
		−(0.079)	

TABLE II

Continued.

Game number	Sincere sequence	Optimal sequence	Canonical sequence
7.4	0.074	0.381	0.333
8.1	0.206	−0.306 −(0.064)	0.758
8.2	0.283	0.077	0.308
8.3	0.455	−0.061	0.492
8.4	0.531	−0.183	0.409
9.1	0.238	0.190	0.428
9.2	0.193	0.285	0.333
9.3	0.118	0.333	0.238
9.4	0.086	−0.361 (0.145)	0.765

• One round in each game contains two optimal sequences.

influence the outcome by the order in which they cast their veto. In both game sets the first voter in the order of voting figured decisively in the course adopted by the group. Subsequent voters may have to change their decision according to the alternatives that were left open to them. In quintet games, although the remaining players enjoyed more room to maneuver, the first player still played a decisive role in the group's capacity to obtain the strategic outcome, thus, limiting the paths open to the other players leading to that alternative. Although the second voter had the largest range of alternatives after the first voter had cast her veto, no appreciable difference was found in the extent of the use of the sophisticated behavior between first-in-line and second-in-line voters (The extent of canonical voting was 26% and 25% respectively out of the 360 triplet game decisions, and 21% and 24% respectively in the quintet games). This finding is contrary to what was speculated. There was no evidence for voting order affecting the voters' strategy. Analyzing the relationship between voting order – the first or the second in line voters – and voters' sophisticated or non-sophisticated choice yields weak and insignificant results ($\phi = 0.0286$, $p > 0.05$).

TABLE III

Del values, quintet games.

Game number	Sincere Sequence	Canonical Sequence
1.1	0.950	0.262
1.2	0.637	0.251
1.3	0.680	0.262
1.4	0.561	0.227
1.5	0.581	0.351
1.6	0.612	0.227
2.1	0.704	0.163
2.2	0.657	−0.034
2.3	0.534	0.135
2.4	0.439	0.135
2.5	0.681	0.258
2.6	0.610	0.139
3.1	0.510	0.375
3.2	0.369	0.327
3.3	0.610	0.067
3.4	0.657	0.071
3.5	0.587	0.320
3.6	0.534	0.375
4.1	0.461	0.179
4.2	0.396	0.469
4.3	0.375	0.474
4.4	0.390	0.058
4.5	0.471	0.320
4.6	0.272	0.660
5.1	0.495	0.372
5.2	0.415	0.199
5.3	0.223	0.341
5.4	0.341	0.446
5.5	0.341	0.447
5.6	0.345	0.463
6.1	0.619	0.167
6.2	0.415	0.146
6.3	0.126	0.493
6.4	0.372	0.488
6.5	0.317	0.512
6.6	0.399	0.534

6. DISCUSSION

This research has addressed the issue of strategic voting under SVV procedure. It focused on three points: assessing the likelihood of strategic voting; understanding the cognitive processes that voters undergo in deciding how to cast their votes (sincere, optimal, or canonical); and determining the conditions that foster sophisticated voting (such as voting order or game's rounds).

The results indicate that, in keeping with earlier studies of this subject (Plott and Levine, 1978; Wilson, 1986; Wilson and Pearson, 1987; Herzberg and Wilson, 1988), subjects select sophisticated voting infrequently. However, these findings depend on the number of voters. The results of this experiment highlighted group size as a key factor in determining the tendency of voters to use strategic voting. Subjects operating in small groups (triplets) were more inclined to strategic voting than subjects operating in larger groups (quintets). This is so due to the fact that under SVV, increasing the number of voters necessarily increases the number of candidates and, consequently, makes strategic voting a complicated calculation, as it presents the participants with many more choices. This is especially true in quintet games, given the need to construct a 120-branch decision tree and then choose the paths leading to the optimal outcome. This course of action not only requires an awareness of the decision-tree solution, but also calls for concentration, relevant skill, and much longer time frames. These results correspond with several studies of strategic voting in binary agendas (Enelow and Koehler, 1980; Wilson and Pearson, 1987; Herzberg and Wilson, 1988), in which the length of an agenda increased the complexity of the decision setting as well as the number of calculations required to reach a strategic vote. Therefore, the more limited the number of elements in an agenda, the more likely participants will make a sophisticated choice.

In the current study, the optimal model is much more complex and complicated than the canonical; hence, the canonical model was clearly dominant in triplet games, while the sincere model enjoyed a decisive advantage over the canonical model in quintet games. Still, as illustrated in Table I above, the 'other' category accounts for between 29 and 36 percent of the outcomes, which could not be classified as sincere, canonical, or optimal voting. These types of decisions are traceable to several factors. In 20% of these decisions, the voters were found to veto the alternative ranked just above their least preferred alternative. For example, when player A realized that his or her least preferred alternative x was also the least preferred alternative of player B, who was slated to move next, player A cast a veto against the alternative ranked just above x in his or her preference ordering, thus nudging B to veto x – the most inferior alternative of both players. This mode of thinking diverges from the canonical in being a short-term strategy. Moreover, the combination of sincere voting by some group members and short-term canonical thinking by the rest of the group accounted for all 36 quintet games and manifested optimal group voting. As noted above, a player was unlikely to adopt an optimal sequence, as such a strategy is complicated. In all of the experimental voting games, not one player was observed to actually attempt a decision tree. In addition, even though no time limit was set for the experiment, it took the quintets between 1 and 2 h to complete 36 rounds of play. It therefore seems reasonable to submit that in quintet games, the optimal outcome resulted not from conscious optimal thinking on the part of all or some of the group members. Rather, it was due to the aforementioned combination of sincere moves by some group members and the implementation of complementary short-term canonical reasoning by the rest. This explanation seems even more compelling when applied to the triplet games. The constraints of time and awareness of a decision-tree solution left their mark in this case as well: no player was observed to construct a decision tree, even though the task, requiring the development of only six different paths, was much simpler than in the quintet games.

This insight suggests that the participants in these experiments acted with limited foresight, as they used only limited calculating abilities in order to reach a sophisticated outcome. Participants appear to be myopic, namely, they tend to use a short-term or incomplete canonical strategy, which is easier than the full canonical sequence and much easier than the optimal sequence. These observations fit well with the empirical cases discussing myopic sophisticated voters (Plott and Levine, 1978; Denzau and Mackay, 1981; Wilson and Pearson, 1987; Herzberg and Wilson, 1988).

Tacit cooperation between group members may also furnish some insight into the size of the 'other' category. As SVV is a consecutive method, the consecutive player tuned his or her voting to the vetoes cast by his or her predecessors. Indeed, tacit inter-group cooperation proved a frequent occurrence (54% of the 360 quintet games and 82% of the 360 triplet games); the group as a whole followed one of the voting sequences. The rest of the games manifested a diversity of voting strategies by the members of the same group. If the first player vetoes alternative x, moving sincerely, and if the second voter plans to adapt canonical reasoning, under these circumstances, the second voter's choice would diverge from the canonical model, since the first player's own divergence has altered the second player's veto vote, but has not altered this player's canonical way of thinking. Thus, while reflecting canonical reasoning on the second player's part, this phenomenon would not be categorized as such during the analysis of the data. The del value also fails somewhat in factoring in this phenomenon. Similarly, and as noted above, while analyzing the quintet games optimal sequence groups, some players where found to vote sincerely – even though, due to the effect of the preceding voters, their move was sometimes incompatible with the prediction of the sincere model.

Therefore, at least part of this 'other' category – including myopic actions and the sequential dependent characteristics of SVV – may extend the amount of sophisticated decision making as well as our understanding of this issue.

Finally, the effect of voting order was also examined in order to assess the degree to which the voting outcome depended on structural dimensions rather than on voters' decisions. This effect was suspicious due to the way in which the order of the agenda's elements specified a sequence of choices to be made by the voters and, hence, constituted a powerful determinant of the ultimate final choice (Ordshook and Schwartz, 1987). Bear in mind the clear differences between the two procedures – in SVV the sequence is between voters, while the sequence of the agenda is between the agenda's elements. In this study no significant differences were found between the way in which the first voter and the second voter cast their veto votes. Being first or second in the voting order did not lead the participants to choose a sophisticated strategy more frequently.

In conclusion, voters operating under SVV manifested both an inclination to sophisticated voting and a capacity to learn how to exercise sophisticated voting once given the opportunity to experience the SVV procedure again. At the same time, the extent of sophisticated voting ran counter to the number of voters, and as the number increased, sophisticated voters opted for the canonical mode and/or the short-term canonical mode. These findings all verified the initial research hypotheses.

Nevertheless, the method of this study still leaves much to be desired. First, it would have been beneficial to keep each of the rounds constant for a longer period and then observe player performance. Yet as noted above, this reiteration of a certain, constant round would have greatly extended the experiment, generating boredom and fatigue among the voters and rendering the monetary reward inadequate a priori. Future researchers may be well advised to attend to these problems of subject rewarding and excessive experiment duration. Second, being the first empirical investigation of SVV ever, the findings of this study offer but an outline of voter behavior and an initial foundation for further major questions not addressed herein, such as voter behavior when the number of alternatives to be selected exceeds 1. This study investigated only situations producing single outcomes, $s = 1$. Yet in real life, multiple-selection situations are common. Similarly, it would be interesting to examine situations where voters outnumber alternatives, and situations where more than one voter is needed to veto an alternative – in other words, voters are bound to form coalitions (tacit or explicit) in order to sort out which alternative to veto. To be sure, an empirical investigation of this last question will become feasible only once a solid theoretical solution is conceived, and that event is yet to come.

APPENDIX

To justify that the existence of an optimal sequence depends crucially on the assumption that the voters' order of preference among the m alternatives does not allow for indifference, consider the following example: Suppose that there are three voters, 1, 2, and 3, who

TABLE IV

The observed results of game 1.1.

Voter	a	b	c	d	Sincere sequence	Optimal sequence	Canonical sequence
A	–	3	2	5	d	c	B
B	5	–	–	5	a	d	D
C	–	3	7	–	c	b	C
Outcome	5	4	1	–	B	A	A
Del value					0.409	0.077	0.308

have to select by SVV one out of four alternatives *a, b, c,* or *d,* and that every voter, when her turn comes, can cast one veto vote against one of the surviving alternatives. Suppose further that voter 1 moves first, voter 2 moves second, and voter 3 moves last. Finally, suppose that the voters' preference orderings among the four alternatives are as follows: voter 1: $a > b \equiv c > d$; voter 2: $c > a \equiv d > b$; voter 3: $c > b > a \equiv d$. Here, ($>$) denotes strict preference, whereas ($\equiv$) denotes indifference. For instance, voter 1 is indifferent regarding b and c, but strictly prefers both to d and prefers a most of all. An easy informal argument shows that if voters 2 and 3 behave rationally, and if voter 1 vetoes a or b or d, then c will be selected; but if voter 1 vetoes *c,* then voter 2 will surely veto b, leaving voter 3 with the dilemma of having to choose between a and *d.* Thus voter 1 is faced with a choice between two possible alternatives: first, she can ensure that c will be selected; second, she can ensure that either a or d will be selected, but she cannot determine which. These two alternatives are mutually incomparable from voter 1's point of view: she can neither prefer one to the other nor express indifference regarding them. For this reason one must assume that the voters' preferences are linear (i.e., strictly ordered).

APPENDIX 2

In order to illustrate the way in which del values should be calculated, the empirical results of game 1.1 are summarized in Table IV.

Table IV displays the first round of the first game, played by ten triplet groups, in the actual order of playing. The first column identifies the voters. The next four columns – a, b, c, d – depict the distribution of voting among players with an identical preference ordering in all ten groups for the given round they played. The predictions of each of the three models – sincere, optimal, and canonical – are presented in the three right-hand columns. For instance, the canonical model predicted that voters A, B, and C would cast a veto vote against candidate *b, d,* and *c* respectively; Furthermore, none of voters A in the ten experimental groups veto candidate *a*; candidates *b*, *c*, and *d* was vetoed by voters A of three groups, two groups and five groups respectively. As there were 10 three-person groups, each voter (A, B or C) appears ten times, and hence each line totals ten.

In order to calculate del, an expected table should construct, according to the margin probabilities of Table V. Note that the observed table shows the distribution of the actual voters' choice, and hence, it is identical to Table IV.

According to Table IV, the *sincere model* predicted that voter A would veto alternative *d,* while empirically only in five out of ten groups did voter A veto *d.* Therefore the model predicted five errors out of ten (in the observed table the error cells are A*a*=0, A*b*=3, and A*c*=2,totaling five errors). Similarly, this model predicted voters B and C would veto alternatives *a* and *c* respectively, while actually they vetoed alternatives *a* and *c* only in five or seven groups respectively. Consequently the sincere model predicted 13 errors out of 30 possible votes. Hence, 13 errors will occupy the numerator of the fraction of the del calculation. The denominator is a summation of the margin probabilities of the certain cells of the expected table, which is parallel to the error cells of the observed table. As derived from the expected table, the denominator is equal to 22.

Consequently, the del value of the sincere model is as follows:

$$\nabla = 1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{k} w_{ij} p_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{k} w_{ij} p_{i\bullet} p_{j\bullet}}$$

TABLE V

Observed table.

Alternative / Voter	*a*	*b*	*c*	*d*	Total
A	0	3	2	5	10
B	5	0	0	5	10
C	0	3	7	0	10
Total	5	6	9	10	30
Expected table					
A	(5x10)/30	(6x10)/30	(9x10)/30	(10x10)/30	10
	1.67	2	3	3.33	
B	(5x10)/30	(6x10)/30	(9x10)/30	(10x10)/30	10
	1.67	2	3	3.33	
C	(5x10)/30	(6x10)/30	(9x10)/30	(10x10)/30	10
	1.67	2	3	3.33	
Total	5	6	9	10	30

$$\begin{aligned}\nabla \text{ sincere} &= 1 \\ &\quad - \frac{(0+2+3+0+0+5+0+3+0)}{(1.67+2+3+2+3+3.33+1.67+2+3.33)} \\ &= \frac{13}{22} = 0.409\end{aligned}$$

Accordingly, del values of optimal and canonical models for game 1.1 are as follows:

$$\nabla \text{ optimal} = 1 - \frac{20}{21.67} = 0.077$$

$$\nabla \text{ canonical} = 1 - \frac{15}{21.67} = 0.308$$

Therefore, according to del values, in this game the sincere model best predicted the empirical voters' choice because it had the largest value of del among the three competing models; that it, its predictions corresponded to the observed behavior of the voters. Indeed, both the sincere and the canonical models predicted voters' behavior much better than did the optimal model. Del value of the optimal model was nearly zero, implying that the optimal model's predictions and the voters' choice were independent.

NOTES

1. This article summarizes a study conducted for completion of the M.A. degree under the inestimable supervision of Professor Dan S. Felsenthal (University of Haifa).
2. A myopic voter is a voter who does not plan several steps ahead, but considers the short term only.
3. A cyclical order of social preference pertains to a situation where most voters prefer alternative A over alternative B, another majority prefers B over C, and yet another prefers C over A (A>B>C>A). In this case, no majoritarian procedure can definitely determine which alternative deserves to win.
4. Alternative X is considered Pareto-optimal when another alternative, preferred by all the voters over X, does not exist.
5. Appendix 1 provides a justification for and an illustration of this assumption.
6. I originally planned to allow six iterations of six games. But while constructing the games, I found that in triplet games, only four voting orders met the conditions set above for a given game. In the remaining two orders the optimal and canonical sequence were one and the same.
7. In triplet groups, during all the 36 rounds each subject occupied one of the three places alternately (being 12 times first, 12 times second, and 12 times third). In quintet groups, each participant occupied each of the five possible places at least once (because of the sixth round).
8. The appendix illustrates the calculation of del by using an example.

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