SUBSIDY AND TAXATION IN ALL-PAY AUCTIONS UNDER INCOMPLETE INFORMATION

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Subsidy and Taxation in All-Pay Auctions under Incomplete Information

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Abstract

We study all-pay auctions under incomplete information with n contestants who have non-linear cost functions. The designer may award two kinds of subsidy (taxation): one that decreases (increases) each contestant's marginal cost of effort and another that increases (decreases) each contestant's value of winning. The designer's expected payoff is the contestants' expected total effort minus the cost of subsidy or, alternatively, plus the tax payment. We show that when the resource of subsidy (the marginal taxation rate) is relatively small and the cost function is concave (convex), the designer's expected payoff in all-pay auctions with both kinds of subsidy (taxation) is higher than in the same contest without any subsidy (taxation). We then compare both kinds of subsidy and demonstrate that if the resource of subsidy is relatively small and the cost functions are concave (convex), the cost subsidy is better than prize subsidy for the designer who wishes to maximize his expected payoff.

Jel Classification: C72, D44, H25

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1 Introduction

Subsidy and taxation are both well common economic policies (see, for example, Sav 2004, Zuniga Vicente et al. 2014, and Bisceglia 2020). Here we focus on the potential of using these economic policies in contests. Our purpose is to show that different forms of subsidy or taxation might be useful for a content designer who wishes to maximize the contestants' efforts (outputs). In order to understand the efficiency of either subsidy or taxation in contests we compare them to the reserve price in auctions. There, when bidders are symmetric and have linear cost function it is well known that the classical auctions (first-price and second-price auctions) with the optimal reserve price are the optimal mechanism to maximize the seller's expected payoff. However, in these auctions an additional bidder is worth more to the seller than the ability to set the optimal reserve price (Bulow and Klemperer 1996). On the other hand, as we will show, an additional contestant in our model might be worth less to the designer than the ability to set the optimal subsidy or the optimal taxation.

We are not the first to deal with the combination of subsidy/taxation and contests, but in contrast to the current literature (see, for example, Glazer and Konrad 1999, Konrad 2000, Person and Sandmo 2005, Fu et al. 2012, Mealem and Nitzan 2014, Carpenter et al. 2016, and Thomas and Wang 2017) who study the role of subsidy/taxation in environments under complete information, we study these policies in all-pay auctions under incomplete information about the contestants' types. The difference of using subsidy/taxation in environments under complete and incomplete information is that while in an environment under complete information the designer can apply a different subsidy/tax for each contestant according to his type (see, for example, Nitzan and Mealem 2014), in an environment under incomplete information the contestants' types are ex-ante identical where each contestant knows his type (which is private information), and therefore the designer has to apply a uniform policy of subsidy/taxation for all the contestants without the ability to discriminate among them.

We study the all-pay auction (contest) with $n \geq 2$ contestants under incomplete information and non-linear cost functions. In other words, our contestants are not risk-neutral and, in particular, the revenue equivalence theorem (see Myerson 1981, and Riley and Samuelson 1981) does not hold in our environment. In such a case, the analysis of the optimal all-pay auction is complex and is generally unknown.

We first consider a cost subsidy where the designer has a monetary resource that can be used to subsidize all the contestants by decreasing their marginal costs of effort (this form of cost subsidy is studied by Glazer and Konrad 1999, and Thomas and Wang 2017 for a rent-seeking model under complete information).[1] For example, Lichtenberg (1990) claimed that the U.S. Department of Defense frequently provides "implicit subsidies" to firms, which help them win its design competitions. He then empirically showed that financial subsidies substantially improve the productivity of private military R&D. In the case of a cost subsidy, the designer's expected payoff is equal to the contestants' expected total effort minus the cost of the subsidy. We show that in the all-pay auction if the resource of subsidy is relatively small, then, if the contestants' cost functions are concave (convex), the designer's expected payoff is larger (smaller) than in the same contest without any subsidy. We show that the optimal cost subsidy might be relatively large, in which case, an additional contestant might be worth less to the designer than the ability to set the optimal subsidy. This finding indicates the effect of the cost subsidy on the designer's payoff might be more significant than the effect of the reserve price on the designer's payoff in the standard all-pay auction with linear cost functions.

Similarly, we consider cost taxation where the designer can tax all the contestants by increasing their marginal costs of effort. Then, the designer's expected payoff is equal to the contestants' expected total effort plus the expected tax payment. We show that in the all-pay auction, if the taxation rate is relatively small and if the contestants' cost functions are convex (concave), the designer's expected payoff is larger (smaller) than in the same contest without any cost taxation. We show that an additional contestant might be worth less to the designer than the ability to set the optimal taxation. It is worth noting that if we combine the above results for cost subsidy and cost taxation we obtain that if the contestants' cost functions are concave the designer should apply a cost subsidy, while if their cost functions are convex, he should apply a cost taxation.

We also consider a different form of subsidy that will be referred to as a prize subsidy. An example for a prize subsidies are 'medal bonuses' (subsidy) which are given in many countries to their Olympic athletes who won medals in the Olympic Games. In the case of prize subsidy, the designer can increase the winner's value of winning by awarding an extra prize. Then, the designer's expected payoff is the contestants' expected

[1] Fu et al. (2012) call this form of subsidy "efficiency-enhancing subsidy."

total effort minus the cost of the prize subsidy. We show that in all-pay auctions, if the contestants' cost functions are strictly concave, there exists a prize subsidy such that the designer's expected payoff is larger than in the same contest without any prize subsidy. Similarly, we consider a prize taxation such that the designer can decrease the winner's value of winning by imposing a tax only on the winner. For example, the U.S. tax system taxes prizes and awards, even Olympic athletes, if the recipient makes $ 1M a year or more. In such a case the $37,500 monetary prize that comes with a gold medal is also going to be taxed. In that case of a prize taxation, the designer's expected payoff is the contestants' expected total effort plus the tax of the winner. We show that if the cost functions are strictly convex, then there exists a positive tax such that the designer's expected payoff is larger than in the same contest without any prize taxation.

The intuition behind the above results is that the efficiency of a subsidy or taxation for the designer depends on whether the marginal increase (decrease) of the contestants' efforts is larger (smaller) than the marginal costs of subsidy (taxation). Since the contestants' expected efforts and the cost of subsidy and taxation are forms of the inverse cost function, the results depend on its curvature. Furthermore, since the amount of either a subsidy or taxation is relatively smaller than the expected contestant's effort, the fact the curvature of the cost function is increasing or decreasing plays a key role and yield the results whether or not subsidy or taxation are efficient for enhancing the designer's expected payoff in our model.

Last, we compare which kind of subsidy is better for the designer. We demonstrate that if the resource of subsidy is sufficiently small then in an all-pay auction with a concave (convex) cost function, the designer's expected payoff is larger (smaller) with a cost subsidy than in the same contest with a prize subsidy. Given our previous results according to which both forms of the subsidy are efficient when the contestants' cost functions are concave, we can conclude that for sufficiently low levels of the resource subsidy, the cost subsidy is better than the prize subsidy for the contest designer. However, as we will show, the optimal value of the subsidy resource could be relatively large, and then the impact of this comparison is quite limited.

As mentioned above, the optimal all-pay auction under incomplete information in which contestants have non-linear cost functions is not known yet. It is not our intention to claim that using a subsidy or taxation are optimal economic policies to maximize the contestants' efforts. Indeed, in the literature on contests we can find several other ways to enhance the contestants' efforts such as limiting the number of contestants by setting a minimum effort level (see Taylor 1995, Fullerton and McAfee 1999, Casson et al. 2010, and Fu et al. 2015), imposing a maximum effort level (see Che and Gale 1998, Gavious et al. 2003, Megidish and Sela 2014, and Olszewski and Siegel 2019), allocating several prizes and punishments (see, Lazear and Rosen 1981, Green and Stokey 1983, Nalebuff and Stiglitz 1983, , Moldovanu and Sela, 2001, 2006, Moldovanu et al. 2012, Olszewski and Siegel 2016, and Sela 2020), allocating head-starts or handicaps (see Kirkegaard 2012, Franke et al. 2013, Segev and Sela 2014, Drugov and Ryvkin 2017, and Fu and Wu 2020), or reimbursing some of the contestants' cost of efforts (see, Cohen and Sela 2005, Matros 2012, Minchuk 2018 and Minchuk and Sela 2020). We do claim, however, that subsidy or taxation, with or without some of the above well-known methods could be a basic component in the (indirect) optimal all-pay auction under incomplete information and non-linear cost functions.

The rest of the paper is organized as follows. In Section 2 we analyze the all-pay auction with a cost subsidy and a cost taxation, and in Section 3 we analyze the all-pay auction with a prize subsidy and a prize taxation. In Section 4 we compare between cost subsidy (taxation) and prize subsidy (taxation). Section 5 concludes. The proofs appear in the Appendix.

2 Cost subsidy and cost taxation

Consider $n \geq 2$ contestants who compete in an all-pay auction for a single prize. Contestant i's value of winning is v_i, $i = 1, .., n$, and is private information. The contestants' values are drawn independently of each other from the interval $[0, 1]$ according to the distribution function F which is common knowledge. We assume that F is continuously differentiable and that $f(x) = F'(x) > 0$ for all $0 \leq x \leq 1$. The contestant with the highest effort wins and and all the contestants pay the cost of their efforts where an effort of x has a cost of $\gamma(x)$ in monetary units. In other words, γ transfers x units of effort to $\gamma(x)$ monetary units. We assume that $\gamma(x)$ satisfies $\gamma' > 0, \gamma(0) = 0$, and $g = \gamma^{-1}$.

The designer has a monetary resource of θ that can be used to subsidize the contestants by decreasing their marginal costs of effort. In such a case, contestant i's cost of effort will be $\beta\gamma(x_i)$ where $0 < \beta \leq 1$ is referred to as the marginal subsidy rate. Since the allocated subsidy is equal to the designer's monetary resource θ we have

$$\theta = n(1-\beta)E(\gamma(x)), \tag{1}$$

where the LHS of (1) is the designer's monetary resource for subsidy, and the RHS of (1) is the expected change of total effort as a result of the cost subsidy. The designer's expected payoff in effort units is

$$R_{cs} = TE - g(\theta), \tag{2}$$

where TE is the contestants' expected total effort, and $g(\theta)$ is the cost of the designer's subsidy in effort units.

Alternatively, the designer can also tax the contestants by increasing their marginal costs of effort. In such a case, contestant i's cost of effort will be $\beta\gamma(x_i)$ where $\beta > 1$ is referred to as the marginal taxation rate. Then, the designer imposes a tax rate of $\beta - 1$ on each effort unit of a contestant in which case, the designer's expected payoff in effort units is

$$R_{ct} = TE + g\left(n(\beta-1)E(\gamma(x))\right), \tag{3}$$

where TE is the contestants' expected total effort, and $g\left(n(\beta-1)E(\gamma(x))\right)$ is the designer's expected profit from taxation in effort units.

2.1 Cost subsidy

We first study the all-pay auction with cost subsidy. If there is a symmetric monotonically increasing equilibrium effort function $x(v_i)$, the utility function of contestant $i, i = 1, .., n$, is

$$U(v_i) = v_i G(v_i) - \beta\gamma(x(v_i)), \tag{4}$$

where $G(v_i) = F^{n-1}(v_i)$ is the probability that the value v_i is the highest among all the n contestants, and the marginal subsidy rate satisfies $0 < \beta \leq 1$. The first order condition (FOC) of the maximization problem of contestant i's expected payoff given by (4) is

$$G'(v_i)v_i - \beta\left(\gamma(x(v_i))\right)' = 0.$$

Rearranging yields

$$\beta\gamma(x(v_i)) = \int_0^{v_i} sG'(s)ds + k.$$

Since $\gamma(x(0) = 0$, we have

$$\gamma(x(v_i)) = \frac{1}{\beta}\int_0^{v_i} sG'(s)sds.$$

Integrating by parts and rearranging yields the equilibrium effort of contestant i, $i = 1, 2, ..., n$ as follows:

$$x_{cs}(v_i) = g\left(\frac{1}{\beta}\left(v_i G(v_i) - \int_0^{v_i} G(s)ds\right)\right). \tag{5}$$

It can be easily verified that the above equilibrium effort is monotonically increasing. Then, contestant i with value v_i has the the following expected payoff:

$$\begin{aligned} U(v_i) &= v_i G(v_i) - \beta\gamma(x) \\ &= \int_0^{v_i} G(s)ds, \end{aligned}$$

which is exactly the contestant's expected payoff in the standard all-pay auction with linear cost functions (see Krishna 2010). Thus, we can conclude that the contestants are indifferent between having or not having a cost subsidy. Contrary to the contestants, we show below that the designer might have an incentive to apply a cost subsidy in all-pay auctions.

Since the allocated subsidy should be equal to the designer's resource θ, we have

$$\begin{aligned} \theta &= n(1-\beta)E(\gamma(x)) \\ &= n(1-\beta)\int_0^1 \frac{1}{\beta}\left(vG(v) - \int_0^{v} G(s)ds\right) f(v)dv = \frac{(1-\beta)}{\beta} R_{lin}, \end{aligned}$$

This yields that that the marginal taxation rate is

$$\beta = \frac{R_{lin}}{\theta + R_{lin}}, \tag{6}$$

where $R_{lin} = n\int_0^1 \left(vG(v) - \int_0^v G(s)ds\right) f(v)dv$ is the designer's expected payoff in the standard all-pay auction with linear cost functions and without subsidy.

The designer's expected payoff in an all-pay auction with a cost subsidy will be denoted by R_{cs} and is equal to the contestants' expected total effort minus the cost of a subsidy in effort units.

$$\begin{aligned} R_{cs} &= n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)f(v)dv - g\left(\theta\right) \qquad (7) \\ &= n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)f(v)dv - g\left(n(1-\beta)\int_0^1 \frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv\right). \end{aligned}$$

Note that the first part of (7) is the contestants' expected total effort, while the second part is the subsidy paid by the designer in effort units. The following result demonstrates the conditions under which a cost subsidy is either profitable or not for the contest designer.

Proposition 1 *In the all-pay auction with a resource subsidy of* $\theta \leq \frac{R_{lin}}{n-1}$, *if the cost function* γ *is concave (convex) then the designer's expected payoff is larger (smaller) than in the same contest without any subsidy.*

Proof. See Appendix. ■

We point out that in the case of linear cost functions, by the revenue equivalent theorem (RET), the designer's expected payoff is the same with and without a subsidy of any θ. However, when the cost functions are non-linear, the RET no longer holds. Then, by Proposition 1, if the cost function γ is concave, a relatively small subsidy increases the designer's expected payoff. The intuition behind this result is that for a subsidy to be profitable for the designer depends on whether the marginal increase in the contestant' expected total effort is larger than the marginal cost of a subsidy. The marginal increase of efforts and the marginal cost of a subsidy are both forms of $g'(x)$ which is the derivative of the inverse cost function g. Since g is convex such that $g'(x)$ is an increasing function and the fact that the expected effort is larger than the level of subsidy, we obtain that by applying a subsidy, the marginal increase of the expected total effort is larger than the marginal increase of the cost of a subsidy. Therefore, we can conclude tat it is profitable for the designer to allocate a subsidy if the cost function γ is concave, and vice versa when it is convex.

It will be verified below that although by Proposition 1 the efficiency of the cost subsidy certainly holds for $\theta \leq \frac{R_{lin}}{n-1}$, the optimal value of the cost subsidy θ^* might be much larger than this upper bound. By (7), the optimal subsidy is obtained by

$$\begin{aligned}\frac{\partial R_{cs}}{\partial \theta} &= -n\frac{1}{\beta^2}\frac{\partial \beta}{\partial \theta}\int_0^1 g'\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv - g'(\theta) \qquad (8)\\ &= \frac{n}{R_{lin}}\int_0^1 g'\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv - g'(\theta) = 0.\end{aligned}$$

In the following example we show that the optimal cost subsidy might be much larger than the upper limit $\frac{R_{lin}}{n-1}$ given by Proposition 1.

Example 1 *Consider an all-pay auction with $n \geq 2$ contestants where each contestant's value is distributed according to $F(v) = v$. The effort cost function is concave and is given by $\gamma(x) = x^{0.5}$. Then, by (8), the optimal subsidy is obtained by*

$$\frac{\partial R_{cs}}{\partial \theta} = 2\left(\frac{(n+1)^2}{n(2n+1)}\left(\theta + \frac{n-1}{n+1}\right) - \theta\right) = 0.$$

Thus, the optimal cost subsidy is

$$\theta^* = \frac{n^2-1}{n^2-n-1},$$

and by (6), the optimal cost subsidy rate is

$$\beta^* = \frac{n^2-n-1}{n(2n+1)}.$$

Substituting these parameters into the designer's expected payoff (7) yields the following designer's optimal expected payoff with a cost subsidy

$$R^*cs = \frac{(n-1)^2}{n^2-n-1}.$$

In the following table we present the optimal cost subsidy θ^*, the oatmeal designer's expected payoff with cost subsidy R^*_{cs}, and his expected payoff without any subsidy R for different numbers of contestants n.

n - contestants	*θ^*- the optimal subsidy*	*R^*_{cs}- payoff with subsidy*	*R- payoff without subsidy*
2	3	1	0.1
3	$\frac{8}{5}$	0.8	0.1905
4	$\frac{15}{11}$	0.818	0.25
5	$\frac{24}{19}$	0.842	0.2909
6	$\frac{35}{29}$	0.862	0.32051

We can see that the optimal cost subsidy θ^* decreases in the number of contestants n, such that for $n = 2$ we get the highest subsidy level of $\theta^* = 3 >> R_{lin}$. Furthermore, the designer's optimal expected payoff when there are two contestants ($R = 1$) is larger than the designer's expected payoff in the same contest without a subsidy but with an additional contestant ($R = 0.1905$). In other words, an additional contestant might be worth less to the designer than the ability to set the optimal subsidy.

2.2 Cost taxation

Now we consider the all-pay auction with cost taxation. As in the previous case, if there is a symmetric monotonically increasing equilibrium effort function $x(v_i)$, the utility function of contestant $i, i = 1, .., n$, is

$$U(v_i) = v_i G(v_i) - \beta\gamma(x(v_i)) \tag{9}$$

where $G(v_i) = F^{n-1}(v_i)$ is the probability that the value v_i is the highest among all the n contestants, and the cost taxation rate satisfies $\beta > 1$. It easy to see that the contestants' equilibrium efforts have the same form as with a cost subsidy except that β has different values. However, the designer's expected payoff in the all-pay auction with a cost taxation (denoted by R_{ct}) has a different form than in the all-pay auction with a cost subsidy and is given by

$$\begin{aligned} R_{ct} &= n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right) f(v)dv + g\left(n(\beta-1)E(\gamma(x))\right) \qquad (10) \\ &= n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right) f(v)dv + g\left(n(\beta-1)\int_0^1 \frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right) f(v)dv\right) \end{aligned}$$

Note that the first part of (10) is the contestants' expected total effort, while the second part is the tax payment in effort units. The following result demonstrates the conditions under which taxation is either profitable or not for the contest designer.

Proposition 2 *In the all-pay auction with a cost taxation rate of $1 < \beta \leq \frac{n+1}{n}$, if the cost function γ is convex (concave), the designer's expected payoff is larger (smaller) than in the same contest without any cost taxation.*

Proof. See Appendix. ■

According to Proposition 2, if the cost taxation rate is sufficiently low and if the contestants' cost functions are convex, taxation will be profitable for the contest designer. The intuition behind this result is that the marginal decrease in the contestants' efforts is smaller than the marginal profit from taxation. Since the tax payment and the expected level of the equilibrium effort are functions of g' (which is a decreasing function) and the the tax payment level is smaller than the expected level of the equilibrium effort, we obtain that by applying the taxation, the marginal decrease of the expected total effort is smaller than the marginal increase of the tax payment. Therefore, it is profitable for the designer to set a tax if the cost function γ is convex, but it is not profitable when the cost function is concave.

Proposition 2 indicates that a cost taxation might be profitable for the designer who wishes to maximize his expected payoff, but the value of the optimal cost taxation rate β^* could be relatively large. This optimal cost taxation is determined by

$$\begin{aligned}\frac{\partial R_{ct}}{\partial \beta} &= -n\frac{1}{\beta^2}\int_0^1 g'\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv \qquad (11)\\ &+n\frac{1}{\beta^2}\left(\int_0^1\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv\right)g'\left(n(\beta-1)\int_0^1\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv\right)=0\end{aligned}$$

In the following example we show that the optimal cost taxation rate might be much larger than the upper limit $\frac{n+1}{n}$ given by Proposition 2.

Example 2 *Consider an all-pay auction with $n \geq 2$ contestants where each contestant's value is distributed according to $F(v) = v$. Each contestant's cost function is convex and is given by $\gamma(x) = x^2$. Then, by (11) we get*

$$\frac{\partial R_{ct}}{\partial \beta} = 0.5\left(-n\frac{1}{\beta^2}\sqrt[2]{\beta}\sqrt[2]{\frac{n-1}{n}}\frac{1}{\frac{n}{2}+1}+\frac{1}{\beta^2}\sqrt[2]{\frac{\beta}{\beta-1}}\sqrt[2]{\frac{n-1}{n+1}}\right)=0$$

Thus, the optimal cost taxation rate is

$$\beta^* = \frac{1}{4n+4n^2}\left(8n+5n^2+4\right).$$

In the following table we present the optimal cost taxation rate β^*, the oatmeal designer's expected payoff with cost taxation R^*_{ct}, and his expected payoff without any taxation R for different numbers of contestants

n.

n - contestants	*β^*- optimal taxation rate*	*R^*_{ct}- revenue with taxation*	*R- revenue without taxation*
2	1.66	0.91287	0.707
3	1.52	1.208	0.979
4	1.45	1.3904	1.1547
5	1.408	1.5613	1.277
6	1.38	1.609	1.369

In contrast to the all-pay auction with an optimal cost subsidy, here the designer's optimal expected payoff increases in the number of contestants. On the other hand, similar to the all-pay auction with a cost subsidy, we can see that the designer's optimal expected payoff with a cost taxation when there are three contestants ($R^*_{ct} = 1.208$) is larger than the designer's expected payoff in the same contest without a taxation but with an additional contestant ($R = 1.1547$). Thus, an additional contestant might be worth less to the designer than the ability to set the optimal taxation.

3 Prize subsidy and prize taxation

Consider $n \geq 2$ contestants who compete in an all-pay auction for a single prize. Contestant i's value of winning is v_i, $i = 1,..,n$, and is private information. The contestants' values are drawn independently of each other from an interval $[0, 1]$ according to the distribution function F which is common knowledge. We assume that F is continuously differentiable and that $f(x) = F'(x) > 0$ for all $0 \leq x \leq 1$. The contestant with the highest effort wins and and all contestants pay for their costs of effort where an effort of x yields a cost of $\gamma(x)$. We assume that $\gamma(x)$ satisfies $\gamma' > 0, \gamma(0) = 0$, and $g = \gamma^{-1}$.

The designer can increase the winner's value of winning by awarding a prize subsidy (bonus) of $\theta > 0$. Then, his expected payoff is

$$R_{ps} = TE - g\left(\theta\right) \tag{12}$$

where TE is the contestants' expected total effort, and $g\left(\theta\right)$ is the cost of subsidy in effort units.

Alternatively, the designer can decrease the winner's value of winning by imposing a tax of $\theta > 0$. Then, his expected payoff is

$$R_{pt} = TE + g\left(\theta\right) \tag{13}$$

where TE is the contestants' expected total effort, and $g\left(\theta\right)$ is the tax payment in effort units. Notice that in contrast to the model with cost subsidy (taxation) in which all the contestants are subsided (taxed), in this model with prize subsidy (taxation) only the winner is subsided (taxed).

3.1 Prize subsidy

We first study the all-pay auction with a prize subsidy in which the designer awards a positive subsidy of $\theta > 0$ for the winner of the contest. If there is a symmetric monotonically increasing equilibrium effort function $x(v_i) : [0,1] \to [0,1]$, the utility function of contestant $i, i = 1, .., n$, is

$$U_i(v_i) = \left(v_i + \theta\right) G(v_i) - \gamma(x_i). \tag{14}$$

where $G(v_i) = F^{n-1}(v_i)$ is the probability that the value v_i is the highest among all n contestants. By a similar analysis of the all-pay auction with cost subsidy, we obtain that the equilibrium effort function is as follows

Proposition 3 *The equilibrium effort function in the all-pay auction with a prize subsidy of θ is*

$$x_{ps}(v) = g\left((v+\theta)G(v) - \int_0^v G(s)ds\right). \tag{15}$$

The designer's expected payoff (denoted by R_{ps}) is the contestants' expected total effort minus the cost of the prize subsidy $g\left(\theta\right)$ in effort units as follows:

$$R_{ps} = n\int_0^1 g\left((v+\theta)G(v) - \int_0^v G(s)ds\right) f(v)dv - g\left(\theta\right). \tag{16}$$

The following result demonstrates the conditions under which a prize subsidy increases the designer's expected payoff.

Proposition 4 *In the all-pay auction, if the cost function γ is strictly concave on $(0,1]$, then there exists a subsidy of $\theta > 0$ such that the designer's expected payoff is larger than in the same contest without any prize subsidy.*

Proof. See Appendix. ■

The intuition behind the result of Proposition 4 according to which prize subsidy is profitable for the contest designer when the cost function is concave is quite similar to the intuition for the result of Proposition 1 according to which the cost subsidy is profitable to the contest designer for every concave cost function.

By the proof of Proposition 4, if the prize subsidy is relatively small then it is profitable to the contest designer. In the following example, we show that the optimal prize subsidy is not necessarily small and its effect on the designer's expected payoff is significant.

Example 3 *Consider an all-pay auction with two contestants where each contestant's value is distributed according to $F(v) = v$. The cost function is concave and is given by $\gamma(x) = x^{0.5}$. Then, by (16), the designer's expected payoff is*

$$\begin{aligned} R_{ps} &= 2\int_0^1 \left((v+\theta)v - \int_0^v sds \right)^2 dv - (\theta)^2 \\ &= -\frac{1}{3}\theta^2 + \frac{1}{2}\theta + \frac{1}{10} \end{aligned}$$

and

$$\frac{dR_{ps}}{d\theta} = -\frac{2}{3}\theta + \frac{1}{2}$$

Thus, the optimal prize subsidy is

$$\theta^* = \frac{3}{4}$$

We can see that the prize subsidy makes the following change in the designer's expected payoff

$$R_{ps}(\theta = \frac{3}{4}) - R(\theta = 0) = 0.1875$$

Note that this difference in the designer's expected payoff is larger than his expected payoff when there is no subsidy ($\theta = 0$).

3.2 Prize taxation

Now we consider the all-pay auction with a prize taxation in which the winner has to pay a tax of $\theta > 0$. In that case, if the tax is larger than contestant i's type, $\theta > v_i$, he will stay out of the contest. Then, If there is a symmetric monotonically increasing equilibrium effort function $x(v_i) : [0,1] \to [0,1]$, the utility function of contestant $i, i = 1, .., n$, is

$$U_i(v_i) = (v_i - \theta)\, G(v_i) - \gamma(x_i). \tag{17}$$

Similar to the previous sections, the symmetric equilibrium effort function is

$$x_{pt}(v) = \begin{cases} 0 & 0 \le v < \theta, \\ g\left((v-\theta)\, G(v) - \int_\theta^v G(s)ds\right) & \theta \le v \le 1\ . \end{cases} \tag{18}$$

Then, the designer's expected payoff is

$$\begin{aligned} R_{pt} &= n \int_\theta^1 x_{pt}(v) f(v)dv + g(\theta) \Pr(\text{there is a winner}) = \\ &= n \int_\theta^1 g\left((v-\theta)\, G(v) - \int_\theta^v G(s)ds\right) f(v)dv + g(\theta)\left(1 - F^n(\theta)\right). \end{aligned} \tag{19}$$

The following result demonstrates the condition under which a prize taxation increases the designer's expected payoff.

Proposition 5 *In the all-pay auction, if the cost function γ is either linear or convex on $(0,1]$, then there exists $\theta > 0$ such that the designer's expected payoff is larger than in the same contest without any prize taxation.*

Proof. See Appendix. ■

The intuition behind the result of Proposition 5 according to which prize taxation is profitable for the designer when the cost function is convex is similar to the intuition for the result of Proposition 2 according to which the cost taxation is profitable for every convex cost function. Note that a prize taxation "serves" also as a reserve price and it is well known that a reserve price is a profitable tool to enhance the contestants' expected total effort.

In the following example, we show that even for a linear cost function the optimal prize subsidy is non-negligible and its effect on the designer's expected payoff is significant.

Example 4 *Consider an all-pay auction with two contestants where each contestant's value is distributed according to $F(v) = v$. The cost function is linear and is given by $\gamma(x) = x$. Then, by (19), the designer's* *expected payoff is*

$$\begin{aligned} R_{st} &= 2\int_{\theta}^{1}\left(\left(v-\theta\right)\left(v\right)-\int_{\theta}^{v} sds\right)dv+\left(\theta\right)\left(1-\left(\theta\right)^{2}\right) \\ &= -\frac{4}{3}\theta^{3}+\theta^{2}+\frac{1}{3}, \end{aligned}$$

and

$$\frac{dR_{ts}}{d\theta}=-2\theta\left(2\theta-1\right).$$

Thus, the optimal prize taxation is

$$\theta^{*}=0.5$$

We can see that the prize taxation makes the following change in the designer's expected payoff

$$R_{ts}(\theta=0.5)-R(\theta=0)=0.083$$

Note that $R(\theta=0)=\frac{1}{3}$ *such that the increase in the designer's expected payoff by the prize taxation is about 25%. For a convex cost function, such an increase will be even larger than for a linear cost function.*

4 Cost subsidy/taxation vs. prize subsidy/taxation

So far we have shown that the designer who wishes to maximize his expected payoff can apply either cost (taxation) subsidy or prize subsidy (taxation). In the following, we compare between a cost subsidy and a prize subsidy.

Proposition 6 *In the all-pay auction, if the resource subsidy of* θ *is sufficiently small, then if the cost function* γ *is concave (convex) the designer's expected payoff with a cost (prize) subsidy is larger than in the same contest with a prize (cost) subsidy.*

Proof. See Appendix ■

Since by Propositions 1 and 4, a subsidy is efficient when the contestants' cost function is concave, Proposition 6 implies that in that case a cost subsidy is better than a prize subsidy if the subsidy resource is sufficiently small. The comparison between a cost taxation and a prize taxation is not clear since in an all-pay auction with a cost taxation there is no tax resource as with a prize subsidy. However, when the cost function is linear this comparison becomes clear since by Proposition 5, for linear cost functions, the designer's expected payoff with a prize taxation is higher than in the same contest without any taxation. On the other hand, for a cost taxation, the revenue equivalence theorem holds, such that the designer's expected payoff is the same as in that contest without any taxation. Thus we have

Corollary 5 *In the all-pay auction with a linear cost function, the designer's expected payoff is larger with prize taxation than in the same contest with cost taxation.*

The intuition behind this result is that in contrast to a cost taxation prize taxation, cost taxation acts also as an entry fee, and this implies a larger expected total effort in the case of a prize taxation as long as the tax is sufficiently small.

5 Conclusion

Lichtenberg (1988) and other researchers raised the question "why does the government provide a subsidy for private military R&D, in addition to establishing prizes for innovation." In order to answer this question we focused on all-pay auctions (contests) with n contestants who have private information about their values of winning and have non-linear cost functions. The optimal structure of such a contest is unknown to a designer who wishes to maximize the contestants' expected total effort. We suggest two forms of a subsidy and a taxation and show that they both make the contest more profitable. The first is a cost subsidy (taxation) that increases (decreases) all the contestants' marginal costs, and the second is a prize subsidy (taxation) that increases (decreases) the winner's value of winning. We showed that in the case of convex cost functions a sufficiently small a taxation of both forms is profitable to the designer, while in the case of concave cost functions, a subsidy of both forms will be profitable. The majority of the considered cost functions in the economics literature are convex, so according to our findings the designer should apply a taxation. On the other hand, for concave cost functions, the designer should apply a subsidy. In addition, we showed that even in the case of a linear cost function, taxation could be a good substitute to other well-known methods for enhancing the designer's expected payoff, and, in particular, the contestants' expected total effort. We also compared both forms of a subsidy and showed that if the resource of subsidy is sufficient small and the contestants' cost functions are concave, then a cost subsidy is superior to a prize subsidy from the designer's point of view. However, since we have shown that the subsidy might be optimal when the level of the subsidy resource is relatively large, this comparison has limited significance.

6 Appendix

6.1 Proof of Proposition 1

If γ is concave and strictly increasing, its inverse function $g = \gamma^{-1}$ is convex. If $0 < \beta \leq 1$, there exists

$$g\left(vG(v) - \int_0^v G(s)ds\right) = g\left(\left(vG(v) - \int_0^v G(s)ds\right)\frac{1}{\beta}\beta\right) \leq g\left(\left(vG(v) - \int_0^v G(s)ds\right)\frac{1}{\beta}\right)\beta. \quad (20)$$

Hence, by (2), and Jensen's inequality, since g is convex we have

$$\begin{aligned}
R_{cs} &= n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv - g(\theta) \\
&= n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv - g\left(n(1-\beta)\int_0^1 \frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)f(v)dv\right) \\
&\geq n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv - \int_0^1 g\left(n\frac{1-\beta}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv.
\end{aligned}$$

Since $\theta \leq \frac{R_{lin}}{n-1}$ where $R_{lin} = \int_0^1 g\left(\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv$ is the expected payoff in the all-pay auction with linear cost functions, by (6) we get $n(1-\beta) \leq 1$. Thus, by (20) we have

$$\begin{aligned}
R_{cs} &\geq n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv - \int_0^1 g\left(n\frac{1-\beta}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv \\
&\geq n\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv - n(1-\beta)\int_0^1 g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv \\
&= n\int_0^1 \beta g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv,
\end{aligned}$$

which implies that

$$R_{cs} \geq n\int_0^1 \beta g\left(\frac{1}{\beta}\left(vG(v) - \int_0^v G(s)ds\right)\right)f(v)dv \geq n\int_0^1 g\left(vG(v) - \int_0^v G(s)ds\right)f(v)dv = R,$$

where R is the contestants 'expected total effort without a subsidy.

Similarly, if γ is convex and strictly increasing, its inverse function $g=\gamma^{-1}$ is concave, and then

$$g\left(vG(v)-\int_0^v G(s)ds\right)=g\left(\left(vG(v)-\int\limits_0^v G(s)ds\right)\frac{1}{\beta}\beta\right)\geq g\left(\left(vG(v)-\int\limits_0^v G(s)ds\right)\frac{1}{\beta}\right)\beta.$$

Likewise, by the same analysis for concave cost functions we obtain the opposite inequality

$$R_{cs}\leq n\int\limits_0^1 g\left(vG(v)-\int_0^v G(s)ds\right)f(v)dv=R.$$

□

6.2 Proof of Proposition 2

If γ is concave and strictly increasing, its inverse function $g=\gamma^{-1}$ is convex. If $\beta>1$, then $\frac{1}{\beta}<1$ and there exists

$$g\left(\left(vG(v)-\int\limits_0^v G(s)ds\right)\frac{1}{\beta}\right)\leq g\left(vG(v)-\int\limits_0^v G(s)ds\right)\frac{1}{\beta}. \tag{21}$$

Hence, by (3), and Jensen's inequality, since g is convex we have

$$\begin{aligned}
R_{ct} &= n\int\limits_0^1 g\left(\frac{1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)\right)f(v)dv+g\left(n(\beta-1)\int\limits_0^1\frac{1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)f(v)dv\right)\\
&\leq n\int\limits_0^1 g\left(\frac{1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)\right)f(v)dv+\int\limits_0^1 g\left(n\frac{\beta-1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)\right)f(v)dv.
\end{aligned}$$

Since $n\left(\beta-1\right)\leq 1$, by (21) we get

$$\begin{aligned}
R_{ct} &\leq n\int\limits_0^1 g\left(\frac{1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)\right)f(v)dv+n\left(\beta-1\right)\int\limits_0^1 g\left(\frac{1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)\right)f(v)dv\\
&= n\int\limits_0^1 \beta g\left(\frac{1}{\beta}\left(vG(v)-\int\limits_0^v G(s)ds\right)\right)f(v)dv\leq n\int\limits_0^1\beta\frac{1}{\beta}g\left(vG(v)-\int\limits_0^v G(s)ds\right)f(v)dv\\
&= n\int\limits_0^1 g\left(vG(v)-\int\limits_0^v G(s)ds\right)f(v)dv=R.
\end{aligned}$$

where R_{all} is the contestants' expected total effort in the all-pay auction without any taxation.

Similarly, if γ is convex and strictly increasing, its inverse function $g=\gamma^{-1}$ is concave, and then

$$g\left(vG(v)-\int\limits_0^v G(s)ds\frac{1}{\beta}\right)\geq g\left(vG(v)-\int\limits_0^v G(s)ds\right)\frac{1}{\beta}.$$

By the same analysis for concave cost functions we have the opposite inequality

$$R_{ct} \geq n\int_0^1 g\left(vG(v) - \int_0^v G(s)ds\right) f(v)dv = R.$$

□

6.3 Proof of Proposition 4

Differentiating the designer's expected payoff (12) with respect to the prize subsidy θ yields

$$\frac{\partial R_{ps}}{\partial \theta} = n\int_0^1 g'\left((v+\theta)G(v) - \int_0^v G(s)ds\right) G(v)f(v)dv - g'(\theta)$$

When θ approaches zero we get

$$\lim_{\theta\to 0} \frac{\partial R_{ps}}{\partial \theta} = n\int_0^1 g'\left(vG(v) - \int_0^v G(s)ds\right) G(v)f(v)dv - g'(0)$$

Note that $n\int_0^1 g'\left(vG(v) - \int_0^v G(s)ds\right) G(v)f(v)dv$ is actually the derivative of the effort function of the contestant with the highest value of winning. Since the highest equilibrium effort is larger than zero, and the inverse cost function function $g = \gamma^{-1}$ is convex (which implies that g' is increasing) we obtain that $\lim_{\theta\to 0} \frac{\partial R_{ps}}{\partial \theta} > 0$. Thus, a relatively small resource of prize subsidy will increase the designer's expected payoff. When the cost function γ is convex we have the opposite result. □

6.4 Proof of Proposition 5

Differentiating (13) with respect to the prize taxation θ we get

$$\frac{\partial R_{pt}}{\partial \theta} = n\int_\theta^1 g'\left((v-\theta)G(v) - \int_\theta^v G(s)ds\right)(G(\theta) - G(v))f(v)dv + g'(\theta)(1 - F^n(\theta)) - g(\theta)nG(\theta)f(\theta).$$

When θ approaches zero we have

$$\lim_{\theta\to 0} \frac{\partial R_{pt}}{\partial \theta} = g'(0) - n\int_0^1 g'\left(vG(v) - \int_0^v G(s)ds\right) G(v)f(v)dv.$$

Note that $n\int_0^1 g'\left(v)G(v) - \int_0^v G(s)ds\right) G(v)f(v)dv$ is actually the derivative of the effort function of the contestant with the highest value of winning. Since the highest equilibrium effort is larger than zero, and the inverse cost function function $g = \gamma^{-1}$ is convex and in particular g' is increasing, we obtain that

$\lim_{\theta\to 0}\frac{\partial R_{ps}}{\partial\theta}<0$. Thus, a relatively small resource of a prize taxation will decrease the designer's expected payoff when the cost function γ is concave. When the cost function γ is convex, we have the opposite result.

Now suppose that γ is linear. Differentiating (13) with respect to θ we get

$$\frac{\partial R_{pt}}{\partial\theta}=nG(\theta)\left(1-F(\theta)-\theta f(\theta)\right).$$

Note that for a sufficiently small θ we get $\frac{\partial R_{pt}}{\partial\theta}>0$. Thus, we obtain that even when the cost functions are linear a relatively small resource of a prize taxation will increase the designer's expected payoff. $\square$

6.5 Proof of Proposition 6

By (2) and (12), the difference between the designer's expected payoff in the all-pay auction with a cost subsidy and a prize subsidy is

$$R_{cs}-R_{ps}=n\int_0^1\left[g\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)-g\left((v+\theta)G(v)-\int_0^v G(s)ds\right)\right]f(v)dv.$$

Since $\beta=\frac{R_{lin}}{\theta+R_{lin}}$, differentiating the designer's expected payoff in the case of a cost subsidy is

$$\begin{aligned}\frac{\partial R_{cs}}{\partial\theta} &= \frac{\partial R_{cs}}{\partial\beta}\frac{\partial\beta}{\partial\theta}=n\int_0^1\left[g'\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)\frac{1}{\beta^2}\left(vG(v)-\int_0^v G(s)ds\right)\frac{\beta}{\theta+R_{lin}}\right]f(v)dv\\ &= n\int_0^1\left[g'\left(\frac{1}{\beta}\left(vG(v)-\int_0^v G(s)ds\right)\right)\frac{vG(v)-\int_0^v G(s)ds}{R_{lin}}\right]f(v)dv\end{aligned}$$

Similarly, differentiating the designer's expected payoff in the case of a prize subsidy is

$$\frac{\partial R_{ps}}{\partial\theta}=n\int_0^1\left[g'\left((v+\theta)G(v)-\int_0^v G(s)ds\right)G(v)\right]f(v)dv.$$

Thus, when θ approaches zero we get

$$\lim_{\theta\to 0}\frac{\partial\left(R_{cs}-R_{ps}\right)}{\partial\theta}=\frac{n}{R_{lin}}\int_0^1\left[g'\left(vG(v)-\int_0^v G(s)ds\right)\left(vG(v)-\int_0^v G(s)ds-G(v)R_{lin}\right)\right]f(v)dv. \quad (22)$$

It can be easily verified that there is $0<v^*<1$ such that $h(v)=vG(v)-\int_0^v G(s)ds-G(v)R_{lin}$ is decreasing for $0<v<v^*$ and is increasing for $v^*\le v\le 1$. In particular, there is $0<v^{**}<1$ such that $h(v)<0$ for $0<v<v^{**}$, and $h(v)\ge 0$ for $v^{**}\le v\le 1$. Thus, if g is concave such that $g'\left(vG(v)-\int_0^v G(s)ds\right)$ is a

decreasing function, we obtain that

$$\begin{aligned}
& \int_0^1 \left[g'\left(vG(v) - \int_0^v G(s)ds \right) \left(vG(v) - \int_0^v G(s)ds - G(v)R_{lin} \right) \right] f(v)dv \\
\leq & \int_0^1 \left[g'\left(v^{**}G(v^{**}) - \int_0^{v^{**}} G(s)ds \right) \left(vG(v) - \int_0^v G(s)ds - G(v)R_{lin} \right) \right] f(v)dv.
\end{aligned}$$

Thus, by (22), we have

$$\begin{aligned}
\frac{\partial \left(R_{cs} - R_{ps} \right)}{\partial \theta}|_{\theta=0} & \leq g'\left(v^{**}G(v^{**}) - \int_0^{v^{**}} G(s)ds \right) \frac{n}{R_{lin}} \int_0^1 \left[\left(\left(vG(v) - \int_0^v G(s)ds \right) - G(v)R_{lin} \right) \right] f(v)dv \\
& = g'\left(v^{**}G(v^{**}) - \int_0^{v^{**}} G(s)ds \right) \frac{n}{R_{lin}} \left[R_{lin} - R_{lin} \right] = 0,
\end{aligned}$$

That is, since $R_{cs}(0) = R_{ps}(0)$ we obtain that if g is concave then for sufficiently small θ, $R_{cs}(\theta) - R_{ps}(\theta) \leq 0$. On the other hand, if g is convex then for sufficiently small θ, $R_{cs}(\theta) - R_{ps}(\theta) \geq 0$. $\square$

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