# USING MERTON'S MODEL: AN EMPIRICAL ASSESSMENT OF ALTERNATIVES

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# *Using Merton model: an empirical assessment of alternatives*

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**Abstract**

It is surprising that although four decades passed since the publication of Merton (1974) model, and despite the development and publications of various extensions and alternative models, the original model is still used extensively by practitioners, and even academics, to assess credit risk. We empirically examine specification alternatives for Merton model and a selection of its variants, concluding that prediction goodness is mainly sensitive to the choice of assets expected return and volatility. A Down-and–Out Option pricing model and a simple naïve model outperform the most common variants of the Merton model, therefore we recommend using the simple model for its easy implementation.

*Keywords:* Credit risk; Default prediction; Merton model; Bankruptcy prediction, Default barrier; Assets volatility; Down and out option

*JEL classification:* G17; G33; G13

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## 1. Introduction

Merton (1974) and Black and Scholes (1973) presented the basic approach for the valuation of stocks and corporate bonds as derivatives on the firm's assets. Merton (1974) is a structural model used for default prediction, viewing the firm's equity as a call option on its assets, because equity holders are entitled to the residual value of the firm after all its obligations are paid. Many theoretical studies suggested models that relax some of the Merton model restrictive assumptions.[1] However, empirical literature mainly focused on the application of the original model. A major benchmark in these studies is the KMV model. KMV was founded in 1989 offering a commercial extension of Merton's model using market-based data. In 2002 it was acquired by Moody's and became Moody's-KMV. KMV published a number of papers which reveal some of its methods (see Keenan and Sobehart, 1999; Keenan, Sobehart and Stein, 2000; Crosbie and Bohn, 2003). Some of the specifications made by KMV were adopted by the academic literature. Vassalou and Xing (2004), Campbell, Hilscher, and Szilagyi (2008), Aretz and Pope (2013) are examples for such studies.

Only a few studies attempted to evaluate the accuracy of Merton's model under these specifications. Hillegeist et al (2004) compared the predictive power of the Merton model to Altman (1968) and Ohlson (1980) models (Z-score and O-score) and came to the conclusion that the Merton model outperforms these models. Duffie et al (2007) showed that macroeconomic variables such as interest rate, historical stock return and historical market return have default prediction ability even after controlling for Merton model's distance to default. Campbell, Hilscher, and Szilagyi (2008), using a hazard model, combined Merton model default probability with other variables relevant to default prediction. They also found that Merton model probabilities have relatively little contribution to the predictive power. Bharath and Shumway (2008) presented a "naïve" application of Merton model that outperformed the iterative application of Merton model (based on presumably Moody's-KMV

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[1] See for example Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001), Hsu Requejo and Santa-Clara (2004), Leland (1994), Leland and Toft (1996), Acharya and Carpenter (2002).

specifications).[2] Another line of literature examined structural models ability to explain credit spreads and concluded that Merton model predictions underestimate market spreads.[3]

In this paper we examine the sensitivity of Merton model default prediction performance to its parameter specifications. We assess the causes for this sensitivity and evaluate the performance of a wide range of model alternatives, including those suggested by other recent studies. We conclude by providing a few prescriptions to enhance the model accuracy and suggesting a very simple model, which provides excellent discriminatory power for a low computation effort. Model wise we evaluate the textbook two-equation Merton model, its down and out (DaO) barrier alternative, the iterative model which is widely believed to be that of KMV, and single equation models and shortcuts including Bharath and Shumway (BhSh) naïve model, Charitou et al. (CDLT), and our simple naïve model (SNM).[4] In each model we focus on its three main components: the default barrier, the expected return on firm assets and the firm assets return volatility (hereafter, asset volatility). For this purpose we construct a sample with annual observations of firms from the merged CRSP/Compustat database during the period 1989 to 2012. We also gather information on default events during 1990 to 2013 from Standard and Poor's (S&P) and Moody's rating agencies reports. After filtering our sample includes 26,579 annual observations of 2,534 firms, of which 306 observations defaulted in the following year.

For each specification of each assessed model, we construct a Receiver Operating Characteristic (ROC) curve. This method is relatively common for the comparison of prediction models since it does not require setting a priori the desired cutoff point between cost of type I error and cost of type II error. Another advantage of using ROC curves, compared to methods used in some prior studies, is that it

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[2] Chava and Purnanandam (2010) used the naïve model as a proxy for credit risk.

[3] See for example, Jones, Mason and Rosenfeld (1984), Huang and Hunag (2003), Eom, Helwege and Huang (2008).

[4] Charitou et al. (2013) is a comprehensive study, similar to this work, aiming to compare various specifications of Merton model. The potential spectrum of methods and specification is too broad to be included in a single paper, hence we regard their work (denoted hereafter by CDLT) and our research as complementary with some essential overlap. This overlap is required to ensure that method comparison is based on identical database. A similar overlap exists also between CDLT, Bharath and Shumway (2008), and other preceding papers, each repeats some of the methods incorporated in its respective prior literature. In the same vein, we include CDLT proposed methodology to estimate asset volatility (σ_CDLT) and asset drift (μ_CDLT) in our study.

enables statistical inference with the non-parametric test suggested by DeLong, DeLong and Clarke-Pearson (2008), testing the statistical significance of the differences between the ROC curves (of two models). For robustness, we also include partial area under the curve (pAUC) calculations and test for pAUC differences, often at a few false positive rate levels. Prior studies, such as Bharath and Shumway (2008), focused mainly on the rate of defaulters within the first deciles of firms (highest predicted default probabilities) and did not offer a robust statistical test for differences between models.

Another approach we use to understand the adequacy of various specifications is the study of firms' characteristics changes on a path to default. For this purpose, we focus on 101 defaulting firms with data available for the five years preceding the default event and compare their level of debt, stock returns, equity volatility and assets volatility to those of a group of 101 non-defaulting firms.

We find that Merton model accuracy is only slightly sensitive to the specification of the default barrier. We explain that this is a result of the calculated assets value and volatility dependence on the default barrier. On one hand, ceteris paribus, a low setting of default barrier for risky firms reduces their probability of default. On the other hand, such misspecification also causes overestimation of assets volatility and underestimation of assets value, thus increasing the default probability. Therefore, a deviation of the default barrier from the common practices has a relatively small effect on the model accuracy.

We also show that using historical equity return as a proxy for expected assets return is questionable.[5] In particular, realized returns for risky firms are low and sometimes negative. While negative stock returns may be a predictive indicator for default, it cannot be a good proxy for forward-looking expected returns. Such a specification simply reduces the precision of the model. There are several ways to minimize the effect of negative returns. Aiming to estimate forward looking expected returns, we present a CAPM based procedure and results. However, we show that setting expected assets return

[5] This specification was used by Bharath and Shumway (2008), in their naïve model. We have similar doubts regarding the use of historical equity returns in the iterative method used by Vassalou and Xing (2004), Bharath and Shumway (2008), and others.

equal to the highest of realized stock return and the risk-free interest rate seems preferable among the alternatives examined in this study, especially for the simplest and powerful naïve model.

Our calculations demonstrate that assets volatility extracted from Black and Scholes (1973) using the historical volatility of equity is under-biased, especially for defaulting firms. This is mainly because the value of equity used for this purpose is up-to-date and forward looking while the backward looking historical volatility of equity is estimated on stock returns that might exhibit mild volatility prior to the deterioration in the financial state of the firm. We show that on average the difference between implied volatility (of stock options) and historical volatility is positive. This difference is larger for defaulting firms than for non-defaulting firms. Hence, model accuracy seems higher using equity volatility than using the theoretical asset volatility calculated by simultaneously solving Black and Scholes (1973) and the volatility relation of Jones, Mason and Rosenfeld (1984).

The rest of the paper is organized as follows. Section 2 describes the Merton model and Section 3 discusses its application. Section 4 presents alternative models. In Section 5 we present the methodology and Section 6 describes the data. In section 7 we present and discuss the results. Section 8 concludes.

## 2. Merton model

Merton model uses the firm equity value, its debt face value, and the volatility of equity returns to evaluate the firm assets and debt. The model assumes that the firm has issued one zero-coupon bond. The firm defaults at the bond maturity (in time *T*) when the value of its assets (*A*) falls below the amount of debt it has to repay (*D*). Otherwise the firm pays its debt in full and the remaining value is its equity $E_T = \max(A_T - D, 0)$. The model assumes that *A* follows a geometric Brownian motion (GBM):

(1) $dA = \mu_A \cdot A \cdot dt + \sigma_A \cdot A \cdot dW$

where $\mu_A$ is the expected continuous-compounded return on $A$, $\sigma_A$ is the volatility of assets returns and $dW$ is the standard Wiener process.[6]

The model applies the Black and Scholes (1973) formula to calculate the value of the firm equity as a call option on its assets with expiration time $T$ and an exercise price equal to the amount of debt ($D$):

$$(2)\quad E = N(d)A - De^{-rT}\, N(d - \sigma_A\sqrt{T})$$

$$(3)\quad d = \frac{\ln(A/D) + [r + 0.5\sigma_A^2]T}{\sigma_A\sqrt{T}}$$

where $E$ is the value of the firm equity, $r$ is the risk free interest rate, and *N(•)* is the cumulative standard normal distribution function.[7] Jones, Mason, and Rosenfeld (1984) show that under the model assumptions the relation between the equity volatility ($\sigma_E$) and the assets volatility ($\sigma_A$) is $\sigma_E = \frac{A}{E} \cdot \frac{\partial E}{\partial A} \cdot \sigma_A$ . Under the Black and Scholes formula it can be shown that $\frac{\partial E}{\partial A} = N(d)$, so the relation between the volatilities is:

$$(4)\quad \sigma_E = \frac{A}{E} N(d)\sigma_A$$

Solving equations (2) and (4) simultaneously results in the values of $A$ and $\sigma_A$ which can be used to calculate a Distance to Default ($DD$) of the firm, defined by:

$$(5)\quad DD = \frac{\ln\left(\frac{A}{D}\right) + [\mu_A - 0.5\sigma_A^2]T}{\sigma_A\sqrt{T}}$$

[6] We omit the subscript t from $A$ and $W$ for convenience. Obviously these vary with time. The drift $\mu_A$ and the volatility $\sigma_A$ are assumed constant in this basic (classical) model.

[7] $E$ and $A$ in (2) and (3) are the values of equity and assets at time t = 0. The risk-free rate $r$ is assumed constant.

*DD* may be regarded as the normalized distance between the firm assets value (*A*) and the face value of its debt (*D*). As the log asset value is normally distributed under the GBM, PD – the probability of default (the probability that the call option is not exercised) is: [8]

(6) $PD = N(-DD)$

## 3. Application of the Merton model

The application of the model in practice requires several refinements. $T$ is usually assumed to be 1 year. The annualized historical volatility of the equity is frequently the choice for $\sigma_E$.[9] It is often estimated over the preceding one year period and we denote it by $\sigma_{E,-1}$. We compare this choice with a mean absolute deviation (MAD) and JP Morgan (RiskMetrics) volatility estimates that we denote by $\sigma_{MAD}$ and $\sigma_{JP}$, respectively. Another issue is the amount of debt that is relevant to a potential default during a one year period. Total debt is inadequate when not all of it is due in one year, as the firm may remain solvent even when the value of its assets falls below its total liabilities. Using the short term debt (debt maturing in one year) for the default barrier *D* would be often wrong, for example, when there are covenants that force the firm to serve other debts when its financial situation deteriorates. Prior studies generally follow KMV (Crosbie and Bohn, 2003) and chose short-term debt plus half of the long term debt for the default barrier.[10] In this work we use $D = STD + k \cdot LTD$ for the default barrier, where $STD$ is the short term debt, $LTD$ is the long term debt and $k$ is the $LTD$ multiplier. We test the predictability power of the model for various values of $k$ and check whether the KMV choice of $k = 0.5$ outperforms the alternatives.

The values of a firm's assets (*A*) and their volatility ($\sigma_A$) are not observed and need to be implied from a model. The textbook method is to simultaneously solve equations (2) and (4). This was originally

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[8] Using the expected returns on the assets ($\mu_A$) in DD, Instead of the risk free rate (r) used in calculating $d_2$ in the Black & Scholes model, results in "real" PD instead of the risk neutral measure, under the model assumption.

[9] A forward looking implied volatility is probably a better choice. However it is not available for many firms and in its extraction from market data is complicated by liquidity and volatility smiles.

[10] For example: Bharath and Shumway (2008), Vassalou and Xing (2004), Duffie, Saita, and Wang (2007), Campbell, Hilscher, and Szilagyi (2008), and Aretz and Pope (2013).

proposed by Merton (1974) and refined by Jones et al (1984), it is also implemented in Hillegeist et al (2004) and Campbell et al (2008). In the next section we present some alternative methods for the estimation of these unobserved variables.

The expected asset return $\mu_A$, has to be estimated separately.[11] Campbell et al. (2008), for example, used a constant market premium and calculated it as $\mu_A = r + 0.06$. In this work we examine several alternatives for $\mu_A$. Under the first two alternatives we apply the CAPM model $\mu_A = r + \beta_A \cdot MP$, where $MP$ is the market premium and $\beta_A$ is the assets beta. First we use daily observations from the previous year on daily stock returns and the CRSP value weighted NYSE-NASDAQ-AMEX index to estimate the equity beta $\beta_E$.[12] Then we use the relation $\beta_A = \beta_E \cdot \frac{\sigma_A}{\sigma_E}$ and the values of $\beta_E, \sigma_A, \sigma_E$ to calculate $\beta_A$.[13] We use two alternative values for *MP*. The first is a constant rate of 6%, which results in $\mu_A = \mu_{MP=0.06} = r + \beta_A \cdot 0.06$. The second assumes a variable market premium which equals the historical excess return of the market index in the previous year. The later results in $\mu_A = \mu_{MP=MKT} = r + \beta_A \cdot (MKT_{-1} - r)$, where $MKT_{-1}$ is the annual rate of return of the market index in the previous year. For our third alternative we simply assume that the expected asset return equals the historical equity return of the preceding year, $r_{E,-1}$. We use this alternative as a benchmark for the other two methods and in accordance to the naïve model of Bharath and Shumway (2008). Historical equity return ($r_{E,-1}$) is sometimes negative. Hence we also examine the possibility that a floor for the assets expected return is $r$ and thus examine the results of $\mu_A = \max(r, r_{E,-1})$. Another alternative is to assume that the assets expected return equals the risk-free rate, $\mu_A = r$. In this case the probability measure that governs the asset and default processes is the risk-neutral measure. We also examine the alternative of a constant asset return $\mu_A = 0.09$.

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[11] Except for the above iterative (KMV) method and CDLT described below.

[12] We refer to the CRSP value weighted NYSE-NASDAQ-AMEX index as the market and designate it by MKT.

[13] The relation between the assets and equity betas is derived from the expression of a Black-Scholes call beta $\beta_E = \frac{A}{E} \cdot N(d) \cdot \beta_A$ where we replace the call option and the underlying by the equity and the assets respectively (see for example Coval and Shumway 2001). We then use equation (4) to replace $\frac{A}{E}N(d)$ by the volatilities ratio $\frac{\sigma_E}{\sigma_A}$.

## 4. Alternative models

In this section we briefly present a few alternative models to those presented in the prior section, and conclude with volatility estimation methods used in this research.

### 4.1 Iterative estimation (KMV)

An approach, allegedly developed and used by KMV, was also used by Bharath and Shumway (2008), Vassalou and Xing (2004), Duffie, Saita, and Wang (2007), and Aretz and Pope (2013), is a calculation intensive iterative procedure. In this process an initial guess value of $\sigma_A$ is used in equation (2) in order to infer the market value of the assets ($A$) for the firm on a daily basis in the prior year. This generates a time series whose volatility is an updated guess of $\sigma_A$, which is used to compute a new time series of the firm's assets. The procedure is repeated until the volatility used to calculate the time series converges to the volatility of the calculated values. Then, the last time series is used to infer the values of $\sigma_A^{KMV}$ and $\mu_A^{KMV}$ which are used in equation (5) of the model. Bharath and Shumway (2008) showed that this approach results are in fact similar or even slightly inferior to the results of the simultaneous solution of equations (2) and (4).

### 4.2 Bharath and Shumway naïve model (BhSh 2008)

Bharath and Shumway (2008) proposed a naïve alternative to Merton model assuming that the asset value is the sum of the default barrier (*D*) and equity (*E*) values: $A = D + E$, where the default barrier is $D = STD + 0.5 \cdot LTD$. The expected return of assets is set equal to the historical return on the firm stock price in the previous year, $\mu_A = r_{E,-1}$. Assets volatility $\sigma_A^{Naive}$ is assumed to be a value-weighted average of historical equity volatility ($\sigma_{E,-1}$) and a "special" value of the debt volatility: [14]

(7) $\sigma_D = 0.05 + 0.25 \cdot \sigma_{E,-1}$

[14] We are not familiar with the foundations and origin of this assumed relation between the debt and equity volatilities.

(8) $$\sigma_A^{Naive} = \frac{E}{E+D}\sigma_{E,-1} + \frac{D}{E+D}(0.05 + 0.25 \cdot \sigma_{E,-1}).$$

The naive Distance to Default is (for T=1 year):

(9) $$DD_{Naive} = \frac{\ln[(E+D)/D)] + r_{E,-1} - 0.5 \cdot (\sigma_A^{Naive})^2}{\sigma_A^{Naive}}$$

and the default probability is: $PD_{Naive} = N(-DD_{Naive})$.

### 4.3 Charitou et al (CDLT 2013)

CDLT proposed to generate a time series of "observable" asset values, each is defined as the sum of market value of equity *E* and the face value of debt *B*. For purposes of asset value estimation *B* is the face value of total liabilities according to CDLT (it calls it "the original default boundary"). The returns of such a time series of asset values are used to calculate the drift ($\mu_A^{CDLT}$) and their annualized standard deviation is $\sigma_A^{CDLT}$. These are then used to calculate the related distance to default ($DD_{CDLT}$) and default probability using the usual formulation of equations (5) and (6).

CDLT used monthly data over a period of 60 months. We apply the model using daily data over a period of one year prior to the point estimation (year-end) date. We acknowledge the benefit of using monthly data for noise considerations, however there is empirical evidence that, on average, volatilities change significantly during the five year prior to default. Furthermore, one year  seems the right choice for our work as the other methods and specifications that we use in this study use the same one year period.

### 4.4 Down and Out call (DaO)

Some prior research, such as Dionne and Laajimi (2012), relaxed the assumption of default only on the year-end date by using a European down and out call option (DaO). The value of such a barrier option, under the GBM process assumption is given by equation (10).[15]

$$(10) \quad E_{DaO} = AN(a) - De^{-rT}N\left(a - \sigma_A\sqrt{T}\right) - A(H/A)^{2\eta}N(b) + De^{-rT}(H/A)^{2\eta-2}N\left(b - \sigma_A\sqrt{T}\right)$$

where A, D, N(·), $\sigma_A$, and T are defined above. When the asset value reaches the barrier H the option expires worthless (E = 0, a default). The variables a, b, and η are defined below.

$$(11) \qquad a = \frac{ln(A/H)}{\sigma_A\sqrt{T}} + \eta\sigma_A\sqrt{T}, \quad b = \frac{ln\left(H^2/AD\right)}{\sigma_A\sqrt{T}} + \eta\sigma_A\sqrt{T}, \quad \eta = r/\sigma_A^2 + 0.5.$$

Similar to the Merton model requiring the simultaneous two-equation solution, equation (10) needs also to satisfy the following relations between the equity and asset volatilities:[16]

$$(12) \qquad \sigma_E = \frac{A}{E}\frac{\partial E_{DaO}}{\partial A}\sigma_A .$$

The default probability is given by:

$$(13) \qquad PD_{DaO} = N\left(\frac{-ln(A/H)-\left(\mu_A-0.5\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right) + exp\left[\frac{-2\mu_A \cdot ln(A/H)}{\sigma_A^2}\right] \cdot N\left(\frac{-ln(A/H)+\left(\mu_A-0.5\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right).$$

For comparability with the other models we analyze in this paper we set H=D.

### 4.5 Single equation models

Bharath and Shumway (2008) found that their naïve model (BhSh) outperforms equations (2) and (4) simultaneous solution (hereafter called *two-equation Merton*) and KMV iterative process. We

[15] Generally a down and out call explicit expression depends on the relation between the barrier H, the exercise price D, and on its rebate. For the case of the risky debt application we assume a zero rebate model and since we explore only the case of D=H the model we present is applicable when D ≥ H. For details see for example Dionne and Laajimi (2012) appendix or any textbook on derivatives such as Hull (2012).

[16] We do not expect $\sigma_A$ of equation (12) to equal that of equation (4). We omit here superscripts to simplify the presentation.

conjecture that the main contributor to the power of BhSh model is their choice of asset volatility ($\sigma_{Naive}$) which depends on the historical equity volatility ($\sigma_{E,-1}$) and the inverse of the book leverage ratio ($E/(D+E)$). As explained in the next section, like prior literature on Merton and similar models, we test the model's power. Assuming that the source for the discrimination performance of BhSh model is $\sigma_{E,-1}$, we examine alternative methods in which we solve a single equation, (2) for Merton model and (10) for the DaO alternative, using equity volatility estimates (either $\sigma_{E,-1}$, $\sigma_{JP}$, or $\sigma_{MAD}$, presented below) for the asset volatility without the formulation of BhSh $\sigma_A^{Naive}$. We call this model *single-equation Merton* and *single-equation DaO* respectively.

### 4.6 Our simple naïve model (SNM)

Inspired by BhSh model we assess the performance of a very simple model, identical to BhSh, except for the choice of the asset volatility. We simply insert equity volatility instead of $\sigma_A^{Naive}$ in equation (9) as follows:

$$(14) \qquad DD_{SNM} = \frac{ln[(E+D)/D)] + \mu_A - 0.5 \cdot (\sigma_{E,-1})^2}{\sigma_{E,-1}}$$

where we allow choosing a proper asset drift $\mu_A$ for flexibility. Our choice is $\mu_A = \max(r_{E,-1}, r)$.[17]

### 4.7 Equity volatility estimation

Like most prior research we use annualized standard deviation of log daily equity gross returns as the basic estimate for historical volatility. We use a whole year daily data of the year preceding each annual observation and denote the volatility estimate $\sigma_{E,-1}$. This common estimate has some drawbacks, two obvious issues are: (i) it is an average of a full year and ignores possible changes during the estimation period; and (ii) standard deviation is sensitive to large deviations that might be caused by outliers.

[17] We find that it provides better results than BhSh choice of $\mu_A = r_{E,-1}$ which could be negative while having a risky asset expected return lower than the risk free rate is counter intuitive and not in line with economic reasoning.

We address the annual averaging matter by using RiskMetrics (1996) exponentially weighted moving average recursive volatility estimate:

(15) $$\sigma_{1,t+1}^2 = \lambda\sigma_{1,t-1}^2 + (1-\lambda)r_{1,t}^2$$

where $\sigma_{1,t}^2$ is the day t estimated daily volatility, $r_{1,t}^2$ is the daily return (logarithm of the price on t to the price on t-1 ratio) squared, and $\lambda$ is a parameter, often set at 0.94. We denote the annualized yearend estimate by $\sigma_{JP}$.[18]

The large outlier effect can be moderated by using absolute deviations instead of squared deviation. A common measure is mean of absolute deviations (MAD):

(16) $$MAD(n)_t = \frac{1}{n}\sum_{j=0}^{n-1}\left|r_{t-j}\right|$$

where n is the number of observations of daily returns (r) until time t, a year in our case. We annualize the MAD and adjust it to normal distribution and denote the estimate $\sigma_{MAD}$:[19]

(17) $$\sigma_{MAD} = \sqrt{\frac{252\pi}{2}} \cdot MAD$$

## 5. Methodology

There are two major challenges in such a study. One is related to the goodness of a model compared to other models and specifications, this is discussed below. The other is the complexity caused by the multidimensionality of the models and their parameters, where comparing all models with all their parameter choices, pair wise or all simultaneously, seems impractical and too fuzzy. Instead, we move

[18] More details about the selection of $\lambda$ can be found in RiskMetrics (1996) and other risk management literature. We simply use the squared return of the first day of the year as the first value in the recursive equation (13). Since there are more than 250 observations per year the initial value does not affect the year end estimate practically.

[19] see for example Ederington and Guan (2006), we omit here t and n for convenience.

our focus sequentially from one issue to the other and finally compare 10 alternatives in a horse race, including the winners of prior steps, the original Merton model, KMV, and CDLT.

Examination of a default model goodness may be of two types. The first is *Model's Power*, the separation capability of the model between observations of default and observations of solvency. The power relates to the goodness of the order in which the model ranks the observations. The second type, *Model's Calibration*, refers to the default probability values produced by the model and how they fit real probabilities. For example, consider a model that results in the following default probabilities (PD) for three companies (A, B, C): $PD_A = 0.1,\ PD_B = 0.05,\ PD_C = 0.01$. The model's power relates to the goodness of the model outcome in ranking the probabilities of default in the right order: $PD_A > PD_B > PD_C$. However, the goodness of the model's calibration relates to the accuracy of the probability values generated by the model. Stein (2002) argues that calibration improves when model power increases. Any calibration method should maintain the ranking order of the model. Hence, we follow prior studies and focus on model power. For this purpose we regard the probabilities (PD) calculated by a model as scores.[20]

Critical values of PD may be used by investors, lenders, or regulators to classify firms to high-risk or low-risk categories. The classification might be inaccurate. A false positive (FP) error relates to a solvent firm classified to the high-risk category, whereas a false negative (FN) error relates to a defaulting firm classified to the low-risk category. These are often referred to, by statisticians, as type I and type II errors, and are often estimated by empirical data of false positive and true positive rates (FPR and TPR respectively), for each critical value of PD, using a database of calculated PD observations and their related default/solvency realizations. Consider a critical value α. TPR, also called hit rate, is the number of defaulting firms classified as high-risk ($PD \geq \alpha$) divided by the total number of defaulting firms. FPR, also called false alarm rate, is the number of non-defaulting firms classified as high-risk ($PD \geq \alpha$) divided by the total number of solvent firms. There is an obvious tradeoff

[20] This is the common practice in the bulk of prior literature and research, yet often the distinction between model power and calibration is not explicitly mentioned.

between these two rates. As one lowers the critical value, he gains in hit rate (TPR) at the cost of higher false alarm rate (FPR).[21]

The Receiver Operating Characteristic (ROC) curve, a graph of TPR versus FPR, is a tool for comparing powers of alternative default models. Figure 1 shows ROC curves demonstrating the tradeoff between hit rates and false alarm rates for all possible critical values. A random model (with no predictability power) is simply the 45 degrees line. Model A is superior to model B when the ROC curve of A is always above the ROC curve of B. When the curves cross, one may compare the Area Under the Curve (AUC) relative to the alternative models. An AUC value is in the range [0, 1] and the AUC of a random model equals 0.5. We use the nonparametric approach of DeLong et al (1988) to test the statistical significance of differences between the AUC of alternative models. This test, which also controls for correlation between examined curves, is considered the most advanced statistics for ROC curves comparison. AUC tests look at the entire sample, which includes mostly non-defaulting firms that are assigned low PD's. Therefore, to enhance the robustness of the test we also focus on the interval of low FPR, where a small increase in FPR causes a large increase in TPR, i.e. $FPR \in (0, x)$. Such interval, for x = 0.25 for example, also seems more valuable for investors, lenders, or regulators then the entire curve (x = 1). This test is known as the Partial AUC (pAUC). In our final comparison of models and specifications (Table 17) we use pAUC with x = 0.5, 0.25, and 0.1 in addition to the entire AUC.

Prior studies such as Bharath and Shumway (2008) measured the accuracy of default models using the defaulting firms' fraction in the lowest-quality deciles among all defaulting firms in the sample. This method is in fact based on particular points on a power curve and does not encompass the information in the entire curve. A power curve shows the cumulative percentage of defaulting firms among all defaulting firms for each percentile of the predicting score. In other words, it shows the percentage of defaulting firms that are detected for each threshold value of the score ($\alpha$ in the above PD example).

[21] Two additional terms that are often used are *sensitivity* for hit rate and 1-*specificity* for false alarm rate.

The Accuracy Ratio (AR) is twice the area between the 45$^{o}$ line and the power curve and it is equivalent to ROC curve comparison, in fact $AR = 2 \cdot AUC - 1$.[22] Hence the deciles comparison method is also a limited snapshot of particular points on the ROC curve. A major advantage of using ROC curves is the availability of statistical inference methods and tests such as that of DeLong et al. (1988).

In addition to ROC curve analysis we also examine changes of selected variables prior to default. Our sample includes 101 defaulting firms with adequate input data for the examined models in each of the five years before the default event. We designate the reported year-end day prior to the year of the default event as time -1. (e.g., for a firm, having its year-end on December 31, that defaulted during the year 2005, time -1 refers to the estimation of 31 December 2004; time -2 denotes the estimation of 31 December 2003 and so on.) We compare the defaulting firms to a control group of 101 non-defaulting firms of the same period.[23]

## 6. Data

The initial sample for this study includes all firms in the merged CRSP-COMPUSTAT database of the period 1989 to 2012 and default events of 1990 to 2013. Daily stock returns and stock prices are taken from CRSP; book value of assets, short-term debt, long-term debt and the numbers of shares outstanding are from COMPUSTAT. For the risk-free interest rate $r$ we use the 1-year Treasury bill rate obtained from the Federal Reserve Board Statistics.

Our sources for default events are the annual default reports of Moody's and S&P for the years 1990-2013. Since these reports exclude unrated firms, we filter out all annual observations of unrated firms. Without such filtering, the sample would have had a large number of observations for which default information is not available, causing an obvious selection bias. Similar to Bharath and Shumway (2008)

[22] See Engelman, Hayden and Tasche (2003)

[23] For a firm which defaulted during the year 2005, we select a non-defaulting firm which operated in the years 2000-2005 in the same industry (2-3 SIC digits). Using the same principle we use for defaulting firm, we mark 31 December 2004 as time -1, 31 December 2003 as time -2 and so forth.

and others we exclude financial firms (SIC Codes 6000-6799). This filtering is needed since financial firms are characterized by high leverage and strict regulations. We also filter out defaulting firms for three years subsequent to a default event.[24] Our final sample contains 26,579 annual observations of 2,534 firms with 306 cases of defaults.

Table 1 shows the distribution of the sample over the years. The number of annual observations starts at 919 in 1990, increases to 1,300 in 1998 and then starts decreasing down to 861 in 2013. The annual number of defaults varies from one (in 2013) to 39 (in 2001).  As expected, default rate vary over time, peaking in 1998-2003 and in 2009. The overall number of defaults (306) seems sufficient for our analysis.

We use stock price data to compute the annual return $r_{E,-1}$, and the three volatilities presented in sub-section 4.7 above,  for each year preceding an annual observation of a company. The beta of stock returns ($\beta_E$) is estimated in a standard technique using the CRSP value-weighted return of NYSE/NASDAQ/AMEX index as the market index. The market value of equity $E$ for each annual observation equals the stock prices times the number of outstanding shares. Table 2 provides some descriptive statistics of the sample. The average market value in our sample is 6,744 million U.S. dollars, which is greater than 808.8 of Bharath and Shumway (2008). We relate this difference to the exclusion of unrated firms from our sample. The annual stock returns is widely dispersed.[25] The average $\beta_E$ in our filtered sample is 0.975, very close to one, as expected from a diverse sample of firms over more than two decades.

[24] For example, if a firm defaults in 2000, we estimate its probability to default on 31 December 1999 and then drop this firm from our sample for the years 2000, 2001 and 2002.

[25] It may seem odd that the minimum value of annual stock return is below -100%. Notice however that $r_{E,-1}$ stands for the continuously-compounded annual return.  e.g. in a rare case, when a stock drops by 80% in a year, its continuous rate of return is *ln*(0.2) = -161% per annum.  Bharath and Shumway (2008) winsorized their sample and hence their minimum value of annual stock return was -85.45%. However, their minimum value for annual asset return was also extremely low: -253.58%.

## 7. Results

We begin by an evaluation of the effects of changes to the default barrier, the expected asset returns, and the asset return volatility, using ROC curves and AUC methods (as discussed above), first on two and single equation Merton models and then on two and single equation DaO models. We then evaluate the performance of 10 models, including the high AUC specifications of two and single equation Merton and DaO models, together with KMV, CDLT, BhSh naïve model, and an alternative specification of even a simpler naïve model (SNM). Along this process, we discuss potential reasons and deductions from the observed results including the examination of volatility and return patterns prior to default of defaulting and non-defaulting firms and their respective leverage ratios.

### 7.1 The default barrier

We estimate the model using seven long-term debt (*LTD*) multipliers (k) values. For that purpose we calculate $A$ and $\sigma_A$ by solving equations (2) and (4) simultaneously (two-equation Merton) and assume that the assets drift $\mu_A = \mu_{MP=0.06} = r_f + \beta_A \cdot 0.06$. Panel a in Table 3 shows the AUC values for the respective ROC curves, they are almost similar (except for k =0) and the largest AUC is for k=0.1. It seems that the pervasive choice of k=0.5 might not be the optimal one. Using DeLong et al. (1988) test shows that the relatively small differences in the AUC values from that of k=0.5 are nevertheless statistically significant. The AUC grows as k becomes smaller, which points to a conclusion that short term debt (STD) is much more critical for predicting a default in one year time frame than the LTD. However, the reduction of AUC when k is reduced from 0.1 to 0 shows that the LTD should not be totally ignored in the estimation of PD. The analysis using pAUC for FPR≤0.25FPR interval results in similar outcomes, though obviously with much lower areas under the curve.

The AUC for the various $k$ specifications is around 0.93, which is equivalent to an Accuracy Ratio of 0.86. Duffie et al. (2007), for example, achieved an AR of 0.87 using a much more complex model. One cannot compare models by comparing their AUC or AR based on different samples, however, this comparison provides some support for the adequacy of our sample and comparability of our findings with prior studies.

We repeat the same assessment for the choice of k in a single-equation Merton model. The results are reported in Panel b of Table 3. In this case k=0.5 provides the highest AUC, though this is not significantly different from that of k = 0.3, 0.7, 0.9, and 1. The pAUC (of 0.25) shows that k = 0.3 is slightly higher than that of 0.5, though the difference is not statistically significant, and k = 0.5 pAUC is higher than that of the others and the difference is statistically significant. This is an interesting result, as the use of k=0.5 origin, to our knowledge, is KMV and its model is essentially based on a single equation Merton, where the asset volatility is estimated using the iterative process. This choice, of k=0.5, which seems optimal for the single equation Merton, is widely adopted by researchers and practitioners for other models, often utilizing two-equation Merton (e.g. Campbell et al., 2008).

In light of the above findings regarding k, it is interesting to further focus on the LTD and its evolution in the years prior to default, for defaulting firms and a control group of non-defaulting firms. Table 4 shows the evolution of $LTD/A$ prior to default, where $A$ is obtained from the two-equation Merton model with $k = 0.5$ for the default barrier. Using t tests and Wilcoxon rank-sum (Mann-Whitney) tests we find statistically significant differences between $LTD/A$ ratio of defaulting and non-defaulting firms. Furthermore, the gap between the two groups increases as firms come nearer to the default event. The average value for the defaulting firms, five years before default is 0.541 in comparison to 0.387 for the non-defaulting firms. As time passes, the $LTD/A$ ratio of the non-defaulting firms increases by less than 40% whereas the ratio for the defaulting firms doubles, on average.[26] A year before default the average ratio for the defaulting firms reaches 1.085 while the average ratio for the non-defaulting firms is 0.533 only.

While it appears that $LTD$ by itself exhibits predictive power, the model power is only slightly sensitive to the $LTD$ multiplier (except for the extreme k=0). This somewhat puzzling behavior is a result of the calculation method of $A$ and $\sigma_A$. The firm equity is regarded as a call option on the firm assets. Hence, an under-specification of the strike price (default barrier) results in an underestimation of the underlying

[26] The increase in the ratio for non-defaulting firms may be associated with systematic risk. This is caused by matching the firms in the control sample to the defaulting firm calendar year of default. Hence, if default risk has a systematic component we may, on average, expect financial deterioration also among the control group firms in the years prior to the defaulting firm default.

assets value ($A$) and overestimation of the assets volatility ($\sigma_A$) in a simultaneous solution of equations (2) and (4). Underestimation of $A$ or overestimation of $\sigma_A$ results in a reduction in the distance to default and thus an increase in the probability of default, hence reducing the sensitivity of PD to changes in *k*. The underestimation of the probability of default caused directly from under-specification of the default barrier is compensated indirectly by underestimation of $A$ and overestimation of $\sigma_A$. This seems to explain the model low sensitivity to the default barrier specification.[27] Table 5 shows the average and median dependence of $A$ and $\sigma_A$ on k for the two and single equation Merton models. As expected, for lower values of the default barrier (small *k*) we find lower mean and median $A$ and higher mean and median $\sigma_A$. Furthermore, we find that the mean and median asset values for the two-equation model are higher than those of the single-equation model, for all k values in the table (compare panels a and c) and these results are statistically significant (see panel c). This supports the above explanation regarding the puzzling effect of k on the two-equation model power. Using t tests and Wilcoxon sign-ranked tests we find that the differences of values resulted from various specifications of *k* compared to the values calculated using *k*=0.5 are statistically significant.

Table 6 shows that PD is highly skewed, as expected. Its mean and median are widely apart under each of the seven specification of the *LTD* multiplier, e.g. for $k = 0.5$ the mean PD is 0.017 and the median is $4.02 \cdot 10^{-9}$. As defaults are rare events (often about one percent of the sample), basing model comparison on deciles (as done in some prior studies) might be misleading. In our sample PD values start to vary substantially only within the highest five percent group.

The seven *LTD* multipliers (*k*) we use yield substantially different probabilities of default. For example, using a high *LTD* multiplier of 0.9 the mean *PD* is 80% larger than the mean *PD* using a low *LTD* multiplier of 0.1. t tests and Wilcoxon sign-ranked tests reveal that the mean (median) probabilities of default for *k*=0, 0.1, 0.3 are lower than those of *k*=0.5, and for *k*=0.7, 0.9, 1 are higher than those of *k*=0.5. This suggests that the calibration of the model is substantially different for each specification.

[27] This somewhat counter intuitive result is caused by the model which maintains the equity value and the equity volatility constant. In “real life” changing the debt level of a firm, or its default barrier, would affect its equity value and probably its equity volatility too.

However, as discussed above, the model's power (the ability to distinguish a defaulting firm from a non-defaulting firm) is relatively less sensitive to *k*.

### 7.2 The expected return on the firm's assets ($\mu_A$)

We examine several alternatives to assess the two and single equation Merton model sensitivity to asset expected returns. In all cases we use *k*=0.5 for the default barrier and $\sigma_{E,-1}$ for the equity volatility. Recall our definitions: $\mu_{MP=0.06} = r + \beta_A \cdot 0.06$ and $\mu_{MP=MKT} = r + \beta_A \cdot (MKT_{-1} - r)$, where $MKT_{-1}$ is the annual rate of return of the market index in the previous year and $\beta_A = \beta_E \cdot \frac{\sigma_A}{\sigma_E}$.

Panel a of Table 7 lists the summary statistics of $\mu_A$ under various specifications. The averages of $\mu_{MP=0.06}$ and $\mu_{MP=MKT}$ are similar. Naturally, the variance of $\mu_{MP=MKT}$ is greater than that of $\mu_{MP=0.06}$. Overall, one would expect average $r_{E,-1}$ to be higher than $\mu_A$ because equity is a leveraged long position on assets, and indeed $r_{E,-1}$ is larger than $\mu_{MP=0.06}$ and $\mu_{MP=MKT}$. Thus, as expected, the mean of $max(r, r_{E,-1})$ is very high 0.390, compared, for example, with 0.082 of $max(r, \mu_{MP=0.06})$.

For the two-equation Merton, panel b in Table 7 shows that using the risk-free rate $\mu_A = r$ results in the largest AUC, though it is only slightly larger than that of $\mu_{MP=0.06}$ and of $max(r, \mu_{MP=0.06})$. The risk-free rate AUC difference from the other alternatives is statistically significant, except for the almost similar performance of the two alternatives related to $\mu_{MP=0.06}$. These relations are supported by the pAUC analysis for FPR≤0.25. The choice of $\mu_A = r$ results in risk-neutral probability of defaults, under the model assumptions. This is obviously not a better calibrated scale than that of the two alternatives related to $\mu_{MP=0.06}$, yet we see that model power can be achieved without PD calibration, and in this case, with the trivial choice of the risk-free rate.

Panel c in Table 7 shows that for the single-equation Merton, using $\mu_A = max(r, r_{E,-1})$ results in the largest AUC, which is only slightly larger than that of $r_{E,-1}$. A few points are worth mentioning here. First, all the AUCs in panel c are higher than those of panel b, suggesting that to enhance the power of the model, a single-equation Merton could be preferable to the "classic" (textbook) two-equation

Merton model. Second, in a single-equation Merton, where we substitute $\sigma_{E,-1}$ for the asset volatility, a corresponding match of equity returns increases the AUC. Third, that the choice of $\mu_A$ has relatively a weak effect on the single-equation Merton model power, with the lowest AUC for $\mu_{MP=MKT}$, which is statistically significant lower than that of $max(r, r_{E,-1})$ and $\mu_{MP=0.06}$. We believe there are two effects causing this result. First, the asset volatility seems to be a prime factor affecting PD and equity volatility ($\sigma_{E,-1}$) seems more reliable than the $\sigma_A$ extracted by the two-equation Merton model (for model discrimination ability). This explains the overall higher AUC of the single-equation Merton model presented in panel c, relative to that of panel b. Second, it seems that the prior year equity returns ($r_{E,-1}$) performs well as a proxy for the firm next year performance and provides almost identical results to that of a CAPM estimator of the asset expected returns, using a fixed 0.06 market premium. Using the prior year market premium ($\mu_{MP=MKT}$) results in the lowest AUC in panel c, because last year market returns are often a poor predictor of next year returns.[28] Yet last year beta seems to perform quite well with a fixed market premium in both panels b and c.

Panel a of Table 8 shows the evolution of the previous year equity return ($r_{E,-1}$) of defaulting firms during the five years preceding the default event. For comparison, the table includes also the data of a control group of 101 non-defaulting firms. During the period, up until two years prior to default, the two groups average and median equity returns are not statistically different. Both average and median returns of defaulting firms decline a year prior to default and their difference from the values of the non-defaulting control group are statistically significant. To calculate real (physical) default probabilities instead of the risk-neutral measure, using equation (6), $\mu_A$ replaces $r$ for the drift in DD (equation 5). Panel b of Table 8 presents the evolution of $\mu_A$ prior to default of defaulting versus the non-defaulting control group and shows that their difference becomes statistically significant in the last two years preceding the default.

[28] For example, a quick test of S&P500 annual returns in the period 1950-2013 shows no significant autocorrelations for lags 1-20 years. Ljung-Box Q-test for autocorrelation cannot reject the null hypothesis of white noise (p-value = 0.8538).

Figure 2a (2b) presents the evolution of the median $r_{E,-1}$ ($\mu_A$) for both groups. It is logical to expect that investors demand higher returns from a riskier firm compared to a safer one. However, $r_{E,-1}$ is the realized historical return (not the forward looking expected return) and its negative value may indicate financial deterioration prior to default. As Table 8 and figure 2 show, defaulting and non-defaulting firms generally have similar returns five years before default. However, when firms approach default, their median equity return falls below that of non-defaulting firms and even becomes negative one year prior to default. This result is consistent with prior papers such as Vassalou and Xing (2004) that discovered a negative equity excess return for credit risk.

On the one hand $r_{E,-1}$ exhibits predictive power, lower $r_{E,-1}$ are observed with higher probabilities of default. On the other hand, historical equity returns of firms approaching default may yield biased estimates for $\mu_A$ and hence harm the precision of the model. It appears that using $\mu_A = \max(r, r_{E,-1})$ mitigates some of the inaccuracy caused by using historical returns, instead of forward-looking returns, by reducing the effect of negative realized returns.[29] This is of practical importance for the single-equation Merton model, where $\mu_A = \max(r, r_{E,-1})$ results in the highest AUC.

### 7.3 The volatility of the assets ($\sigma_A$)

As a firm approaches a default event often, both equity volatility and leverage increase. These two processes affect the calculation of asset volatility in opposite directions. We examine changes in equity and asset volatilities as the firms approach their default event. We use a sample including the 101 defaulting firms and a comparison group of 101 non-defaulting firms in parallel years, as explained earlier. Table 9 panel a shows that the mean of historical equity volatility of defaulting firms increases from 0.504 five years before default to 0.959 a year before default. In the same period, the average volatility of equity for the non-defaulting group increases slightly from 0.395 to 0.556. t tests and Wilcoxon rank-sum tests reveal that in all these years, equity volatility is statistically significant higher

[29] This work is not the first to use risk free rate for the lower bound of expected asset returns, though in somewhat other specifications. Prior papers generally use a single specification in each paper, examples include Hillegeist et al (2004) and Charitou et al. (2013). However, we believe this is the first study of a wide range of alternative asset returns specifications.

for defaulting firms than for non-defaulting firms. Figure 3a shows graphically the evolution of median equity volatility for the two groups.

This development in historical equity volatility is expected. However, this is not the case for the historical assets volatility.[30] Panel b in Table 9 demonstrates that contrary to the non-defaulting firms, historical assets volatility of defaulting firms, calculated by Merton Model, decreases, on average, as the time to the default event becomes shorter. Five years prior to default the mean of historical assets volatility is 0.349 while a year before default it is 0.295. In the same period, the mean of historical assets volatility of non-defaulting firms increases from 0.299 to 0.360. Whereas the historical assets volatility difference between defaulting firms and non-defaulting firms five years prior to default is statistically insignificant, it becomes negative and statistically significant in the year before default. The evolution of median historical asset volatilities (see Figure 3b) and Wilcoxon rank-sum tests portray a similar picture and hence it seems that this pattern is not caused by outliers.

We suspect that our findings regarding assets volatility are related to our use of historical equity volatility rather than expected volatility. Historical volatility of equity is computed using prior year data whereas the equity value is current. As a firm approaches default, its equity value decreases and its equity volatility increases. Hence using up-to-date equity value jointly with out-of-date equity volatility value causes an underestimation of assets volatility. To assess this hypothesis, we examine assets volatility calculated by the model, using equity volatility implied by stock options market prices as input, instead of historical equity volatility. Implied volatility is forward-looking by nature. Bharath and Shumway (2008) showed that using implied volatility substantially improves Merton model results. We now examine the source of this improvement. It should also be noted that using implied volatility substantially reduces model’s applicability since stock options are not available for all the firms. Therefore, this examination is merely intended to assess the goodness of current practices in Merton model application.

[30] By historical assets volatility we refer to the $\sigma_A$ estimated by the two-equation Merton, using historical equity volatility as the input to the model. We later discuss implied assets volatility, calculated similarly, using implied equity volatility as the input.

Following Bharath and Shumway (2008), for each firm we select the implied volatility of at-the-money 30-day call option on its stocks ($\sigma^{imp}$). We use Optionmetrics data which is available since 1996. Thus, for observations of 1995 year-end we use the data of the first trading day in 1996 (a single trading day shift) and for other observations we simply use data for the year-end date. Due to the limited availability of implied volatility data, our sample decreases from 26,579 annual observations to 14,490 and the number of defaults diminishes from 306 to 88.

Figure 4 shows the average implied volatility in our sample alongside the average historical volatility ($\sigma_{E,-1}$) throughout the period 1995-2012. Though one should be careful drawing conclusions from a chart of averages, there are a few observations that seem worth mentioning. First, the two curves look similar in pattern and value. Second, most of the time it seems that the historical volatility lags the implied volatility. The lag in 2009 is most prominent. The deep drop in implied volatility is accompanied by a much milder drop in historical volatility and the gap closes in the following year. Third, it seems that since 2010 the historical volatility lag vanishes, though no conclusions can be drawn from this short period. The relation between the two charts that Figure 4 demonstrate, support prior research (such as Bharath and Shumway 2008) preference for the use of implied volatilities.

Table 10 panel a shows the historical volatility and implied volatility of equity and assets for 14,490 annual observations for the years 1995-2012. Implied and historical volatilities of equity are $\sigma^{imp}$ and $\sigma_{E-1}$ respectively. Assets volatility is calculated by solving the two-equation Merton model, assuming the default barrier is $D = STD + 0.5 \cdot LTD$. Implied and historical assets volatilities are calculated using $\sigma^{imp}$ and $\sigma_{E-1}$, respectively, as the input to the model. P values are for differences between historical volatility and implied volatility. It appears that implied volatility is greater, on average, than historical volatility for defaulting firms, for both equity and assets; the differences are statistically significant using t-tests or Wilcoxon rank-sum tests. However, these differences are miniscule for non-defaulting firms (and generally may statistically be rejected).

We conjecture that the comparison between historical and empirical volatilities is significantly affected by the great and irregular difference between these values in 2009. Panel b shows similar information and results using data that excludes 2009 observations (end of 2009 data and 2010 defaults). Here the differences between implied and historical equity and asset volatilities are statistically significant for both defaulting and non-defaulting firms. Although only a few percent of the observations are excluded from panel b compared to a, the single omitted year changes the statistical significance of non-defaulting firms. We also observe that one year before default equity volatility is larger for defaulting firms than for non-defaulting firms, as expected. However, surprisingly, assets volatility is larger for non-defaulting firms than for defaulting firms.

These results suggest that the use of historical volatility (rather than expected volatility) might harm Merton model applications. This practice apparently causes an underestimation of assets volatility. We explore below the use of a dynamic volatility estimation model (JP) as a practical substitution for the implied volatility.

Table 11 compares the AUC (and pAUC at the 0.25 FPR level) for eight specifications of assets volatility. The first is our benchmark model, in which assets volatility ($\sigma_A^{2eqM}$) is calculated using the two-equation Merton model, assuming the equity volatility equals the historical volatility ($\sigma_E = \sigma_{E,-1}$). The specifications $\sigma_A^{2eqM}(\sigma_E = \sigma_{JP})$ and $\sigma_A^{2eqM}(\sigma_E = \sigma_{MAD})$ are also calculated using the two-equation Merton model but assume that the equity volatility equals the JP-Morgan volatility ($\sigma_{JP}$) and MAD volatility ($\sigma_{MAD}$) respectively. The three subsequent models are of a single-equation Merton type, where assets volatility is set equal to either the equity volatility ($\sigma_{E,-1}$), the JP-Morgan volatility ($\sigma_{JP}$), or the MAD volatility ($\sigma_{MAD}$) respectively. In these six models the default barrier is $D = STD + 0.5 \cdot LTD$ and the expected returns on the firm's assets is set to $\mu_A = \mu_{MP=0.06}$ (based on the $\beta_A$ of the assets calculated from historical $\beta_E$ of equity and assuming the market premium equals 0.06). Model 7 $\sigma_A^{KMV}$ is based on the iterative method presented in subsection 4.1 and model 8 $\sigma_A^{CDLT}$ is based on the Charitou et al. (2013) model presented in subsection 4.3. P values are of DeLong, et al. (1988) test for the difference between the AUC of the alternative specifications and the partial AUC (at the 0.25 FPR level) respectively.

Model 5, the single-equation Merton model using JP-Morgan volatility ($\sigma_{JP}$) for the asset volatility, results in the highest AUC. This result is statistically significant compared to all other models, except for the two other single-equation models using $\sigma_{E,-1}$ and $\sigma_{MAD}$ (models 4 and 6 respectively). The pAUC tests results lead to compatible conclusions. It should be noted that equity volatility is always larger than assets volatility calculated using a two-equation model, for all firms either defaulting or non-defaulting. This is a cross-firm effect that may be adjusted in the calibration process of the model. However, the calculated assets volatility ($\sigma_A^{2eqM.}$), using historical equity volatility in equation (2) and (4) underestimates assets volatility mainly for defaulting firms and hence reduces model's power. Model 7 using KMV iterative model results in intermediate AUC between those of the two-equation and single-equation models. Model 8, based on CDLT as we describe it in subsection 4.3, results in the lowest AUC, presumably because its volatility does not capture the riskiness of defaulting firms, in contrast with $\sigma_{JP}$ which responses fast to recent changes in equity volatility.

### 7.4 Down and out (DaO) call alternatives

From the above it appears that simpler models result in a better explanatory power than more complex calculations, we explore here the more complex DaO model, which relaxes the assumption that default may occur only at the end of the examined horizon, at time T. Instead, DaO allows a default to occur at any time $t \in [0, T]$. The details of the model are presented in subsection 4.4.[31]

Table 12 shows AUC and pAUC (at the 0.25 FPR level) for several specifications of the two-equation Down and Out (DaO) model. The LTD multipliers ($k$) in the default barrier specification ($D = STD + k \cdot LTD$) varies in panel a and is set to $k = 0.5$ in the other panels. The expected return on the firm's assets varies in panel b and is set to $\mu_A = \mu_{MP=0.06}$ in the other panels (see $\mu_A$ specification details in subsection 7.2 and Table 7). Equity volatility specification varies in panel c and is set to $\sigma_{E,-1}$ in the other panels, see alternative equity volatility details in subsection 4.7. Asset volatility and asset value

[31] Some prior research, such as Dionne & Laajimi (2012) and Brockman, & Turtle (2003), use DaO to find an implied barrier (H), adjusting the book asset value by the market equity, yet ignoring credit risk effect on the assets. This assumption allowed them to find an implied barrier as a proportion of their assumed asset value and to analyze H≠D. In this paper, on the other hand, we assume that the barrier equals D = STD + K*LTD, consistent with the other models evaluated in this work, and thus we calculate the asset value which is implied by the model.

are the solution of the two-equation DaO model presented in subsection 4.4. P-values are of DeLong, et al. (1988) test for the difference between the AUC of the various specifications.

Panel a of Table 12 shows similar results to that of panel a in Table 3, i.e., the highest AUC is for k=0.1 and relatively a low sensitivity to k except for a decline when k=0. Thus, in two-equation DaO, similar to two-equation Merton model, the exact coefficient of LTD is not crucial for model power; however, LTD should not be entirely ignored in the model. Panel b of Table 12 displays similar results to that of panel b in Table 7. The two-equation DaO highest AUC is for $\mu_A = r$ and it is only slightly larger than that of $\mu_{MP=0.06}$ and of $max(r, \mu_{MP=0.06})$. The risk-free rate AUC difference from the other alternatives is statistically significant, except for the almost similar performance of the two alternatives related to $\mu_{MP=0.06}$. These relations are supported by the pAUC analysis for FPR≤0.25. Panel c of Table 12 presents similar results to models 1-3 of Table 11, with the highest AUC using $\sigma_{JP}$, followed by $\sigma_{MAD}$, and lastly $\sigma_{E,-1}$, however, the differences are not statistically significant.

To conclude the DaO analysis we repeat the above evaluation for the single-equation DaO model, summarizing the results in Table 13. Panel a shows, that very similar to single-equation Merton (Table 3 panel b), k=0.5 provides the highest AUC, though this is not significantly different from that of k = 0.3, 0.7, 0.9, and 1. The pAUC (of 0.25) shows that k = 0.3 is slightly higher than that of 0.5, though the difference is not statistically significant, and k = 0.5 pAUC is higher than that of the others and the difference is statistically significant, except for k = 0.7. Table 13 panel b shows similar results to those of the single-equation Merton (panel c in Table 7), i.e. that for the single-equation DaO, using $\mu_A = r_{E,-1}$ results in the largest AUC and the result is statistically significant, except for the AUC of $max(r, r_{E,-1})$ which is only slightly lower than that of $r_{E,-1}$.[32] Similar results hold for the pAUC, though the p-values are lower than those of the AUC. Lastly, Table 13 panel c shows similar results to those of the single-equation Merton (Table 11 models 4-6), the single-equation DaO model using JP-

[32] For the single-equation Merton, the choice of $\mu_A = max(r, r_{E,-1})$ results in a slightly higher AUC than that of $r_{E,-1}$.

Morgan volatility ($\sigma_{JP}$) for the asset volatility, results in the highest AUC compared to the two other single-equation models using $\sigma_{E,-1}$ and $\sigma_{MAD}$. The pAUC tests results supports those of the AUC.

### 7.5 Model alternative comparison

At this final stage of the analysis we compare 10 representative models. Model 1 is the textbook Merton model. Models 2-5 are two and single equation Merton and DaO models, using volatility and asset expected returns specifications, which result the highest AUC above. BhSh naïve model is model 6 (see details in subsection 4.2). This model inspires our simple naïve model (SNM) for which we present results for two choices of volatility, $\sigma_{E,-1}$ and $\sigma_{JP}$, which we call model 7 and 8 respectively (see details in subsection 4.6). KMV and CDLT results (models 9 and 10 respectively) are included for comparison. We start by comparing the 10 models' AUCs for the entire sample. Then we compare AUCs dividing the sample to two periods, until 2000 and after it. Next we look at industry samples and finally, for robustness check, we look at the pAUC of the entire sample for three levels of FPR, 0.5, 0.25, and 0.1. In the tables we present DeLong et al (1988) p-values for AUC differences from models 1, 5, and 7 because model 1 is widely used as a reference model and models 5 and 7 are leading in model power in this work.

Panel a in Table 14 summarizes the 10 models' results for the entire sample. It shows that model 5, the single-equation DaO, using $\sigma_{JP}$ for asset volatility and $r_{E,-1}$ for asset expected returns, results in the highest AUC and this result is statistically significant, over all other models, except for model 7, which is ranked second by its AUC. Model 7 is probably the simplest possible application of a Merton type model, simpler than BhSh naïve and CDLT models. Yet, using $\sigma_{E,-1}$ for asset volatility and $max(r, r_{E,-1})$ for asset expected returns, model 7 results in the highest AUC (except for model 5) and this result is statistically significant, over all other models, except for model 5 and 3. Panels b and c support these results, showing that models 5 and 7 AUC surpasses the others when we look at each period, 1990-2000 and 2001-2013, and in the later period model 5 and 7 result in equal AUCs.

Table 15 follows panel a in Table 14, looking at three industry related sub-samples: manufacturing (panel a); transportation, communications, electric, gas, and sanitary services (panel b); and others (panel c).[33] In the manufacturing division, model 7 (SNM), the simplest model, has the highest AUC, larger even that that of model 5 which comes second, and its AUC is statistically significant (p-value ≤ 5%) higher than that of models 1, 6, 8 and 10 only. In the other two industry groups model 5 has the highest AUC, model 3 is second and model 7 is third. The statistical significance of model 5 includes models 2, 4, 6, 9, and 10 (and model 8 for other industries). Overall, the superiority of model 5 (DaO using $\sigma_{JP}$) and model 7 (SNM) appears to be industry independent.

Table 16 shows pAUC results for FPR levels of 0.5, 0.25, and 0.1 in panels a, b, and c, respectively. Model 5 leads with the highest pAUC in 0.25 and 0.1 levels where model 7 has the second highest pAUC and the positions of leader and follower reverse for the 0.5 FPR level. In all three levels the pAUC differences between these two models are not statistically significant. In all three levels model 5 pAUC differences from all other models (except model 7) is statistically significant.

We summarize the "horse race" of the 10 models in Table 17, showing the ranking in 1-10 in the comparisons of Tables 14-16, 1 is assigned to the highest AUC (pAUC). For visual clarity we use dark shading for the first place and light for the second larger AUC, both with larger and bold font. From the above analysis there is no surprise that model 5 is the highest ranked, on average, followed by model 7. Except for the manufacturing subsample where it is ranked eighth, model 10 is ranked last in all the tables. Probably our greatest surprise is model 6 which almost always takes the ninth position, with two exceptions (until 2000 and other industries) in the eighth and once (manufacturing) in the tenth position. The other surprise is model 9, the laborious KMV iterative model, which most often occupies the seventh position, except for the fifth place in manufacturing and other industries, and the subsample prior to 2000, whereas after 2000 it occupies the eighth place. Lastly, we assume equal weight to all ordinal ranking in Table 17 and find an average ordinal ranking over the nine columns. The result is

[33] The manufacturing division includes all firms with SIC code 2000-3999. The transportation, communications, electric, gas, and sanitary services division includes all firms with SIC codes 4000-4999. The other divisions included each a small number of default observations and therefore had to be merged into a single division which we named 'others'.

identical to that of the entire sample AUC of Table 14 panel a, except for a swap in the fifth (two-equation Merton, model 2) and sixth (SNM, model 8) positions.

Table 17 shows that KMV, model 9, performs better than BhSh naïve model 6 in all 9 columns. We thus conclude that the superiority of the BhSh naïve model (presented in their paper) over the KMV model is not robust. Looking at DeLong et al (1988) test p-values, the differences between model 1 and 6 are not statistically significant, except for the period after 2000 where model 1 is ranked fifth and model 6 is ranked ninth. This result is puzzling, especially given the better performance of model 7 (SNM) which is almost identical to that of BhSh except for the choice of asset volatility and expected returns. Whereas BhSh assumes that the assets volatility is a weighted average of the equity volatility and an enigmatic debt volatility (see equations 7 and 8), model 7 simply uses the equity volatility ($\sigma_A = \sigma_{E,-1}$). While BhSh uses the equity returns over the prior year for the expected asset returns ($\mu_A = r_{E,-1}$), model 7 ensures the value of the expected returns is not lower than the risk free rate ($\mu_A = max(r, r_{E,-1})$). The power of the simple naïve model 7 is higher than that of BhSh naïve model in all nine columns of Table 17 and the results are statistically significant. Furthermore, a slight change in SNM, using JP volatility for the asset volatility ($\sigma_A = \sigma_{JP}$) in model 8, results in superior AUC compared to BhSh in all nine columns. However, this change results in inferior performance compared to model 7 and the results are statistically significant in most tables. This ranking of very similar models: $6 \prec 8 \prec 7$, is robust in all our tests in this work including subsamples of period and industry for AUC and pAUC.

The most powerful model is the single-equation DaO, model 5. However, the close second place SNM, model 7, is the simplest to calculate. Since their AUCs are not statistically different and model 7 is much simpler than model 5, model 7, the simple naïve model, is our practical recommended choice, when model discriminatory power is the goal.

## 8. Conclusions

This paper is motivated by the fact that Merton (1974) model is (surprisingly) still used extensively by practitioners, and even academics, to assess credit risk. Although four decades passed since its official introduction, and despite the development and publications of various extensions and alternative models, the original model is still very useful. In this paper we examine the sensitivity of Merton model default prediction performance to its parameter specifications. We assess the causes for this sensitivity and evaluate the performance of a wide range of model alternatives, including those suggested by other recent studies. We conclude by providing a few prescriptions to enhance the model accuracy and suggesting a very simple model, which provides excellent discriminatory power for a low computation effort.

This work compares various alternatives for the application of the Merton model in default prediction. For this purpose, we compare the Area Under the Curve (AUC) of Receiver Operating Characteristic (ROC) curves and use the DeLong et al. (1988) nonparametric test to measure the statistical differences between the ROC curves. For robustness, we also include partial AUC (pAUC) calculations and test for pAUC differences, often at a few levels. We also examine how key inputs evolve over time prior to default, of defaulting and non-defaulting firms. The alternatives we consider are of model type and of specification details. Model types include the textbook two-equation Merton model, its down and out (DaO) barrier alternative, the iterative model which is widely believed to be that of KMV, and single equation models and shortcuts including Bharath and Shumway (BhSh) naïve model, Charitou et al. (CDLT), and our simple naïve model (SNM). We focus on three main specification details: the default barrier expressed by the long-term debt (LTD) multiplier k, asset expected return ($\mu$) specification, and volatility specification and estimation method.

Our results conform to those by Jessen and Lando (2014) who provided an interesting perspective on Merton model. They focused on the functional robustness of the distance to default (DD, presented in section 2) to model misspecifications. . Using simulations they showed that, in general, DD is successful in ranking firms' default probabilities, even if the underlying model assumptions are altered.

Our paper, on the other hand, uses empirical data to analyze which underlying model assumptions can be altered to enhance the model power and even simplify the model application.

Overall we find that simplified applications of the model have superior model power compared to more complex and computational intensive methods. The setting of the default barrier appears to have a small impact on the separation ability of the model. However, the specification of assets expected return and assets volatility is important. We use DaO in a comparable method to that of the textbook Merton model and find that its single-equation alternative, using k=0.5, prior year return on equity for its drift ($\mu = r_{E,-1}$), and JP equity volatility ($\sigma_{JP}$), provides the highest AUC and pAUC which are statistically significant, compared to all models, except for the simple naïve model that we propose, which comes a close second to the best, however it is much simpler to apply than the DaO. We find that these two models outperform model suggested in previous studies.[34]

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[34] We acknowledge that the CDLT model may be superior in estimating the probability of default in longer horizons than one year. We leave this matter for future research.

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# Tables

## Table 1: sample distribution over time

This table reports the observations' distribution in the sample period. The table presents the number of firms observed, the number of default events during the year and the ratio between them for each year in the sample.

| Year | Number of observations | Number of default events (during the Year) | Ratio of default events to the observations |
|---|---|---|---|
| **1990** | 919 | 11 | **1.20%** |
| **1991** | 943 | 11 | **1.17%** |
| **1992** | 958 | 7 | **0.73%** |
| **1993** | 1,016 | 4 | **0.39%** |
| **1994** | 1,085 | 5 | **0.46%** |
| **1995** | 1,178 | 8 | **0.68%** |
| **1996** | 1,231 | 6 | **0.49%** |
| **1997** | 1,250 | 9 | **0.72%** |
| **1998** | 1,300 | 14 | **1.08%** |
| **1999** | 1,237 | 21 | **1.70%** |
| **2000** | 1,210 | 23 | **1.90%** |
| **2001** | 1,229 | 39 | **3.17%** |
| **2002** | 1,183 | 35 | **2.96%** |
| **2003** | 1,204 | 20 | **1.66%** |
| **2004** | 1,186 | 11 | **0.93%** |
| **2005** | 1,158 | 12 | **1.04%** |
| **2006** | 1,138 | 4 | **0.35%** |
| **2007** | 1,113 | 2 | **0.18%** |
| **2008** | 1,099 | 8 | **0.73%** |
| **2009** | 1,091 | 32 | **2.93%** |
| **2010** | 1,029 | 9 | **0.87%** |
| **2011** | 991 | 7 | **0.71%** |
| **2012** | 970 | 7 | **0.72%** |
| **2013** | 861 | 1 | **0.12%** |
| **Total** | **26,579** | **306** | **1.15%** |

## Table 2: Model variables - summary statistics

This table reports summary statistics for all the variables used in the Merton model. BA (Book Assets) is the book value of *Total Assets*; LTD is the *Long Term Debt* ; STD is the *Short Term Debt*; E is the firm's market value of equity (the product of the price per share times the number of outstanding shares); $\boldsymbol{r_{E,-1}}$ is the annual firm's equity return (the average daily equity return times the number of trading days); $\sigma_{E,-1}$ is the annual firm's stock return volatility (the standard deviation of daily stock returns times the square root of the trading days in a year); $\beta_E$ is the beta computed from daily return and the value-weighted CRSP index (NYSE/NASDAQ/ AMEX). BA, LTD, STD and E are measured in millions of US dollars. The other variables are presented in decimal fractions. The data is as of the end of each fiscal year for the period 1989-2012 for default prediction (and observations) in the period 1990-2013

| | Variable | Mean | Std. dev. | Min | Max |
|---|---|---|---|---|---|
| BA | Book value of assets | 6460 | 18984 | 2 | 371933 |
| LTD | Long term debt | 1440 | 3866 | 0 | 77927 |
| STD | Short term debt | 333 | 2363 | 0 | 134136 |
| E | Market value of equity | 6744 | 20792 | 2 | 504240 |
| $\boldsymbol{r_{E,-1}}$ | Stock return | 0.326 | 0.571 | -2.272 | 18.796 |
| $\boldsymbol{\sigma_{E,-1}}$ | Stock return volatility | 0.463 | 0.257 | 0.060 | 4.274 |
| $\boldsymbol{\beta_E}$ | Beta of stock return | 0.975 | 0.570 | -2.174 | 4.567 |

## Table 3: Area under ROC curves for various specifications of the default barrier

This table shows AUC (area under the ROC curve) and pAUC (partial area under the ROC curve at the 0.25 level of type I error) for different values of the $LTD$ multipliers ($k$) in the default barrier specification ($D = STD + k \cdot LTD$), where $STD$ is short-term debt and $LTD$ is the long-term debt. The expected return on the firm's assets is set to be $\mu_A = \mu_{MP=0.06}$; i.e. based on the $\beta_A$ of the assets extracted from historical $\beta_E$ of equity and the assumption that the market premium equals 0.06. P-values are of DeLong, et al. (1988) test for the difference between the AUC of the particular k and the AUC of k=0.5.

**Panel a: two-equation Merton model**

| K | AUC | P value for difference from AUC for k=0.5 | pAUC (0.25) | P value for difference from pAUC for k=0.5 |
|---|---|---|---|---|
| **0.0** | 0.8720 | 0.000 | 0.1754 | 0.000 |
| **0.1** | 0.9317 | 0.001 | 0.1938 | 0.000 |
| **0.3** | 0.9300 | 0.000 | 0.1920 | 0.000 |
| **0.5** | 0.9277 | - | 0.1899 | - |
| **0.7** | 0.9256 | 0.000 | 0.1881 | 0.000 |
| **0.9** | 0.9241 | 0.000 | 0.1866 | 0.000 |
| **1.0** | 0.9233 | 0.000 | 0.1859 | 0.000 |

**Panel b: single-equation Merton model**

| K | AUC | P value for difference from AUC for k=0.5 | pAUC (0.25) | P value for difference from pAUC for k=0.5 |
|---|---|---|---|---|
| **0.0** | 0.8713 | 0.000 | 0.1760 | 0.000 |
| **0.1** | 0.9388 | 0.010 | 0.2007 | 0.030 |
| **0.3** | 0.9417 | 0.314 | 0.2032 | 0.870 |
| **0.5** | 0.9422 | - | 0.2031 | - |
| **0.7** | 0.9420 | 0.438 | 0.2027 | 0.047 |
| **0.9** | 0.9416 | 0.254 | 0.2021 | 0.015 |
| **1.0** | 0.9414 | 0.193 | 0.2018 | 0.009 |

## Table 4: Evolution of long-term debt to assets ratio prior to default

This table shows the evolution of LTD/A for 101 firms on December 31 for each of the 5 years preceding the default year. A is the value of assets extracted from the two-equation Merton model assuming a default barrier of $D = STD + 0.5 \cdot LTD$ where STD is short-term debt and LTD is long-term debt and using $\sigma_{E,-1}$ in equation (4). A control group of 101 non-defaulting firms is added for comparison. P values are for differences between the group of defaulting firms and non-defaulting firms.

| Years before default | Defaulting | | | Non-Defaulting | | | P value for difference | |
|---|---|---|---|---|---|---|---|---|
| | Obs. | Mean | Median | Obs. | Mean | Median | t test | Wilcoxon rank- sum (Mann-Whitney) |
| **5** | 101 | 0.541 | 0.451 | 101 | 0.387 | 0.261 | 0.000 | 0.002 |
| **4** | 101 | 0.641 | 0.593 | 101 | 0.437 | 0.319 | 0.000 | 0.000 |
| **3** | 101 | 0.742 | 0.701 | 101 | 0.443 | 0.312 | 0.000 | 0.000 |
| **2** | 101 | 0.892 | 0.882 | 101 | 0.482 | 0.373 | 0.000 | 0.000 |
| **1** | 101 | 1.085 | 0.996 | 101 | 0.533 | 0.410 | 0.001 | 0.000 |

## Table 5: Model's results for various specifications of the default barrier

This table shows the summary statistics for assets value ($A$) and assets volatility ($\sigma_A$) in Merton model under different specifications of the $LTD$ multipliers ($k$) used for the default barrier value ($D = STD + k \cdot LTD$), where $STD$ is the short-term debt and $LTD$ is the long-term debt. P values are listed for t tests and Wilcoxon sign-ranked tests. Panels a and b are for the case that assets volatility results from the two-equation Merton. Panel c is for the case that assets volatility is assumed to be equal to equity volatility in the previous year ($\sigma_{E,-1}$), using the single-equation Merton model.

**Panel a – Assets value (A) in two-equation Merton model**

| K | Obs. | Mean | Median | P value for difference from result for k=0.5 | |
|---|---|---|---|---|---|
| | | | | t test | Sign-ranked test |
| **0.0** | 26579 | 7067 | 1233 | 0.000 | 0.000 |
| **0.1** | 26579 | 7207 | 1287 | 0.000 | 0.000 |
| **0.3** | 26579 | 7487 | 1390 | 0.000 | 0.000 |
| **0.5** | 26579 | 7768 | 1492 | - | - |
| **0.7** | 26579 | 8048 | 1587 | 0.000 | 0.000 |
| **0.9** | 26579 | 8328 | 1682 | 0.000 | 0.000 |
| **1.0** | 26579 | 8468 | 1730 | 0.000 | 0.000 |

**Panel b – Assets volatility ($\sigma_A$) in two-equation Merton model**

| K | Obs. | Mean | Median | P value for difference from result for k=0.5 | |
|---|---|---|---|---|---|
| | | | | t test | Sign-ranked test |
| **0.0** | 26579 | 0.429 | 0.372 | 0.000 | 0.000 |
| **0.1** | 26579 | 0.404 | 0.353 | 0.000 | 0.000 |
| **0.3** | 26579 | 0.371 | 0.323 | 0.000 | 0.000 |
| **0.5** | 26579 | 0.348 | 0.300 | - | - |
| **0.7** | 26579 | 0.329 | 0.282 | 0.000 | 0.000 |
| **0.9** | 26579 | 0.314 | 0.268 | 0.000 | 0.000 |
| **1.0** | 26579 | 0.307 | 0.261 | 0.000 | 0.000 |

**Panel c – Assets value (A) in single-equation Merton model**

| K | Obs. | Mean | Median | P value for difference from result for k=0.5 | | P value for difference from two-equations model | |
|---|---|---|---|---|---|---|---|
| | | | | t test | Sign-ranked test | t test | Sign-ranked test |
| **0.0** | 26579 | 7063 | 1227 | 0.000 | 0.000 | 0.000 | 0.000 |
| **0.1** | 26579 | 7202 | 1281 | 0.000 | 0.000 | 0.000 | 0.000 |
| **0.3** | 26579 | 7477 | 1379 | 0.000 | 0.000 | 0.000 | 0.000 |
| **0.5** | 26579 | 7748 | 1474 | - | - | 0.000 | 0.000 |
| **0.7** | 26579 | 8018 | 1564 | 0.000 | 0.000 | 0.000 | 0.000 |
| **0.9** | 26579 | 8285 | 1652 | 0.000 | 0.000 | 0.000 | 0.000 |
| **1.0** | 26579 | 8418 | 1695 | 0.000 | 0.000 | 0.000 | 0.000 |

## Table 6: Estimated probabilities of default for various specifications of the default barrier for the two equation Merton model – summary of statistics

This table presents descriptive statistics for the estimated probability of default, using two equation Merton model, under various specifications of k, the $LTD$ multiplier used to calculate the default barrier, $D = STD + k \cdot LTD$, where $STD$ is the short-term debt and $LTD$ is the long-term debt. The expected return on the firm's assets is set to $\mu_A = \mu_{MP=0.06}$ (using $\beta_A$ of the assets based on historical $\beta_E$ of equity and the assumption that the market premium equals 0.06), and the equity volatility is $\sigma_{E,-1}$.

| K | Obs. | Mean | Median | 5% percentile | 95% percentile | P value for difference from result of k=0.5 | |
|---|---|---|---|---|---|---|---|
| | | | | | | t test | Sign-ranked test |
| **0.0** | 26579 | 0.007 | $2.57 \cdot 10^{-28}$ | 0.000 | 0.008 | 0.000 | 0.00 |
| **0.1** | 26579 | 0.011 | $1.96 \cdot 10^{-15}$ | 0.000 | 0.039 | 0.000 | 0.00 |
| **0.3** | 26579 | 0.015 | $6.00 \cdot 10^{-11}$ | 0.000 | 0.073 | 0.000 | 0.00 |
| **0.5** | 26579 | 0.017 | $4.02 \cdot 10^{-9}$ | 0.000 | 0.093 | - | - |
| **0.7** | 26579 | 0.019 | $4.57 \cdot 10^{-8}$ | 0.000 | 0.109 | 0.000 | 0.00 |
| **0.9** | 26579 | 0.020 | $2.25 \cdot 10^{-7}$ | 0.000 | 0.120 | 0.000 | 0.00 |
| **1.0** | 26579 | 0.021 | $4.15 \cdot 10^{-7}$ | 0.000 | 0.125 | 0.000 | 0.00 |

## Table 7: Area under the ROC curve for various specification of firm's asset expected return ($\boldsymbol{\mu_A}$)

This table shows the results of the model for $\mu_A$ alternatives - the expected return on the firm's assets. $\mu_{MP=0.06}$ is calculated for each firm-year observation, using $\mu_{MP=0.06} = r + \beta_A \cdot 0.06$, where $r$ is the risk free interest rate (1-year treasury bills yield to maturity) and $\beta_A$ is the beta of the firm's assets. $\boldsymbol{r_{E,-1}}$ is the annual equity return for the previous year. $\mu_{MP=MKT}$ is calculated using $\mu_{MP=S\&P} = r + \beta_A \cdot (MKT_{-1} - r)$, where $MKT_{-1}$ is the annual rate of return of the CRSP value-weighted return of NYSE/NASDAQ/AMEX index in the previous year. For reference, we added a fixed (arbitrary) expected return of 0.09. In this table use $\sigma_{E,-1}$ (prior year standard deviation of daily returns) for equity volatility and $D = STD + 0.5 \cdot LTD$ for the default barrier, where *STD* is the short-term debt and *LTD* is the long-term debt. The sample includes 26,579 observations of which 306 are defaults. Panel a shows $\mu_A$ descriptive statistics, panels b and c present the results for two-equation and single-equation Merton models respectively.

**Panel a – Expected asset return**

| Specification | Obs. | Mean | Median | Std. dev. | Min | Max |
|---|---|---|---|---|---|---|
| $\boldsymbol{\mu_{MP=0.06}}$ | 26579 | 0.081 | 0.077 | 0.034 | -0.050 | 0.263 |
| $\boldsymbol{r_{E,-1}}$ | 26579 | 0.326 | 0.176 | 0.571 | -2.272 | 18.796 |
| $\boldsymbol{\mu_{MP=MKT}}$ | 26579 | 0.093 | 0.095 | 0.157 | -0.968 | 1.144 |
| $r$ | 26579 | 0.035 | 0.038 | 0.021 | 0.001 | 0.077 |
| $\boldsymbol{max}(r, \boldsymbol{\mu_{MP=0.06}})$ | 26579 | 0.082 | 0.077 | 0.034 | 0.001 | 0.263 |
| $\boldsymbol{max}(r, \boldsymbol{r_{E,-1}})$ | 26579 | 0.390 | 0.176 | 0.503 | 0.001 | 18.796 |
| $\boldsymbol{max}(r, \boldsymbol{\mu_{MP=MKT}})$ | 26579 | 0.152 | 0.135 | 0.114 | 0.002 | 1.148 |
| $\boldsymbol{0.09}$ | 26579 | 0.090 | 0.090 | 0.000 | 0.090 | 0.090 |

**Panel b – AUC (Area under the ROC curve) for the two-equation Merton model**

| Specification | AUC | P-Value for difference from $\mu_{MP=0.06}$ | P-Value for difference from $r$ | pAUC (0.25) | P-Value for difference from $\mu_{MP=0.06}$ | P-Value for difference from $r$ |
|---|---|---|---|---|---|---|
| $\mu_{MP=0.06}$ | 0.9277 | - | 0.122 | 0.1899 | - | 0.130 |
| $r_{E,-1}$ | 0.9005 | 0.004 | 0.000 | 0.1796 | 0.043 | 0.000 |
| $\mu_{MP=MKT}$ | 0.9223 | 0.000 | 0.000 | 0.1854 | 0.000 | 0.000 |
| $r$ | 0.9280 | 0.122 | - | 0.1902 | 0.130 | - |
| $max(r, \mu_{MP=0.06})$ | 0.9277 | 0.064 | 0.141 | 0.1899 | 0.110 | 0.150 |
| $max(r, r_{E,-1})$ | 0.9017 | 0.003 | 0.000 | 0.1805 | 0.022 | 0.000 |
| $max(r, \mu_{MP=MKT})$ | 0.9201 | 0.000 | 0.000 | 0.1835 | 0.000 | 0.000 |
| $0.09$ | 0.9191 | 0.000 | 0.000 | 0.1813 | 0.000 | 0.760 |

**Panel c – AUC (Area under the ROC curve) for the single equation Merton model**

| Specification | AUC | P-Value for difference from $\mu_{MP=0.06}$ | P-Value for difference from $max(r, r_{E,-1})$ | pAUC (0.25) | P-Value for difference from $\mu_{MP=0.06}$ | P-Value for difference from $max(r, r_{E,-1})$ |
|---|---|---|---|---|---|---|
| $\mu_{MP=0.06}$ | 0.9422 | - | 0.051 | 0.2031 | - | 0.076 |
| $r_{E,-1}$ | 0.9450 | 0.279 | 0.496 | 0.2050 | 0.440 | 0.320 |
| $\mu_{MP=MKT}$ | 0.9380 | 0.000 | 0.000 | 0.1991 | 0.000 | 0.000 |
| $r$ | 0.9426 | 0.024 | 0.080 | 0.2036 | 0.003 | 0.140 |
| $max(r, \mu_{MP=0.06})$ | 0.9422 | 0.002 | 0.053 | 0.2032 | 0.005 | 0.079 |
| $max(r, r_{E,-1})$ | 0.9458 | 0.051 | - | 0.2060 | 0.065 | - |
| $max(r, \mu_{MP=MKT})$ | 0.9403 | 0.007 | 0.004 | 0.2014 | 0.006 | 0.007 |
| $0.09$ | 0.9427 | 0.011 | 0.090 | 0.2037 | 0.002 | 0.140 |

## Table 8: Evolution of equity and assets returns prior to default

This table shows the evolution of the previous year equity return ($r_{E,-1}$) and expected asset returns ($\mu_A = \mu_{MP=0.06}$) for 101 firms on December 31 for each of the 5 years preceding the default year. $\mu_{MP=0.06}$ is based on the $\beta_A$ (assets beta) calculated from historical $\beta_E$ (equity beta) and assuming the market premium equals 0.06. A control group of 101 non-defaulting firms is used for comparison. P values are for differences between the group of defaulting firms and non-defaulting firms.

**Panel a - Annualized equity return for the previous year ($r_{E,-1}$)**

| Years before default | Defaulting | | | Non-Defaulting | | | P value for difference | |
|---|---|---|---|---|---|---|---|---|
| | Obs. | Mean | Median | Obs. | Mean | Median | t test | Wilcoxon rank- sum (Mann-Whitney) |
| **-5** | 101 | 0.443 | 0.367 | 101 | 0.332 | 0.280 | 0.184 | 0.476 |
| **-4** | 101 | 0.185 | 0.131 | 101 | 0.232 | 0.177 | 0.565 | 0.352 |
| **-3** | 101 | 0.319 | 0.255 | 101 | 0.385 | 0.282 | 0.275 | 0.315 |
| **-2** | 101 | 0.194 | 0.104 | 101 | 0.231 | 0.222 | 0.525 | 0.213 |
| **-1** | 101 | -0.029 | -0.346 | 101 | 0.263 | 0.204 | 0.170 | 0.000 |

**Panel b – Expected asset return $\mu_A = \mu_{MP=0.06}$**

| Years before default | Defaulting | | | Non-Defaulting | | | P value for difference | |
|---|---|---|---|---|---|---|---|---|
| | Obs. | Mean | Median | Obs. | Mean | Median | t test | Wilcoxon rank- sum (Mann-Whitney) |
| **-5** | 101 | 0.079 | 0.079 | 101 | 0.082 | 0.077 | 0.409 | 0.999 |
| **-4** | 101 | 0.081 | 0.078 | 101 | 0.080 | 0.074 | 0.861 | 0.947 |
| **-3** | 101 | 0.077 | 0.078 | 101 | 0.082 | 0.082 | 0.137 | 0.077 |
| **-2** | 101 | 0.068 | 0.064 | 101 | 0.075 | 0.074 | 0.024 | 0.014 |
| **-1** | 101 | 0.047 | 0.049 | 101 | 0.063 | 0.060 | 0.000 | 0.000 |

## Table 9: Evolution of volatility prior to default

This table shows the evolution of equity's volatility ($\sigma_{E,-1}$) and asset's volatility ($\sigma_A$) for 101 firms for each of the 5 years preceding the default year. $\sigma_{E,-1}$ is the annualized standard deviation of daily stock returns in the year before. $\sigma_A$ is extracted from the two-equation Merton model, assuming the default barrier is $D = STD + 0.5 \cdot LTD$ ($STD$ is short-term debt and $LTD$ is the long-term debt), and expected assets return is $\mu_A = \mu_{MP=0.06}$ that is based on the $\beta_A$ (assets beta) calculated from historical $\beta_E$ (equity beta) using 0.06 for the market premium. A control group of 101 non-defaulting firms is used for comparison. P values are for differences between the group of defaulting firms and non-defaulting firms.

**Panel a – Equity volatility ($\sigma_{E,-1}$)**

| Years before default | Defaulting | | | Non-Defaulting | | | P value for difference | |
|---|---|---|---|---|---|---|---|---|
| | Obs. | Mean | Median | Obs. | Mean | Median | t test | Wilcoxon rank- sum (Mann-Whitney) |
| 5 | 101 | 0.504 | 0.415 | 101 | 0.395 | 0.362 | 0.000 | 0.000 |
| 4 | 101 | 0.538 | 0.470 | 101 | 0.441 | 0.367 | 0.000 | 0.000 |
| 3 | 101 | 0.602 | 0.537 | 101 | 0.476 | 0.392 | 0.000 | 0.000 |
| 2 | 101 | 0.639 | 0.579 | 101 | 0.466 | 0.432 | 0.000 | 0.000 |
| 1 | 101 | 0.959 | 0.850 | 101 | 0.556 | 0.473 | 0.000 | 0.000 |

**Panel b – Assets volatility ($\sigma_A$)**

| Years before default | Defaulting | | | Non-Defaulting | | | P value for difference | |
|---|---|---|---|---|---|---|---|---|
| | Obs. | Mean | Median | Obs. | Mean | Median | t test | Wilcoxon rank- sum (Mann-Whitney) |
| 5 | 101 | 0.349 | 0.277 | 101 | 0.299 | 0.283 | 0.075 | 0.093 |
| 4 | 101 | 0.338 | 0.268 | 101 | 0.323 | 0.281 | 0.534 | 0.561 |
| 3 | 101 | 0.334 | 0.300 | 101 | 0.342 | 0.296 | 0.724 | 0.646 |
| 2 | 101 | 0.322 | 0.256 | 101 | 0.324 | 0.315 | 0.959 | 0.425 |
| 1 | 101 | 0.295 | 0.199 | 101 | 0.360 | 0.323 | 0.163 | 0.000 |

# Table 10: Historical volatility vs. implied volatility

This table shows the historical volatility and implied volatility of equity and assets for 14,490 annual observations for the years 1995-2012. Implied volatility of equity is the annualized implied volatility of the 30-days at-the-money call option on the firms stocks. Historical volatility of equity is the annualized standard deviation of daily stock returns in the year preceding the annual observation. Assets volatility is calculated by solving the two-equation Merton model, assuming the default barrier is $D = STD + 0.5 \cdot LTD$, where $STD$ is short-term debt and $LTD$ is the long-term debt. Historical assets volatility is calculated using historical equity volatility as an input to the model, and implied assets volatility is calculated using implied equity volatility. P values are for differences between historical volatility and implied volatility.

**Panel a: All years**

| Group | Obs. | Historical volatility | | Implied volatility | | P value for difference | | Obs. Implied> Historical (%) |
|---|---|---|---|---|---|---|---|---|
| | | Mean | Median | Mean | Median | t test | Wilcoxon rank-sum (Mann-Whitney) | |
| Non-defaulting | | | | | | | | |
| Equity | 14402 | 0.438 | 0.386 | 0.437 | 0.384 | 0.081 | 0.409 | 50.2 |
| Assets | 14402 | 0.354 | 0.310 | 0.352 | 0.309 | 0.024 | 0.193 | 50.2 |
| Defaulting | | | | | | | | |
| Equity | 88 | 0.775 | 0.699 | 0.896 | 0.881 | 0.000 | 0.000 | 67.0 |
| Assets | 88 | 0.314 | 0.255 | 0.384 | 0.346 | 0.001 | 0.000 | 67.0 |

**Panel b: Excluding 2009***

| Group | Obs. | Historical volatility | | Implied volatility | | P value for difference | | Obs. Implied> Historical (%) |
|---|---|---|---|---|---|---|---|---|
| | | Mean | Median | Mean | Median | t test | Wilcoxon rank-sum (Mann-Whitney) | |
| Non-defaulting | | | | | | | | |
| Equity | 13528 | 0.429 | 0.378 | 0.437 | 0.382 | 0.000 | 0.000 | 52.7 |
| Assets | 13528 | 0.347 | 0.304 | 0.353 | 0.309 | 0.000 | 0.000 | 52.7 |
| Defaulting | | | | | | | | |
| Equity | 85 | 0.761 | 0.686 | 0.895 | 0.886 | 0.000 | 0.000 | 68.2 |
| Assets | 85 | 0.305 | 0.251 | 0.384 | 0.344 | 0.000 | 0.000 | 68.2 |

* excluded are end of 2009 observations (regarding defaults of 2010)

**Table 11: Area under the ROC curve for various specifications of assets volatility ($\sigma_A$)**

This table shows AUC (area under the ROC curve) and pAUC (partial area under the ROC curve at the 0.25 FPR level) for eight specifications of assets volatility. The first is the benchmark model in which assets volatility ($\sigma_A^{2eqM}$) is calculated using the two-equation Merton model, assuming the equity volatility equals the historical volatility ($\sigma_E = \sigma_{E,-1}$). The specifications $\sigma_A^{2eqM}(\sigma_E = \sigma_{JP})$ and $\sigma_A^{2eqM}(\sigma_E = \sigma_{MAD})$ are also calculated using the two-equation Merton model but assuming the equity volatility equals the JP-Morgan volatility ($\sigma_{JP}$) and MAD volatility ($\sigma_{MAD}$) respectively. The three subsequent models are of a single-equation Merton type, where assets volatility is set equal to either the equity volatility ($\sigma_{E,-1}$), the JP-Morgan volatility ($\sigma_{JP}$), or the MAD volatility ($\sigma_{MAD}$) respectively. In these six models, the default barrier is $D = STD + 0.5 \cdot LTD$, where $STD$ is short-term debt and $LTD$ is the long-term debt. The expected return on the firm's assets is set to $\mu_A = \mu_{MP=0.06}$ (based on the $\beta_A$ of the assets calculated from historical $\beta_E$ of equity and assuming the market premium equals 0.06). Model 7 $\sigma_A^{KMV}$ is based on the iterative method presented in subsection 4.1 and model 8 $\sigma_A^{CDLT}$ is based on the Charitou et al. (2013) model presented in subsection 4.3. P values and pAUC are of DeLong, et al. (1988) test for the difference between the AUC of the alternative specifications and the partial AUC (at the 0.25 FPR level) respectively.

| Model | $\sigma_A$ | AUC | P value for difference from AUC of model 1 | P value for difference from AUC of model 5 | pAUC (0.25) | P value for difference from pAUC of model 1 | P value for difference from pAUC of model 5 |
|---|---|---|---|---|---|---|---|
| 1 | $\sigma_A^{2eqM}(\sigma_E = \sigma_{E,-1})$ | 0.9277 | - | 0.000 | 0.1899 | - | 0.000 |
| 2 | $\sigma_A^{2eqM}(\sigma_E = \sigma_{JP})$ | 0.9319 | 0.141 | 0.000 | 0.1923 | 0.310 | 0.000 |
| 3 | $\sigma_A^{2eqM}(\sigma_E = \sigma_{MAD})$ | 0.9287 | 0.000 | 0.000 | 0.1906 | 0.000 | 0.000 |
| 4 | $\sigma_{E,-1}$ | 0.9422 | 0.000 | 0.162 | 0.2031 | 0.000 | 0.440 |
| 5 | $\sigma_{JP}$ | 0.9449 | 0.000 | - | 0.2042 | 0.000 | - |
| 6 | $\sigma_{MAD}$ | 0.9424 | 0.000 | 0.208 | 0.2034 | 0.000 | 0.540 |
| 7 | $\sigma_A^{KMV}$ | 0.9340 | 0.074 | 0.001 | 0.1956 | 0.056 | 0.000 |
| 8 | $\sigma_A^{CDLT}$ | 0.9032 | 0.000 | 0.000 | 0.1781 | 0.002 | 0.000 |

## Table 12: Two-equation down and out (DaO) model

This table shows AUC and pAUC (partial area under the curve at the 0.25 level of FPR) for several specifications of the two-equation DaO model. The LTD multipliers ($k$) in the default barrier specification ($D = STD + k \cdot LTD$) varies in panel a and is set to $k = 0.5$ in the other panels. The expected return on the firm's assets varies in panel b and is set to $\mu_A = \mu_{MP=0.06}$ in the other panels (see $\mu_A$ specification details in subsection 7.2 and Table 7). Equity volatility specification varies in panel c and is set to $\sigma_{E,-1}$ in the other panels, see alternative equity volatility details in subsection 4.7. Asset volatility and asset value are the solution of the two-equation DaO model presented in subsection 4.4. P-values are of DeLong, et al. (1988) test for the difference between the AUC of the various specifications.

### Panel a: Various specification of the default barrier

| K | AUC | P value for difference from result for k=0.5 | pAUC (0.25) | P value for difference from result for k=0.5 |
|---|---|---|---|---|
| **0.0** | 0.8694 | 0.000 | 0.1745 | 0.005 |
| **0.1** | 0.9281 | 0.008 | 0.1927 | 0.002 |
| **0.3** | 0.9249 | 0.030 | 0.1899 | 0.040 |
| **0.5** | 0.9161 | - | 0.1866 | - |
| **0.7** | 0.9166 | 0.912 | 0.1854 | 0.540 |
| **0.9** | 0.9142 | 0.758 | 0.1846 | 0.440 |
| **1.0** | 0.9166 | 0.934 | 0.1853 | 0.570 |

### Panel b: Various specification of the assets expected return

| Specification | AUC | P-Value for difference from $\mu_{MP=0.06}$ | P-Value for difference from $r$ | pAUC (0.25) | P-Value for difference from $\mu_{MP=0.06}$ | P-Value for difference from $r$ |
|---|---|---|---|---|---|---|
| $\mu_{MP=0.06}$ | 0.9179 | - | 0.707 | 0.1872 | - | 0.250 |
| $r_{E,-1}$ | 0.8941 | 0.004 | 0.004 | 0.1748 | 0.021 | 0.020 |
| $\mu_{MP=MKT}$ | 0.9127 | 0.000 | 0.000 | 0.1820 | 0.000 | 0.000 |
| $r$ | 0.9180 | 0.707 | - | 0.1868 | 0.270 | - |
| $max(r, \mu_{MP=0.06})$ | 0.9180 | 0.119 | 0.764 | 0.1866 | 0.180 | 0.280 |
| $max(r, r_{E,-1})$ | 0.8934 | 0.001 | 0.001 | 0.1736 | 0.004 | 0.005 |
| $max(r, \mu_{MP=MKT})$ | 0.9082 | 0.000 | 0.000 | 0.1804 | 0.000 | 0.000 |
| $0.09$ | 0.9026 | 0.000 | 0.000 | 0.1751 | 0.000 | 0.000 |

**Panel c: Various specification of the equity volatility**

| $\sigma_E$ | AUC | P-value for the difference from results for $\sigma_{E,-1}$ | P-value for the difference from results for $\sigma_{JP}$ | pAUC | P-value for the difference from results for $\sigma_{E,-1}$ | P-value for the difference from results for $\sigma_{JP}$ |
|---|---|---|---|---|---|---|
| $\sigma_{E,-1}$ | 0.9161 | - | 0.280 | 0.1866 | - | 0.320 |
| $\sigma_{JP}$ | 0.9242 | 0.280 | - | 0.1898 | 0.320 | - |
| $\sigma_{MAD}$ | 0.9233 | 0.200 | 0.866 | 0.1894 | 0.130 | 0.880 |

## Table 13: Single-equation down and out (DaO) model

This table shows AUC and pAUC (partial area under the curve at the 0.25 level of FPR) for several specifications of the single-equation DaO model. The LTD multipliers ($k$) in the default barrier specification ($D = STD + k \cdot LTD$) varies in panel a and is set to $k = 0.5$ in the other panels. The expected return on the firm's assets varies in panel b and is set to $\mu_A = \mu_{MP=0.06}$ in the other panels (see $\mu_A$ specification details in subsection 7.2 and Table 7). Equity volatility specification varies in panel c and is set to $\sigma_{E,-1}$ in the other panels, see alternative equity volatility details in subsection 4.7. Assets values are the solution of equations (10) substituting equity volatility for assets volatility. P-values are of DeLong, et al. (1988) test for the difference between the AUC of the various specifications.

**Panel a: Various specifications of the default barrier**

| K | AUC | P value for difference from result for k=0.5 | pAUC (0.25) | P value for difference from result for k=0.5 |
|---|---|---|---|---|
| **0.0** | 0.8712 | 0.000 | 0.1759 | 0.000 |
| **0.1** | 0.9393 | 0.004 | 0.2011 | 0.015 |
| **0.3** | 0.9425 | 0.220 | 0.2040 | 0.870 |
| **0.5** | 0.9431 | - | 0.2041 | - |
| **0.7** | 0.9429 | 0.529 | 0.2036 | 0.084 |
| **0.9** | 0.9425 | 0.292 | 0.2030 | 0.026 |
| **1.0** | 0.9423 | 0.224 | 0.2027 | 0.013 |

**Panel b: Various specification of the assets expected return**

| Specification | AUC | P-Value for difference from $\boldsymbol{\mu_{MP=0.06}}$ | P-Value for difference from $\boldsymbol{r_{E,-1}}$ | pAUC (0.25) | P-Value for difference from $\boldsymbol{\mu_{MP=0.06}}$ | P-Value for difference from $\boldsymbol{r_{E,-1}}$ |
|---|---|---|---|---|---|---|
| $\boldsymbol{\mu_{MP=0.06}}$ | 0.9431 | - | 0.029 | 0.2040 | - | 0.033 |
| $\boldsymbol{r_{E,-1}}$ | 0.9478 | 0.029 | - | 0.2077 | 0.036 | - |
| $\boldsymbol{\mu_{MP=S\&P500}}$ | 0.9401 | 0.000 | 0.000 | 0.2012 | 0.000 | 0.001 |
| $r$ | 0.9434 | 0.065 | 0.038 | 0.2044 | 0.010 | 0.061 |
| $\boldsymbol{max}(r, \boldsymbol{\mu_{MP=0.06}})$ | 0.9431 | 0.016 | 0.029 | 0.2040 | 0.027 | 0.048 |
| $\boldsymbol{max}(r, \boldsymbol{r_{E,-1}})$ | 0.9473 | 0.003 | 0.661 | 0.2075 | 0.001 | 0.870 |
| $\boldsymbol{max}(r, \boldsymbol{\mu_{MP=S\&P500}})$ | 0.9419 | 0.028 | 0.007 | 0.2029 | 0.028 | 0.008 |
| $\boldsymbol{0.09}$ | 0.9435 | 0.032 | 0.042 | 0.2045 | 0.003 | 0.067 |

**Panel c: Various specification of the equity volatility**

| $\sigma_E$ | AUC | P-value for the difference from results for $\sigma_{E,-1}$ | P-value for the difference from results for $\sigma_{JP}$ | pAUC | P-value for the difference from results for $\sigma_{E,-1}$ | P-value for the difference from results for $\sigma_{JP}$ |
|---|---|---|---|---|---|---|
| $\sigma_{E,-1}$ | 0.9431 | - | 0.137 | 0.2040 | - | 0.380 |
| $\sigma_{JP}$ | 0.9458 | 0.137 | - | 0.2051 | 0.380 | - |
| $\sigma_{MAD}$ | 0.9431 | 0.949 | 0.154 | 0.2041 | 0.033 | 0.420 |

## Table 14: Area Under Curve (AUC) for alternative model specifications

This table shows AUC for 10 specifications of the model listed in subsection 7.7. The default barrier is *STD* +k·*LTD* where *STD* and *LTD* is the short and long-term debt respectively, except for model 10 where it is *STL* +k·*LT*L, and *STL* and *LTL* are short and long-term liabilities respectively. The assets expected returns alternatives include: $\mu_A = \mu_{MP=0.06}$ based on $\beta_A$ of the assets calculated using historical $\beta_E$ of equity, assuming the market premium equals 0.06; or $r_{E,-1}$ the equity return in the previous year; or $r$ (the risk-free interest rate, 1-year treasury bills yield to maturity); or the larger of $r_{E,-1}$ and $r$. $\sigma_{E,-1}$ is the annualized volatility of daily equity return in the previous year, $\sigma_{JP}$ is JP-Morgan annualized year end estimate of equity volatility and $\sigma_A^{Naive}$ is based on Bharath and Shumway specification of assets volatility. The CDLT model is based on Charitou et al. (2013). P-values are DeLong, et al. (1988) test results for the difference between the AUCs of the alternative specifications. Panel a shows the results for the entire sample period and panels b and c for 1990-2000 and 2001-2013 respectively.

**Panel a: Area Under the Curve (AUC) for the entire sample**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 | Merton (textbook) | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.9277 | - | 0.000 | 0.000 |
| 2 | 2-equation Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.9354 | 0.011 | 0.004 | 0.008 |
| 3 | 1-equation Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9375 | 0.096 | 0.025 | 0.117 |
| 4 | 2-equation DaO | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.9357 | 0.009 | 0.006 | 0.010 |
| **5** | **1-equation DaO** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.9443** | **0.000** | **-** | **0.806** |
| 6 | BhSh (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.9201 | 0.251 | 0.000 | 0.000 |
| **7** | **SNM (naïve)** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.9437** | **0.000** | **0.806** | **-** |
| 8 | SNM (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9348 | 0.233 | 0.002 | 0.026 |
| 9 | KMV | $STD + 0.5 \cdot LTD$ | Merton** | iterative | - | iterative | 0.9340 | 0.074 | 0.001 | 0.000 |
| 10 | CDLT | $STL + LTL$ | E+L | return of E+L | - | σ(E+L) | 0.9032 | 0.000 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

**Panel b: Area Under the Curve (AUC) for 1990-2000**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 | Merton (textbook) | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.9264 | - | 0.000 | 0.000 |
| 2 | 2-equation Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.9381 | 0.006 | 0.006 | 0.011 |
| 3 | 1-equation Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9486 | 0.002 | 0.067 | 0.711 |
| 4 | 2-equation DaO | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.9387 | 0.004 | 0.008 | 0.015 |
| **5** | **1-equation DaO** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO**** ** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.9526** | **0.000** | **-** | **0.455** |
| 6 | BhSh (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.9319 | 0.536 | 0.000 | 0.001 |
| **7** | **SNM (naïve)** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.9502** | **0.000** | **0.455** | **-** |
| 8 | SNM (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9469 | 0.005 | 0.016 | 0.451 |
| 9 | KMV | $STD + 0.5 \cdot LTD$ | Merton** | iterative | - | Iterative | 0.9452 | 0.000 | 0.054 | 0.068 |
| 10 | CDLT | $STL + LTL$ | E+L | return of E+L | - | σ(E+L) | 0.9060 | 0.035 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

**Panel c: Area Under the Curve (AUC) for 2001-2013**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 | Merton (textbook) | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.9289 | - | 0.163 | 0.041 |
| 2 | 2-equation Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.9336 | 0.274 | 0.314 | 0.396 |
| 3 | 1-equation Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9278 | 0.906 | 0.102 | 0.163 |
| 4 | 2-equation DaO | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.9335 | 0.293 | 0.303 | 0.380 |
| **5** | **1-equation DaO** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.9373** | **0.163** | **-** | **0.991** |
| 6 | BhSh (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.9083 | 0.031 | 0.000 | 0.000 |
| **7** | **SNM (naïve)** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.9373** | **0.041** | **0.991** | **-** |
| 8 | SNM (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9242 | 0.612 | 0.025 | 0.054 |
| 9 | KMV | $STD + 0.5 \cdot LTD$ | Merton** | iterative | - | Iterative | 0.9226 | 0.170 | 0.006 | 0.000 |
| 10 | CDLT | $STL + LTL$ | E+L | return of E+L | - | σ(E+L) | 0.9009 | 0.001 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

**Table 15: Model accuracy across industries**

This table shows AUC for 10 specifications of the model listed in subsection 7.7. The default barrier is $STD$ +k·$LTD$ where $STD$ and $LTD$ is the short and long-term debt respectively, except for model 10 where it is $STL$ +k·$LT$L, and $STL$ and $LTL$ are short and long-term liabilities respectively. The assets expected returns alternatives include: $\mu_A = \mu_{MP=0.06}$ based on $\beta_A$ of the assets calculated using historical $\beta_E$ of equity, assuming the market premium equals 0.06; or $r_{E,-1}$ the equity return in the previous year; or $r$ (the risk-free interest rate, 1-year treasury bills yield to maturity); or the larger of $r_{E,-1}$ and $r$. $\sigma_{E,-1}$ is the annualized volatility of daily equity return in the previous year, $\sigma_{JP}$ is JP-Morgan annualized year end estimate of equity volatility and $\sigma_A^{Naive}$ is based on Bharath and Shumway specification of assets volatility. The CDLT model is based on Charitou et al. (2013). P values are based on DeLong, et al. (1988) test for the difference between the AUC of the alternative specifications. Panel a shows the manufacturing division (SIC code 2000-3999), panel b is for Transportation, Communications, Electric, Gas and Sanitary service (SIC codes 4000-4999) and panel c for all other industries.

**Panel a: Manufacturing (13,141 observations, 117 defaults)**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 | Merton (textbook) | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.9539 | - | 0.422 | 0.029 |
| 2 | 2-equation Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.9592 | 0.188 | 0.938 | 0.286 |
| 3 | 1-equation Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9465 | 0.544 | 0.087 | 0.058 |
| 4 | 2-equation DaO | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.9589 | 0.230 | 0.883 | 0.251 |
| **5** | **1-equation DaO** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.9597** | **0.422** | **-** | **0.288** |
| 6 | BhSh (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.9376 | 0.150 | 0.006 | 0.001 |
| **7** | **SNM (naïve)** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.9636** | **0.029** | **0.288** | **-** |
| 8 | SNM (naïve) | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9442 | 0.432 | 0.046 | 0.033 |
| 9 | KMV | $STD + 0.5 \cdot LTD$ | Merton** | iterative | - | Iterative | 0.9558 | 0.729 | 0.470 | 0.052 |
| 10 | CDLT | $STL + LTL$ | E+L | return of E+L | - | σ(E+L) | 0.9449 | 0.222 | 0.036 | 0.002 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

**Panel b: Transportation, Communications, Electric, Gas, and Sanitary services (3723 observations, 74 defaults)**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 (original) | Merton | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.8977 | - | 0.014 | 0.002 |
| 2 | Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.9074 | 0.186 | 0.021 | 0.214 |
| 3 | Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9189 | 0.012 | 0.936 | 0.745 |
| 4 | Down and Out | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.9084 | 0.147 | 0.030 | 0.258 |
| **5** | **Down and Out** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.9191** | **0.014** | **-** | **0.735** |
| 6 (Bh&Sh) | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.8849 | 0.334 | 0.002 | 0.000 |
| **7** | **Naïve** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.9171** | **0.002** | **0.735** | **-** |
| 8 | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9155 | 0.041 | 0.138 | 0.793 |
| 9 (Iterative) | Merton | $STD + 0.5 \cdot LTD$ | Merton** | Iterative | - | Iterative | 0.9024 | 0.524 | 0.025 | 0.014 |
| 10 (CDLT) | Merton | $STL + LTL$ | E+D | Iterative | - | Iterative | 0.8631 | 0.023 | 0.001 | 0.001 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

**Panel c: Other (9715 observations, 115 defaults)**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 (original) | Merton | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.9094 | - | 0.000 | 0.000 |
| 2 | Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.9201 | 0.047 | 0.001 | 0.048 |
| 3 | Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9320 | 0.002 | 0.003 | 0.959 |
| 4 | Down and Out | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.9209 | 0.035 | 0.002 | 0.060 |
| **5** | **Down and Out** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.9374** | **0.000** | **-** | **0.126** |
| 6 (Bh&Sh) | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.9158 | 0.548 | 0.001 | 0.008 |
| **7** | **Naïve** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.9318** | **0.000** | **0.126** | **-** |
| 8 | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.9292 | 0.008 | 0.000 | 0.478 |
| 9 (Iterative) | Merton | $STD + 0.5 \cdot LTD$ | Merton** | Iterative | - | Iterative | 0.9244 | 0.019 | 0.002 | 0.020 |
| 10 (CDLT) | Merton | $STL + LTL$ | E+D | Iterative | - | Iterative | 0.8799 | 0.017 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

## Table 16: Partial Area Under the Curve (pAUC) for alternative model specifications

This table shows pAUC for 10 specifications of the model listed in subsection 7.7. The default barrier is *STD* +k·*LTD* where *STD* and *LTD* is the short and long-term debt respectively, except for model 10 where it is *STL* +k·*LT*L, and *STL* and *LTL* are short and long-term liabilities respectively. The assets expected returns alternatives include: $\mu_A = \mu_{MP=0.06}$ based on $\beta_A$ of the assets calculated using historical $\beta_E$ of equity, assuming the market premium equals 0.06; or $r_{E,-1}$ the equity return in the previous year; or $r$ (the risk-free interest rate, 1-year treasury bills yield to maturity); or the larger of $r_{E,-1}$ and $r$. $\sigma_{E,-1}$ is the annualized volatility of daily equity return in the previous year, $\sigma_{JP}$ is JP-Morgan annualized year end estimate of equity volatility and $\sigma_A^{Naive}$ is based on Bharath and Shumway specification of assets volatility. The CDLT model is based on Charitou et al. (2013). P values are based on DeLong, et al. (1988) test for the difference between the pAUCs of the alternative specifications. Each panel shows pAUC at a different type I error. Panel a at 0.5, panel b at 0.25 and panel c at 0.1.

**Panel a: Partial Area Under the Curve (pAUC) at 0.5 level of Type I error**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | pAUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 (original) | Merton | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.4328 | - | 0.001 | 0.000 |
| 2 | Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.4388 | 0.028 | 0.006 | 0.003 |
| 3 | Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.4430 | 0.026 | 0.009 | 0.085 |
| 4 | Down and Out | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.4391 | 0.025 | 0.006 | 0.004 |
| **5** | **Down and Out** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.4474** | **0.001** | **-** | **0.920** |
| 6 (Bh&Sh) | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.4255 | 0.230 | 0.000 | 0.000 |
| **7** | **Naïve** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.4476** | **0.000** | **0.920** | **-** |
| 8 | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.4404 | 0.120 | 0.000 | 0.007 |
| 9 (Iterative) | Merton | $STD + 0.5 \cdot LTD$ | Merton** | Iterative | - | Iterative | 0.4387 | 0.089 | 0.004 | 0.000 |
| 10 (CDLT) | Merton | $STL + LTL$ | E+D | Iterative | - | Iterative | 0.4114 | 0.000 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)
** refers to the solution of equation (2)
*** refers to the simultaneous solution of equations (10) and (12)
**** refers to the solution of equation (10)

**Panel b: Partial Area Under the Curve (pAUC) at 0.25 level of Type I error**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | pAUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 (original) | Merton | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.1899 | - | 0.000 | 0.000 |
| 2 | Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.1960 | 0.005 | 0.001 | 0.001 |
| 3 | Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.2017 | 0.001 | 0.006 | 0.130 |
| 4 | Down and Out | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.1962 | 0.007 | 0.001 | 0.001 |
| **5** | **Down and Out** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.2044** | **0.000** | **-** | **0.740** |
| 6 (Bh&Sh) | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.1855 | 0.370 | 0.000 | 0.000 |
| **7** | **Naïve** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.2039** | **0.000** | **0.740** | **-** |
| 8 | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.1991 | 0.007 | 0.000 | 0.001 |
| 9 (Iterative) | Merton | $STD + 0.5 \cdot LTD$ | Merton** | Iterative | - | Iterative | 0.1956 | 0.056 | 0.000 | 0.000 |
| 10 (CDLT) | Merton | $STL + LTL$ | E+D | Iterative | - | Iterative | 0.1781 | 0.002 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)

** refers to the solution of equation (2)

*** refers to the simultaneous solution of equations (10) and (12)

**** refers to the solution of equation (10)

**Panel c: Partial Area Under the Curve (pAUC) at 0.1 level of Type I error**

| Model | Type | Default barrier (D) | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | pAUC | P value for difference | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | From model 1 | From model 5 | From model 7 |
| 1 (original) | Merton | $STD + 0.5 \cdot LTD$ | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 0.0572 | - | 0.000 | 0.000 |
| 2 | Merton | $STD + 0.1 \cdot LTD$ | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 0.0614 | 0.001 | 0.000 | 0.004 |
| 3 | Merton | $STD + 0.5 \cdot LTD$ | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.0655 | 0.000 | 0.021 | 0.770 |
| 4 | Down and Out | $STD + 0.1 \cdot LTD$ | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 0.0616 | 0.001 | 0.000 | 0.009 |
| **5** | **Down and Out** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **DaO****** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **0.0670** | **0.000** | **-** | **0.210** |
| 6 (Bh&Sh) | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 0.0569 | 0.920 | 0.000 | 0.000 |
| **7** | **Naïve** | $\boldsymbol{STD + 0.5 \cdot LTD}$ | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **0.0657** | **0.000** | **0.210** | **-** |
| 8 | Naïve | $STD + 0.5 \cdot LTD$ | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 0.0631 | 0.002 | 0.000 | 0.006 |
| 9 (Iterative) | Merton | $STD + 0.5 \cdot LTD$ | Merton** | Iterative | - | Iterative | 0.0604 | 0.087 | 0.000 | 0.000 |
| 10 (CDLT) | Merton | $STL + LTL$ | E+D | Iterative | - | Iterative | 0.0543 | 0.160 | 0.000 | 0.000 |

* refers to the simultaneous solution of equations (2) and (4)

** refers to the solution of equation (2)

*** refers to the simultaneous solution of equations (10) and (12)

**** refers to the solution of equation (10)

## Table 17: Model power competition results

This table summarizes model ranking by AUC/pAUC of tables 14-16 for 10 specifications of the model listed in subsection 7.7. The default barrier is *STD* +k·*LTD* where *STD* and *LTD* is the short and long-term debt respectively, except for model 10 where it is *STL* +k·*LT*L, and *STL* and *LTL* are short and long-term liabilities respectively. The assets expected returns alternatives include: $\mu_A = \mu_{MP=0.06}$ based on $\beta_A$ of the assets calculated using historical $\beta_E$ of equity, assuming the market premium equals 0.06; or $r_{E,-1}$ the equity return in the previous year; or $r$ (the risk-free interest rate, 1-year treasury bills yield to maturity); or the larger of $r_{E,-1}$ and $r$. $\sigma_{E,-1}$ is the annualized volatility of daily equity return in the previous year, $\sigma_{JP}$ is JP-Morgan annualized year end estimate of equity volatility and $\sigma_A^{Naive}$ is based on Bharath and Shumway specification of assets volatility. The CDLT model is based on Charitou et al. (2013). To help visualize the AUC ranking we use special font and shading for places 1 and 2 in each column.

| Model | Type | Default barrier k | Value of assets ($A$) | Expected return on assets ($\mu_A$) | Equity Volatility ($\sigma_E$) | Assets volatility ($\sigma_A$) | AUC entire sample | AUC until 2000 | AUC after 2000 | AUC manufacturing | AUC trans. utility,.. | AUC other indust. | pAUC 0.5 level | pAUC 0.25 level | pAUC 0.1 level |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Merton (textbook) | 0.5 | Merton* | $\mu_{MP=0.06}$ | $\sigma_{E,-1}$ | Merton* | 8 | 9 | 5 | 6 | 8 | 9 | 8 | 8 | 8 |
| 2 | 2-equation Merton | 0.1 | Merton* | $r$ | $\sigma_{JP}$ | Merton* | 5 | 7 | 3 | 3 | 6 | 7 | 6 | 6 | 6 |
| 3 | 1-equation Merton | 0.5 | Merton** | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 3 | 3 | 6 | 7 | **2** | **2** | 3 | 3 | 3 |
| 4 | 2-equation DaO | 0.1 | DaO*** | $r$ | $\sigma_{JP}$ | DaO*** | 4 | 6 | 4 | 4 | 5 | 6 | 5 | 5 | 5 |
| **5** | **1-equation DaO** | **0.5** | **DaO******** | $\boldsymbol{r_{E,-1}}$ | **-** | $\boldsymbol{\sigma_{JP}}$ | **1** | **1** | **1** | **2** | **1** | **1** | **2** | **1** | **1** |
| 6 | BhSh (naïve) | 0.5 | E+D | $r_{E,-1}$ | $\sigma_{E,-1}$ | $\sigma_A^{Naive}$ | 9 | 8 | 9 | 10 | 9 | 8 | 9 | 9 | 9 |
| **7** | **SNM (naïve)** | **0.5** | **E+D** | $\boldsymbol{\max(r_{E,-1}, r)}$ | **-** | $\boldsymbol{\sigma_{E,-1}}$ | **2** | **2** | **2** | **1** | 3 | 3 | **1** | **2** | **2** |
| 8 | SNM (naïve) | 0.5 | E+D | $\max(r_{E,-1}, r)$ | - | $\sigma_{JP}$ | 6 | 4 | 7 | 9 | 4 | 4 | 4 | 4 | 4 |
| 9 | KMV | 0.5 | Merton** | iterative | - | iterative | 7 | 5 | 8 | 5 | 7 | 5 | 7 | 7 | 7 |
| 10 | CDLT | 1♦ | E+L | return of E+L | - | σ(E+L) | 10 | 10 | 10 | 8 | 10 | 10 | 10 | 10 | 10 |
| | | | | | | Source (Table No. and panel): | 14a | 14b | 14b | 15a | 15b | 15c | 16a | 16b | 16c |

* refers to the simultaneous solution of equations (2) and (4)

** refers to the solution of equation (2)

*** refers to the simultaneous solution of equations (10) and (12)

**** refers to the solution of equation (10)

♦ refers to liabilities (*STL* +1·*LTL*)

## Figures

**Figure 1**: Illustration of ROC curves of true positive rate (TPR) versus false positive rate (FPR) for two models and for a random order

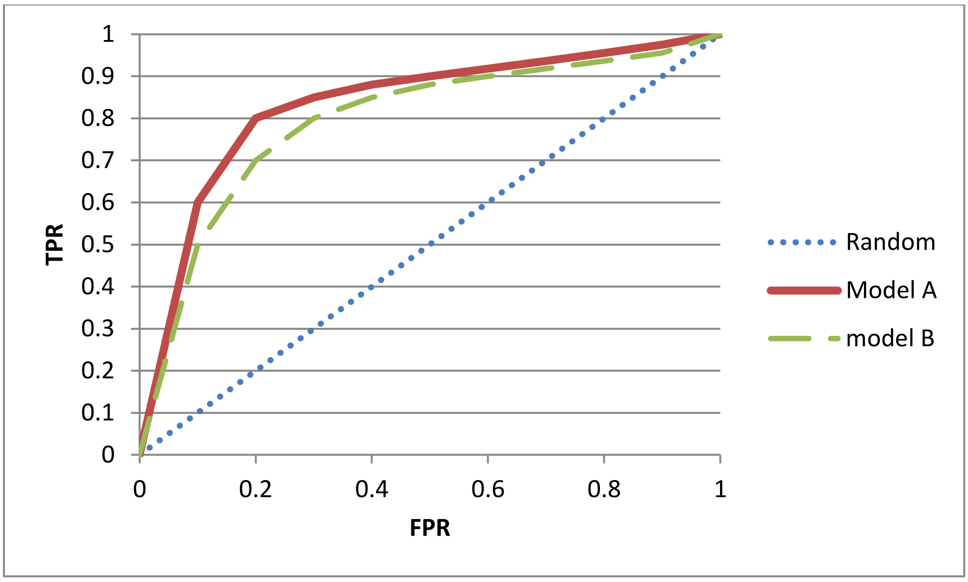

**Figure 2**: The median annual returns on the firm's equity in the previous year ($r_{E,-1}$) and the median expected asset returns ($\mu_A$) while the defaulting firms (101 firms) approach the default event. $\mu_A = \mu_{MP=0.06}$ (based on the $\beta_A$ of the assets calculated from historical $\beta_E$ of equity and assuming the market premium equals 0.06). A control group of 101 non-defaulting firms is used for comparison.

Figure 2a: Median annualized stock return ($r_{E,-1}$)

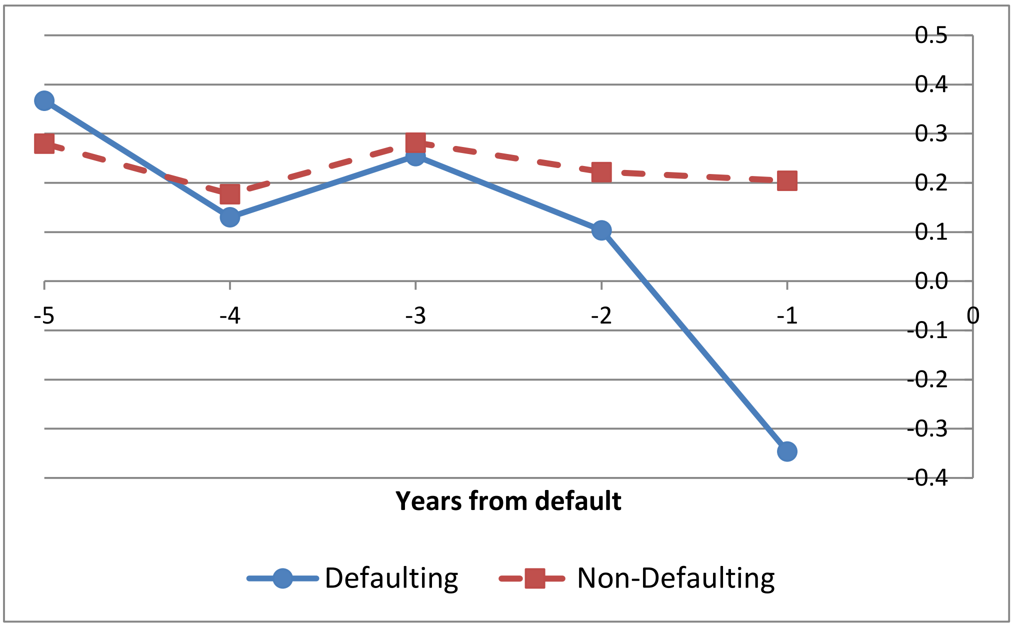


Figure 2b: Median expected assets return ($\mu_A = \mu_{MP=0.06}$)

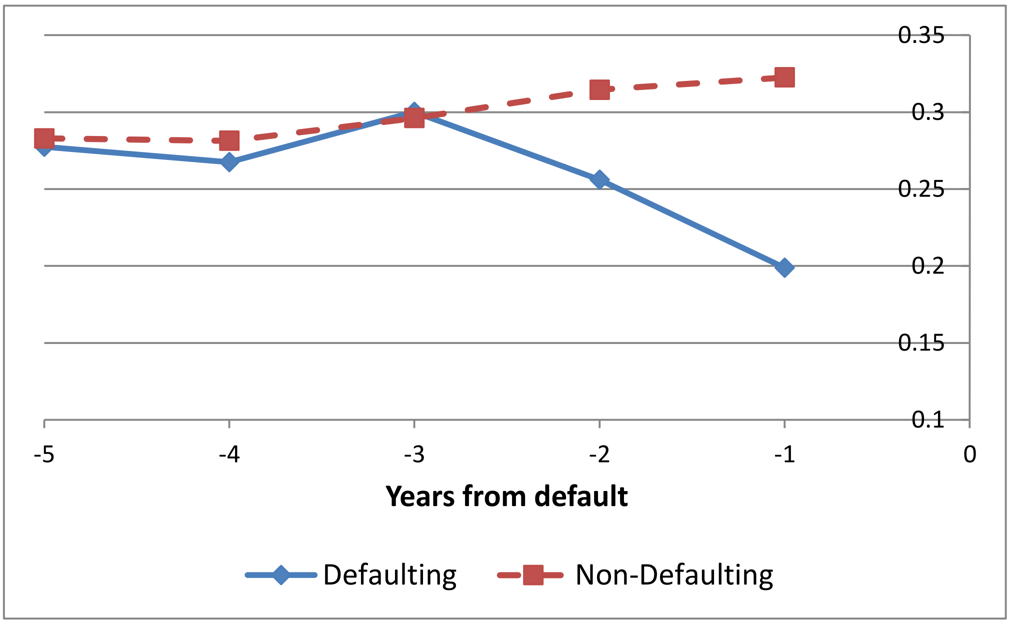

**Figure 3**: The median equity return volatility ($\sigma_{E-1}$) and median asset return volatility ($\sigma_A$) while the defaulting firms (101 firms) approach the default event. A control group of 101 non-defaulting firms is used for comparison. For data and model specifications see Table 9.

Figure 3a: Median Equity returns volatility ($\sigma_{E-1}$)

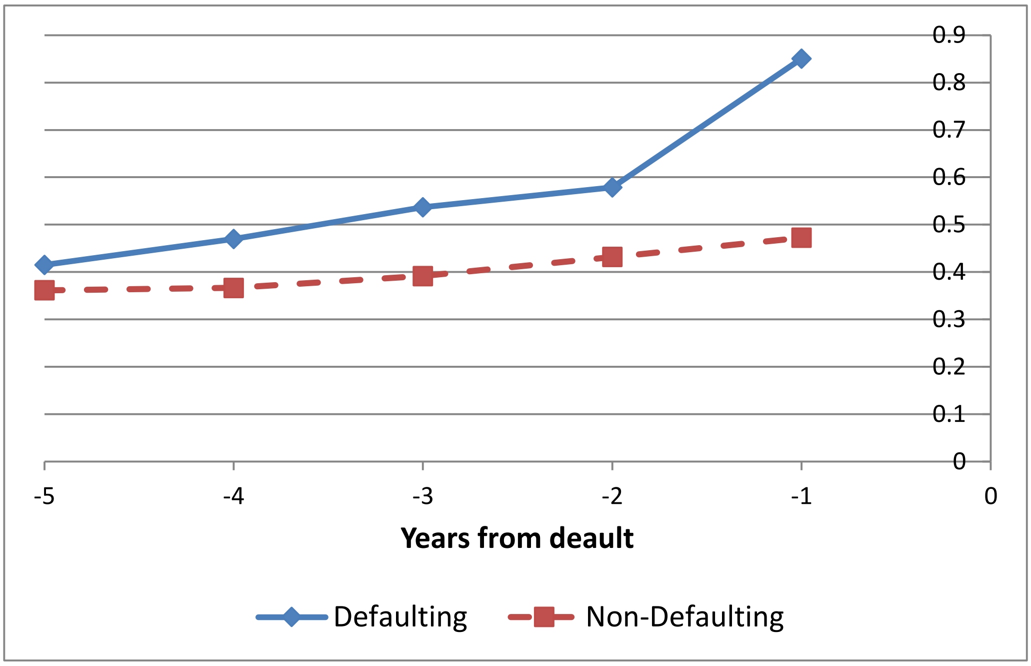


Figure 3b: Median Assets returns volatility ($\sigma_A$)

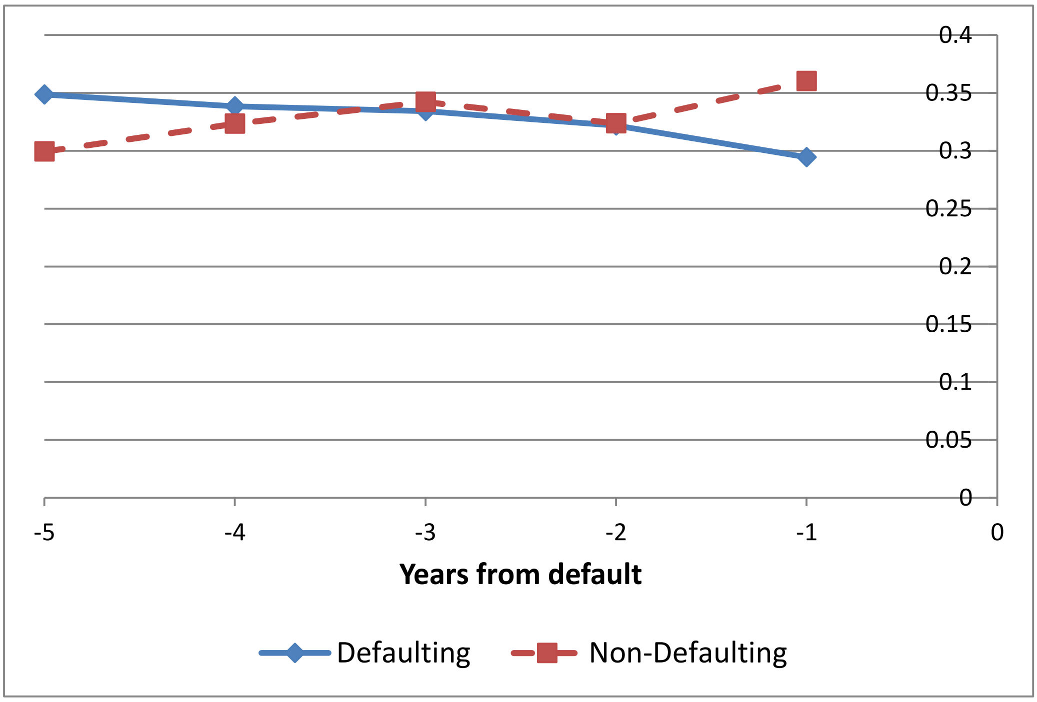

**Figure 4**: The evolution of average equity return volatility ($\sigma_{E-1}$) and average implied volatility ($\sigma_E^{implied}$) throughout the period 1995-2012.