OPTIMAL STRATEGY OF MULTI-PRODUCT RETAILERS WITH RELATIVE THINKING AND REFERENCE PRICES

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Abstract

Experimental evidence suggests that consumers are affected by reference prices and by relative price differences ("relative thinking"). A linear-city model of two retailers that sell two goods suggests how this consumer behavior affects firm strategy and market outcomes. A simple model analyzes the case in which all consumers want to buy both goods. An extended version adds consumers who want only one good. Relative thinking leads firms to increase the markup on the good with the higher reference price and decrease the markup on the other good, possibly to a negative markup. Stronger relative thinking increases the firms' profits.

JEL codes: L13, D43, M31, M20, D11

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1. Introduction

Experimental evidence suggests that people are affected by relative price differences even when only absolute price differences should matter. The seminal experiment on this issue was conducted by Tversky and Kahneman (1981), who asked people to answer one of two versions of the following question:

> Imagine that you are about to purchase a jacket for ($125) [$15], and a calculator for ($15) [$125]. The calculator salesman informs you that the calculator you wish to buy is on sale for ($10) [$120] at the other branch of the store, located 20 minutes drive away. Would you make the trip to the other store?

The responses in the two treatments were significantly different: 68 percent of the respondents were willing to make the trip to save $5 on a $15 calculator, but only 29 percent were willing to exert the same effort for the same savings when the calculator's price was $125. Notice that this not only implies that the respondents considered the relative savings, but also that they compared the savings to the price of the good on which the discount is given and not to the bundle's price.[1]

Later, similar results were obtained also by others. Mowen and Mowen (1986) show that the effect holds similarly for students and for business managers. Frisch (1993) suggests that the effect holds also when only a calculator is being purchased, and Ranyard and Abdel-Nabi (1993) vary the price of the other good (the jacket) and get similar results. Darke and Freedman (1993) find that both the amount of money and the percentage of the base price that can be saved affect consumers' decisions. Azar (2004) showed, in an experiment with nine different price treatments,

[1] The bundle's price remains the same ($140) in both treatments, and therefore considering the savings relative to the bundle's price should not trigger different behavior in the two treatments.

that when subjects can purchase a certain good either in a store they currently visit or in a remote store, the minimal price difference for which they are willing to travel to the remote store is an increasing function of the good's price. Azar (2011a, 2011b) showed that people consider relative price differences also when choosing between substitute goods that differ in quality, even when only absolute price differences are relevant.

The phenomenon that people are affected by relative price differences even when these should be irrelevant was sometimes described as "mental accounting," but more recently the term "relative thinking" was offered instead (Azar, 2004), and here I use the latter. Relative thinking has important implications for firm strategy, one of which is pricing by multi-product firms.

Relative thinking is not the only psychological aspect that should affect pricing, however. Another important consideration of consumers when making purchase decisions is the perceived fairness of prices, which in turn depends on a comparison between the firm's prices and some reference prices (Kahneman, Knetsch and Thaler, 1986a).[2] A rich literature, mostly in marketing, studies price fairness perceptions and reference prices, and how these are determined (see for example Thaler, 1985; Kahneman, Knetsch and Thaler, 1986b; Urbany, Bearden and Weilbaker, 1988; and Bolton, Warlop and Alba, 2003).[3]

Dodonova and Khoroshilov (2004), who examine empirical data from the auction website Bidz.com, provide additional evidence for the importance of reference prices. They find that

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[2] Kahneman, Knetsch and Thaler (1986a, p. 729-730) write, "A central concept in analyzing the fairness of actions in which a firm sets the terms of future exchanges is the *reference transaction*, a relevant precedent that is characterized by a reference price or wage…"

[3] Reviewing this literature in detail is beyond the scope of this article. The interested reader is referred to Xia, Monroe and Cox (2004) for a review and conceptual framework of price fairness perceptions, and Mazumdar, Raj and Sinha (2005) for a literature review on reference price research.

people bid more for the same item when its posted "buy now" price is higher, suggesting that the reference "buy now" price affects buyers' valuation of a good. Other studies (e.g., Koszegi and Rabin, 2006; Heidhues and Koszegi, 2008) offer analysis of reference prices and how they affect pricing by firms when consumers are loss averse, i.e., attach a greater weight to losses than to gains (relative to some reference point).

Another psychological aspect that has implications for pricing is the utility that consumers may derive from finding a good bargain, beyond the utility that can be obtained from using the money saved for additional consumption. Darke and Freedman (1995), for example, find that subjects enjoyed bargains regardless of any financial gain, implying that non-financial motives might also be involved. In addition, they report that bargains acquired through skill were not enjoyed more than bargains achieved because of luck, suggesting that achievement motives could not explain why subjects enjoyed bargains when there was no associated financial gain.

The psychological evidence mentioned above has important implications for optimal pricing strategy in general, and in particular for pricing of multi-product firms. However, models of multi-product firms' pricing (e.g., DeGraba, 2006; Doraszelski and Draganska, 2006) have not yet considered these implications. The purpose of this article is to model how incorporating reference prices and relative thinking affects optimal pricing in the presence of multiple goods, thus contributing to the literature on pricing of multi-product firms. The article also contributes to the growing literature that addresses the effects of psychological biases on industrial organization and firm strategy.[4]

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[4] For theoretical models in this area, see DellaVigna and Malmendier (2004), Gabaix and Laibson (2006), and Ho, Lim and Camerer (2006). DellaVigna and Malmendier (2006) analyze empirically the case of gym pricing. For literature reviews, see Ellison (2006) and Camerer and Malmendier (2007).

The article presents a duopoly model of retailers that are located at the endpoints of a linear city and sell two goods, L and H, with H being the good with the higher reference price.[5] The firms take into account that consumers exhibit relative thinking and are affected by reference prices. This is captured in the model by assuming that consumers minimize not only the usual total costs (the goods' prices plus the transportation costs), but rather a combination of these total costs and an expression that involves the ratio of the prices to some reference prices.[6]

Section 2 presents a simple model in which all consumers buy one unit of each of the two goods sold by the firms. Assuming that it is impossible to choose negative prices, the firms give good L for free. Nevertheless, their profits are higher than those obtained with consumers who do not exhibit relative thinking, and the more relative thinking exists, the higher are the firms' profits.

Section 3 then extends the model by adding also consumers who are interested in only one of the two goods. In equilibrium, both firms choose the same prices. The markup on L might be negative, corresponding to the practice of loss-leader pricing.[7] The markup on H, however, is always positive. Compared to a model without consumers who buy both goods (henceforth "B consumers," B standing for "both-goods"), the markup on H may be increased or decreased, but

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[5] See Hotelling (1929) for the original model that used the linear city framework.

[6] In what follows, for the sake of brevity and to avoid too many cumbersome sentences, I sometimes use "relative thinking" to describe this consumer behavior of considering the ratio between the prices and the reference prices. This behavior captures both the idea that consumers pay attention to relative price differences (and not only to absolute differences) and the idea that they are affected by reference prices. When I mention stronger relative thinking, this means more emphasis of consumers on the expression that involves the ratio of the prices to the reference prices.

[7] The next section discusses this practice and the related literature in more detail.

the markup on L is always reduced. This outcome results from the combination of two main effects that affect the firms' pricing compared to the pricing in the absence of B consumers.

The first effect is the relative-thinking effect: when considering the profits from B consumers, each firm has an incentive to decrease the price of L and increase the price of H by the same amount, because this makes the firm more attractive to B consumers as a result of relative thinking. The relative-thinking effect becomes stronger (and consequently reduces the markup on L and increases the markup on H further) when the tendency of consumers to relative thinking is stronger and when the ratio between the reference prices of H and L is higher.

The second effect that affects pricing is the "attracting B consumers" effect. In the absence of B consumers, the firms charge a markup of t (the transportation cost parameter) on each good. However, when we add B consumers, the total markup on the two goods becomes too high, and the firms have an incentive to reduce prices in order to increase their profits from the B consumers. This effect is unrelated to relative thinking. Because lowering the markups on L and H reduces the profits from consumers who buy only L or H, the decision how much to reduce each markup depends on the size of the consumer segments.

Because both effects reduce the markup on L, it is unambiguously lower than its level without B consumers (which is t). In the pricing of H, the two effects act in opposite direction, and therefore the markup on H might be smaller or higher than t. Equilibrium profits are higher when relative thinking is stronger, implying that the firms are able to exploit relative thinking despite the competition between them. As we add more H consumers, profits increase, but when we add more L consumers, profits may increase or decrease. Interestingly, while a negative markup on L is a necessary condition for profits to decrease (following an increase in the number of L consumers), it is not a sufficient condition.

2. All consumers buy two goods

To explore how relative thinking and reference prices affect pricing of multi-product firms, I use the framework of the linear-city model. The length of the line is normalized to one. The firms are located at the endpoints of the line (firm 1 is on the left, located at 0, and firm 2 is on the right, located at 1). Each firm sells two goods, which are denoted as goods L and H. Denote the price charged by firm i (i = 1, 2) for good j (j = L, H) as P_{ij}. There are no fixed costs, and the marginal cost of the goods is C_L for good L and C_H for good H for both firms. Other than the location differentiation, the goods sold by the two firms are identical.

All consumers are interested in buying one unit of L and one unit of H, and their willingness to pay is high enough that in equilibrium everyone buys both goods. Consumers purchase both goods from the same firm.[8] They are distributed uniformly over the city line and their mass is normalized to one. The total transportation cost is linear in the distance traveled to the firm (denoted by d), and the transportation cost per unit of distance is strictly positive and is denoted as t.

In a traditional linear-city model, consumers go to the firm that minimizes their total costs, which consist of the good's price and the transportation cost. With two goods, the consumer chooses the firm i that offers him the lower value of $P_{iL} + P_{iH} + td$. This, however, does not take into account relative thinking and reference prices. To account for these issues, I assume that the consumer chooses the firm that minimizes a somewhat different expression,

$$\alpha\left(\frac{P_{iL}}{R_L}+\frac{P_{iH}}{R_H}\right)+(1-\alpha)(P_{iL}+P_{iH})+td, \text{ where } 0 \le \alpha \le 1.$$

[8] In equilibrium prices of the firms are equal and therefore indeed no consumer has an incentive to incur larger transportation costs in order to buy each good from a different firm.

This expression implies that the customer takes into account transportation costs as usual, but his treatment of the prices is not limited to considering only the total price of the bundle. To some extent, measured by the coefficient $(1-\alpha)$, he still considers the total price he pays. However, the term $\alpha(P_{iL}/R_L + P_{iH}/R_H)$ captures the idea that the consumer also considers the prices relative to some reference prices he has in mind, R_L for good L and R_H for good H. Let us denote the good for which the reference price is higher as H, so $R_H \geq R_L$, and denote the ratio between the two reference prices as β, i.e., $\beta = R_H/R_L$. There are many possible sources for such reference prices, for example the manufacturer suggested retail price (MSRP) for cars, the list prices of books (which are printed on the books by the publishers), prices the customer obtained when searching for the good online before going to the store, etc. The experimental evidence on relative thinking suggests that people are affected by the percentage price difference between two prices, which implies that the natural way to model the treatment of the reference price is by the ratio of the seller's price to the reference price. The parameter α measures the extent of the consumer's tendency to relative thinking; a higher α captures stronger tendency. The extreme case of $\alpha = 0$ represents the standard consumer that is assumed in traditional models.

The consideration of the relative prices (P_{iL}/R_L and P_{iH}/R_H) can be the result of each of several reasons. First, it is consistent with the experimental finding that people make more effort to save the same amount when the savings are higher relative to the good's price, i.e., when the good's price is lower, as Tversky and Kahneman (1981) and others have found. While sometimes this behavior could be explained by a desire to avoid unfair prices or to find bargains, it was documented also when fairness and the utility from finding a good bargain were not relevant (Azar, 2004). This behavior might be, for example, a result of a decision-making bias according to which people have a tendency to consider relative magnitudes (either instead or in addition to considering absolute magnitudes) even when relative magnitudes should be ignored.[9]

Another reason that the relative prices might matter is the reluctance of consumers to pay unfair prices. As discussed above, price fairness is determined by comparing the price to some reference price. A third reason why consumers might be affected by relative prices is that consumers sometimes derive psychological utility from finding a good bargain. To determine whether the price indeed reflects a good bargain, the consumer compares the price to some reference price he has in mind for the good.

We have a set of prices (C_L, C_H, P_L, P_H, R_L, R_H, t), and as with any system of prices, multiplying all prices by a constant does not change anything real. Therefore we can normalize these prices by multiplying them such that we get $R_L = 1$, which implies that $R_H = \beta \geq 1$. Then, the expression that the consumer minimizes (by choosing from which firm to buy) becomes:

$$P_{iL} + (1 - \alpha + \frac{\alpha}{\beta})P_{iH} + td.$$

To simplify the analysis below, let us define $\gamma \equiv 1 - \alpha(1 - \beta^{-1})$. It follows that the consumer buys from the firm that offers him the lower value of $P_{iL} + \gamma P_{iH} + td$. Because $0 \leq \alpha \leq 1$ and $\beta \geq 1$, it follows that $0 < \gamma \leq 1$. We can see that γ is decreasing in α whenever $\beta > 1$, and γ is decreasing in β whenever $\alpha > 0$.

The parameter γ is inversely related to the level of what we may call "actual relative thinking," which can be defined as the level of relative thinking exhibited by the consumer in a particular scenario. This level depends on the consumer's innate tendency to relative thinking,

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[9] Indeed, this is in line with the Weber-Fechner Law, which suggests that our ability to distinguish between stimuli depends on the relative difference between them and not on the absolute difference.

captured by α, and on the environment. The crucial element in the environment that affects the extent of actual relative thinking is β, the ratio between the two reference prices; the larger is β, the more prominent the actual relative thinking becomes. Notice that either $\alpha = 0$ or $\beta = 1$ imply no actual relative thinking, and indeed in these cases we get $\gamma = 1$ and the consumer minimizes $P_{iL} + P_{iH} + td$, just as he would do in a traditional linear-city model with two goods.

Now we can turn to analyze the outcome in the Nash equilibrium of the game between the two firms. Proposition 1 characterizes equilibrium prices and profits.

Proposition 1.

(a) Assume that prices cannot be negative, $\beta > 1$ and $\alpha > 0$. Then equilibrium prices are $P_{1L} = P_{2L} = 0$ and $P_{1H} = P_{2H} = C_L + C_H + t/\gamma$, and equilibrium profits are $\Pi_1 = \Pi_2 = t/2\gamma$.

(b) If $\beta = 1$ or $\alpha = 0$, then equilibrium prices are not unique but they satisfy $P_{1L} + P_{1H} = P_{2L} + P_{2H} = C_L + C_H + t$, and equilibrium profits are $\Pi_1 = \Pi_2 = t/2$.

Proof: See the Appendix.

Proposition 1 implies that when consumers exhibit relative thinking, the firms can exploit this and earn higher profits, despite the competition between the firms that mitigates their ability to use manipulative pricing schemes. Because consumers attach a greater significance to saving a dollar on a cheap good than on an expensive good, the firms find it optimal to lower the price of L as much as possible – down to zero – and compensate for this by raising the price of H.

The result that the firms lose on each unit of L is consistent with loss-leader pricing, a practice that is sometimes used by retailers to price certain items below cost (see for example Walters and MacKenzie, 1988; Chevalier, Kashyap, and Rossi, 2003). The model implies that the best candidates to be loss leaders are goods with low reference prices, because then the firm can offer a high percentage discount at a relatively low cost for itself (and because consumers are affected also by the relative discount even though only the absolute savings should matter to them). This result is consistent with the observation that loss leaders are often carbonated drinks, white bread, flour, and eggs (Nagle, 1987).

Several theoretical models of loss-leader pricing were developed in the literature before. Hess and Gerstner (1987) present a two-period model with stores that sell one shopping good (the leader product) and a selection of "impulse goods" (products that are bought on sight without price comparisons across stores). They analyze the issue of loss leaders in conjunction with rain check policy that ensures that if the leader product is out of stock, the customer will be allowed to buy one unit of it at its current price when he comes to the store again. Lal and Matutes (1994) develop a model with uninformed consumers who do not know the prices unless these are advertised by the firms. They find that firms advertise prices below marginal cost to attract consumers into the store and profit from other goods that the consumers plan to purchase at the store.

The usual explanation for loss leaders says that customers choose where to shop based on the price of one good (the loss leader), and when they come to the store, they buy also additional goods (DeGraba, 2006). DeGraba, however, presents a model with a different explanation for loss-leader pricing, in which loss leaders are products that are bought primarily by more-profitable consumers (people who buy large quantities of other goods), and loss-leader pricing is a way to price discriminate between customers.

Here, however, the result of loss-leader pricing is different from the results in the theoretical models mentioned above. Pricing below cost here is a result of the firm's response to relative thinking, which was not considered in the previous models. In addition, the result of negative markups in this model is different from that in Lal and Matutes (1994) and Hess and Gerstner (1987), because here consumers consider all prices before they decide to which store to go. It also differs from the explanation of DeGraba (2006) because here the reason for pricing below cost is not to attract consumers who buy large quantities of other goods, as in DeGraba's model.

The result that the firms go as far as offering good L for free seems too strong to be reasonable in most real-world scenarios of multi-product firms. One of the main reasons for this discrepancy between this result and what we might expect to see in practice is the simplifying assumption that every consumer buys both goods. Therefore, a natural way to extend the model and make it more realistic is to add consumers who buy only one of the two goods. This is what the next section does.

3. Consumers buy either one or two goods

To model the possibility that some consumers are interested in purchasing only one of the two goods, I adopt the same linear-city framework of the previous section. Now, however, in addition to a mass of consumers (whose size is normalized to one) who want to purchase both goods, there is a mass of $\lambda > 0$ consumers who want to purchase only good L, and a mass of $\eta > 0$ consumers who want to buy only H. In what follows, let us refer to these consumers as types L, H and B (consumers interested in both goods). All three types of consumers are distributed uniformly on the city line. Consumers who purchase only one good choose the firm that offers them the lower total cost (the good's price plus transportation costs).

Before analyzing the equilibrium with the three consumer segments, it is worthwhile to mention what happens when only one type of consumers exists. The case where all consumers want to purchase both goods was already analyzed in the previous section. What happens when all consumers are interested in only one good? This is a standard linear-city model, in which case the equilibrium markup is equal to the transportation cost parameter (see for example Tirole, 1988, p. 279–280). That is, with only L consumers (with a mass of λ) the prices are $P_{1L} = P_{2L} = C_L + t$ and equilibrium profit is $\lambda t/2$. Similarly, with only H consumers (with a mass of η) the prices are $P_{1H} = P_{2H} = C_H + t$ and equilibrium profit is $\eta t/2$.

When the three consumer segments exist together, it turns out that the second-order conditions are no longer satisfied for all possible parameter values. To ensure that the second-order sufficient conditions are satisfied, we have to assume the following: [10]

Assumption 1. $(2+4\lambda)\gamma + 4\eta(1+\lambda) - 1 - \gamma^2 > 0$.

As Proposition 2 below implies, the markup on good L might be negative. To avoid having to analyze corner solutions (which will complicate the analysis considerably with no apparent benefit), I assume that at least one of the following two conditions holds: (1) the parameter values are such that the negative markup (in absolute value) is weakly smaller than C_L, and therefore the price of L is non-negative despite the negative markup; or (2) the firms can charge negative prices.[11] Now we can state Proposition 2, which characterizes the equilibrium prices.

[10] Because the left-hand side of this inequality is increasing in γ for the possible parameter values in the model, a sufficient condition for this inequality to hold for any value of $\gamma \in [0, 1]$ is that this inequality holds for $\gamma = 0$; that is, the condition $4\eta(1+\lambda) - 1 > 0$ ensures that the second-order sufficient conditions are satisfied for any value of γ.

[11] Of course, in such a case the firms would like to limit purchases to one per customer; they can enforce it for example by conditioning the benefit of the negative price on registration that allows the firm to verify that the customer does not use the offer again. In the model such a limitation does not change the analysis because consumers anyway buy no more than one unit. In the previous section negative prices were not allowed because with only B

Proposition 2.

(a) In equilibrium the firms' prices are equal, $P_{1L} = P_{2L}$ and $P_{1H} = P_{2H}$.

(b) Let us define $M_L \equiv P_{1L} - C_L = P_{2L} - C_L$ and $M_H \equiv P_{1H} - C_H = P_{2H} - C_H$. Equilibrium markups are then given by $M_L = t\dfrac{\lambda(\gamma+\eta)+\gamma-1}{\lambda(\gamma+\eta)+\eta}$ and $M_H = t\dfrac{(1+\lambda)(1+\eta-\gamma)}{\lambda(\gamma+\eta)+\eta}$.

Proof: See the Appendix.

[Table 1 here]

To get a more concrete feeling for how the equilibrium looks like, Table 1 presents the equilibrium markups and profits with several possible parameter values.[12] Proposition 2 now allows us to obtain a few additional results, stated in Corollary 1.

Corollary 1.

(a) When $\lambda \leq \eta$, we get $M_H > M_L$, except for the knife-edge case in which both $\lambda = \eta$ and $\gamma = 1$ (in that case, $M_H = M_L$).

When $\lambda > \eta$, M_L might be smaller than, equal to, or larger than M_H.

(b) In equilibrium, there might be a negative markup on good L ($M_L < 0$), but not on good H.

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consumers, allowing negative prices results in no equilibrium: the firms always want to lower M_L further and increase M_H. Here this does not happen because of the consumers who want to purchase only one good.

[12] To find equilibrium profits, notice that because equilibrium prices of the two firms are equal, each firm sells to half of the market and earns profits of $0.5[(1+\lambda)M_L + (1+\eta)M_H]$.

(c) M_L is lower than its value in the absence of B consumers, which is equal to t. M_H may be lower than, equal to, or higher than its value in the absence of B consumers (which is also equal to t).

(d) $M_H > t$ is a necessary but not a sufficient condition for $M_L < 0$.

Proof: See the Appendix.

The intuition for the results stated in Corollary 1 is that the firms have two main effects that affect their pricing compared to the pricing of one good (or two goods where each consumer only buys one of them, which gives the same optimal prices). One effect can be called the "relative thinking effect." Because B consumers are affected by relative prices (the ratio between the prices and the reference prices), a decrease in the price of L and an equivalent increase in the price of H make the firm more attractive to B consumers, because the savings due to this change are on L (and therefore are high relative to the good's reference price) while the price increase is on H and therefore relative to the reference price it seems smaller. This effect was also present in the model in Section 2, but here the existence of L and H consumers mitigates the price changes in response to relative thinking.

The second effect can be explained as follows. In the absence of B consumers, the firms charge a markup of t on each good. However, with such markups, when we introduce also consumers who buy both goods, they have to pay a total markup of $2t$, above the equilibrium markup when only B consumers exist (which is also t). The firms then find it optimal to reduce prices in order to increase their profits from B consumers. At least with small changes in prices this must increase total profits because the effect of reducing each markup a little (from t) is negligible (because with markups being equal to t, the first-order derivative of profits from L and H consumers with respect to the prices of L and H is zero). This is another effect of introducing B consumers, which we can call the "attracting-B-consumers effect," because the essence of this is that in order to attract the B consumers the firm lowers both prices. It is unrelated to relative thinking and exists also when $\gamma = 1$. Because lowering the markups on L and H reduces the profits from L and H consumers, the decision which markup to reduce more depends on which consumer segment is larger – i.e., on the comparison between λ and η.

Notice that both effects act to reduce the markup on L compared to its level without B consumers (which is t), but the two effects act in opposite directions on the markup on H (the relative thinking effect acts to increase it while the attracting-B-consumers effect acts to decrease it). This explains why M_L is always smaller than t, but M_H might be smaller or higher than t (Corollary 1c). It also helps to understand why M_L can be reduced so much that it becomes negative, while M_H is always positive (Corollary 1b). When the segment of H consumers is larger than the segment of L consumers ($\eta > \lambda$), the attracting-B-consumers effect requires a larger decrease in M_L than in M_H (in order to increase profits from B consumers while minimizing the profit loss from L and H consumers). Because the relative-thinking effect increases M_H and decreases M_L, it is clear why $M_H > M_L$ (Corollary 1a). When $\eta < \lambda$, however, the attracting-B-consumers effect necessitates a larger decrease in M_H than in M_L ; depending on the parameters, either this or the relative-thinking effect dominates, and therefore M_L might be smaller than, equal to, or larger than M_H (Corollary 1a).

Finally, to understand why $M_L < 0$ necessarily implies $M_H > t$ (Corollary 1d), notice that $M_H \leq t$ and $M_L < 0$ together cannot be optimal, because in this case the total markup on L and H together is smaller than optimal to maximize profits from the B consumers, and yet increasing M_L also increases the profits from L consumers (and if $M_H < t$, also increasing M_H increases profits from H consumers). Consequently, if $M_H \leq t$ and $M_L < 0$ the firm should increase one of the markups or both.

Corollary 1b establishes that pricing below cost might be optimal, but only on the good with the low reference price. We obtained a similar result in Section 2; however, the introduction of consumers who buy only one good creates two important differences. First, the negative markup here might be small and the price might not go all the way to zero. Second, here pricing below cost is possible but does not always occur, depending on the parameter values.

How are equilibrium markups affected by the size of the consumer segments and by the extent of relative thinking? Proposition 3 provides the answers.

Proposition 3.

(a) $\partial M_L/\partial\gamma > 0$ and $\partial M_H/\partial\gamma < 0$. This implies that $\partial M_L/\partial\alpha < 0$, $\partial M_L/\partial\beta < 0$, $\partial M_H/\partial\alpha > 0$, and $\partial M_H/\partial\beta > 0$.

(b) $\partial M_L/\partial\lambda > 0$ and $\partial M_H/\partial\lambda < 0$.

(c) $\partial M_L/\partial\eta$ has the same sign as $(M_H - t)$; $\partial M_H/\partial\eta$ has the opposite sign.

Proof: See the Appendix.

The intuition for the results in part (a) is that when relative thinking is stronger (lower γ) – due to more relative thinking tendency (higher α) or to a larger gap in the two goods' prices (higher β) – this reinforces the relative-thinking effect on the firms' pricing, reducing the markup on L and increasing the markup on H. The intuition behind part (b) is that when the number of L consumers increases, the profit loss from L consumers due to decreasing M_L (relative to its level in the absence of B consumers, which is t) becomes more costly, and therefore the firm chooses a higher M_L. To compensate the B consumers, at least partially, the firm reduces the markup on H.

The effect of the number of H consumers (η) depends on the comparison between M_H and t. When $M_H > t$, the firm chooses a markup on H that is higher than the markup that would maximize profits from H consumers (t), because it is beneficial in order to maximize profits from the B consumers. Then, when the size of the segment of H consumers increases, this sacrifice of profits from H consumers becomes more costly, so the firm lowers M_H. The reduced markup on H also reduces the total price paid by the B consumers and this allows the firm to increase the markup on L. When $M_H < t$, the opposite happens. The markup on H is too low compared to the level that maximizes profits from H consumers, so when the number of these consumers increases, the firm raises the markup on H and compensates the B consumers by reducing the markup on L.

So far we focused on equilibrium markups. Proposition 4 turns to equilibrium profits and present a few results about how profits are affected by relative thinking and by the size of the consumer segments.

Proposition 4.

(a) $\partial\Pi_i/\partial\gamma < 0$.

(b) $\partial\Pi_i/\partial\eta > 0$.

(c) The sign of $\partial\Pi_i/\partial\lambda$ is the same as the sign of $(\lambda\gamma+\lambda\eta+\eta)^2-\eta\gamma(\eta+1-\gamma)$. It follows that:

If $M_L > 0$, then $\partial\Pi_i/\partial\lambda > 0$.

If $M_L = 0$, then $\partial\Pi_i/\partial\lambda > 0$ if $\gamma < 1$ and $\partial\Pi_i/\partial\lambda = 0$ if $\gamma = 1$.

If $M_L < 0$, then $\partial\Pi_i/\partial\lambda$ may have any sign.

Proof: See the Appendix.

Proposition 4a implies that stronger relative thinking increases the firms' profits. In particular, profits with consumers' relative thinking ($\gamma < 1$) are higher than without ($\gamma = 1$); the firms price their goods in a way that exploits consumers' relative thinking, and doing so increases profits despite the competition between the firms. Regarding the effect of additional consumers, it is worthwhile to mention that in a similar model without B consumers, adding consumers increases the firms' profits (it does not change equilibrium prices but it increases the quantity sold). Here, it turns out that increasing the number of H consumers indeed unambiguously increases profits, but more L consumers might sometimes reduce profits. While a negative markup on L is a necessary condition for profits to be decreasing in λ, however, it is not a sufficient condition; that is, we might have a negative markup on L and yet profits will increase when more L consumers are added – despite making loss on each such additional consumer.[13]

4. Conclusion

The article presents two models that analyze the optimal pricing strategy of multi-product retailers who take into account that consumers are affected by the ratio of prices to some reference prices. This behavior is consistent with the findings of many experiments, and can result from a decision making bias, from a desire to avoid prices that are perceived as unfair, or from utility that consumers derive when they find a good bargain (beyond the utility from using the money saved for additional consumption).

[13] The explanation how this is possible is that when λ increases, M_L also increases; thus, adding L consumers when $M_L < 0$ increases the number of consumers on whom the firm loses, but the loss on each such consumer decreases.

The first model assumes that all consumers buy one unit of each of the two goods sold by the firms. Assuming that negative prices are not feasible, in equilibrium the firms give the good with the low reference price for free. Nevertheless, their profits are higher than with consumers who do not exhibit relative thinking; and the more relative thinking exists, the higher are the firms' profits.

The second model then extends this framework by adding also consumers who are interested in only one of the two goods. In equilibrium, both firms choose the same prices. The markup on L (the good with the low reference price) might be negative, corresponding to the practice of loss-leader pricing. The markup on H (the good with the high reference price), however, is always positive.

The introduction of B consumers (consumers who buy both goods) creates two effects on pricing. The first is the relative-thinking effect: the firms should decrease the price of L and increase the price of H to exploit relative thinking of B consumers. The second effect is the "attracting-B-consumers effect": to increase profits from B consumers (and total profits), the firms have to reduce the markups compared to the ones they would choose in the absence of B consumers. The combination of these two effects implies that the introduction of B consumers reduces the markup on L but may increase or decrease the markup on H.

Equilibrium profits are higher when relative thinking is stronger, implying that the firms are able to exploit relative thinking despite the competition between them. As we add H consumers, profits increase, but when we add L consumers, profits may increase or decrease. Interestingly, while a negative markup on L is a necessary condition for profits to decrease (following an increase in the number of L consumers), it is not a sufficient condition.

While the model assumes two firms and two goods for the sake of simplicity and tractability, the main results are relevant also when more firms and more goods exist. In particular, the relative-thinking effect implies that when consumers buy several goods together, the firm can benefit from decreasing the prices of the cheaper items (possibly even below cost) and increasing the prices of the expensive items (compared to optimal prices without consideration of relative thinking). The attracting-B-consumers effect implies that the existence of consumers who buy many goods reduces prices.

Appendix: Proofs of Propositions

Proof of Proposition 1. (a) Denote the location of the consumer who is indifferent between the two firms by A. The value of A has to satisfy $P_{1L} + \gamma P_{1H} + tA = P_{2L} + \gamma P_{2H} + t(1–A)$, from which it follows that $A = 0.5 + [P_{2L} - P_{1L} + \gamma(P_{2H} - P_{1H})]/2t$. Everyone to the left of the indifferent consumer buys from firm 1 while everyone to his right buys from firm 2. This implies that the demand of firm 1 is equal to A and the demand of firm 2 is $1–A$. The assumption in the Proposition that $\beta > 1$ and $\alpha > 0$ implies that $\gamma < 1$. This implies that in equilibrium we must have $P_{1L} = 0$. To see why, notice that if $P_{1L} > 0$, then firm 1 can increase its profits by lowering P_{1L} and increasing P_{1H} by the same amount. This increases the number of consumers who buy from firm 1 and yet leaves the markups earned on each consumer unchanged. Consequently, $P_{1L} > 0$ cannot be part of an equilibrium, and because prices cannot be negative, in equilibrium we have $P_{1L} = 0$. By a similar argument, in equilibrium we also have $P_{2L} = 0$. This implies that $A = 0.5 + \gamma(P_{2H} - P_{1H})/2t$. The profit of firm 1 is then given by $\Pi_1 = [0.5 + \gamma(P_{2H} - P_{1H})/2t](P_{1H} - C_L - C_H)$. Differentiating this function with respect to P_{1H} gives the following first-order condition for profit maximization:

$\frac{\partial \Pi_1}{\partial P_{1H}} = \frac{1}{2} + \frac{\gamma(P_{2H} - P_{1H})}{2t} - \frac{\gamma}{2t}(P_{1H} - C_L - C_H) = 0,$ which after multiplying by $2t/\gamma$ becomes $t/\gamma + P_{2H} - 2P_{1H} + C_L + C_H = 0$. It is easy to see that the second-order derivative of Π_1 with respect to P_{1H} is negative, satisfying the second-order condition for maximum. Similarly, the first-order condition of firm 2 yields after further manipulations $t/\gamma + P_{1H} - 2P_{2H} + C_L + C_H = 0$. The Nash equilibrium of the game is then obtained by solving the two equations simultaneously. Doing so gives $P_{1H} = P_{2H} = C_L + C_H + t/\gamma$. The profit firm 1 makes from each consumer is equal to $P_{1L} + P_{1H} - C_L - C_H = t/\gamma$. Because the total mass of consumers is normalized to 1 and the two firms have the same prices and therefore share the market equally, each firm obtains equilibrium profits of $t/2\gamma$.

(b) When $\beta = 1$ or $\alpha = 0$ then $\gamma = 1$, from which it follows by part (a) that $A = 0.5 + (P_{2L} - P_{1L} + P_{2H} - P_{1H})/2t$. The profit of firm 1 is then given by $\Pi_1 = [0.5 + (P_{2L} - P_{1L} + P_{2H} - P_{1H})/2t](P_{1L} + P_{1H} - C_L - C_H)$. The first-order conditions of this function with respect to P_{1L} and P_{1H} are identical, $\partial\Pi_1/\partial P_{1L} = \partial\Pi_1/\partial P_{1H} = 0.5 + (P_{2L} - P_{1L} + P_{2H} - P_{1H})/2t - (P_{1L} + P_{1H} - C_L - C_H)/2t = 0$. Similarly, the first-order conditions for firm 2 yield $\partial\Pi_2/\partial P_{2L} = \partial\Pi_2/\partial P_{2H} = 0.5 + (P_{1L} - P_{2L} + P_{1H} - P_{2H})/2t - (P_{2L} + P_{2H} - C_L - C_H)/2t = 0$. This gives us two different equations in four unknown variables and therefore it is clear that we cannot obtain unique equilibrium prices. However, we can multiply both equations by $2t$ to obtain $t + P_{2L} + P_{2H} - 2P_{1L} - 2P_{1H} + C_L + C_H = 0$ and $t + P_{1L} + P_{1H} - 2P_{2L} - 2P_{2H} + C_L + C_H = 0$. Multiplying the second equation by 2 and adding the result to the first equation gives $3t - 3P_{2L} - 3P_{2H} + 3C_L + 3C_H = 0$. Rearranging and substituting again in one of the equations above then show that $P_{1L} + P_{1H} = P_{2L} + P_{2H} = C_L + C_H + t$. This implies that the firms make a profit of t on each consumer. Because consumers are interested in the total price they pay for the two goods and not in the separate prices, and because this total price is identical in the two firms, consumers are split equally between the two firms, and therefore each firm's profits are $t/2$. Q.E.D.

Proof of Proposition 2. (a) First, we should find the location of the indifferent consumer for each consumer segment. For the B customers, we found in Proposition 1 that the indifferent consumer is at $A = 0.5 + [P_{2L} - P_{1L} + \gamma(P_{2H} - P_{1H})]/2t$. The indifferent consumer in the L segment is located in a point D that satisfies $P_{1L} + Dt = P_{2L} + (1-D)t$, from which it follows that $D = 0.5 + (P_{2L} - P_{1L})/2t$. Similarly, the indifferent consumer in the H segment is located at $E = 0.5 + (P_{2H} - P_{1H})/2t$. Consequently, the profit of firm 1 is given by $\Pi_1 = (A + \lambda D)(P_{1L} - C_L) + (A + \eta E)(P_{1H} - C_H)$. Substituting A, D, and E and simplifying we get:

$$\Pi_1 = \left[\gamma\frac{P_{2H}-P_{1H}}{2t} + (1+\lambda)\left(0.5+\frac{P_{2L}-P_{1L}}{2t}\right)\right](P_{1L}-C_L) + \left[\frac{P_{2L}-P_{1L}}{2t} + 0.5(1+\eta) + (\gamma+\eta)\left(\frac{P_{2H}-P_{1H}}{2t}\right)\right](P_{1H}-C_H).$$ The first-order condition of Π_1 with respect to P_{1L} then gives:

$$\frac{\partial \Pi_1}{\partial P_{1L}} = \gamma\frac{P_{2H}-P_{1H}}{2t} + (1+\lambda)\left(0.5+\frac{P_{2L}-2P_{1L}+C_L}{2t}\right) + \frac{C_H-P_{1H}}{2t} = 0,$$ which after multiplying by $2t$ and rearranging becomes:

(1) $$(1+\lambda)(t+C_L) + C_H = \gamma(P_{1H}-P_{2H}) + (1+\lambda)(2P_{1L}-P_{2L}) + P_{1H}.$$

The first-order condition of Π_1 with respect to P_{1H} yields:

$$\frac{\partial \Pi_1}{\partial P_{1H}} = \gamma\frac{C_L-P_{1L}}{2t} + \frac{P_{2L}-P_{1L}}{2t} + 0.5(1+\eta) + (\gamma+\eta)\left(\frac{P_{2H}-2P_{1H}+C_H}{2t}\right) = 0,$$ which after multiplying by $2t$ and rearranging becomes:

(2) $$\gamma C_L + t(1+\eta) + (\gamma+\eta)C_H = \gamma P_{1L} + P_{1L} - P_{2L} + (\gamma+\eta)(2P_{1H}-P_{2H}).$$

It is easy to verify that the second-order conditions for maximum are satisfied:

$\partial^2\Pi_1/\partial P_{1L}^2 = -(1+\lambda)/t < 0$;

$\partial^2\Pi_1/\partial P_{1H}^2 = -(\gamma + \eta)/t < 0$;

and $(\partial^2\Pi_1/\partial P_{1L}^2)(\partial^2\Pi_1/\partial P_{1H}^2) - (\partial^2\Pi_1/\partial P_{1H}\partial P_{1L})^2 = (1+\lambda)(\gamma + \eta)/t^2 - (1+\gamma)^2/4t^2 = [(2 + 4\lambda)\gamma + 4\eta(1+\lambda) - 1 - \gamma^2]/4t^2 > 0$ (the inequality follows from Assumption 1).

The profit of firm 2 is given by $\Pi_2 = [1-A + \lambda(1-D)](P_{2L} - C_L) + [1-A + \eta(1-E)](P_{2H} - C_H)$, which after substitution of A, D, and E and simplification becomes:

$$\Pi_2 = \left[\gamma\frac{P_{1H}-P_{2H}}{2t} + (1+\lambda)\left(0.5 + \frac{P_{1L}-P_{2L}}{2t}\right)\right](P_{2L}-C_L) + \left[\frac{P_{1L}-P_{2L}}{2t} + 0.5(1+\eta) + (\gamma+\eta)\left(\frac{P_{1H}-P_{2H}}{2t}\right)\right](P_{2H}-C_H).$$

The first-order condition with respect to P_{2L}, after multiplication by $2t$ and rearranging, is:

(3) $$(1+\lambda)\left(t + C_L\right) + C_H = \gamma(P_{2H} - P_{1H}) + (1+\lambda)\left(2P_{2L} - P_{1L}\right) + P_{2H}.$$

The first-order condition with respect to P_{2H}, after multiplication by $2t$ and rearranging, yields:

(4) $$\gamma C_L + t(1+\eta) + (\gamma+\eta)C_H = \gamma P_{2L} + P_{2L} - P_{1L} + (\gamma+\eta)(2P_{2H} - P_{1H}).$$

Notice that the left-hand side of equations (1) and (3) is identical. Consequently, the right-hand sides of the two equations are also equal:

$$\gamma(P_{1H} - P_{2H}) + (1+\lambda)\left(2P_{1L} - P_{2L}\right) + P_{1H} = \gamma(P_{2H} - P_{1H}) + (1+\lambda)\left(2P_{2L} - P_{1L}\right) + P_{2H}.$$

Rearranging this equation gives:

(5) $$P_{2L} - P_{1L} = \frac{(1+2\gamma)(P_{1H} - P_{2H})}{3(1+\lambda)}.$$

Because the left-hand side of (2) is equal to that of (4), we also have:

$$\gamma P_{1L} + P_{1L} - P_{2L} + (\gamma+\eta)(2P_{1H} - P_{2H}) = \gamma P_{2L} + P_{2L} - P_{1L} + (\gamma+\eta)(2P_{2H} - P_{1H}),$$

which after rearranging becomes:

(6) $$P_{2L} - P_{1L} = \frac{3(\gamma+\eta)(P_{1H} - P_{2H})}{(2+\gamma)}.$$

Equations (5) and (6) imply that $\frac{(1+2\gamma)(P_{1H}-P_{2H})}{3(1+\lambda)}=\frac{3(\gamma+\eta)(P_{1H}-P_{2H})}{(2+\gamma)}$. This implies that in equilibrium we must have $P_{1H} = P_{2H}$; there are no parameter values for which $P_{1H} \neq P_{2H}$. To see why, notice that if $P_{1H} \neq P_{2H}$, then we must have $\frac{(1+2\gamma)}{3(1+\lambda)}=\frac{3(\gamma+\eta)}{(2+\gamma)}$. Rearranging this equation shows that it is equivalent to $9(\eta + \lambda\gamma + \lambda\eta) - 2 - 2\gamma^2 + 4\gamma = 0$. However, Assumption 1 implies that this equality cannot hold, because we have $9(\eta + \lambda\gamma + \lambda\eta) - 2 - 2\gamma^2 + 4\gamma = \eta + \lambda\gamma + \lambda\eta + 2[-\gamma^2 + (2+4\lambda)\gamma + 4\eta(1+\lambda) - 1] > 0$. Substituting $P_{1H} = P_{2H}$ in (5) then shows that also $P_{1L} = P_{2L}$. That is, the only equilibrium is symmetric, with the firms' prices being equal.

(b) To proceed, let us denote $P_L \equiv P_{1L} = P_{2L}$ and $P_H \equiv P_{1H} = P_{2H}$. Equations (1) and (2) can then be rearranged as follows:

$$\begin{bmatrix}1+\lambda & 1\\ \gamma & \gamma+\eta\end{bmatrix}\begin{bmatrix}P_L-C_L\\ P_H-C_H\end{bmatrix}=\begin{bmatrix}(1+\lambda)t\\ (1+\eta)t\end{bmatrix}.$$

Solving this system of equations and using $M_L \equiv P_L - C_L$ and $M_H \equiv P_H - C_H$ gives:

$$M_L = t\frac{\lambda(\gamma+\eta)+\gamma-1}{\lambda(\gamma+\eta)+\eta} \text{ and } M_H = t\frac{(1+\lambda)(1+\eta-\gamma)}{\lambda(\gamma+\eta)+\eta}. \qquad \text{Q.E.D.}$$

Proof of Corollary 1. (a) It is easy to see using Proposition 2b that $M_H > M_L$ if and only if $(1+\lambda)(1+\eta-\gamma) > \lambda(\gamma+\eta)+\gamma-1$, which is equivalent to $(2+\lambda)(1-\gamma)+\eta-\lambda\gamma>0$. When both $\lambda = \eta$ and $\gamma = 1$ the left-hand side of this latter inequality is equal to zero and therefore $M_H = M_L$. When $\lambda < \eta$, this inequality always holds. When $\lambda = \eta$ and $\gamma < 1$, the inequality also holds.

However, when $\lambda > \eta$, the inequality can hold, or it can be reversed (and because the left-hand side is continuous in the parameters, it can also hold with equality). To see this, notice that when

γ approaches zero the left-hand side approaches 2 + λ + $\eta > 0$, but when γ approaches one, the left-hand side approaches $\eta - \lambda < 0$.

(b) The markup on L is negative if and only if $\lambda(\gamma+\eta)+\gamma-1 < 0$. This happens for certain parameter values, for example when λ is sufficiently small and $\gamma < 1$. To see that the markup on H is always positive, notice that $(1+\lambda)(1+\eta-\gamma) > 0$ for all relevant parameter values.

(c) To see why $M_L < t$, notice that $\lambda(\gamma+\eta)+\gamma-1 < \lambda(\gamma+\eta)+\eta$. However, $M_H > t$ if and only if $(1+\lambda)(1+\eta-\gamma) > \lambda(\gamma+\eta)+\eta$, which is equivalent to $1-\gamma+\lambda-2\lambda\gamma > 0$. This inequality might hold (e.g., when γ is small enough), but might also be reversed (e.g., when γ is close to 1), implying that M_H may be lower than, equal to, or higher than t.

(d) To show that $M_H > t$ is a necessary condition for $M_L < 0$, we should show that $M_L < 0$ implies $M_H > t$. Rearranging the inequality in part (b) implies that $M_L < 0$ if and only if $1 - \gamma - \lambda\gamma - \lambda\eta > 0$. Because $\lambda(1-\gamma) \geq 0 > -\lambda\eta$, we then have $1-\gamma+\lambda-2\lambda\gamma > 1 - \gamma - \lambda\gamma - \lambda\eta > 0$. Part (c) then implies that $M_H > t$. To see that $M_H > t$ is not a sufficient condition for $M_L < 0$, notice that when γ approaches zero, we have $M_H > t$, but M_L might still be positive (e.g., consider any λ and η such that $\lambda\eta > 1$). Q.E.D.

Proof of Proposition 3. (a) The derivative of M_L with respect to γ is:

$$\frac{\partial M_L}{\partial \gamma} = t\frac{(\lambda+1)[\lambda(\gamma+\eta)+\eta]-[\lambda(\gamma+\eta)+\gamma-1]\lambda}{[\lambda(\gamma+\eta)+\eta]^2}$$, which after simplification becomes:

$$\frac{\partial M_L}{\partial \gamma} = t\frac{2\lambda\eta+\eta+\lambda}{[\lambda(\gamma+\eta)+\eta]^2} > 0.$$

Regarding M_H, it is easy to see that $\partial M_H/\partial\gamma < 0$, because the numerator of M_H is decreasing in γ and the denominator of M_H is increasing in γ. More formally, differentiating M_H with respect to

γ gives $\frac{\partial M_H}{\partial \gamma} = t\frac{-(1+\lambda)[\lambda(\gamma+\eta)+\eta]-(1+\lambda)(1+\eta-\gamma)\lambda}{[\lambda(\gamma+\eta)+\eta]^2} < 0$. The signs of the derivatives with respect to α and β follow immediately because γ is decreasing in α and in β.

(b) The derivative of M_L with respect to λ is:

$\frac{\partial M_L}{\partial \lambda} = t\frac{(\gamma+\eta)[\lambda(\gamma+\eta)+\eta]-[\lambda(\gamma+\eta)+\gamma-1](\gamma+\eta)}{[\lambda(\gamma+\eta)+\eta]^2}$, which after simplification becomes:

$$\frac{\partial M_L}{\partial \lambda} = t\frac{(\gamma+\eta)(\eta+1-\gamma)}{[\lambda(\gamma+\eta)+\eta]^2} > 0.$$

Differentiating M_H with respect to λ yields:

$\frac{\partial M_H}{\partial \lambda} = t\frac{(1+\eta-\gamma)[\lambda(\gamma+\eta)+\eta]-(1+\lambda)(1+\eta-\gamma)(\gamma+\eta)}{[\lambda(\gamma+\eta)+\eta]^2}$, which after simplification gives:

$$\frac{\partial M_H}{\partial \lambda} = -t\frac{\gamma(1+\eta-\gamma)}{[\lambda(\gamma+\eta)+\eta]^2} < 0.$$

(c) The derivative of M_L with respect to η is:

$\frac{\partial M_L}{\partial \eta} = t\frac{\lambda[\lambda(\gamma+\eta)+\eta]-[\lambda(\gamma+\eta)+\gamma-1](1+\lambda)}{[\lambda(\gamma+\eta)+\eta]^2}$, which after simplification becomes:

$\frac{\partial M_L}{\partial \eta} = t\frac{1-\gamma+\lambda-2\lambda\gamma}{[\lambda(\gamma+\eta)+\eta]^2}$. The sign of this derivative is equal to the sign of $1-\gamma+\lambda-2\lambda\gamma$, and from the proof of Corollary 1c it follows that this expression has the same sign as $(M_H - t)$.

Differentiating M_H with respect to η yields:

$\frac{\partial M_H}{\partial \eta} = t\frac{(1+\lambda)[\lambda(\gamma+\eta)+\eta]-(1+\lambda)(1+\eta-\gamma)(1+\lambda)}{[\lambda(\gamma+\eta)+\eta]^2}$, which after simplification gives:

$\frac{\partial M_H}{\partial \eta} = -t\frac{(1+\lambda)(1-\gamma+\lambda-2\lambda\gamma)}{[\lambda(\gamma+\eta)+\eta]^2}$, which has the opposite sign of $\partial M_L/\partial \eta$ and $(M_H - t)$. Q.E.D.

Proof of Proposition 4. (a) Because the consumers are divided equally between the firms in equilibrium, each firm earns profits of $0.5[(1+\lambda)M_L+(1+\eta)M_H]$. Substituting for M_L and M_H and simplifying we then obtain $\Pi_i=\frac{t(1+\lambda)[\lambda\gamma+\eta(2-\gamma+\lambda+\eta)]}{2[\lambda(\gamma+\eta)+\eta]}$. Taking the derivative with respect to γ we get $\frac{\partial\Pi_i}{\partial\gamma}=\frac{t(1+\lambda)}{2}\;\frac{(\lambda-\eta)[\lambda(\gamma+\eta)+\eta]-\lambda[\lambda\gamma+\eta(2-\gamma+\lambda+\eta)]}{[\lambda(\gamma+\eta)+\eta]^2}$,

which after simplification becomes $\frac{\partial\Pi_i}{\partial\gamma}=-\frac{t(1+\lambda)(2\lambda\eta+\eta+\lambda)\eta}{2[\lambda(\gamma+\eta)+\eta]^2}<0.$

(b) Differentiating Π_i with respect to η yields:

$$\frac{\partial\Pi_i}{\partial\eta}=\frac{t(1+\lambda)}{2}\;\frac{(2-\gamma+\lambda+2\eta)[\lambda(\gamma+\eta)+\eta]-[\lambda\gamma+\eta(2-\gamma+\lambda+\eta)](1+\lambda)}{[\lambda(\gamma+\eta)+\eta]^2},$$

which after simplification gives $\frac{\partial\Pi_i}{\partial\eta}=\frac{t(1+\lambda)[\lambda\gamma(1-\gamma+2\eta)+\eta^2(1+\lambda)]}{2[\lambda(\gamma+\eta)+\eta]^2}>0.$

(c) Differentiating Π_i with respect to λ gives:

$$\frac{\partial\Pi_i}{\partial\lambda}=\frac{t}{2}\;\frac{[2\lambda\gamma+\gamma+\eta(3-\gamma+2\lambda+\eta)][\lambda(\gamma+\eta)+\eta]-(1+\lambda)[\lambda\gamma+\eta(2-\gamma+\lambda+\eta)](\gamma+\eta)}{[\lambda(\gamma+\eta)+\eta]^2},$$ which

after simplification yields $\frac{\partial\Pi_i}{\partial\lambda}=\frac{t(\lambda\gamma+\lambda\eta+\eta)^2-\eta\gamma(\eta+1-\gamma)}{2[\lambda(\gamma+\eta)+\eta]^2}$. When $M_L \geq 0$, we have $\lambda\gamma+\lambda\eta\geq 1-\gamma$. This implies that we also have:

$$(\lambda\gamma+\lambda\eta+\eta)^2-\eta\gamma(\eta+1-\gamma)\geq(\eta+1-\gamma)^2-\eta\gamma(\eta+1-\gamma)=(\eta+1-\gamma)(1-\gamma)(1+\eta)\geq 0,$$

from which it follows that $\partial\Pi_i/\partial\lambda \geq 0$. For the two weak inequalities above to hold with equality, both $M_L = 0$ and $\gamma = 1$ should hold; when this is not the case, we get $\partial\Pi_i/\partial\lambda > 0$. To see that $M_L <$ 0 allows for both positive and negative values of $\partial\Pi_i/\partial\lambda$, notice that when $\lambda = 0.25$ and $\eta = \gamma =$ 0.5, then $M_L = -t/3$ and $\partial\Pi_i/\partial\lambda = 0.278t$; and when $\gamma = 0.5$ and $\eta = \gamma = 0.1$, then $\partial\Pi_i/\partial\lambda =$

$-0.0859t$ and $M_L = -2.75t$. Because $\partial\Pi_i/\partial\lambda$ is continuous in the parameters, it is clear that it can also be equal to zero when $M_L < 0$. Q.E.D.

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Table 1: Equilibrium markups and profits

γ / $\lambda=\eta$	**0.01**	**0.05**	**0.10**	**0.20**	**0.35**	**0.50**	**0.65**	**0.80**	**0.90**	**0.95**	**1.00**
						M_L/t					
0.01								-10.60	-4.76	-2.06	0.50
0.05						-6.10	-3.71	-1.70	-0.54	0.00	0.51
0.25	-2.94	-2.69	-2.41	-1.90	-1.25	-0.71	-0.26	0.12	0.35	0.45	0.56
0.5	-0.97	-0.87	-0.75	-0.53	-0.24	0.00	0.21	0.39	0.50	0.55	0.60
1	0.01	0.05	0.10	0.18	0.30	0.40	0.49	0.57	0.62	0.64	0.67
2	0.50	0.52	0.53	0.56	0.60	0.64	0.68	0.71	0.73	0.74	0.75
5	0.80	0.80	0.81	0.81	0.82	0.83	0.84	0.85	0.85	0.85	0.86
10	0.90	0.90	0.90	0.90	0.91	0.91	0.91	0.91	0.92	0.92	0.92
50	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
						M_H/t					
0.01								11.72	5.82	3.09	0.50
0.05						7.45	4.94	2.84	1.62	1.05	0.51
0.25	4.92	4.62	4.26	3.62	2.81	2.14	1.58	1.10	0.81	0.68	0.56
0.5	2.96	2.81	2.63	2.29	1.86	1.50	1.19	0.91	0.75	0.67	0.60
1	1.98	1.90	1.81	1.64	1.40	1.20	1.02	0.86	0.76	0.71	0.67
2	1.49	1.45	1.40	1.31	1.19	1.07	0.97	0.87	0.81	0.78	0.75
5	1.20	1.18	1.16	1.12	1.07	1.02	0.97	0.92	0.89	0.87	0.86
10	1.10	1.09	1.08	1.06	1.03	1.00	0.98	0.95	0.93	0.93	0.92
50	1.02	1.02	1.02	1.01	1.01	1.00	0.99	0.99	0.98	0.98	0.98
						Π_i/t					
0.01								0.56	0.53	0.52	0.51
0.05						0.71	0.65	0.60	0.57	0.55	0.54
0.25	1.24	1.20	1.16	1.08	0.98	0.89	0.82	0.76	0.73	0.71	0.69
0.5	1.49	1.45	1.41	1.32	1.22	1.13	1.05	0.98	0.94	0.92	0.90
1	1.99	1.95	1.90	1.82	1.70	1.60	1.51	1.43	1.38	1.36	1.33
2	2.99	2.95	2.90	2.81	2.69	2.57	2.47	2.37	2.31	2.28	2.25
5	5.99	5.95	5.90	5.81	5.67	5.54	5.41	5.29	5.22	5.18	5.14
10	10.99	10.95	10.90	10.80	10.66	10.52	10.39	10.25	10.17	10.13	10.08
50	50.99	50.95	50.90	50.80	50.65	50.50	50.36	50.21	50.12	50.07	50.02

The table presents the coefficients (that multiply t) of the equilibrium markups and profits for various parameter values (always assuming $\lambda=\eta$). Blank cells represent parameter values for which Assumption 1, and therefore also the second-order conditions, do not hold.