USING THE MERTON MODEL: AN EMPIRICAL ASSESSMENT OF ALTERNATIVES

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Using Merton model: an empirical assessment of alternatives

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Abstract

Merton (1974) suggested a structural model for default prediction which allows using timely information from the equity market. The literature describes several specifications to the application of the model, including methods presumably used by practitioners. However, recent studies demonstrate that these methods result in inferior estimates compared to simpler substitutes. We empirically examine various specification alternatives and find that the prediction goodness is only slightly sensitive to different choices of default barrier, whereas the choice of assets expected return and assets volatility is significant. Equity historical return and historical volatility produce underbiased estimates for assets expected return and assets volatility, especially for defaulting firms. Acknowledging these characteristics we suggest specifications that improve the model accuracy.

Keywords: Credit risk; Default prediction; Merton model; Bankruptcy prediction, Default barrier; Assets volatility

JEL classification: G17; G33; G13

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Introduction

Merton (1974) and Black and Scholes (1973) presented the basic approach for the valuation of stocks and corporate bonds as derivatives on the firm's assets. Merton (1974) is thus also used as a structural model for default prediction, viewing the firm's equity as a call option on its assets, because equity holders are entitled to the residual value of the firm after all its obligations are paid. Many theoretical studies suggested models that relax some of the restrictive assumptions in the Merton model.[1] However, empirical literature mainly focused on the application of the original model.[2] A major benchmark in these studies is the KMV model. KMV was founded in 1989 offering a commercial extension of Merton's model using market-based data. In 2002 it was acquired by Moody's and became Moody's-KMV. KMV published a number of papers which reveal some of its methods (see Keenan and Sobehart, 1999; Keenan, Sobehart and Stein, 2000; Crosbie and Bohn, 2003). Some of the specifications made by KMV were adopted by the academic literature. Vassalou and Xing (2004), Campbell, Hilscher, and Szilagyi (2007) are examples for such studies.

Only a few studies attempted to evaluate the accuracy of Merton's model under these specifications. Hillegeist, Keating, Cram, and Lundstedt (2004) compared the predictive power of the Merton model to Altman (1968) and Ohlson (1980) models (Z-score and O-score) and came to the conclusion that the Merton model outperforms these models. Duffie, Saita, and Wang (2007) showed that macroeconomic variables such as interest rate, historical stock return and historical market return have default prediction ability even after controlling for Merton model's distance to default. Campbell, Hilscher, and Szilagyi (2007), using a hazard model, combined Merton model probability of default with other variables relevant to default prediction. They also found that Merton model probabilities have relatively little contribution to the predictive power. Bharath and Shumway (2008) presented a "naïve" application of Merton model that outperformed the complex application of

[1] See for example Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001), Hsu Requejo and Santa-Clara (2004), Leland (1994), Leland and Toft (1996), Acharya and Carpenter (2002).

[2] An exceptional line of empirical studies (such as Brockman and Turtle, 2003) deals with the application of models in which the equity is viewed as a Down-and-out call option on the firm's assets. These models are not in the scope of this study.

Merton model (based on presumably Moody's-KMV specifications).[3] Another line of literature examined structural models ability to explain credit spreads and concluded that Merton model predictions underestimate market spreads.[4]

In this paper we examine the sensitivity of Merton model's default predictability to its parameter specifications. We assess the causes for this sensitivity and for prior studies lukewarm performance and conclude by providing a few prescriptions to enhance the model accuracy. We focus on the three main components of the model: the default barrier, the expected return on firm assets and the firm assets return volatility (hereafter, asset volatility). For this purpose we construct a sample with annual observations of firms from the merged CRSP/Compustat database during the period 1988-2008. We also gather information on default events during 1988-2009 from Standard and Poor's (S&P) and Moody's rating agencies reports. After filtering our sample includes 41,831 annual observations of 5,845 firms, of which 322 observations defaulted in the following year.

For each specification of the model we construct a Receiver Operating Characteristic (ROC) curve. This method is relatively common in comparison of prediction models since it does not require setting a priori the desired cutoff point between cost of type I error and cost of type II error. Another advantage of using ROC curves, compared to methods used in some prior studies, is that it enables statistical inference with the non-parametric test suggested by DeLong, DeLong and Clarke-Pearson (2008). One can test whether the differences between two models accuracy is statistically significant. Prior studies, such as Bharath and Shumway (2008), focused mainly on the rate of defaulters within the first deciles of firms (highest predicted default probabilities) and did not offer a robust statistical test for differences between models.

Another approach we use to understand the adequacy of various specifications is the study of firms' characteristics changes on a path to default. For this purpose we focus on 137 defaulting firms with

[3] Chava and Purnanandam (2010) used the naïve model as a proxy for credit risk.
[4] See for example, Jones, Mason and Rosenfeld (1984), Huang and Hunag (2003), Eom, Helwege and Huang (2008).

data available for the five years preceding the default event and compare their level of debt, stock returns, equity volatility and assets volatility to those of a group of 137 non-defaulting firms.

We find that Merton model accuracy is only slightly sensitive to the specification of the default barrier. We show that this is a result of the calculated assets value and volatility dependence on the default barrier. On one hand, ceteris paribus, a low setting of default barrier for risky firms reduces their probability of default. On the other hand, such misspecification also causes overestimation of assets volatility and underestimation of assets value, thus increasing the default probability. Therefore, the misspecification of the default barrier (relative to common practices) has a relatively small effect on the model accuracy.

We also show that using historical equity return as a proxy for expected assets return is questionable.[5] In particular, realized returns for risky firms are low and sometimes negative. While negative stock returns may be a predictive indicator for default, it cannot be a good proxy for forward-looking expected returns. Such a specification simply reduces the precision of the model. There are several ways to minimize the effect of negative returns. We show that setting expected assets return equal to the highest of realized stock return and the risk-free interest rate seems preferable among the alternatives examined in this study.

Our calculations demonstrate that assets volatility extracted from Black and Scholes (1973) using the historical volatility of equity is underbiased, especially for defaulting firms. This is mainly because the value of equity used for this purpose is up-to-date and forward looking while the backward looking historical volatility of equity is estimated on stock returns that might exhibit mild volatility prior to the deterioration in the financial state of the firm. We show that on average the difference between implied volatility (of stock options) and historical volatility is positive. This difference is larger for defaulting firms than for non-defaulting firms. Hence, model accuracy seems better using

[5] e.g. Bharath and Shumway (2008), in their naïve model. We have similar doubts regarding the use of historical equity returns in the iterative method used by Vassalou and Xing (2004), Bharath and Shumway (2008), and others.

equity volatility than using the theoretical asset volatility calculated by simultaneously solving Black and Scholes (1973) and the volatility relation of Jones, Mason and Rosenfeld (1984).

Finally we analyze Bharath and Shumway (2008) naïve model. We show that the superiority of this model to the more computationally intensive Merton model is due to its special "estimation" of assets volatility. Hence, following our analysis of various alternatives, we suggest a specification of Merton model that outperforms the naïve model.

The organization of the paper is as follows. Section 1 describes the Merton model. Section 2 discusses the difficulties and common practices in the application of Merton model. In section 3 we present the methodology and particularly the way we compare model accuracy under various specifications. Section 4 describes the data. In section 5 we present and discuss the results. Section 6 concludes.

1. Merton model

Merton Model uses the firm equity value, its debt face value, and the volatility of equity returns to evaluate the firm assets and debt. It also allows the calculations of the Probability of Default (*PD*).[6] The model assumes that the firm has issued one zero-coupon bond. The firm defaults at the bond maturity (in time T) when the value of its assets (A) falls below the amount of debt it has to repay (D). Otherwise the firm pays its debt in full and the remaining value is its equity $E_T = \max(A_T - D, 0)$. The model assumes that A follows a geometric Brownian motion (GBM):

(1) $dA = \mu_A \cdot A \cdot dt + \sigma_A \cdot A \cdot dW$

where μ_A is the expected continuous-compounded return on A, σ_A is the volatility of assets returns and dW is the standard Wiener process.[7] The model applies the Black and Scholes (1973) formula to calculate the value of the firm equity as a call option on its assets with expiration time T and an exercise price equal to the amount of debt (D):

[6] Under the model assumptions, as described below, that the asset follows a diffusion process and that the asset drift is known (for the calculation of PD under the physical measure).
[7] We omitted the subscript t from A and W for convenience. Obviously these vary with time. The drift μ_A and the volatility σ_A are assumed constant in this basic (classical) model.

(2) $E = N(d)A - De^{-rT}\, N(d - \sigma_A\sqrt{T})$

(3) $d = \frac{\ln(A/D)+[r+0.5\sigma_A^2]T}{\sigma_A\sqrt{T}}$

where E is the value of the firm equity, *r* is the risk free interest rate, and *N(•)* is the cumulative standard normal distribution function.[8] Jones, Mason, and Rosenfeld (1984) show that under the model assumptions the relation between the equity volatility (σ_E) and the assets volatility (σ_A) is $\sigma_E = \frac{A}{E} \cdot \frac{\partial E}{\partial A} \cdot \sigma_A$. Under the Black and Scholes formula it can be shown that $\frac{\partial E}{\partial A} = N(d)$, so the relation between the volatilities is:

(4) $\sigma_E = \frac{A}{E} N(d)\sigma_A$

Solving equations (2) and (4) simultaneously results in the values of *A* and σ_A which can be used to calculate a Distance to Default (*DD*) of the firm, defined by:

(5) $DD = \frac{\ln\left(\frac{A}{D}\right)+[\mu_A-0.5\sigma_A^2]T}{\sigma_A\sqrt{T}}$

DD may be regarded as the normalized distance between the firm assets value (*A*) and the face value of its debt (*D*).[9] DD is normally distributed under the GBM, and hence PD – the probability of default (the probability that the call option is not exercised) is:

(6) $PD = N(-DD)$

2. Application of the Merton model

The application of the model in practice needs several refinements. *T* is usually assumed to be 1 year. The annualized historical volatility of the equity is often the choice for σ_E.[10] It is often estimated over

[8] *E* and *A* in (2) and (3) are the values of equity and assets at time t = 0. The risk-free rate *r* is assumed constant.

[9] Taking into account the expected returns on the assets.

[10] A forward looking implied volatility is probably a better choice. However it is not available for many firms and in its extraction from market data is complicated by liquidity and volatility smiles.

the preceding one year period and we denote it by $\sigma_{E,-1}$. Another issue is the amount of debt that is relevant to a potential default during a one year period. Total debt is inadequate when not all of it is due in one year, as the firm may remain solvent even when the value of assets falls below its total liabilities. Using the short term debt (debt maturing in one year) for the default barrier D would be often wrong, for example, when there are covenants that force the firm to serve other debts when its financial situation deteriorates. Prior studies generally follow KMV (Crosbie and Bohn, 2003) and chose short-term debt plus half of the long term debt for the default barrier.[11] In this work we use $D = STD + k \cdot LTD$ for the default barrier, where STD is the short term debt, LTD is the long term debt and k is the LTD multiplier. We test the predictability power of the model for various values of k and check whether the KMV choice of $k = 0.5$ outperforms the alternatives.

Since the values of firm's assets (A) and their volatility (σ_A) are not observed, we solve equations (2) and (4) simultaneously.[12] This method was originally proposed by Merton (1974) and refined by Jones et al (1984), it is also implemented in Hillegeist, Keating, Cram, and Lundstedt (2004) and Campbell, Hilscher and Szilagyi (2008). The expected asset return μ_A, has to be estimated separately. Campbell et al. (2008), for example, used a constant market premium and calculated it as $\mu_A = r + 0.06$. In this work we examine several alternatives for μ_A. Under the first two alternatives we apply the CAPM model $\mu_A = r + \beta_A \cdot MP$, where MP is the market premium and β_A is the assets beta. First we use daily observations from the previous year on daily stock returns and the CRSP value weighted NYSE-NASDAQ-AMEX index to estimate the equity beta β_E. Then we use the relation

[11] For example: Bharath and Shumway (2008), Vassalou and Xing (2004), Duffie, Saita, and Wang (2007), Campbell, Hilscher, and Szilagyi (2008).

[12] Another approach, used by Bharath and Shumway (2008), Vassalou and Xing (2004) and Duffie, Saita, and Wang (2007)) is a complicated iterative procedure. In this process an initial guess value of σ_A is used in equation (2) in order to infer the market value of the assets (A) for the firm on a daily basis in the prior year. This generates a time series which is used to derive an "updated" σ_A. This new σ_A is used to compute a new time series of the firm's assets. The procedure repeats itself until the volatility used to calculate the time series converges to the volatility of the calculated values. Then, the last time series is used to infer the values of σ_A and μ_A which are used in equation (5) of the model. Bharath and Shumway (2008) showed that this approach results are in fact similar or even slightly inferior to the results of the simultaneous approach implemented in this paper.

$\beta_A = \beta_E \cdot \frac{\sigma_A}{\sigma_E}$ and the values of $\beta_E, \sigma_A, \sigma_E$ to calculate β_A.[13] We use two alternative values for *MP*. The first is a constant rate of 6%, which results in $\mu_A = \mu_{MP=0.06} = r + \beta_A \cdot 0.06$. The second assumes a variable market premium which equals the historical excess return of the S&P500 index in the previous year. The later results in $\mu_A = \mu_{MP=S\&P} = r + \beta_A \cdot (S\&P500_{-1} - r)$, where $S\&P500_{-1}$ is the annual rate of return of the S&P500 index in the previous year. For our third alternative we simply assume that the expected asset return equals the historical equity return of the preceding year, $r_{E,-1}$. We use this alternative as a benchmark for the other two methods and in accordance to the naïve model of Bharath and Shumway (2008). Historical equity return ($r_{E,-1}$) is sometimes negative. Hence we also examine the possibility that a floor for the assets expected return is r and thus examine the results of $\mu_A = \max(r, r_{E,-1})$. Another alternative is to assume that the assets expected return equals the risk-free rate, $\mu_A = r$. In this case the probability measure that governs the asset and default processes is the risk-neutral measure. We also examine the alternative of a constant asset return $\mu_A = 0.09$.

For comparison, we use the naïve alternative of Bharath and Shumway (2008) for Merton model. In this naïve model the default barrier *D* is $D = STD + 0.5 \cdot LTD$. The value of assets is set to be the sum of the default barrier and equity values: $A = D + E$. The expected return of assets is set equal to the historical return on the firm stock price in the previous year: $\mu_A = r_{E,-1}$. Assets volatility σ_{Naive} is assumed to be a value-weighted average of historical equity volatility ($\sigma_{E,-1}$) and a "special" value of the debt volatility: [14]

(7) $\sigma_D = 0.05 + 0.25 \cdot \sigma_{E,-1}$

(8) $\sigma_{Naive} = \frac{E}{E+D}\sigma_{E,-1} + \frac{D}{E+D}(0.05 + 0.25 \cdot \sigma_{E,-1})$.

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[13] The relation between the assets and equity betas is derived from the expression of a Black-Scholes call beta $\beta_E = \frac{A}{E} \cdot N(d) \cdot \beta_A$ where we replace the call option and the underlying by the equity and the assets respectively (see for example Coval and Shumway 2001). We then use equation (4) to replace $\frac{A}{E}N(d)$ by the volatilities ratio $\frac{\sigma_E}{\sigma_A}$.

[14] We are not familiar with the foundations and origin of this assumed relation between the debt and equity volatilities.

The naive Distance to Default is (for T=1 year):

$$DD_{Naive} = \frac{\ln[(E+D)/D)] + r_{E,-1} - 0.5\cdot(\sigma_{Naive})^2}{\sigma_{Naive}},$$

and the default probability is: $PD_{Naive} = N(-DD_{Naive})$.

3. Methodology

Examination of default model goodness may be of two types. The first is *Model's Power*, the separation capability of the model between observations of default and observations of solvency. This power relates to the goodness of the order in which the model ranks the observations. The second type, *Model's Calibration*, refers to the default probability values produced by the model and how they fit real probabilities. For example, consider a model that results in the following default probabilities (PD) for three companies (A, B, C): $PD_A = 0.1,\ PD_B = 0.05,\ PD_C = 0.01$. The model's power relates to the goodness of the model outcome in ranking the probabilities of default in the right order: $PD_A > PD_B > PD_C$. However, the goodness of the model's calibration relates to the accuracy of the probability values generated by the model. Stein (2002) argues that calibration improves when model power increases. Any calibration method should maintain the ranking order of the model. Hence, we follow previous studies and focus on model power. For this purpose we regard the probabilities (PD) calculated by a model as scores.[15]

Critical values of PD may be used by investors, lenders, or regulators to classify firms to high-risk or low-risk categories. The classification might be inaccurate. Type I error relates to default of a firm classified to the low-risk category while a type II error relates to the solvency of a firm classified to the high-risk category (a "false alarm"). It is customary to estimate the type I and type II error rates for each critical value of PD using a database of PD observations and their related default/solvency realizations. Consider a critical value α. Type I error rate is the number of defaulting firms classified low-risk ($PD < \alpha$) divided by the total number of defaulting firms. Type II error rate is the number of non-defaulting firms classified high-risk ($PD \geq \alpha$) divided by the total number of solvent firms.

[15] This is the common practice in the bulk of prior literature and research, yet often this issue is not explicitly mentioned.

There is an obvious tradeoff between type I and type II errors. As one increases the critical value, the rate of type I error increases and rate of type II error decreases.

The Receiver Operating Characteristic (ROC) curve is a tool for comparing powers of alternative default models. Figure 1 shows ROC curves demonstrating the tradeoff between type I error and type II error for all possible critical values. The observations are ordered by type I error rate and then sensitivity rate (1 minus type II error rate) is plotted versus type I error rate (1 minus specifity rate). A random model (with no predictability power) is simply the 45 degrees line. Model A is superior to model B when the ROC curve of A is always above ROC curve of B. When curves are crossed, one may compare the Area Under the Curve (AUC) related to the alternative models. An AUC value is in the range [0, 1] and the AUC of a random model equals 0.5. We use the nonparametric approach of DeLong, DeLong and Clarke-Pearson (1988) to test the statistical significance of differences between the AUC of alternative models. This test which also controls for correlation between examined curves is considered the most advanced statistics for ROC curves comparison.

Prior studies such as Bharath and Shumway (2008) measured the accuracy of default models using the defaulting firms' fraction in the lowest-quality deciles among all defaulting firms in the sample. This method is in fact based on particular points on a power curve and does not encompass the information in the entire curve. A power curve shows the cumulative percentage of defaulting firms among all defaulting firms for each percentile of the predicting score. In other words, it shows the percentage of defaulting firms that are detected for each threshold value of the score (α in the above PD example). The Accuracy Ratio (AR) is twice the area between the 45^{o} line and the power curve and it is equivalent to ROC curve comparison, in fact $AR = 2 \cdot AUC - 1$.[16] Hence the deciles comparison method is also a restricted snapshot of particular points on the ROC curve. A major advantage of using ROC curves is the availability of statistical inference methods and tests such as that of DeLong et al. (1988).

[16] See Engelman, Hayden and Tasche (2003)

In addition to ROC curve analysis we also examine changes of selected variables prior to default. Our sample includes 137 defaulting firms with adequate input data for Merton model in each of the five years before the default event. We designate December 31 day prior to the year of the default event as time -1. (e.g., for a firm that defaulted during the year 2005, time -1 refers to the estimation of 31 December 2004; time -2 denotes the estimation of 31 December 2003 and so on.) We compare the defaulting firms to a control group of 137 non-defaulting firms of the same period.[17]

4. Data

The sample for this study includes all firms in the merged CRSP-COMPUSTAT database of the period 1988 to 2008. Daily stock returns and stock prices are taken from CRSP; book value of assets, short-term debt, long-term debt and the numbers of shares outstanding are from COMPUSTAT. For the risk-free interest rate r we use the 1-year Treasury bill rate obtained from the Federal Reserve Board Statistics.

Our sources for default events are the annual default reports of Moody's and S&P for the years 1989-2009. Since these reports mainly relate to large firms, we filter out all annual observations of firms with book assets value bellow 150 million USD.[18] Then we drop all firms with fiscal year that does not end in December 31.[19] Similar to Bharath and Shumway (2008) and others we exclude financial firms (SIC Codes: 6021, 6022, 6029, 6035, and 6036). This filtering is needed since financial firms are characterized by high leverage and strict regulations. We also filter out defaulting firms for three

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[17] For a firm which defaulted during the year 2005, we randomly select a non-defaulting firm which operated in the years 2000-2005. Using the same principle we used for defaulting firm, we mark 31 December 2004 as time -1, 31 December 2003 as time -2 and so forth.

[18] These reports cover firms that have been rated sometime and hence tend to be biased toward large firms. Therefore our default information regarding small firms is not reliable and we filter them out of the sample.

[19] An estimation of distance to default (DD) uses financial data which is published on annual or quarterly basis and market data which is updated daily. A monthly estimation of DD, for example, is prone to over-emphasize market data influence since accounting data are constant for 3 subsequent months (quarterly update) or 12 months (annual update) while market data changes daily usually. Therefore we choose to estimate the model on an annual basis using accounting data from the annual report only. In order to reduce time unobserved heterogeneity across observations, we choose using a specific day in every year, and therefore our final sample includes only firms with fiscal year ending on December 31.

years subsequent to a default event.[20] Our final sample contains 41,831 annual observations of 5,846 firms with 322 cases of defaults.

Table 1 shows the distribution of the sample across the years. The number of annual observations varies from 1,234 (in 1989) to 2,423 (in 2006) and the annual number of defaults varies from 0 (in 1995) to 60 (in 2001). As expected, defaults vary over time, peaking in 1999-2003 and in 2009. The overall number of defaults (322) seems sufficient for our analysis.

We use daily stock returns to compute the annualized average daily return, $r_{E,-1}$, and the annualized standard deviation of returns, $\sigma_{E,-1}$ for each year preceding an annual observation of a company. The beta of stock returns (β_E) is estimated in a standard technique using the CRSP value-weighted return of NYSE/NASDAQ/AMEX index as the market index. The market value of equity E for each annual observation equals the stock prices times the number of outstanding shares. Using a MATLAB program we simultaneously solve equations (2) and (4) for each annual observation and compute the assets value and volatility (A and σ_A). Table 2 provides descriptive statistics of the sample. The average market value in our sample is 4,750 which is greater than 808.8 of Bharath and Shumway (2008). We relate this difference to the exclusion of small firms from our sample. Nevertheless, the average annual stock returns of both samples are similar, 14.0 percent and 13.75 percent respectively.[21] The average β_E in our sample is 0.805. Among other possible reasons, we can relate the fact that the average β_E is smaller than 1 to the selection of our sample which excludes small firms.

5. Results

We begin by an evaluation of the effects of changes to the default barrier, the expected asset return and the assets returns volatility, using ROC curves and AUC methods (as discussed above). We then

[20] For example, if a firm defaults in 2000, we estimate its probability to default in 31 December 1999 and then drop this firm from our sample for the years 2000, 2001 and 2002.

[21] It may seem odd that the minimum value of annual stock return is below -100%. Notice however that $r_{E,-1}$ stands for the continuously-compounded annual return. e.g. in a rare case, when a stock drops by 80% in a year, its continuous rate of return is *ln*(0.2) = -161% per annum. Bharath and Shumway (2008) winsorized their sample and hence their minimum value of annual stock return was -85.45%. However, their minimum value for annual asset return was also extremely low: -253.58%.

study the properties of Bharath and Shummway (2008) naïve model and examine what appears to make it more accurate than the complex application of the Merton model.[22] Then we suggest a specification that seems to outperform the complex model and the naïve model of Bharath and Shumway (2008).

5.1 The default barrier

We estimate the model using five long-term debt (LTD) multipliers (k) values. Doing this, we assume that the assets drift $\mu_A = \mu_{MP=0.06} = r_f + \beta_A \cdot 0.06$ and extract A and σ_A by solving equations (2) and (4) simultaneously. The ROC curve (Figure 2) illustrates that the predictive powers of the five default barriers are very similar. Analyzing the AUC values for these ROC curves shows that although the greatest AUC is for k=0.5, the largest gap between two AUC values is merely 0.001. DeLong et al. (1988) test in Table 3 reveals that one cannot reject the hypothesis that the AUCs for k=0.1, k=0.3, k=0.7 equal the AUC of k=0.5.

The AUC under the various specifications is around 0.92 which is equivalent to an Accuracy Ratio of 0.84. Duffie et al. (2007) for example achieved an AR of 0.87 using a much more complex model. One cannot compare models by comparing their AUC or AR based on different samples, however, this comparison may support the adequacy of our sample.

Table 4 shows the evolution of LTD/A prior to default, where A is obtained from the simultaneous solution of equations (2) and (4) with $k = 0.5$ for the default barrier. Using t tests and Wilcoxon rank-sum (Mann-Whitney) tests we find statistically significant differences between LTD/A ratio of defaulting and non-defaulting firms. Furthermore, the gap between the two groups increases as firms come nearer to the default event. The average value for the defaulting firms, five years before default is 0.433 in comparison to 0.269 for the non-defaulting firms. As time passes, the LTD/A ratio of the

[22] Lacking a better word, we use 'complex' to refer to applications of Merton model that do not utilize short-cuts such as that of the naïve model of Bharath and Shummway (2008), our simplified solution, etc. The complex application of this paper is the simultaneous solution of equations (2) and (4) with its various specification alternatives. As presented earlier, Bharath and Shummway (2008) also compared their results to another, more complex application of the model, utilizing the iterative method presumably developed and used by KMV.

non-defaulting firms slightly increases whereas the ratio for the defaulting firms rises dramatically.[23] A year before default the average ratio for the defaulting firms reaches 0.754 while the average ratio for the non-defaulting firms is 0.325 only.

It appears that the model power is only slightly sensitive to the LTD multiplier while LTD by itself exhibits predictive power. This somewhat puzzling behavior results in from the calculation method of A and σ_A. The firm equity is regarded as a call option on the firm assets. Hence, an under-specification of the strike price (default barrier) results in an underestimation of the underlying assets value (A) or overestimation of the assets volatility (σ_A) or both. Underestimation of A or overestimation of σ_A results in a reduction in the distance to default and thus an increase in the probability of default, hence reducing the sensitivity of PD to changes in k. The underestimation of the probability of default caused directly from under-specification of the default barrier is compensated indirectly by underestimation of A and overestimation of σ_A. This seems to explain the model low sensitivity to the default barrier specification.[24] Table 5 shows the properties of A and σ_A. As expected, for lower values of the default barrier (small k) we find lower mean and median A and higher mean and median σ_A. We use t tests and Wilcoxon sign-ranked tests and find that the values resulted from various specifications of k are statistically significant from the values calculated using k=0.5.

Table 6 shows that PD is highly skewed, as expected. Its mean and median are widely apart under each of the five specification of the LTD multiplier, e.g. for $k = 0.5$ the mean PD is 0.0196 and the median is $4.67 \cdot 10^{-7}$. Moreover, even the 95% percentile is very small and lower than the mean PD for any k we used. As defaults are rare events (often about 1 percent of the sample), basing model comparison on deciles (as done in some prior studies) might be misleading. In our sample PD values start to vary substantially only within the highest five percent group. Similarly DD (distance to

[23] The increase in the ratio for non-defaulting firms may be associated with systematic risk. This is due to the fact that the firms in the control sample are matched to the defaulting firm calendar year of default. Hence, if default risk has a systematic component we may, on average, expect financial deterioration also among the control group firms in the years prior to the defaulting firm default.

[24] This somewhat counter intuitive result is caused when we maintain the equity value and the equity volatility constant. In “real life” changing the debt level of a firm, or its default barrier, would affect its equity level and probably its equity volatility too.

default) is skewed, its mean and median are 5.466 and 4.905 respectively. Skewness and Kurtosis statistical tests reject (in 1 percent significance level) the hypothesis that DD is normally distributed.

The five *LTD* multipliers (*k*) we use yield substantially different probabilities of default. For example, using the highest *LTD* multiplier (0.9) the mean *PD* is 30% larger than the mean *PD* using the lowest *LTD* multiplier (0.1). t tests and Wilcoxon sign-ranked tests reveal that the mean (median) probabilities of default for k=0.1, k=0.3 are smaller than those of k=0.5, and for k=0.7, k=0.9 are larger than those of k=0.5. This suggests that the calibration of the model is substantially different for each specification. However, as discussed above, the model's power (the ability to distinguish a defaulting firm from a non-defaulting firm) is relatively insensitive to k.

5.2 The expected return on the firm's assets (μ_A)

We examine several alternatives to assess the model sensitivity to assets expected returns. In all cases we use k=0.5 and solve simultaneously equations (2) and (4) for A and σ_A. Recall our definitions: $\mu_{MP=0.06} = r + \beta_A \cdot 0.06$ and $\mu_{MP=S\&P} = r + \beta_A \cdot (S\&P500_{-1} - r)$, where $S\&P500_{-1}$ is the annual rate of return of the S&P500 index in the previous year and $\beta_A = \beta_E \cdot \frac{\sigma_A}{\sigma_E}$.

Panel a of Table 7 lists the summary statistics of μ_A under various specifications. The average values of $\mu_{MP=0.06}$ and $\mu_{MP=S\&P500}$ are similar. This is not surprising since the average annual excess return of the *S&P500* index in our sample period is approximately 0.06. Naturally, the volatility of $\mu_{MP=S\&P500}$ is greater than the volatility of $\mu_{MP=0.06}$. Overall, one would expect average $r_{E,-1}$ to be higher than μ_A because equity is a leveraged long position on assets, and indeed $r_{E,-1}$ is larger than $\mu_{MP=0.06}$ and $\mu_{MP=S\&P500}$. However, as expected, the mean of $max\ (r, r_{E,-1})$ is very high (0.274).

Panel b in Table 7 shows that the largest AUC is achieved using $\mu_A = max\ (r, r_{E,-1})$. The AUC of $\mu_A = r$ is slightly greater than of $\mu_{MP=0.06}$ and the latter is slightly greater than of $r_{E,-1}$. The fixed $\mu_A = 0.09$ and $\mu_A = \mu_{MP=0.06}$ result in the smallest AUC. However, most of these differences are statistically insignificant. For example, AUC for $\mu_A = r_{E,-1}$ (0.9184) is only slightly smaller than the AUC for $\mu_A = \mu_{MP=0.06}$ (0.9207) and the difference in statistically insignificant. The difference between the AUC of $\mu_A = max\,(r, r_{E,-1})$ (0.9235) and that of $\mu_A = \mu_{MP=0.06}$ (0.9207) is also statistically insignificant. ROC curve in Figure 3 show the predictive power of three alternatives: $\mu_A = \mu_{MP=0.06}$, $\mu_{MP=S\&P500}$ and $\mu_A = r_{E,-1}$. The figure demonstrates that the model's power is apparently insensitive to the specification sets we used.

It is interesting to point out that the AUC of $\mu_A = \mu_{MP=0.06}$ is close to the highest, demonstrating that in this case β seems to outperform the predictability offered by historical equity returns $\mu_A = r_{E,-1}$. Although the differences are not statistically significant, in comparing the performance of $\mu_{MP=0.06}$ and $\mu_{MP=S\&P500}$, the constant market premium of 0.06 outperforms the predictability of prior year (historical) S&P500 market premium.

Table 8 and Figure 4 present the average rate of return on the firms' stock price, $r_{E,-1}$ and $\mu_A = \mu_{MP=0.06}$ when firms approach default. The average rate of return for the non-defaulting firms is positive at all time points, while the average rate of return for the defaulting firms is near zero at time -3 and is negative closer to default (time -2 and time -1). When defaulting firms approach the default event, the rate of return decreases dramatically averaging -0.68 one year prior to default.

To calculate real default probabilities by $PD = N(-DD)$, instead of risk-neutral probabilities, μ_A replaces r for the drift in DD (equation 5). It is logical to expect that investors demand higher returns from a riskier firm compared to a safer one. However, $r_{E,-1}$ is the realized historical return (not the forward looking expected return) and its negative value may indicate financial deterioration prior to default. This is supported by our data and results, see Table 8 and figure 4 which show the values of μ_A and $r_{E,-1}$ in the years prior to default. Defaulting and non-defaulting firms have on average similar returns five years before default. However, when firms approach default, their average equity returns fall beneath those of non-defaulting firms and even become negative in the two years prior to default. This result is consistent with prior papers such as Vassalou and Xing (2004) that discovered a negative equity excess return for credit risk.

On one hand $r_{E,-1}$ exhibits predictive power, lower $r_{E,-1}$ are observed with higher probabilities of default. On the other hand, historical equity return of firms approaching default may yield biased estimates for μ_A and hence harm the precision of the model. It appears that using $\mu_A = \max(r, r_{E,-1})$ mitigates some of the inaccuracy caused by using historical returns, instead of forward-looking returns, by reducing the effect of negative realized returns.

5.3 The volatility of the assets (σ_A)

As a firm approaches a default event often both equity volatility and leverage increase. These two processes affect the calculation of assets volatility in opposite directions. We examine changes in equity and assets volatilities as the firms approach a default event. We use a sample including the 137 defaulting firms and a comparison group of randomly selected 137 non-defaulting firms in parallel years, as explained earlier. Table 9 panel a and Figure 5a show that the mean of historical equity volatility increases from 0.440 five years before default to 0.956 a year before default. In the same period, the average volatility of equity for the non-defaulting group increases slightly from 0.377 to 0.482. t tests and Wilcoxon rank-sum tests (panel a in Table 9) reveal that in all these years, equity volatility is statistically significant higher for defaulting firms than for non-defaulting firms.

This development in historical equity volatility is expected. However, this is not the case for historical assets volatility.[25] Figure 5a and panel b in Table 9 demonstrate that contrary to the non-defaulting firms, historical assets volatility of defaulting firms, calculated by Merton Model, decreases, on average, as the time to the default event becomes shorter. Five years prior to default the mean of historical assets volatility is 0.249 while a year before default it is 0.206. In the same period, the mean of historical assets volatility of non-defaulting firms increases from 0.259 to 0.295. Whereas the historical assets volatility difference between defaulting firms and non-defaulting firms five years prior to default is statistically insignificant, it becomes negative and statistically significant in the year before default. The development in the median of historical assets volatility and Wilcoxon rank-sum tests portrait a similar picture and hence it seems that this pattern is not caused by outliers.

[25] By historical assets volatility we refer to the volatility estimated by solving equations (2) and (4) simultaneously, using historical equity volatility as the input to the model. We later discuss implied assets volatility, calculated similarly, using implied equity volatility as the input.

We suspect that this behavior is related to the fact that we use historical equity volatility rather than expected volatility. Historical volatility of equity is computed using prior year data whereas the equity value is current. As a firm approaches default, its equity value decreases and its equity volatility increases. Hence using up-to-date equity value jointly with out-of-date equity volatility value causes an underestimation of assets volatility. To assess this hypothesis, we examine assets volatility calculated by the model, using equity volatility implied by stock options market prices as input, instead of historical equity volatility. Implied volatility is forward-looking by nature. Bharath and Shumway (2008) showed that using implied volatility substantially improves Merton model results. We now examine the source of this improvement. It should also be noted that using implied volatility substantially reduces model's applicability since stock options are only available for a fraction of firms. Therefore this examination is merely intended to assess the goodness of current practices in Merton model application.

Following Bharath and Shumway (2008), for each firm we select the implied volatility of at-the-money 30-day call option on its stocks. We use Optionmetrics data which is available since 1996. Thus, for observations of 1995 we use the data of the first trading day in 1996 and for other observations we simply use data from the last trading day of the year. Due to the limited availability of implied volatility data, our sample decreases from 41,831 annual observations to 14,003 and the number of defaults diminished from 322 to 95.

Table 10 and Figure 6 show the development in implied volatility of 40 defaulting firms and 40 non-defaulting firms (the control group) in the five years preceding default.[26] It appears that for both defaulting firms and non-defaulting firms, every year in our sample, the mean and median implied equity volatility is greater than that of historical equity volatility (panels a and b). The same holds for assets volatility calculated using implied volatility, compared to assets volatility calculated using historical volatility (panels c and d). However, the difference between implied equity (assets) volatility and historical equity (assets) volatility is statistically significant only for defaulting firms one year prior to default. More interestingly, the mean of implied assets volatility of defaulting firms

[26] Of the 95 default events we found sufficient data of 5 years prior to the default event for 40 firms only.

remains relatively steady when these firms approach default, rising moderately from 0.241 five years prior to default to 0.245 one year prior to default. These results demonstrate the distortion of asset volatilities calculated using historical equity volatilities

Since this finding is based on a limited sample of 80 firms only we also compare historical volatility to implied volatility for the entire sample (Table 11). It appears that implied volatility is greater on average than historical volatility for defaulting and non-defaulting firms, for both equity and assets. The differences are statistically significant using t tests or Wilcoxon rank-sum tests. However, differences are larger for defaulting firms compared to non-defaulting ones. For example, while the mean of historical assets volatility is quite similar for defaulting and non-defaulting firms (0.319 and 0.320 respectively), the implied assets volatility of defaulting firms is much larger than that for non-defaulting firms (0.429 and 0.329 respectively). It is observed that for 68.4 percent of defaulting firms implied volatility (either equity or assets) is greater than historical volatility, compared to 53.5 percent only among non-defaulting firms.

These results suggest that the use of historical volatility (rather than expected volatility) might harm Merton model applications. This practice causes an underestimation of assets volatility. Table 12 compares the AUC for two alternatives. The first is the benchmark model in which assets volatility ($\sigma_A^{simul.}$) is calculated by solving equations (2) and (4) simultaneously. In the alternative model the assets volatility is set equal to the equity volatility (σ_{E-1}) and the value of assets (A) is calculated by solving equation (2). The AUC for the simultaneous equation (0.9207) is lower than the one for the volatility of equity (0.9280). The AUC difference is statistically significant using DeLong, et al. (1988) test. It should be noted that equity volatility is always larger than assets volatility for all firms either defaulting or non-defaulting. This is a cross-firm effect that may be adjusted in the calibration process of the model. However, the calculated assets volatility ($\sigma_A^{simul.}$), using historical equity volatility in equation (2) and (4) underestimates assets volatility mainly for defaulting firms and hence reduces model's power.

5.4 Simplified model alternatives

Bharath and Shumway (2008) suggested a naïve model, a simplified model that beats the complex application of the Merton model.[27] The naïve model uses a default barrier identical to that of the original model ($D = STD + 0.5 \cdot LTD$) and differs in three other parameters: assets value (A), assets volatility (σ_A) and expected assets returns (μ_A). The naïve model assumes that the assets value is the sum of equity and the default barrier ($A = E + D$); that the expected assets return equals the equity return over the prior year ($\mu_A = r_{E,-1}$); and that the assets volatility is a weighted average of the equity volatility and an enigmatic debt volatility (see equations 7 and 8). The AUC for this naïve model in our sample is 0.9223 which is higher than that of the "complex" model (0.9207). p value for this difference using DeLong et al. (1988) test is 0.698. Hence, the attractiveness of the naïve model is mainly its simplicity, not its power. We now examine why it performs so well despite its simplicity. We first show that the 'naïve' choice of asset value A = D+E and the asset drift $\mu_A = r_{E,-1}$, although easy to use, do not enhance the power of the model and are in fact inferior to the choices of the complex model.

The value of assets in the naïve model is simply assumed to be the sum of market value of equity (E) and the default barrier (D). Table 13 shows the mean values of ln (A/D) used in the complex model and ln $[\frac{E+D}{D}]$ used in the naïve model, for various LTD multipliers (k) of the default barrier equation. Both terms are inverse of leverage. As expected, inverse value of leverage is substantially smaller for defaulting firms compared to non-defaulting firms. For the non-defaulting firms the inverse-leverage mean of the complex model is similar to that of the naïve model (less than 2% differences). However, the mean values for defaulting firms are significantly larger in the naïve model than in the complex model (19-42% difference, increasing with k). The leverage difference between defaulting and non-defaulting firms is smaller in the naïve model for all k values. This is not surprising since the naïve model implicitly assumes that the debt value equals its accounting value (instead of its market value). This assumption ignores credit risk effect which substantially reduces the asset value of defaulting firms compared to non-defaulting ones. This indicates that the firm's assets value calculated by the

[27] There is no standard terminology for naming these models. In this section we refer to the simultaneous solution of equations (2) and (4) as the 'complex' model (and as 'Merton') to distinguish it from the naïve method of Bharath and Shumway (2008) and other simplified methods.

simultaneous solution provides stronger discriminative power for distinguishing defaulting firms from non-defaulting firms. Thus it seems that the method for computing the firm's assets value harms the naïve model discriminative power compared to the Merton model.

The expected assets return in the naïve model equals the equity return in the prior year ($\mu_A = r_{E,-1}$), we now compare it to a Merton model application using a CAPM estimate ($\mu_A = \mu_{MP=0.06}$). In section 5.2 we show that using $\mu_A = r_{E,-1}$ has mixed effects on the discriminating power of the model. In fact one can use $\mu_A = r_{E,-1}$ also in the complex application. We show in table 7 (panel b) that the AUC for this approximation is 0.9184 which smaller than 0.9207, the AUC of using $\mu_A = \mu_{MP=0.06}$. Therefore it seems that the discriminative power of the naïve model is not a result of its assets expected return (μ_A) specification.

The last remaining potential source of the naïve model discriminative power is its assets volatility specification. Figure 7 shows the evolution of σ_A^{Naive} for 137 defaulting firms as they approach default together with a control group of 137 non-defaulting firms. We can see that contrary to σ_A calculated in the simultaneous solution of (2) and (4), σ_A^{Naive} increases when firms approach default. Additionally, σ_A^{Naive} of defaulting firms is slightly higher than that of non-defaulting firms. These results suggest that indeed the naïve model formulation of assets volatility enhances its predictive power compared to the complex model. On the other hand, we argue above that the naïve model choices of assets value (A) and assets expected return (μ_A) seem simplistic and inferior to the Merton model. Hence, we examine several additional alternative models, summarized in Table 14. It shows that the naïve model is inferior to an alternative model (model 3) in which assets value (A) is calculated by solving equation (2), expected assets return is $\mu_A = max\ (r_f, r_{E,-1})$ and assets volatility equals the historical equity volatility ($\sigma_A = \sigma_{E,-1}$). Replacing assets value calculated from equation (2) with a simple accounting value ($E + D$) in model (4) slightly affects the model power, reducing the AUC from 0.9338 to 0.9337. As demonstrated in model (5) of Table 14, the naïve model choice of assets volatility (σ_A^{Naive}) seems to have no advantage over setting the assets volatility equal to historical equity volatility. This causes only a small drop of the AUC from 0.9338 to 0.9333.

Comparing model (4) to model (6) we find that setting assets expected returns to $\mu_A = max\ (r_f, r_{E,-1})$ is significant. Setting $\mu_A = r_{E,-1}$ reduces model's power, the AUC drops from 0.9337 in model (4) to 0.9263 in model (6). However, the AUC of model (6) is greater than the AUC of the naïve model and the difference is statistically significant. This result supports our finding that although the specification of assets volatility in the naïve model is better than in the simultaneous solution of equations (2) and (4), it is not optimal and it is inferior to an alternative such as simply using historical equity volatility.

6. Conclusions

In this paper we examine the sensitivity of Merton model default predictability to its parameter specifications. We assess the causes for this sensitivity and for prior studies lukewarm performance. We conclude by providing a few prescriptions to enhance the model accuracy.

Several alternatives to apply the Merton model in default prediction are explored. For this purpose, we compare the Area Under the Curve (AUC) of Receiver Operating Characteristic (ROC) curves and use the DeLong et al. (1988) nonparametric test to measure the statistical differences between the ROC curves. We also examine how key inputs evolve over time prior to default, of defaulting and non-defaulting firms. Overall we find that the complex application of the Merton model as carried out in previous studies is inferior to other somewhat simplified applications of the model. The setting of the default barrier appears to have a small impact on the separation ability of the model. However, the specification of assets expected return and assets volatility is important. The current practice of using realized (historical) values of equity returns and equity volatility rather than forward-looking values substantially reduces the models' ability to distinguish between defaulting and non-defaulting firms. This is mainly because of two reasons. First, historical returns result in under-biased estimates of equity volatility especially for defaulting firms. Second, realized past returns of defaulting firms are substantially low while one would expect riskier stocks to offer higher expected return. We conclude by offering a specification that beats both the complex application and the naïve application (Bharath and Shumway, 2008) of the Merton model.

This study attempts to re-evaluate the current practices in the application of the Merton model. It appears that the reliance of academic literature on sketchy descriptions by practitioners has its disadvantages. Future research on this topic may have various directions not necessarily in line with what is commonly known as practitioners practice. This study demonstrates that enhancements in the estimation of the expected assets return and assets volatility may significantly improve the quality of the model output.

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Tables

Table 1: sample distribution over time

This table reports the distribution of the sample's observations in the sampled period. The table presents the number of firms observed, the number of default events in the following year and the ratio between them for each year in the sample.

Year	Number of observations	Number of default events (during the subsequent Year)	Ratio of default events to the observations
1988	1238	3	**0.24%**
1989	1234	7	**0.57%**
1990	1247	8	**0.64%**
1991	1268	5	**0.39%**
1992	1323	4	**0.30%**
1993	1617	3	**0.19%**
1994	1778	0	**0.00%**
1995	1903	5	**0.26%**
1996	2129	12	**0.56%**
1997	2268	18	**0.79%**
1998	2369	31	**1.31%**
1999	2438	23	**0.94%**
2000	2469	60	**2.43%**
2001	2304	38	**1.65%**
2002	2235	27	**1.21%**
2003	2241	9	**0.40%**
2004	2322	11	**0.47%**
2005	2378	1	**0.04%**
2006	2423	5	**0.21%**
2007	2387	16	**0.67%**
2008	2260	36	**1.59%**
Total	**41831**	**322**	**0.77%**

Table 2: Model variables - summary statistics

This table reports summary statistics for all the variables used in the Merton model. BA (Book Assets) is the book value of *Total Assets*; LTD is the *Long Term Debt* ; STD is the *Short Term Debt*; E is the firm's market value of equity (the product of the price per share times the number of outstanding shares); $\boldsymbol{r_{E,-1}}$ is the annual firm's equity return (the average daily equity return times the number of trading days); $\sigma_{E,-1}$ is the annual firm's stock return volatility (the standard deviation of daily stock returns times the square root of the trading days in a year); β_E is the beta computed from daily return and the value-weighted CRSP index (NYSE/NASDAQ/ AMEX). BA, LTD, STD and E are measured in millions of US dollars. The other variables are presented in decimal fractions. The data is as of December 31 of each year for the period 1988-2008.

	Variable	Mean	Std. dev.	Min	Max
BA	Book value of assets	11,877	80,243	150	3,771,200
LTD	Long term debt	1,911	12,676	0.001	486,876
STD	Short term debt	2,142	16,411	0.001	575,319
E	Market value of equity	4,750	17,171	0.893	508,329
$\boldsymbol{r_{E,-1}}$	Stock return	0.140	0.554	-10.234	12.625
$\boldsymbol{\sigma_{E,-1}}$	Stock return volatility	0.434	0.265	0.046	4.807
$\boldsymbol{\beta_E}$	Beta of stock return	0.805	0.599	-6.557	7.850

Table 3: Area under ROC curves for various specifications of the default barrier

This table shows AUC (area under the ROC curve) for five different values of the LTD multipliers (k) in the default barrier specification ($D = STD + k \cdot LTD$), where STD is short-term debt and LTD is the long-term debt . The expected return on the firm's assets is set to be $\mu_A = \mu_{MP=0.06}$; i.e. based on the β_A of the assets extracted from historical β_E of equity and the assumption that the market premium equals 0.06. P values are of DeLong, et al. (1988) test for the difference between the AUC of the particular k and the AUC of k=0.5.

K	AUC	P value for difference from result for k=0.5
0.1	0.9197	0.349
0.3	0.9206	0.995
0.5	0.9207	-
0.7	0.9203	0.156
0.9	0.9198	0.062

Table 4: Evolution of long-term debt to assets ratio prior to default

This table shows the evolution of LTD/A for 137 firms on December 31 for each of the 5 years preceding the default year. A is the value of assets extracted from Merton model assuming a default barrier of $D = STD + 0.5 \cdot LTD$ where STD is short-term debt and LTD is long-term debt. Asset's expected return is assumed to be $\mu_A = \mu_{MP=0.06}$; i.e. based on the β_A of the assets extracted from historical β_E of equity and the assumption that the market premium equals 0.06. A control group of 137 non-defaulting firms is added for comparison. P values are for differences between the group of defaulting firms and non-defaulting firms.

Years before default	Defaulting			Non-Defaulting			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	137	0.433	0.357	137	0.269	0.190	0.000	0.000
4	137	0.489	0.457	137	0.249	0.191	0.000	0.000
3	137	0.576	0.509	137	0.277	0.210	0.000	0.000
2	137	0.703	0.687	137	0.275	0.227	0.000	0.000
1	137	0.754	0.727	137	0.325	0.247	0.000	0.000

Table 5: Model's results for various specifications of the default barrier

This table shows the summary statistics for assets value (A) and assets volatility (σ_A) in Merton model under the five different specifications of the LTD multipliers (k) used for the default barrier value ($D = STD + k \cdot LTD$), where STD is the short-term debt and LTD is the long-term debt. The expected return on the firm's assets is set to $\mu_A = \mu_{MP=0.06}$; i.e. based on the β_A of the assets extracted from historical β_E of equity and assuming the market premium equals 0.06. . P values are listed for t tests and Wilcoxon sign-ranked tests.

Panel a – Assets value (A)

K	Obs.	Mean	Median	P value for difference from result for k=0.5	
				t test	Sign-ranked test
0.1	41831	6987	989	0.000	0.000
0.3	41831	7354	1052	0.000	0.000
0.5	41831	7721	1118	-	-
0.7	41831	8087	1179	0.000	0.000
0.9	41831	8455	1239	0.000	0.000

Panel b – Assets volatility (σ_A)

K	Obs.	Mean	Median	P value for difference from result for k=0.5	
				t test	Sign-ranked test
0.1	41831	0.308	0.259	0.000	0.000
0.3	41831	0.290	0.243	0.000	0.000
0.5	41831	0.276	0.230	-	-
0.7	41831	0.264	0.218	0.000	0.000
0.9	41831	0.254	0.208	0.000	0.000

Table 6: Estimated probabilities of default for various specifications of the default barrier – summary of statistics

This table presents descriptive statistics for the estimated probability of default under various specifications of k, the LTD multiplier used the default barrier equation ($D = STD + k \cdot LTD$), where STD is the short-term debt and LTD is the long-term debt. The expected return on the firm's assets is set to $\mu_A = \mu_{MP=0.06}$; i.e. based on the β_A of the assets calculated using historical β_E of equity and assuming the market premium equals 0.06.

K	Obs.	Mean	Median	5% percentile	95% percentile	P value for difference from result of k=0.5	
						t test	Sign-ranked test
0.1	41831	0.0166	$2.26 \cdot 10^{-8}$	$1.81 \cdot 10^{-8}$	$2.84 \cdot 10^{-8}$	0.000	0.000
0.3	41831	0.0183	$1.44 \cdot 10^{-7}$	$1.20 \cdot 10^{-7}$	$1.74 \cdot 10^{-7}$	0.000	0.000
0.5	41831	0.0196	$4.67 \cdot 10^{-7}$	$3.98 \cdot 10^{-7}$	$5.58 \cdot 10^{-7}$	-	-
0.7	41831	0.0205	$1.12 \cdot 10^{-6}$	$9.64 \cdot 10^{-7}$	$1.32 \cdot 10^{-6}$	0.000	0.000
0.9	41831	0.0214	$2.18 \cdot 10^{-6}$	$1.84 \cdot 10^{-6}$	$2.49 \cdot 10^{-6}$	0.000	0.000

Table 7: Area under the ROC curve for various specification of firm's asset expected return (μ_A)

This table shows the results of the model for three alternatives of the expected return on the firm's assets (μ_A). $\mu_{MP=0.06}$ is calculated for each firm using: $\mu_{MP=0.06} = r + \beta_A \cdot 0.06$. where r is the risk free interest rate (1-year treasury bills yield to maturity) and β_A is the beta of the firm's assets. $\boldsymbol{r_{E,-1}}$ is the annual equity return for the previous year. $\mu_{MP=S\&P500}$ is calculated using $\mu_{MP=S\&P} = r + \beta_A \cdot (S\&P500_{-1} - r)$, where $S\&P500_{-1}$ is the annual rate of return of the S&P500 index in the previous year. For reference we added a fixed expected return of 0.09. In this table use $D = STD + 0.5 \cdot LTD$ for the default barrier, where *STD* is the short-term debt and *LTD* is the long-term debt. The sample includes 41,831 observations of which 322 are defaults.

Panel a – Expected asset return

Specification	Obs.	Mean	Median	Std. dev.	Min	Max
$\boldsymbol{\mu_{MP=0.06}}$	41831	0.077	0.074	0.033	-0.299	0.440
$\boldsymbol{r_{E,-1}}$	41831	0.140	0.145	0.554	-10.234	12.625
$\boldsymbol{\mu_{MP=S\&P500}}$	41831	0.071	0.063	0.135	-1.011	1.702
r	41831	0.045	0.050	0.019	0.012	0.084
$\boldsymbol{max}(r, \boldsymbol{r_{E,-1}})$	41831	0.274	0.145	0.386	0.012	12.625
$\boldsymbol{max}(r, \boldsymbol{\mu_{MP=0.06}})$	41831	0.078	0.074	0.032	0.012	0.440
$\boldsymbol{0.09}$	41831	0.090	0.090	0.000	0.090	0.090

Panel b – AUC (Area under the ROC curve)

Specification	AUC	P-Value for difference from $\boldsymbol{\mu_{MP=0.06}}$	P-Value for difference from $\boldsymbol{r_{E,-1}}$
$\boldsymbol{\mu_{MP=0.06}}$	0.9207	-	0.6614
$\boldsymbol{r_{E,-1}}$	0.9184	0.6614	-
$\boldsymbol{\mu_{MP=S\&P500}}$	0.9105	0.0000	0.1351
r	0.9210	0.1241	0.6076
$\boldsymbol{max}(r, \boldsymbol{r_{E,-1}})$	0.9235	0.4704	0.0386
$\boldsymbol{max}(r, \boldsymbol{\mu_{MP=0.06}})$	0.9207	0.0000	0.6527
$\boldsymbol{0.09}$	0.9095	0.0000	0.1332

Table 8: Evolution of equity and assets returns prior to default

This table shows the evolution of historical equity return ($r_{E,-1}$) and expected asset returns ($\mu_A = \mu_{MP=0.06}$) for 137 firms on December 31 for each of the 5 years preceding the default year. $\mu_{MP=0.06}$ is based on the β_A (assets beta) calculated from historical β_E (equity beta) and assuming the market premium equals 0.06. A control group of 137 non-defaulting firms is used for comparison. P values are for differences between the group of defaulting firms and non-defaulting firms.

Panel a - Annualized equity return for the previous year ($r_{E,-1}$)

Years before default	Defaulting			Non-Defaulting			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	137	0.207	0.218	137	0.249	0.178	0.479	0.573
4	137	0.069	0.063	137	0.230	0.167	0.002	0.001
3	137	0.004	0.014	137	0.103	0.089	0.060	0.056
2	137	-0.152	-0.119	137	0.145	0.144	0.000	0.000
1	137	-0.679	-0.613	137	0.068	0.130	0.000	0.000

Panel b – Expected asset return $\mu_A = \mu_{MP=0.06}$

Years before default	Defaulting			Non-Defaulting			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	137	0.074	0.072	137	0.076	0.073	0.611	0.627
4	137	0.078	0.071	137	0.081	0.077	0.325	0.285
3	137	0.077	0.075	137	0.082	0.080	0.087	0.167
2	137	0.068	0.065	137	0.078	0.074	0.003	0.001
1	137	0.050	0.050	137	0.065	0.065	0.000	0.000

Table 9: Evolution of volatility prior to default

This table shows the evolution of equity's volatility ($\sigma_{E,-1}$) and asset's volatility (σ_A) for 137 firms on December 31 for each of the 5 years preceding the default year. $\sigma_{E,-1}$ is the annualized standard deviation of daily stock returns in the year before. σ_A is extracted from simultaneous solution of equations (2) and (4) for the Black-Scholes model when assuming default barrier is $D = STD + 0.5 \cdot LTD$ and expected assets return is $\mu_A = \mu_{MP=0.06}$; i.e. based on the β_A of the assets extracted from historical β_E of equity and assumption on the market premium to be equal to 0.06. *STD* is short-term debt and *LTD* is the long-term debt. A control group of 137 non-defaulting firms was used for comparison. P values are for differences between the group of defaulting firms and non-defaulting firms.

Panel a – Volatility of equity (σ_E)

Years before default	Defaulting			Non-Defaulting			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	137	0.440	0.388	137	0.377	0.323	0.013	0.000
4	137	0.463	0.431	137	0.374	0.312	0.000	0.000
3	137	0.516	0.483	137	0.399	0.375	0.000	0.000
2	137	0.588	0.519	137	0.405	0.365	0.000	0.000
1	137	0.956	0.888	137	0.482	0.413	0.000	0.000

Panel b – Assets volatility (σ_A)

Years before default	Defaulting			Non-Defaulting			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	137	0.249	0.210	137	0.259	0.219	0.630	0.726
4	137	0.243	0.206	137	0.264	0.215	0.305	0.277
3	137	0.233	0.196	137	0.267	0.235	0.063	0.013
2	137	0.229	0.164	137	0.269	0.235	0.063	0.000
1	137	0.206	0.149	137	0.295	0.263	0.000	0.000

Table 10: Evolution of implied volatility of equity and assets prior to default

This table shows the evolution of implied volatility of equity ($\sigma_E^{implied}$) and the derived assets volatility ($\sigma_A^{implied}$) for 40 firms on December 31 for each of the 5 years preceding the default year. $\sigma_E^{implied}$ is the annualized implied volatility of at-the money call options on firms stocks. $\sigma_{E,-1}$ is the annualized standard deviation of daily stock returns in the prior year. Assets volatility is calculated by solving equations (2) and (4) simultaneously, assuming the default barrier is $D = STD + 0.5 \cdot LTD$, where *STD* and LTD is the short-term and the long-term debt respectively. $\sigma_{A,-1}$ is calculated using historical volatility ($\sigma_{E,-1}$) as an input for the model, and similarly $\sigma_A^{implied}$ is calculated using implied equity volatility ($\sigma_E^{implied}$). A control group of 40 non-defaulting firms is used for comparison. P values are for differences between results achieved when using $\sigma_E^{implied}$ instead of $\sigma_{E,-1}$.

Panel a: Implied equity volatility ($\sigma_E^{implied}$) vs. historical equity volatility ($\sigma_{E,-1}$) of defaulting firms

Years before default	Implied volatility			Historical volatility			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	40	0.410	0.388	40	0.411	0.348	0.513	0.872
4	40	0.453	0.405	40	0.422	0.368	0.029	0.116
3	40	0.491	0.384	40	0.486	0.402	0.387	0.563
2	40	0.540	0.539	40	0.507	0.449	0.154	0.313
1	40	0.903	0.855	40	0.793	0.733	0.008	0.017

Panel b: Implied equity volatility ($\sigma_E^{implied}$) vs. historical equity volatility ($\sigma_{E,-1}$) of non-defaulting firms

Years before default	Implied volatility			Historical volatility			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank-sum (Mann-Whitney)
5	40	0.454	0.448	40	0.429	0.410	0.116	0.452
4	40	0.431	0.410	40	0.426	0.392	0.398	0.914
3	40	0.463	0.406	40	0.446	0.415	0.162	0.452
2	40	0.432	0.403	40	0.432	0.405	0.493	0.957
1	40	0.537	0.525	40	0.548	0.483	0.669	0.788

Panel c: Implied assets volatility ($\sigma_A^{implied}$) vs. historical assets volatility ($\sigma_{A,-1}$) of defaulting firms

Years before default	Implied volatility			Historical volatility			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	40	0.241	0.203	40	0.242	0.212	0.930	0.893
4	40	0.232	0.169	40	0.214	0.170	0.065	0.098
3	40	0.215	0.166	40	0.207	0.171	0.422	0.375
2	40	0.228	0.183	40	0.210	0.161	0.292	0.282
1	40	0.245	0.197	40	0.202	0.128	0.028	0.035

Panel d: Implied assets volatility ($\sigma_A^{implied}$) vs. historical assets volatility ($\sigma_{A,-1}$) of non-defaulting firms

Years before default	Implied volatility			Historical volatility			P value for difference	
	Obs.	Mean	Median	Obs.	Mean	Median	t test	Wilcoxon rank- sum (Mann-Whitney)
5	40	0.338	0.312	40	0.311	0.272	0.093	0.320
4	40	0.322	0.282	40	0.320	0.292	0.877	0.872
3	40	0.327	0.268	40	0.315	0.284	0.372	0.528
2	40	0.309	0.278	40	0.309	0.290	0.982	0.717
1	40	0.333	0.323	40	0.333	0.323	0.996	0.819

Table 11: Historical volatility vs. implied volatility

This table shows the historical volatility and implied volatility of equity and assets for 14,003 annual observations for the years 1995-2008. Implied volatility of equity is the annualized implied volatility of the 30-days at-the-money call option on the firms stocks. Historical volatility of equity is the annualized standard deviation of daily stock returns in the year preceding the annual observation. Assets volatility is calculated by solving equations (2) and (4) simultaneously, assuming the default barrier is $D = STD + 0.5 \cdot LTD$. STD is short-term debt and LTD is the long-term debt. Historical assets volatility is calculated using historical equity volatility as an input to the model, and implied assets volatility is calculated using implied equity volatility. P values are for differences between historical volatility and implied volatility.

Group	Obs.	Historical volatility		Implied volatility		P value for difference		Obs. Implied> Historical (%)
		Mean	Median	Mean	Median	t test	Wilcoxon rank-sum (Mann-Whitney)	
Non-defaulting								
Equity	13908	0.452	0.389	0.466	0.399	0.000	0.000	53.5
Assets	13908	0.320	0.270	0.329	0.279	0.000	0.000	53.5
Defaulting								
Equity	95	0.836	0.791	0.996	0.906	0.000	0.000	68.4
Assets	95	0.319	0.273	0.429	0.346	0.000	0.000	68.4

Table 12: Area under the ROC curve for various specifications of assets volatility (σ_A)

This table shows AUC (area under the ROC curve) for two alternatives. The first is the benchmark model in which assets volatility ($\sigma_A^{simul.}$) is calculated from simultaneous solution of equations (2) and (4) and the other is a model in which assets volatility is set equal to the equity volatility ($\sigma_{E,-1}$) and the assets value (A) is calculated by solving equation (2). $\sigma_{E,-1}$ is the annualized average equity return in the previous year. The default barrier is $D = STD + k \cdot LTD$, where STD is short-term debt and LTD is the long-term debt. The expected return on the firm's assets is set to $\mu_A = \mu_{MP=0.06}$ (based on the β_A of the assets calculated from historical β_E of equity and assuming the market premium equals 0.06). P values are of DeLong, et al. (1988) test for the difference between the AUC of the alternative specifications.

σ_A	AUC	P value for difference from result for $\sigma_A^{Simul.}$
$\sigma_A^{Simul.}$	0.9207	-
$\sigma_{E,-1}$	0.9280	0.012

Table 13: Debt ratio comparison

This table shows the average values for ln(*A*/*D*) of the Merton model and the respective variable from the naïve model for five *LTD* multiplier (*k*) values. The fourth column lists the difference between the second column and the third column. The sample included 322 defaulting observations and 41,509 non-defaulting observations.

LTD multiplier (k)	mean ln(A/D) (Merton model)		mean ln[(E+D)/D] (naive model)		relative difference (naïve – Merton)	
	Defaulting	**Non-defaulting**	**Defaulting**	**Non-defaulting**	**Defaulting**	**Non-defaulting**
0.1	0.442	1.574	0.525	1.590	18.7%	1.0%
0.3	0.330	1.380	0.415	1.397	25.7%	1.2%
0.5	0.269	1.256	0.353	1.275	31.2%	1.5%
0.7	0.228	1.164	0.312	1.185	36.8%	1.8%
0.9	0.198	1.092	0.281	1.113	41.9%	1.9%

Table 14: Alternative model specification AUCs

This table shows AUC (area under the ROC curve) for several specifications of the Merton model. In all specifications the default barrier is $D = STD + 0.5 \cdot LTD$, where STD and LTD is the short and long-term debt respectively. The assets value is either a solution of equations (2) and (4) or simply the sum of market value of equity (E) and the default barrier (D). The assets expected returns alternatives include: $\mu_A = \mu_{MP=0.06}$ based on β_A of the assets calculated using historical β_E of equity, assuming the market premium equals 0.06; or $r_{E,-1}$ the annualized average daily return of equity in the previous year; or the larger of $r_{E,-1}$ and r (the risk-free interest rate, 1-year treasury bills yield to maturity). Assets volatility is either a solution of equations (2) and (4); or the annualized volatility of daily equity return in the previous year ($\sigma_{E,-1}$); or is based on Bharath and Shumway specification of assets volatility (σ_A^{Naive}). P values are based on DeLong, et al. (1988) test for the difference between the AUCs of the alternative specifications.

Model	Value of assets (A)	Expected return on assets (μ_A)	Assets volatility (σ_A)	AUC	P value for difference	
					From model 1 (complex)	From model 2 (naïve)
1 (complex)	Merton*	$\mu_{MP=0.06}$	Merton*	0.9207	-	0.698
2 (naïve)	E+D	$r_{E,-1}$	σ_A^{Naive}	0.9223	0.698	-
3 (modified)	Merton**	max $(r_{E,-1}, r)$	$\sigma_{E,-1}$	0.9338	0.000	0.000
4	E+D	max $(r_{E,-1}, r)$	$\sigma_{E,-1}$	0.9337	0.000	0.000
5	E+D	max $(r_{E,-1}, r)$	σ_A^{Naive}	0.9333	0.000	0.000
6	E+D	$r_{E,-1}$	$\sigma_{E,-1}$	0.9263	0.145	0.003

* refers to the simultaneous solution of equations (2) and (4)

**refers to the solution of equation (2)

Figures

Figure 1: Illustration of ROC curves for two models and for a random order

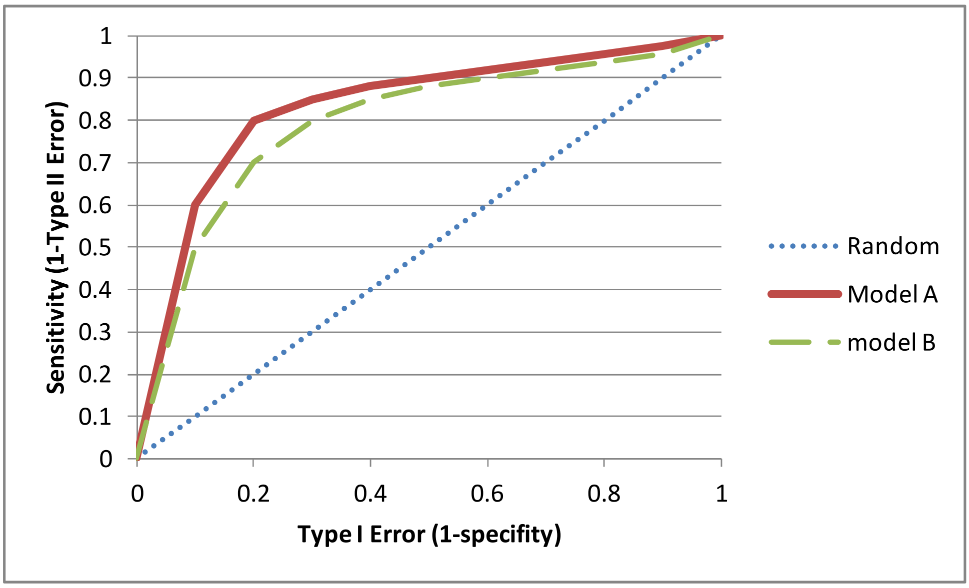

Figure 2: ROC curves computed from the Merton model's for five *LTD* multipliers (k), using $\mu_A = \mu_{MP=0.06} = r_f + \beta_A \cdot 0.06$ and A and $\sigma_{A,-1}$ of the simultaneous solution of (2) and (4)

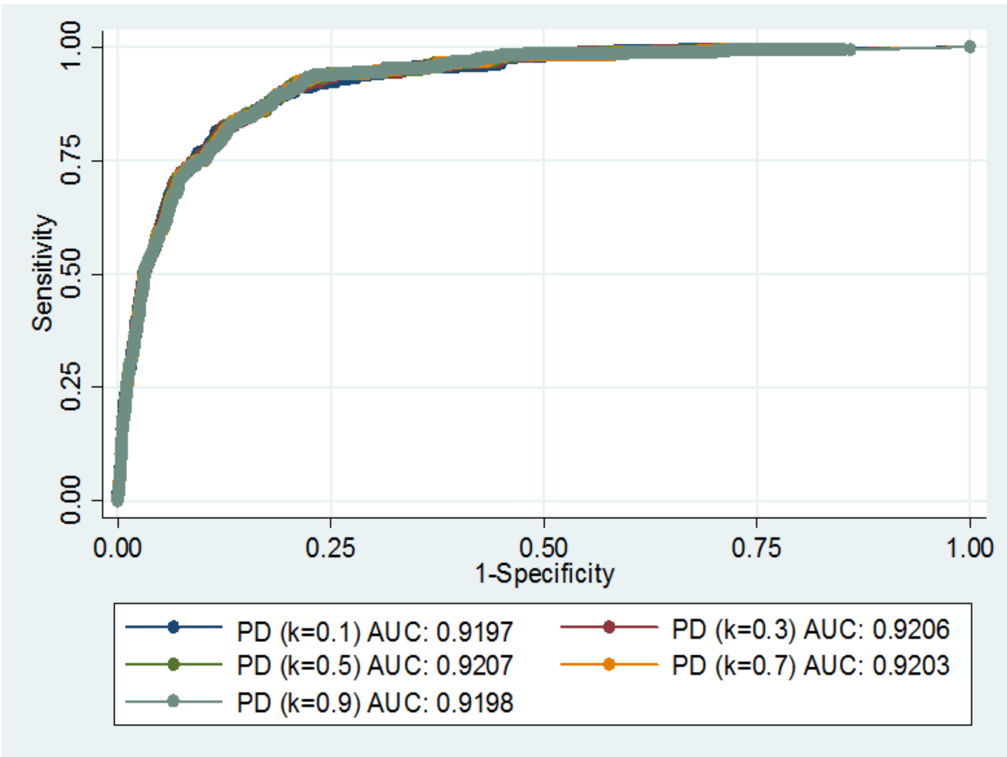

Figure 3: ROC curves computed for Merton model's PD rankings of three μ specifications: $\mu_A = \mu_{MP=0.06}$, $\mu_{MP=S\&P500}$, and $\mu_A = r_{E,-1}$. (k=0.5 for the three curves).

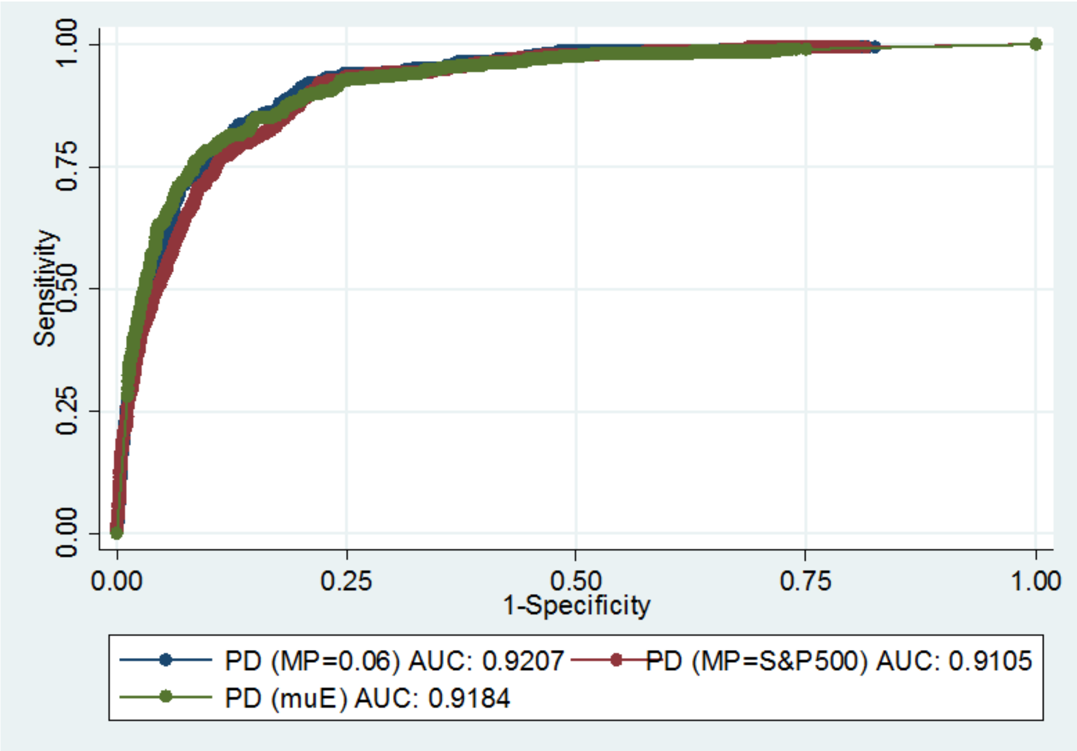

Figure 4: The average annual rate of return on the firm's equity in the previous year ($r_{E,-1}$) and the average expected assets return (μ_A) while the defaulting firms (137 firms) approach the default event. $\mu_A = \mu_{MP=0.06}$ (based on the β_A of the assets calculated from historical β_E of equity and assuming the market premium equals 0.06). A control group of 137 non-defaulting firms is used for comparison.

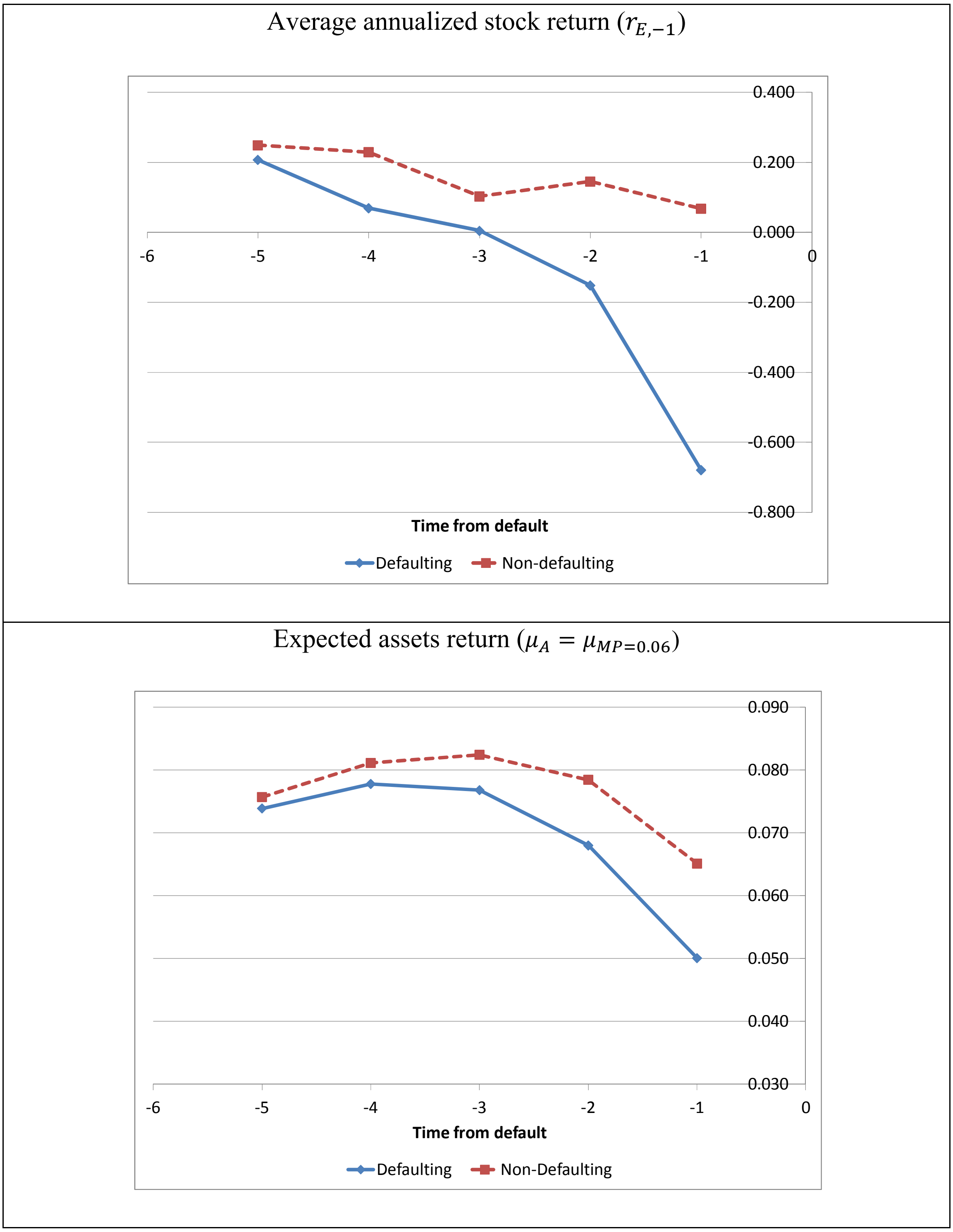

Figure 5: The average equity return volatility (σ_{E-1}) and average assets return volatility (σ_A) while the defaulting firms (137 firms) approach the default event. A control group of 137 non-defaulting firms is used for comparison. For data and model specifications see Table 9.

Figure 5a: Equity returns volatility (σ_{E-1})

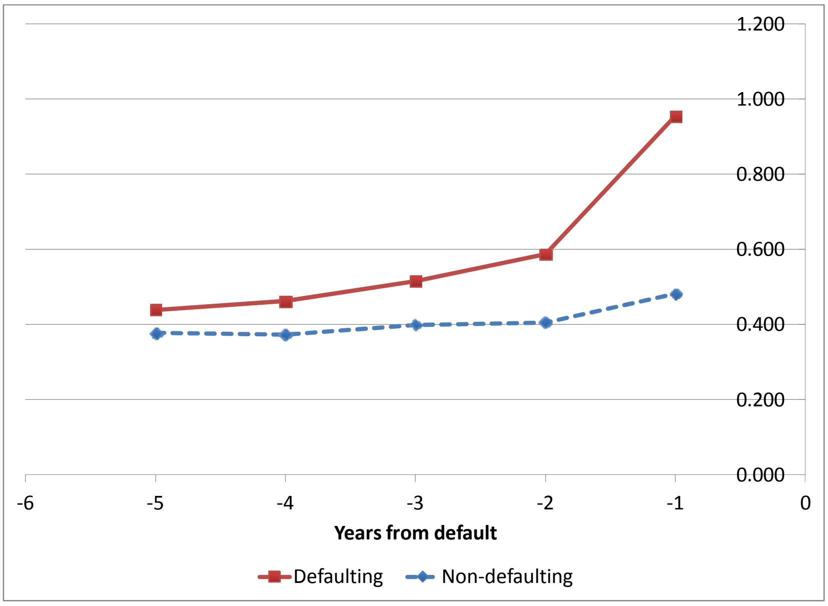

Figure 5b: Assets returns volatility (σ_A)

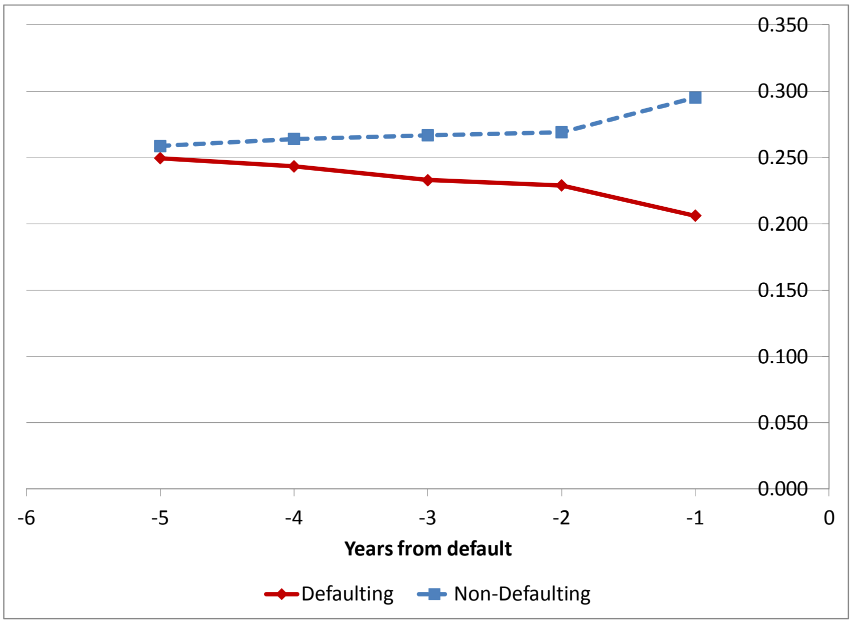

Figure 6: The average equity return volatility (σ_{E-1}) and average assets return volatility (σ_A) while the defaulting firms (40 firms) approach the default event. A control group of 40 non-defaulting firms is used for comparison. See Table 10 for the data and model specifications.

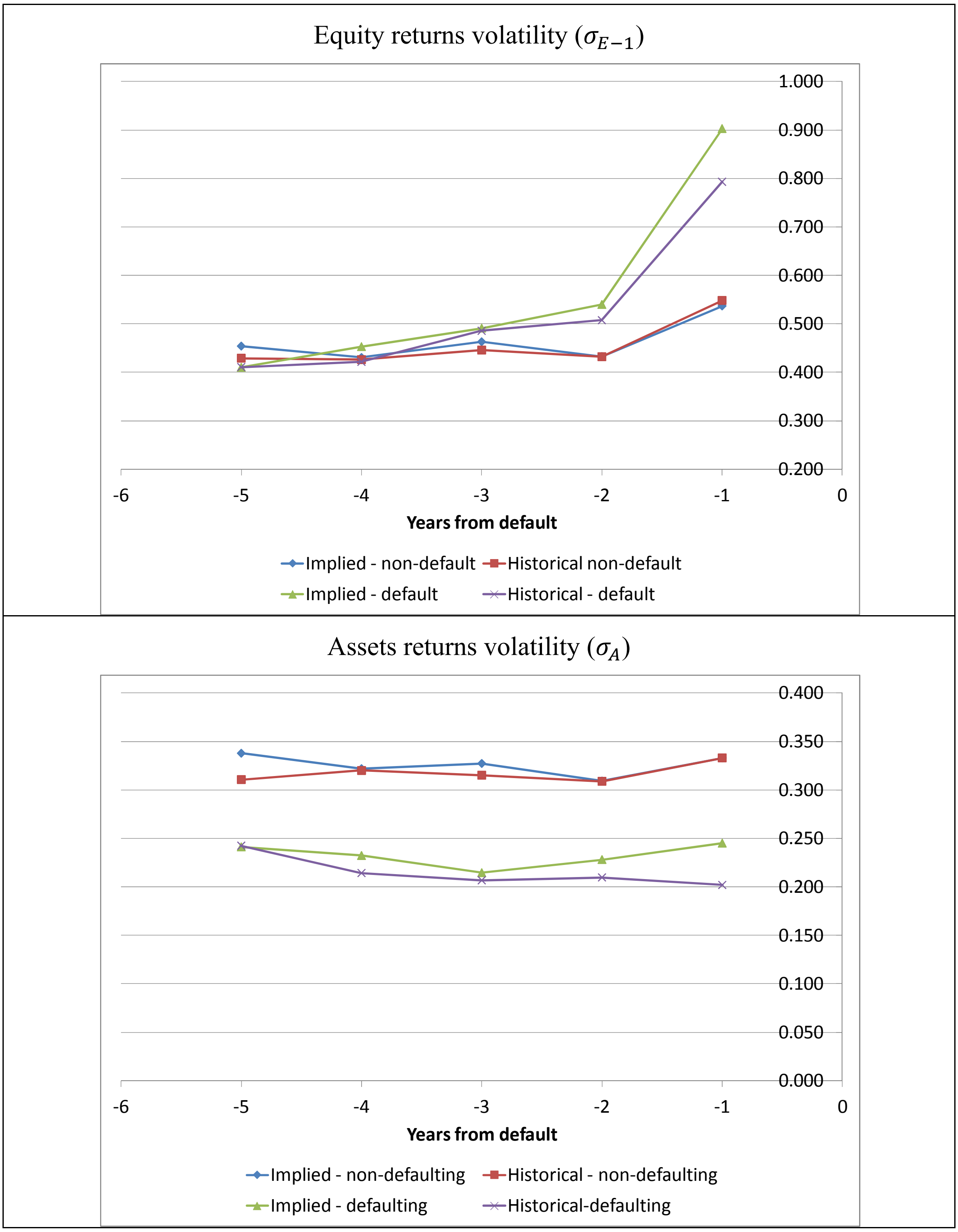

Figure 7: The values of the annual assets volatility, calculated by the naïve model (σ_A^{Naive}), for defaulting and non-defaulting firms, versus years prior to default, using k=0.5. Each of the curves represents averages of 137 firms (averaged by year to default). One curve shows the defaulting firms and the other the non-defaulting firms (a control group selected randomly in matching periods).