WHAT IT TAKES TO BE A LEADER: LEADERSHIP AND CHARISMA IN A CITIZEN-CANDIDATE MODEL

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What It Takes to Be a Leader: Leadership and Charisma in a Citizen-Candidate Model

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Abstract

This paper analyses leadership and charisma within the framework of social choice. In societies that lack formal institutional authorities, the power of leaders to coerce is limited. Under such conditions we find that social outcomes will depend not only on policy preferences but also on individuals' abilities to transform voluntary efforts into some communal public good. The paper has three central results: (1) Leaders might credibly compromise on policies they favor in order to elicit more social efforts, while society members might be willing to compromise on favorable policies in order to gain better leaders. (2) Under imperfect information regarding individuals' abilities, social choice might be biased toward less competent but more charismatic leaders. (3) Less-competent, more charismatic leaders can achieve more in terms of social goals than competent non-charismatic ones.

JEL Classification: D7, D72, H4, H41

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1. Introduction

The theory of political economy has often addressed the question of how constituencies elect representatives to choose a policy under a variety of situations concerning elections and voters. To tackle this issue, political economists have often adopted the view that after a candidate is elected for office he receives complete authorization to implement his chosen policy as an office holder. This view accords with typical situations of public economic choice in which policies are observable by society members and are enforceable by communal authorities. Yet, while this view seems reasonable enough in the framework of public economic choice, it is somewhat misleading in a more natural and broader context in which the power of leaders to coerce is limited. Under such conditions, leaders are often required to create post-election incentives to magnetize the *voluntary* efforts of society members by means of leadership that have not yet been analyzed. It is the purpose of this paper to examine these means by removing the standard assumption that the chosen leader can automatically enforce his policy via authorized power, and assuming instead that leaders are required to use (among other things) personal abilities as well as charisma to recruit voluntary efforts. This exchange of assumptions establishes new tradeoffs between policy, effort, leadership abilities and charisma thereby providing considerable insight into social choice and behavior.

The paper is set by the following preliminary observation: *leadership* and *charisma* though contextually related are not formally identical. Whereas the notion of *leadership* can be interpreted as the competence of individuals to transform communal effort into some shared goal, *charisma* is more related to the talent of individuals to recruit these efforts. From an economic perspective, the distinction between these two notions can be associated with two separate environments: one with perfect information and the other with imperfect information. In the case of perfect information, leadership and charisma coincide. This is due to the fact that the abilities of individuals to transform social resources into some shared goal are fully observed and therefore in themselves might be an important factor in the decision making of individuals regarding whether or not to grant support and effort. In the case of imperfect information however, charisma and leadership might mismatch as leadership abilities are not fully observed, and therefore, society members are forced to consider other attributes of potential leaders in order to decide whether or not to provide them with support and effort. Hence, under imperfect information, charisma can be viewed as personal attributes (such as self-confidence, charm, poise etc.) that are not perfectly correlated with leadership abilities but are completely observable. Hence, in the absence of information the notion of charisma becomes essential in leadership allocation as society members are forced to use these personal attributes as external signals to evaluate candidates' abilities.

The paper has three main results. First, in contrast to the results of Osborne and Slivinski (1996) and Besley and Coate (1997), leaders might *credibly* compromise on favorable policies.[2] The rationale of this result is that by compromising policies, society members might gain better leaders while leaders might attain additional social efforts. This result stems directly from the new tradeoffs between policy, effort and leadership ability that our paper establishes. Second, in the face of imperfect information, society members may possibly choose less worthy leaders while providing them with extra efforts. This result stems from the fact that when leadership abilities are not perfectly observable, society members are forced to use candidates' personal attributes as "external -signals" in order to evaluate their leadership abilities. These outer signals may very well mislead society members to over-evaluate some charismatic non-competent leaders while under-evaluating non-charismatic but competent ones. We call this misallocation of resources *the charisma bias*. Third, in the face of imperfect information, this charisma bias may lead to situations where non-competent charismatic leaders can accomplish considerably more in terms of social goals than competent non-charismatic ones. Hence, a charismatic advantage might

[2] Like in Osborne and Slivinski (1996) and Besley and Coate (1997) candidates cannot commit to positions at all.

counterbalance a leadership disadvantage due to an extensive collection of efforts that eventually offsets leadership inefficiencies.

The idea of the paper is presented by two models that follow the citizen-candidate approach pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997). The first model focuses on leadership per se, and hence is set in a perfect information environment. This model examines how societies allocate policy and choose leadership when members vary in their leadership abilities as well as in their preferences over communal agendas. The second model examines the idea of charisma and is set in an imperfect information environment. In this model we relax the assumption that society members have different preferences over policies but we add the assumption that society members cannot fully observe the abilities of others but observe some external signals that are imperfectly correlated with leadership abilities. In both models we set an additional stage in which society members voluntarily exert efforts that are provided for the production of some public good. The results of the paper can be explained by the strategic decisions of society members stemming from this additional stage. In the first model (with perfect information and divers preferences over policies), the decision of how much effort to exert is a function of the leader's chosen policy and his leadership ability. Potential leaders will take this into account when choosing optimal policies. Society members anticipate the decisions of potential leaders at earlier stages and decide to which candidate they will grant support. Potential leaders anticipate this and decide whether or not to offer candidacy. These considerations lead to allocations of leadership and policy in a subgame perfect Nash equilibrium that reproduces our first result - leaders might credibly compromise on policies. In the second model (with imperfect information and individuals' identical preferences), the decision of how much effort to exert is a function of the leader's leadership ability *as evaluated by other society members*. Potential leaders anticipate this and decide whether or not to offer candidacy. This scheme leads to an inefficient equilibrium that might be biased toward charismatic non-competent leaders with excess supply of efforts.

The paper can be related to two different lines of research in the economic literature: one that examines social choice in political setting and is rooted in the traditional electoral competition theory, and another that examines leadership within a framework of the theory of incentives. The first line of research which is largely based on Downs (1957) model with its numerous extensions (see Wittman (1977, 1983), Calvert (1985), Alesina and Spear (1988) and Alesina (1988)) has been recently extended to models that describe political equilibria in situations where citizens can endogenously offer candidacy under a plurality rule.[3,4]In recent years these models where implemented in other works such as the study of politicians' quality (see Caselli and Morelli) and lobbying (see Besley and Coate (2001)). Yet, while this whole literature provides considerable insight into a variety of situations of public choice with differing assumptions concerning voters and elections, it generally ignores the role of governance in creating post election incentives and its effect on political equilibria. The second approach pioneered by Rotemberg and Saloner (1993) and Hermalin (1998), views leadership as a device that creates incentives in organizations under the conditions of asymmetric information and incomplete contracting environment.[5,6] This approach provides new insights into leader-organization interactions, but ignores the questions of

[3]Downs viewed policy as a means to winning elections, while Wittman (1977, 1983), Calvert (1985), Alesina and Spear (1988) and Alesina (1988) analyzed political equilibria with a fixed number of candidates who have policy preferences.

[4] See Osborne and Slivinski (1996) and Besley and Coate (1997).

[5] Rotemberg and Saloner (1993) examine how leadership style might affect firms' profits under the conditions of asymmetric information and incomplete contracting environment. They show that leaders who empathize with their employee adopt a participatory leadership style that might improve profitability when the firm has the potential of exploiting relatively many innovating ideas. Their model is based on the assumption that empathy of leaders with employee is common knowledge among organization's members and therefore can be served as a commitment device.

[6] In his seminal paper Hermalin (1998) emphasizes the role of leaders in transmitting information to followers under the conditions of asymmetric information. According to Hermalin leaders can transmit information to followers and convince them that the information is indeed true by leading by example: the leader himself exerts high level of effort. Followers observe the leader's effort and are therefore convinced that the leader considers the activities to be truly worthwhile. This motivates followers to exert effort as well.

how leadership is formed and why certain individuals become leaders rather than others.[7] Our paper combines elements from both literatures by analyzing the formation of leadership in conjunction with the role of leaders in creating incentives among society members.[8]

The paper can also be related to theoretical and empirical studies outside the economic discipline, especially in the social psychology trait approach.[9] The trait approach relates leadership to personal attributes of leaders.[10] In the context of our paper these traits should be classified according to taxonomy in which traits reflect the ability of leaders to transform effort into some public good - thereby manifesting leadership ability, or whether these traits are considered by followers as external signals and cause them to exert more effort, thereby reflecting charisma.

The rest of the paper is organized as follows. Section 2 sets up the basic model of leadership with perfect information and provides examples to demonstrate the results. Section 3 presents an imperfect information model of charisma and leadership. Section 4 provides examples to demonstrate the charisma bias and the tradeoff between leadership ability and charisma. Section 5 concludes and the main mathematical proofs appear in the appendix.

2. The Basic Model of Leadership with Perfect Information.

Consider a society inhabited by a finite number of individuals of different type, labeled $i \in N = \{1,...,n\}$. Each member $i \in N$ is endowed by a power index θ_i which represents his

[7] This line of research usually takes the identity of leaders as exogenous, and emphasizes the role of leaders in creating incentives and on leadership style in the face of asymmetric information.

[8] The incentives that leaders create in our paper are different from that of Hermalin and Rotemberg and Saloner, as we focus on the provision of public good and not on output in organizations (like firms), and we do not emphasize the role of leaders in transmitting information.

[9] The phenomenon of leadership by and large has puzzled researchers from a wide range of disciplines for nearly a century. Books and articles in sociology, social psychology, political sciences and management are numbered in the thousands, while additional works are still being published at a rapid rate (see Yukl and Van Fleet (1991), Northouse (1997) and Yukl (1998) for surveys). Most of the leadership studies can be classified according to whether the primary focus is on leaders' traits, behavior, power and influence or situational factors.

[10] During the 1930s and 1940s a numerous of trait studies were conducted to discover leaders qualities. These studies found differences between leaders and non leaders on some traits type basis but have not succeeded to classify leaders' performance on this basis as well (see Gibb (1954), Mann (1959) and Stogdill (1948)). Since the 1970s however, trait researchers have used more advanced methods, and their studies have shown that some traits increase the likelihood of successes as leaders (see Bray, Campbell and Grant (1974), Howard and Bray (1988))(See also Bass (1990) for survey).

relative power among the rest of the society members.[11] There are two distinct types of goods: menu of policies Q and a public good g. We assume that in a certain society, only one policy $q \in Q$ can exist at a time, and therefore, elements in Q are mutually exclusive. We also assume that preferences on ideas differ among individuals and therefore may be a source of controversy among society members. [12] For the sake of simplicity it is also assumed that Q is an open interval in $\Re$ (or alternatively that $Q = \Re$). Society is facing a decision problem of choosing a policy q out of a menu Q of types of policies.

Preferences

Each individual gains utility (or disutility) from both the policy q that his society implements, and from the quantity of the public good g that his society provides. We assume that the more individual i identifies with the policy q, the more he enjoys consuming the public good g.[13] Individuals may also bear some non-monetary costs $c(e)$, if they decide to exert some efforts e in producing the public good g. We will assume that the non-monetary effort cost function $c : \Re_+ \to \Re_+$ is three times continuously differentiable with $c(0) = 0$, and that for any $e>0$ $c'(e) > 0$ and $c''(e) > 0$ (i.e. $c(e)$ is monotonically increasing and convex). Furthermore, in order to avoid corner solutions and cases of multiple Nash equilibria, we also assume that $c'(0) = 0$. Each individual $i \in N$ has a utility function:

$$u_i = v_i(q) \cdot g - c(e_i) \tag{1}$$

[11] In the usual electoral competition context all individuals have identical index power, however as this paper explore leadership in a broader context in which societies do not necessarily have formal institutions, this index may represent different categories of power in different societies, such as: individuals' relative physical strength, individuals' relative wealth, and in a tribal society it may also represent the number of individuals in each tribe.

[12] In ordinary models of political economy, the space Q usually applies to a "policy space", in this model however, We will often use more general terms as "ideology", "ideas" "political agenda" interchangeably in order to address general circumstances.

[13] This assumption can be justified by the following argument: Once a political agenda q is implemented, all the society members "consume" it (for better or worse) and hence as such, can be interpreted as a "political public good" or an "ideological public good". We can therefore view the space of ideas Q as a variety of mutually exclusive public goods - once a public good $q \in Q$ was chosen, a quantity of g symbolizes the extent of q's production. Hence, policy and public good exhibit complementarities.

where g is the total quantity of the public good that individual i consumes, e_i is the total effort individual i devotes to the production of the public good g, and $v_i(q)$ is a singled peaked twice continuously differentiable function with nonempty support which represents individual i's private attitude toward political agendas (his idiosyncratic valuation of policies). We refer to $v_i(q)$ as the value function or the idea function of individual i. Whenever $v_i(q) > 0$, individual i perceives the policy q as an economic good, whereas for any policy q with $v_i(q) < 0$ individual i perceives the policy q as an economic bad. Note also that the idiosyncratic value of policy is complement with the quantity of the public good consumed.

We will take another crucial assumption that the most un-preferable position for any individual is to lead the society under a political agenda that he considers as bad. Hence, if some individual $j \in N$ was chosen to lead the society, and the implemented policy q is such that $v_j(q) < 0$ then individual j's utility is $-\infty$.[14]

Production

The production process of the public good g requires two inputs: leadership technology and total community efforts. Each individual $i \in N$ is endowed with an innate leadership technology $K(i) \in R_{++}$. Once individual $j \in N$ is chosen to lead, the total output of the public good is given by:[15]

$$g = K(j) \cdot \varphi(E) \qquad (2)$$

[14] Hence if for some individual j $v_j(q'') < v_j(q') < 0$ then he prefers to be led under political agenda q'' than to lead the society with political agenda q' regardless the quantity of the public good g he consumes.

[15] The ability of leaders to transform effort into some public good might also depend on their policy choice such that $K = K(j,q)$. The rationale of this is that the ability of leaders to implement a policy might depend on the policy itself. In our model however, we abstract from this dependency in order to focus on the tradeoffs between effort, ability, and policy only through the channel of individuals' preferences.

where g is the total output of the public good, E is the total effort exerted by the participants of the production process, and $\varphi(\bullet)$ is a monotonically increasing weakly concave function where $\varphi(0)=0$, $\varphi'(\bullet)>0$ and $\varphi''(\bullet)\le 0$.

Society

All the fundamentals of a certain society are common knowledge among the society members and are fully described by the five-tuple $\left(N,\langle\theta_i\rangle_{i\in N},\langle v_i(q)\rangle_{i\in N},\langle K(i)\rangle_{i\in N},\varphi\right)$.

2.1 The Mechanism

The mechanism of this model is constructed parallel to that of Besley and Coate (1997) except for one fundamental assumption. We add to their political process an additional stage in which after the leader is chosen to make his political choice, society members voluntarily exert efforts to manufacture the public good.

From the assumption it follows that when individuals decide whether to cast support to some individual $j\in N$ to lead them, they take into account not only his valuation of political ideas $v_j(q)$ but also two fundamental factors: his leadership technology $K(j)$ and the social effort that he may eventually recruit. Furthermore, every potential leader $j\in N$ knows that the level of social effort he can recruit depends not only on his leadership technology $K(j)$ but also on the policy q he chooses to implement. Hence, a potential leader j may choose a policy q which differs from his original bliss point ($\bar{q}_j=\arg\max v_j(q)$) so as to recruit more social efforts.

A leader is chosen in the following manner. Each individual can costlessly declare his desire to be a leader (offer his candidacy). Subsequently, each member casts support at most to one candidate. The candidate who receives the most powerful set of supporters (which is weighted by power indices of society members) is chosen to lead the society. In the case where more than one candidate receives maximum support, the leader is chosen according to a uniform lottery on the set of winning candidates. A chosen leader declares a policy, and at the final stage society members voluntarily chose a level of effort to exert in the production of the public good. To sum up, the political process has four stages: At stage 1, members declare their desire to lead. At stage 2, society members grant support to potential leaders. At stage 3, the chosen leader makes a policy choice *q*. At the final stage, society members voluntarily exert efforts to manufacture the public good. The model is solved backward.

2.11 Optimal Effort Decisions given Leadership and Policy

Suppose that some individual $j \in N$ with a leadership technology *K*(*j*) is the society's chosen leader and suppose that he chooses a policy $q \in Q$. Then, the optimal effort of any individual $i \in N$ for any given level of effort exerted by the rest of the community members is given by:

$$e_i = \arg\max_{0 \le e_i} [v_i(q)K(j)\varphi(E_{-i} + e_i) - c(e_i)] \tag{3}$$

where E_{-i}, is the total effort exerted by all the society members excluding individual *i*.[16] There are two possible conditions for individual *i* under which he decides the level of effort to exert. If the leader's chosen policy *q* is an "economic bad" in the viewpoint of individual *i* (i.e. $v_i(q) \le 0$) then individual *i* will exert no effort in producing the public good *g* (thus e_i=0). Otherwise if $v_i(q) > 0$, optimum considerations lead to a first order condition, given by:

$$v_i(q)K(j)\varphi'(E) = c'(e_i) \tag{4}$$

Equation (4) implies that for any given leader *j* and any selected ideology *q* where $v_i(q) > 0$, individual *i*'s optimal action corresponds to the total level of efforts exerted by the rest of the

[16]One may suggest another setting where in Stackelberg fashion, the leader announces his level of effort before the society members decide their own level of efforts. This setting will add another sub-stage to the model and provide similar results.

society members, and therefore, a Nash equilibrium profile of efforts $\left(e_1^j * (q),...,e_n^j * (q)\right)$ exists if for each society member $i \in N$ the following equation holds:

$$e_i = \begin{cases} c'^{-1}\left[v_i(q)K(j)\varphi'(E)\right] & if \quad v_i(q) > 0 \\ & , \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

where $E = \sum_{i=1}^{n} e_i$. Equation (5) can be therefore interpreted as an implicit function of player *i*'s best response.

Lemma 1: The analytical assumptions on the cost function $c(e)$ and the function $\varphi(E)$ ensure that an efforts' Nash equilibrium profile indeed exists and is unique.

Proof: Assume that some individual *j* is the leader and that he chooses a political agenda *q*. Let $M \subset N$ be the set of society members who view the idea *q* as a positive political agenda (i.e., $M = \{i \in N : v_i(q) > 0\}$). A sufficient condition for a Nash equilibrium to exist is that equation (5) will hold for each society member $i \in N$. As all the society members not in subset *M* will choose the nil strategy e_i=0. A summation of equation (5) over the subset M of the society members leads to:

$$E = \sum_{i \in M} c'^{-1}\left[v_i(q)K(j)\varphi'(E)\right]. \tag{6}$$

The analytical assumptions on $c(e)$ and $\varphi(E)$ ensure that the summation $\sum_{i \in M} c'^{-1}\left[v_i(q)K(j)\varphi'(E)\right]$ is a continuously positive monotonically non-increasing function of

E. Thus, the intermediate value theorem implies that equation (6) will hold for a unique $E^* > 0$.[17]

Substituting E^* into equation (5) for each society member $i \in N$ yields a Nash equilibrium profile of efforts $\left(e_1^j*(q),...,e_j^j*(q),...,e_n^j*(q)\right)$, as required.□

For a given leader j and ideology q, we denote the society Nash equilibrium's total effort by $E^j*(q) = \sum_{i=1}^{n} e_i^j*(q)$.

It is useful to describe the above Nash equilibrium by manipulating equation (4) into the following expression:

$$K(j)\varphi'(E^j*(q)) = \frac{c'(e_{\gamma_1}^j*(q))}{v_{\gamma_1}(q)} = ... = \frac{c'(e_{\gamma_m}^j*(q))}{v_{\gamma_m}(q)}, \tag{7}$$

where $\gamma_1,...,\gamma_m$ are the members of the subset $M \subset N$. We can therefore display the Nash equilibrium by plotting the society's total effort next to M individuals' efforts in one graph (see Figure 1).

[Insert Figure 1 about here]

Given that individual j is the leader and that his chosen policy is q, the indirect utility of individual $i \in N$ which is derived in the final stage Nash equilibrium is given by:

$$u_i^j(q) = v_i(q)K(j)\varphi(E^j*(q)) - c(e_i^j*(q)) \tag{8}$$

[17] The derivation of summation (6) provides $\sum_{i \in M} {c'^{-1}}' \left[v_i(q)K(j)\varphi'(E)\right]\varphi''(E)$, and as $\varphi'' \le 0$ and $c', c'' > 0$ we get that $\sum_{i \in M} {c'^{-1}}' [\bullet]\varphi''(E) \le 0$.

Lemma 2: Suppose that the elected leader is individual j, and suppose that the chosen policy is q. The total effort E^* in the fourth stage Nash equilibrium is a non-decreasing function of the leader's technology $K(j)$ (i.e. $dE^*(q) / dK(j) \geq 0$).

Proof: See the Appendix.

2.12 The Political Choice of the Leader

Suppose that individual $j \in N$ was chosen to lead. He then faces a problem of political choice, where he seeks to maximize his indirect utility function in the fourth stage Nash equilibrium, given by equation (8).[18] The policy choice problem of the leader is therefore given by:

$$q^* = \arg\max_{q \in Q} \left[v_j(q) K(j) \varphi(E^j * (q)) - c(e_j^j * (q)) \right]. \tag{9}$$

The solution to this problem is characterized by the following proposition.

Proposition 1: Suppose that for every individual $i \in N$ the idiosyncratic value functions $v_i = v_i(q)$ are continuously twice differentiable and single peaked functions, and suppose that individual j was chosen to lead the society. A necessary condition for q^* to be the leader's optimal policy is that at least one of the following two conditions holds:

(I) $v_j(q^*) > 0$ and $-\mathbf{e}_{v_j,q} = \mathbf{e}_{g,E} \cdot \hat{\mathbf{e}}_{E_{-j},q}$ [19]

(II) $v_j(q^*) = 0$.

[18] Under the assumption that leadership abilities depend on policy choice such that $K = K(j,q)$, the leader's policy choice problem becomes $q^* = \arg\max_{q \in Q} \left[v_j(q) K(j,q) \varphi(E^j * (q)) - c(e_j^j * (q)) \right]$.

[19] Where the letter **e** indicates elasticity, hence:

$$\mathbf{e}_{v_j,q} = \frac{(dv_j / dq)}{v_j(q)} \cdot q \quad , \quad \mathbf{e}_{g,E} = \frac{(d\varphi(E)/dE)}{\varphi(E)} \cdot E \quad \text{and} \quad \hat{\mathbf{e}}_{E_{-j},q} = \frac{(dE_{-j}^j / dq)}{E^j *} \cdot q .$$

Specifically, if $v_j(q^*) > 0$ the policy q^* is optimal in the viewpoint of the leader only if the minus elasticity of the leader's value with respect to q^* is equal to the elasticity of the total contribution of the rest of the society members with respect to q^*.[20]

Proof: See the Appendix.

Proposition 1 highlights the leader's tradeoff between choosing his favorite policy and his goal to assemble social collaboration and efforts. Condition (I) claims that whenever $v_j(q) > 0$, the leader will be better off by compromising on his policy as long as the additional percentage change in social collaboration exceeds the percentage drop in the idiosyncratic value form ideas. We denote the optimal policy of individual *j* (once he was chosen to lead) by q_j*, and the vector of optimal political choices of each individual (as potential leaders) by $q^* = (q_1*,...,q_n*)$. Due to the common knowledge and perfect information assumptions, it is clear that the vector of optimal ideologies q^* is correctly calculated by all society members and is taken into account by individuals in the previous stages of the political leadership game.

2.13 Choosing the Leader (*Voting*)

Suppose that the set of candidates is $\Im \subset N$. Each individual may cast his support to any candidate in $\Im$ or to abstain. Let $\alpha_i \in \Im \cup \{0\}$ denote individual *i*'s decision (if $\alpha_i = j$, where $j \in \Im$ then individual *i* supports the leadership of individual *j*, and if $\alpha_i = 0$ he abstains. We denote the supporting vector (voting vector) by $\alpha = (\alpha_1,...,\alpha_n)$ and the set of wining candidates where voting decisions are α by $W(\Im,\alpha)$ where:

[20] Under the assumption that leadership abilities depend on policy choice such that $K = K(j,q)$, equation (I) in proposition 1 becomes $-\mathbf{e}_{v_j,q} = \mathbf{e}_{g,E} \cdot \hat{\mathbf{e}}_{E_{-j},q} + \mathbf{e}_{K,q}$ where $\mathbf{e}_{K,q}$ is the elasticity of leadership ability with respect to his own policy choice. This implies that leaders might lake into account also changes in their own ability when choosing an optimal policy. As mentioned above (footnote 15) the assumption that leadership abilities depend on policy choice adds very little to our understanding of the central tradeoffs.

$$W(\Im,\alpha)=\left\{\forall\, l\in\Im:\ \sum_{\{i\in N:\alpha_i=l\}}\theta_i\geq\sum_{\{j\in N:\alpha_i=k\}}\theta_j\qquad\forall k\neq j\text{ where }\quad k\in\Im\right\}\tag{10}$$

If $W(\Im,\alpha)=\{j\}$ for some $j\in\Im$ then j is automatically chosen to be the leader. If $\#W(\Im,\alpha)>1$ then, a leader is chosen by a uniformly distributed lottery that assigns probability $P^l(\Im,\alpha)=\dfrac{1}{\#W(\Im,\alpha)}$ to each candidate $l\in W(\Im,\alpha)$. Of course, if candidate $l\in\Im$ wins, he chooses a policy $q_l{*}\in Q$ that maximizes his own indirect utility in Nash equilibrium, by solving problem (9) and by applying proposition 1. Again, as all the fundamentals of the model are common knowledge, all the society members correctly calculate $q_l\,*$ for each candidate in $\Im$. Moreover, the result of any individual's action depends on the actions of the rest of the society members and hence the decision whether to support a candidate or not is strategic. A supporting equilibrium is thus a vector $(\alpha_1{*},...,\alpha_n{*})$ such that for each individual *i*, $\alpha_i\,*$ is the optimal reaction to $\alpha_{-i}\,*$, namely:

$$\alpha_i{*}\in\arg\max\left\{\sum_{l\in\Im}P^l\left(\Im,(\alpha_i{*},\alpha_{-i}{*})\right)u_i^l(q_l^*):\alpha_i\in\Im\cup\{r\}\right\}\ ,\tag{11}$$

where $\alpha_i\,*$ is not a weakly dominated supporting strategy.[21]

Proposition 2: For any nonempty candidates' set $\Im$, a supporting equilibrium exists.

Proof: see the Appendix.

Clearly, multiple supporting equilibria may exist when there are more than two candidates.

2.14 Declaring Candidacy (*Entry*)

Each society member must decide whether or not to declare candidacy. Of course, the result of the entry stage depends on the set of candidates, and therefore, the decision whether or

[21] As in Besley and Coate, ruling out weakly dominated strategies implies a sincere supporting strategy in the case where there are two candidates for leadership (though not necessarily for more than two).

not to declare candidacy is strategic. Let $s=(s^1,...,s^n)$ be the pure strategic entry profile, where $s^i \in \{0,1\}$ and s^i=1 denotes entry. Given the strategic profile *s*, the set of candidates is $\Im(s)=\{\forall i \in N : s^i=1\}$. Given a function α(•) that assigns a supporting vector (voting vector) to each candidate configuration, the expected payoff of individual *i* from the pure strategic profile *s* is given by:

$$U^i(s,\alpha(\cdot)) = \sum_{l\in\Im(s)} P^l\big(\Im(s),\alpha(s)\big)\cdot u_i^l(q_l^*) \tag{12}$$

Given a function α(•) that assigns a supporting vector to each candidate configuration, *an equilibrium of pure strategies of the entry game* (if exists) is a profile $s=(s^1,...,s^n)$ such that s^i is the best response to s^{-i} for each society member *i*. Of course, equilibrium in pure strategies does not always exist. We therefore permit society members to use mixed strategies of entry decisions. Each society member *i* may choose an entry probability $\chi_i \in [0,1]$. A mixed strategy profile is denoted by $\mathrm{X}=(\chi_1,...,\chi_n)$. Given a function α(•) that assigns supporting vectors to all candidates' configurations, the expected payoff of individual *i* from the mixed strategy X is given by:

$$U^i(X,\alpha(\cdot)) = \sum_{s\in 2^n}\left\{\prod_{k=1}^{n}\left[\chi_k^{s^k}(1-\chi_k)^{(1-s^k)}\right]U^i(s,\alpha(\cdot))\right\} \tag{13}$$

Given a function α(•) that assigns a supporting vector to each candidate configuration, *an equilibrium of mixed strategies of the entry game* is a profile of mixed strategies $\mathrm{X}=(\chi_1,...,\chi_n)$ such that χ^i is the best response with respect to χ^{-i} for each member *i*.

2.15 Equilibrium

A Sub-Game Perfect Nash Equilibrium of the entire leadership game is constructed by incorporating the analysis of the four stages described above. Hence, leadership equilibrium can be fully described by the quartet $\left(X^*, \alpha^*(\cdot), q^*, \left\langle e^j{}^*(q)\right\rangle_{j\in N}\right)$ of an entry decision profile X, a function α(·) that assigns a supporting vector to each non empty candidate configuration, an *n*-tuple vector *q** of policies and *n* vectors $\left\langle e^j{}^*(q)\right\rangle_{j\in N}$ of effort functions depending on ideologies for each possible leader $j \in N$ where:

A) X* is an equilibrium (mixed or pure) of the entry game, given the function α*(·).

B) $\alpha^*(\Im)$ is a function that assigns to each possible non empty candidate set $\Im$, a supporting (voting) *equilibrium*, given the vector of policies q^*.

C) $q^* = (q_1^*,...,q_n^*)$ is a vector of optimal policies in the viewpoint of the *n* potential leaders, given the society members efforts $\left\langle e^j{}^*(q)\right\rangle_{j\in N}$.

D) $\left\langle e^j{}^*(q)\right\rangle_{j\in N}$ are Nash equilibrium efforts' profiles depending on ideologies for each possible leader $j \in N$.

Proposition 3: There exists a sub-game perfect equilibrium to the leadership game described above.

Proof: The construction above (with lemma 1 and propositions 1 and 2) fully defines a sub-game perfect Nash equilibrium.□[22]

[22] Similar to Besley and Coate (1997), there are two necessary and sufficient conditions for a pure strategic equilibrium *s** of the entry stage to exist given a supporting function $\alpha(\cdot)$:

First, for all $i \in \Im(s^*)$, $\sum_{j\in\Im(s^*)} P^j(\Im(s^*), \alpha(\Im(s^*))) \cdot u_i^j(q_j^*) \geq \sum_{j\in\Im(s^*)\setminus\{i\}} P^j(\Im(s^*), \alpha(\Im(s^*))) \cdot u_i^j(q_j^*)$

where $\Im(s^*)\setminus\{i\}$ is the candidate set with individual i removed.

Second, for all $i \notin \Im(s)$,

$$\sum_{j\in\Im(s)} P^j(\Im(s), \alpha(\Im(s))) \cdot u_i^j(q_j^*) \geq \sum_{j\in\Im(s)\bigcup\{i\}} P^j(\Im(s)\bigcup\{i\}, \alpha(\Im(s)\bigcup\{i\})) \cdot u_i^j(q_j^*)$$

The first inequality says that each candidate must be willing to run given who else is in the race. Note that the event where no leader is chosen is omitted from the inequality above as the utility from this event is zero. The second inequality says that all individuals not in the race will not be better-off by entering the race (an entry proof condition). Following theses inequalities, the equilibria with pure strategies can be characterized by dividing the society into sincere partitions. For details see Besley and Coate (1997)'s section on pure strategic equilibrium. B&C's results can be adjusted to our model by substituting $u_i^j(q_j^*)$ into the appropriate utility functions subsequent to the third and the fourth stages.

3. What it takes to be a Leader: Some Expository Examples

In this section we provide three examples of how societies choose their leaders when members are perfectly informed about the fundamentals of their community. The first example demonstrates a consensual society in which social decision on leadership is solely based on the division of leadership abilities ($K(j)$s). In the second example, the society is separated into two groups that exhibit preference homogeneity within each group but extreme heterogeneity between groups. In this example, the social decision on leadership is based on the division of relative power between groups, on one hand, and on the other, on the division of leadership abilities within the powerful group. In the third example, the society is separated into two subsets that exhibit preferences homogeneity within each subset and heterogeneity between subsets, however, the heterogeneity between subsets is relatively minor and therefore potential leaders have an incentive to compromise on political agenda in order to heave voluntary effort from members of the contender subset, while society members may face two contradicting effects: a "leadership gap" effect that motivate them to support a candidate with the highest leadership technology and a "quantity-effort" effect which motivate them to support a candidate from the largest subset from which the leader drags massive effort. These two effects will be demonstrated in the third example in cases where the conditions are such that society members prefer to support a candidate of the other group rather than a candidate of their own.

In order to ensure tractability of the examples we substitute our general functions with simple explicit ones. We assume that the cost function of each society member is $c(e) = ce^2$ and that the production function net of leadership input is $\varphi(E) = aE$ where $c,a>0$ are constant parameters. This choice of functions ensures that the optimal effort of each society member is independent of the other members' efforts (i.e., the effort Nash equilibrium profile consists of strictly dominate strategies).[23]

Before providing our three examples we briefly calculate the results of the fourth and the third stages of the leadership game under the above specification and then we present our examples.

The Fourth Stage (*with* $c(e) = ce^2$ *and* $\varphi(E) = aE$)

Given that the leader is individual *j* and that he chooses a policy *q*, each society member $i \in N$ with $v_i(q) > 0$ exerts $e_i = \frac{a}{2c} v_i(q) K(j)$ level of effort and zero otherwise. Hence, the total effort that individual *j* can magnetize from society members as a leader is $E = \frac{a}{2c} K(j) \sum_{i \in M} v_i(q)$ (where *M* is the set of all society members who valuate *q* positively i.e., $M = \{i \in N : v_i(q) > 0\}$). Given a policy *q*, the private Nash equilibrium indirect utility function of individual *i* when *j* is the leader is $u_i^j(q) = \frac{a^2}{2c} v_i(q) \left(K(j)\right)^2 \left[\sum_{u \in M} v_u(q) - \frac{v_i(q)}{2} \right]$ and the private indirect utility of individual *j* when he himself is the leader is given by: $u_j^j(q) = \frac{a^2}{2c} v_j(q) \left(K(j)\right)^2 \left[\sum_{u \in M} v_u(q) - \frac{v_j(q)}{2} \right]$.

The Third Stage (The Leader Chooses a Policy q):

[23] $(e_1*,...,e_n*)$ is an Nash equilibrium of strictly dominate strategies if $\forall i \in n$ $u_i(e_{-i}, e_i*) > u_i(e_{-i}, e_i)$ for all e_{-i} and e_i.

A necessary condition for q_j* to be individual *j*'s optimal policy had he won the leadership game is $\frac{d}{dq}u_j^j(q_j*)=0$. Since *K*(*j*)>0 for each society member, this condition implies that $\left\{v_j'(q)\left[\sum_{u\in M}v_u(q)\right]+v_j(q)\left[\sum_{u\in M\setminus\{j\}}v_u'(q)\right]\right\}=0$

(or put differently $\frac{v_j'(q)}{v_j(q)}=-\left(\frac{\sum_{u\in M\setminus\{j\}}v_u'(q)}{\sum_{u\in M}v_u(q)}\right)$).[24]

We now demonstrate the forces that affect the result of the leadership game for each example.

*3.1 Example*1: *Consensual society* (*all society members have the same ideological bliss point*):

Consider a society whose members share the same ideological bliss points (i.e., $\bar{q}=\arg\max v_i(q)\quad \forall i\in N$). The individual with the highest level of leadership technology *K*(*j*) will be elected and the leader will choose the political agenda $q*=\bar{q}$. In this trivial example, the power indices of society members are irrelevant since there is an intrinsic consensus between society members over political ideas.

3.2 Example 2 (*Domination of one group*):

This example demonstrates a case where one powerful group imposes all the significant social decisions (leadership and policy) due to large preferences heterogeneity *between* groups.

Suppose that the society consists of two subsets of individuals (groups) N_1 and N_2 such that $N=N_1\cup N_2$ and $N_1\cap N_2=\phi$ (where n_1 and n_2 are the sizes of each subset). In each

[24] Note that this result is consistent with proposition 1 which states that $-\mathbf{e}_{v_j,q}=\mathbf{e}_{z_g,E}\cdot\hat{\mathbf{e}}_{E_{-j},q}$. However the above condition does not contain leader leadership technology *K(j)*, due to our construction of φ which is linear in total effort E.

subset, all individuals have the same preferences; however there is large preferences heterogeneity *between* these groups such that the ideas' function supports $Supp(v_i(q))$ of members from each subset are disjoint (See figure 2)[25].

[Insert figure 2 about here]

It is easy to see that in equilibrium one group dominates the entire society. Denote the subset with the highest power by N_d (where $d = \arg\max\left\{\sum_{l\in N_1}\theta_l, \sum_{k\in N_2}\theta_k\right\}$). All members of N_d agree on their decisions on leadership and effort and they all choose a leader $j \in N_d$ such that $j = \arg\max\{K(i)\}_{i\in N_d}$. The leader j will obviously choose the political agenda $q_d{*} = \arg\max v_j(q)$ (which also maximizes the ideas' functions of the members of N_d). The members of the subset N_d dominate the whole society by imposing their leader and his policy on the entire society. It is important to emphasize that even when the highest leadership technology belongs to the non-dominate group and exceeds by far the leadership technology of individual *j*, the dominant group will not choose a leader from the second group due to the disagreement between groups on any political agenda.

Solving the model with the specified functions above yield that all N_d's members will exert $e_i = \frac{a}{2c}v^d(q)K(j)$ effort (where $v^d(q) = v_i(q) \quad \forall i \in N_d$) and their indirect utility function in Nash equilibrium is $u_d(q_d{*}) = \frac{a^2}{2c}\left(n_d - \tfrac{1}{2}\right)\left[v_d(q_d{*})K(j)\right]^2$, while the indirect utility of the rest

[25] The support of $v_i(q)$ is defined as the closure of the domain of its positive values (i.e. $Supp(v_i(q))=Closure\{\forall q\in Q: v_i(q)>0\}$. The above assumptions imply that $v_i(q)=v_i(q) \quad \forall i,j\in N_1$ and $v_i(q) = v_j(q) \quad \forall i, j \in N_2$, but $Supp(v_i(q)) \cap Supp(v_j(q)) = \phi \quad \forall i \in N_1$ and $\forall j \in N_2$.

of the society members is $u_{-d}(q_d*) = \frac{[aK(j)]^2}{2c}\left(n_d - \tfrac{1}{2}\right)v_d(q_d*)v_{-d}(q_d*)$.[26] Note that the second group's utility in equilibrium is negative.

It is also important to emphasize that though the dominate group is more powerful than the second it is not necessarily the biggest in size. In our framework power superiority ensures the group's domination while its size determines the total effort it exerts.

3.3 Example 3 The Tradeoff between Leadership Technology Quantity and Policies.

The aim of this example is to show that individuals may optimally compromise on policies or leadership even when they have power superiority or leadership advantage. As in the previous example we still assume that the society consists of two subsets of individuals where individuals in each subset are homogenous in their preferences over ideas. However, in contrast to the previous example we now assume that the preference heterogeneity between groups is relatively minor and hence individuals may agree on ideas at least to some extent. Specifically, the society consists of two subsets of individuals N_1 and N_2 such that $N = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi$ (where n_1 and n_2 are the sizes of each subset), however, now we construct a pattern where the intersection of the ideas' function supports $Supp(v_i(q))$ contain the maximum points of each member. Assume that the members of subset N_i (i=1,2) have an ideas' function which is a "translation operator" of some function $v(q)$, explicitly $\forall i \in N$ $v_i(q) = v(q - \overline{q}_i)$ where $\overline{q}_i$ is the maximum point of the ideas' function of member i, and where the function $v(q)$ exhibits the following properties:

- Single Peaked at Zero: for all $q' < q'' < 0$, and for all $0 > q'' > q'$, $v(q') < v(q'') < v(0)$.
- Symmetry: $v(q)=v(-q)$.
- Concavity: $v(q)$ is strictly concave and continuously twice differentiable.

[26] Note that n_d is the size of the dominant group and q_d* is the political agenda that maximizes the ideas function $v_d(q)$ of individuals in the dominant group.

The construction above implies that the idea's function of each society member $i \in N$ exhibits symmetry around the bliss point $\overline{q}_i$. Individuals in each subset are homogenous in their preferences and hence we can denote the ideas functions of individuals from subsets N_1 and N_2 by $v_1(q)$ and $v_2(q)$ respectively. We also denote the maximum points of the ideas' functions for individuals in subset N_1 and N_2 by $\overline{q}_1$ and $\overline{q}_2$ respectively.[27]. In order to ensure that the intersection of the ideas' functions' supports contains the maximum points of each member we assume that $\overline{q}_1 < \overline{q}_2$ and that $\overline{q}_2 - \overline{q}_1 \in Supp(v(q))$[28] .

By construction, $v_1(q)$ and $v_2(q)$ intersect at $\tilde{q} = \dfrac{\overline{q}_1 + \overline{q}_2}{2}$ (i.e. $v_1(\tilde{q}) = v_2(\tilde{q})$), and the symmetry property ensures that $v_1'(\tilde{q}) = -v_2'(\tilde{q})$[29]. This construction is demonstrated in Figures 3.

[Insert Figure 3]

In order to achieve tractable results we employ our specific cost and production functions again.

Lemma 3: Each potential leader $j(1)$ in subset N_1 ($j(2)$ in subset N_2) will choose a policy $q_{j(1)}*$ such that $\overline{q}_1 < q_{j(1)}* < \tilde{q}$ (will choose a policy $q_{j(2)}*$ in $\tilde{q} < q_{j(2)}* < \overline{q}_2$).

Furthermore, $q_{j(1)}*$ approaches $\overline{q}_1$ when n_1/n increases (i.e. $\dfrac{\partial q_{j(1)}*}{\partial(n_1/n)} < 0$ and $q_{j(1)}* \underset{n_1/n\to 1}{\rightarrow} \overline{q}_1$)

and $q_{j(2)}*$ approaches $\overline{q}_2$ when n_2/n increases (i.e. and $\dfrac{\partial q_{j(2)}*}{\partial(n_2/n)} > 0$ and $q_{j(2)}* \underset{n_2/n\to 1}{\rightarrow} \overline{q}_2$).

Proof: See the Appendix

Lemma 3 states that the optimal policy of a leader from subset N_1 (N_2) will always be bounded in the open interval $(\overline{q}_1, \tilde{q})$ $\left((\tilde{q}, \overline{q}_2)\right)$ and the larger is the relative size of subset N_1 (N_2) the

[27] $v_2(q) = v_1(q - (\overline{q}_2 - \overline{q}_1))$.

[28] Hence $\overline{q}_1, \overline{q}_2 \in Supp(v_i(q)) \cap Supp(v_j(q))$ for all $i \in N_1$ and $j \in N_2$.

[29] The symmetry of $v(q)$ ensures that $v'(q) = -v'(-q)$.

closer is the choice of a leader from N_1 (N_2) to his bliss point $\bar{q}_1$ ($\bar{q}_2$) (see the thick arrows in figure 4).

[insert figure 4 here]

3.31 Choosing the Leader

We denote the individuals with the highest leadership technology in subsets N_1 and N_2 by $j(1)$ and $j(2)$ respectively.[30] Certainly one of them will be chosen to lead.[31] Define a variable $\alpha_1 = \frac{n_1}{n}$ that represents the ratio between the size of subset N_1 and the size of the entire society, and define two functions: $B(1,\alpha_1)$ that represents $j(1)$'s welfare gap between a position when $j(2)$ is leading and when $j(1)$ himself is leading, and $B(2,\alpha_1)$ that represents j(2)'s welfare gap between a position when $j(2)$ himself is leading and when $j(1)$ is leading. $B(1,\alpha_1)$ and $B(2,\alpha_1)$ are given by:

$$B(1,\alpha_1) = \frac{v_1(q_2*)\left[(\alpha_1 - \tfrac{1}{2n})v_1(q_2*) + (1-\alpha_1)v_2(q_2*)\right]}{v_1(q_1*)\left[(\alpha_1 - \tfrac{1}{2n})v_1(q_1*) + (1-\alpha_1)v_2(q_1*)\right]}$$

$$B(2,\alpha_1) = \frac{v_2(q_2*)\left[(1-\alpha_1 - \tfrac{1}{2n})v_2(q_2*) + \alpha_1 v_1(q_2*)\right]}{v_2(q_1*)\left[(1-\alpha_1 - \tfrac{1}{2n})v_2(q_1*) + \alpha_1 v_1(q_1*)\right]}$$

Our assumptions lead to the following conditions: The members of subset N_1 will choose $j(1)$ if and only if $\left(\frac{K(j(1))}{K(j(2))}\right)^2 > B(1,\alpha_1)$, and the members of subset N_2 will choose $j(2)$ if and only if $\left(\frac{K(j(1))}{K(j(2))}\right)^2 < B(2,\alpha_1)$.

[30] i.e. $j(1) = \arg\max\{K(i)\}_{i\in N_1}$ and $j(2) = \arg\max\{K(i)\}_{i\in N_2}$ (generally there may be more than two).

[31] If the society members are going to choose one of the candidates of the subset N_l (l=1,2) they will choose the one with the highest leadership technology.

These conditions imply three possible types of pure strategic voting (supporting) equilibira with sincere partition which we will entitle as Events: In Event $E(1,1)$ members of N_1 and N_2 choose individual $j(1)$, In Event $E(1,2)$ members of N_1 choose $j(1)$and members of N_2 choose $j(2)$ and in Event $E(2,2)$ members of subset N_1 and N_2 choose $j(2)$[32]. See table 1.

Event E(1,1)	$\left(\frac{K(j(1))}{K(j(2))}\right)^2 > B(1,\alpha_1)$ $\left(\frac{K(j(1))}{K(j(2))}\right)^2 > B(2,\alpha_1)$	$j(1)$ is chosen
Event E(1,2)	$\left(\frac{K(j(1))}{K(j(2))}\right)^2 > B(1,\alpha_1)$, $\left(\frac{K(j(1))}{K(j(2))}\right)^2 < B(2,\alpha_1)$	If $\sum_{u\in N_1}\theta_u > \sum_{u\in N_2}\theta_u$ Then $j(1)$ is chosen. If $\sum_{u\in N_1}\theta_u < \sum_{u\in N_2}\theta_u$ Then $j(2)$ is chosen. (In a case of tie each is chosen with probability 1/2)
Event E(2,2)	$\left(\frac{K(j(1))}{K(j(2))}\right)^2 < B(1,\alpha_1)$ $\left(\frac{K(j(1))}{K(j(2))}\right)^2 < B(2,\alpha_1)$	j (2) is chosen.

The event $E(1,2)$ represents a situation where members of each subset prefer a leader of their own. This event may possibly occur when the ideas functions of individuals from distinct subsets are relatively remote as in Figure 5, or when there is no significant leadership

[32]The event $E(2,1)$ (i.e. "individuals from subset N_1 choose $j(2)$and individuals from N_2 choose $j(1)$") is not possible. It is easy to see that if individuals in subset N_1 choose $j(2)$, then $j(2)$ leadership technology exceeds the leadership technology of $j(1)$ by far, hence, the members of N_2 will choose j(2) as well. See also the inequalities corresponding to $E(2,2)$ in Table 1.

advantage or size overabundance of one subset over the other. The events $E(1,1)$ and $E(2,2)$, on the other hand, represent situations where all society members prefer the same leader. These events may arise from two possibly distinct sources: a leadership gap effect and a quantity effect.

A leadership gap effect arises when leadership advantage of one group over the other may possibly instigate members of the other group to support the advantageous candidate. A quantity effect arises when the size of one group is exceedingly larger than that of the other such that the total effort the leader can assemble is relatively high and may offset other disadvantageous.

We now demonstrate a tradeoff between leadership technology, quantity and policy, in a case where the subset N_2 has a leadership advantage over the subset N_1 however *all society members will unanimously choose a leader from* N_1 despite the leadership advantage of subset N_2, as the "quantity effect" of subset N_1 offsets the leadership advantage of subset N_2. We assume that the society is sufficiently large and that the ideas' functions of society members are as depicted in Figure 3.

Proposition 5: If the gap between leadership technology of $j(1)$ and $j(2)$ is such that $\xi/\eta < \frac{K(j(1))}{K(j(2))} < 1$ and the ideas functions are such that $\sigma > \xi^4/\eta^3$, there exists a threshold ratio $0 < \hat{\alpha} < 1$ such that for any society with $n_1/n > \hat{\alpha}$ all the society members choose $j(1)$ (i.e., the $E(1,1)$ case will prevail).

Proof: See the Appendix

Proposition 5 demonstrates a situation where leadership advantage is offset by quantity effect namely the proposition states that if the ideas functions of members of subsets N_1 and N_2 are sufficiently close, and if the size of subset N_1 is sufficiently bigger than that of N_2, then despite the $j(2)$'s leadership advantage over $j(1)$ all society members (including those of N_2) will sincerely choose $j(1)$ as their leader.

Note that under the conditions of proposition (5), if the ideas functions are such that $\sigma < \xi^4/\eta^3$, then all the society members who belong to N_2 will choose individual $j(2)$ as their leader even when n_1/n approaches 1. This is due to the fact that $j(1)$'s optimal policy choice q_1* will never be sufficiently valuable for members of N_2 that is required to offset the quantity effect of N_1's members over $j(2)$'s leadership advantage.[33]

4. Charisma - a Leadership Model with Imperfect Information

In the previous sections we showed that when society members observe that some potential leader is endowed with high leadership technology, they are ready to support him, and to exert high levels of efforts conditioned on his policy taste. Hence in perfect information environment, leadership ability in itself serves as an important factor in recruiting social effort. In this section we describe a mechanism of leadership formation when society members cannot fully observe leadership abilities, but rather observe some positively correlated external signals of leadership abilities. These external signals might mislead society members to exert more effort than they would if they knew the true candidates' leadership ability. Hence, charisma in this model can be viewed as personal traits that enable them to project extra leadership abilities

[33] Note that the E(1,2) case will prevail and the candidate of the most powerful subset will be chosen to lead. Hence, $j(1)$ will be chosen to lead if and only if $\sum_{u\in N_1}\theta_u > \sum_{u\in N_2}\theta_u$, and $j(2)$ will be chosen to lead if and only if $\sum_{u\in N_1}\theta_u < \sum_{u\in N_2}\theta_u$ and in a case of tie one of them will be chosen with probability 1/2.

than they really posses and therefore to recruit more effort by misleading other society members.

The important results of this section are that charismatic individuals might even succeed as leaders to assemble more effort from society members than other potential leaders with higher leadership abilities. This enable them to produce more quantity of the public good by gathering a high level of communal efforts even though they may possibly be less efficient as leaders than other non-charismatic potential leaders. Furthermore, We introduce an example where even when some individual with high leadership technology but no charisma *who is well informed about all the society members' leadership technologies* may cast his support *to another individual* with less leadership ability but high charisma, since he knows that the advantage of the charismatic individual in heaving social efforts is higher than his advantage in leadership ability. The implication of this result is that when information about leadership abilities is imperfect there may be a tradeoff between relatively high leadership ability and no charisma and between relatively low leadership ability and high charisma. We now describe a mechanism of the leadership game with imperfect information while emphasizing the quality of charisma as an important factor in the determination of leadership and its quality.

4.1 Nature's Random Moves and the Structure of Information

In this section we will assume that before the leadership game is launched, Nature makes three sequential moves:

First: Each individual $i \in N$ is endowed (by Nature) with leadership technology $K(i)$ which is drawn from a certain probability distribution P which is known to all individuals.

Second: For each individual $i \in N$, Nature draws an independent identically Bernoulli distributed lottery T_i such that:

$$T_i = \begin{cases} d & \text{with probability} \quad \Theta \\ 0 & \text{with probability} \quad 1-\Theta \end{cases}$$

The realization of T_i will reflect individual *i*'s "deception parameter".

Third: Following the realization of the two previous lotteries, Nature reveals partial information to society members as follows:

(i) Each individual $i \in N$ observes the realization of his own leadership ability and the realization of his own deception parameter (i.e. $(K(i),T_i)$),

(ii) Nature transmits to each individual $i \in N$ a "charisma vector" of external signals $(a_1,...,a_n)$ such that $a_i = K(i) + T_i$.

After Nature makes its moves the leadership game is launched.

4.2 The Mechanism of the Leadership Game with Imperfect Information

In order to focus on charisma and for the sake of simplicity we will assume that all community members have the same preferences over political agendas (i.e. $v(q) = v_1(q) = ... = v_n(q)$).[34] The implication of this assumption is that we can omit the third stage of the original leadership game with perfect information presented previously as now it is trivially solved.[35]

Following the realization of Nature's lottery and the allocation of information among society members, the leadership game will be carried out in three sequential stages: At stage 1, members declare their desire to lead (offer candidacy). At stage 2 society members grant

[34] Keeping the assumption that individuals differ in their attitude toward political agendas will create a strategic dependency between political choices of leaders and some extra information about leadership abilities that can be gained by choosing a certain political agenda. This strategic dependency will complicate the model with adding very little to our comprehension of the leadership-charisma phenomenon.

[35] Note that due to this assumption, any chosen leader will decide on the political agenda q*=argmax $v(q)$. With no loss of generality we will also assume that max $v(q)$=1.

support to potential leaders. At the final stage, after the leader is elected, society members voluntarily exert efforts in the production process of the public good.

The model is solved backwards.

4.21 Optimum Effort Decisions given Leadership

Suppose that some individual $j \in N$ who is endowed with a leadership technology $K(j)$ and a charisma parameter T_j was chosen to lead a society. As the rest of society members do not know j's leadership technology, they use their observation on the outer-signal a_j (where $a_j = K(j) + T_j$) in order to decide on how much effort to allocate to the production of the public good. Note that as the leadership technology $K(j)$ is unknown to individuals in $N \setminus \{j\}$, they perceive $K(j)$ as a random variable such that $K(j) = a_j$ with probability 1-Θ and $K(j) = a_j - d$ with probability Θ. The ex-ante objective of each individual is to maximize the expected utility function $\mathsf{E}^{\,i}\left(u_i^j \mid a_j\right)$ conditioned on the leader's observable parameter a_j.

We denote by e_i the efforts that individual $i \in N$ devotes to the production of the public good and we will denote by $E_{-\{i,j\}}$ the total efforts of society members excluding individual i and the leader j.[36]

Each individual $i \in N \setminus \{j\}$ does not know j's leadership abilities and hence calculates his best response by maximizing his expected utility u_i^j given $E_{-\{i,j\}}$, and the leader's efforts conditioned on his unknown leadership abilities $e_j\left[K(j)\right]$. Individual i therefore solve the following optimization problem:

$$e_i = \arg\max_{e_i > 0} \mathsf{E}^{\,i}\left(u_i^j\left(e_j\left[K(j)\right] + E_{-\{i,j\}} + e_i\right) \mid a_j\right) \qquad (3')$$

Substituting the parameters of the model and calculating the expected utility will provide:

[36] That is $E_{-\{i,j\}} = \sum_{l \in N \setminus \{i,j\}} e_l$.

$$e_i = \arg\max_{e_i>0}\begin{bmatrix}(1-\Theta)\cdot a_j \cdot \varphi\left(E_{-\{i,j\}} + e_j[K(j)=a_j] + e_i\right) \\ + \Theta\cdot(a_j - d)\cdot\varphi\left(E_{-\{i,j\}} + e_j[K(j)=a_j - d] + e_i\right) - c(e_i)\end{bmatrix}$$

where $e_j[K(j)=a_j]$ is the leader's effort if his leadership technology is $K(j)=a_j$, and $e_j[K(j)=a_j - d]$ is the leader's effort if his leadership technology is $K(j)=a_j - d$.

The first order condition of this problem is therefore given by:

$$c'(e_i) = \begin{bmatrix}(1-\Theta)\cdot a_j \cdot \varphi'\left(E_{-\{i,j\}} + e_j[K(j)=a_j] + e_i\right) \\ + \Theta\cdot(a_j - d)\cdot\varphi'\left(E_{-\{i,j\}} + e_j[K(j)=a_j - d] + e_i\right)\end{bmatrix} \tag{4'}$$

And in Nash equilibrium each individual *i*'s optimal effort (where $i \in N \setminus \{j\}$) is:

$$e_i = c'^{-1}\begin{bmatrix}(1-\Theta)\cdot a_j \cdot \varphi'\left(E_{-\{i,j\}} + e_j[K(j)=a_j] + e_i\right) \\ + \Theta\cdot(a_j - d)\cdot\varphi'\left(E_{-\{i,j\}} + e_j[K(j)=a_j - d] + e_i\right)\end{bmatrix} \tag{5'}$$

The leader *j* calculates his optimal effort knowing that the rest of the society members do not know his leadership ability while he knows it. Hence, the leader solves the optimization problem:

$$e_j = \arg\max_{e_i>0}\left[K(j)\cdot\varphi(E_{-j} + e_j) - c(e_j)\right] \tag{3''}$$

Lemma 4: A society members' profile of efforts equilibrium exists and is unique. Furthermore, in equilibrium, all non-leader individuals exert the same level of effort. furthermore, the level of efforts and the ex-ante indirect utility function of all individuals (as well as the leader's) increases with a_j.

Proof: the existence and uniqueness of equilibrium follow immediately by using the same considerations of Lemma 1 in section 2 in two different stages. In the first stage equation (5') is applied and we obtain:

$$E_{-j}*(a_j) = \sum_{i\in N\setminus\{j\}} c'^{-1}\begin{bmatrix}(1-\Theta)\cdot a_j \cdot \varphi'\left(E_{-j} + e_j[K(j)=a_j]\right) \\ + \Theta\cdot(a_j - d)\cdot\varphi'\left(E_{-j} + e_j[K(j)=a_j - d]\right)\end{bmatrix} \tag{6'}$$

In the second stage, $E_{-j}*(a_j)$ is plugged into equation (3'') by the leader. Note that the non-leader individuals are identical in their target functions and hence have the same best response efforts. From using the implicit function theorem on equation (6') we get that $E_{-j}*(a_j)$ increases with a_j. Finally, Apply Lemma 2 on equation (6') complete the prove as required □

4.22 Choosing the Leader (*Voting*)

Suppose that the set of candidate leaders is $\Im \subset N$. Then each individual may cast his support to any candidate in $\Im$. As now all the individuals have the same preferences over ideas, the rejection alternative of the game with perfect information is no longer valid.

Again, we denote the supporting (voting) decision of individual i by α_i. and the supporting vector by $\alpha = (\alpha_1, ..., \alpha_n)$. The set of winning candidates (i.e., those who receive the majority of supporters weighted by their index power) where voting decisions are α, is $W(\Im, \alpha)$ where:

$$W(\Im,\alpha) = \left\{ \forall\, l \in \Im : \sum_{\{i \in N:\alpha_i = l\}} \theta_i \geq \sum_{\{j \in N:\alpha_i = k\}} \theta_j \qquad \forall k \neq j \right\} \tag{10'}$$

If $W(\Im,\alpha) = \{j\}$ for some $j \in \Im$, then j is automatically chosen to be the leader. If $\#W(\Im,\alpha) > 1$, then a leader is chosen by a uniformly distributed lottery that assigns probability $P^l(\Im,\alpha) = \dfrac{1}{\#W(\Im,\alpha)}$ to each candidate $l \in W(\Im,\alpha)$.

As the result of individuals' actions depend on the actions of the rest of the society members the decision whether to support a candidate or not is strategic. A supporting equilibrium is thus a vector $(\alpha_1 *, ..., \alpha_n *)$ such that for each individual i, $\alpha_i *$ is the optimal reaction to $\alpha_{-i} *$, namely:

$$\alpha_i * \in \arg\max \left\{ \sum_{l \in \Im} P^l\left(\Im, (\alpha_i *, \alpha_{-i} *)\right) E^i\left(u_i^l \mid a_l\right) : \alpha_i \in \Im \right\}\ , \tag{11'}$$

and α_i * is not a weakly dominated supporting strategy.

Proposition 6: The supporting strategy profile $\alpha = (\alpha_1,...,\alpha_n)$ where $\alpha_i = \arg\max\{E^i(u_i^j \mid a_j)\}_{j\in\Im}$ for each individual $i \in N$, is a strategic Nash equilibrium profile.

Proof: Note that no society member can benefit ex-ante by deviating from the equilibrium strategy alone.□

4.23 Declaring Candidacy (*Entry*)

Each individual in the community should decide whether or not to declare candidacy. Again the result of the entry stage (and the individuals' payoff) depends on the set of candidates, and therefore, the decision whether to declare candidacy is strategic. We use the same notations as in the game of leadership with perfect information where $s = (s^1,..., s^n)$ is the pure strategic entry profile ($s^i \in \{0,1\}$) and $\Im(s) = \{\forall i \in N : s^i = 1\}$. Given a function α(•) that assigns a supporting vector (voting vector) to each candidate configuration, the expected payoff of individual *i* from the pure strategic profile *s* is given by:

$$U^i(s,\alpha(\cdot)) = \sum_{l\in\Im(s)} P^l\left(\Im(s),\alpha(s)\right)\cdot E^i\left(u_i^l \mid a_l\right) \qquad (12')$$

Note that in the leadership game with perfect information, a default case where none of the community members offer candidacy may occur due to substantial differences in their idiosyncratic valuations of policies. Hence, we took the assumption that in the default case the society dismantled and all community members are left with zero utility. In this model of imperfect information however, individuals have identical valuations of policies but have different information about leadership abilities of community members, hence, the default case where none of the community members offer candidacy may possibly occur only due to the imperfect information structure of the game. We therefore make the assumption that if none of the society members declare candidacy, Nature chooses the one with the highest a_i, if there is more than one society member with the highest a_i then one of them is chosen by a uniform lottery[37]. Given a function $\alpha(\bullet)$ that assigns a supporting vector (voting vector) to each candidate configuration, *an equilibrium of pure strategies of the entry game* is a profile $s = (s^1,...,s^n)$ such that s^i is the best response to s^{-i} for each society member *i*.

Proposition 7: A pure profile of entry strategies in which each individual $j \in N$ enters if and only if the condition that $u_j^j \geq \mathrm{E}\left(u_j^l \mid a_l\right)$ holds for every $l \in N \setminus \{j\}$ is a Nash equilibrium profile in the entry stage.[38]

Proof: Note that no society member can benefit ex-ante by deviating alone.□

4.3 The Tradeoff between Charisma and Leadership under Incomplete Information An Example

Again for the sake of simplicity we will assume that for each individual $i \in N$ the non-monetary cost function from exerting effort is given by $c(e) = c \cdot e^2$ where c>0 is constant, and suppose that the production function of the public good net of leadership input is linear in total society effort and is given by $\varphi(E) = a \cdot E$ where a>0 is constant. From now on until the end of this example we will assume that for each society member $l \in N$ the parameters are such that $1 < d < a_l$.

4.31 Solving the Model Backwards:

[37] The pure Nash equilibrium strategic profile where no community member offers candidacy may occur due to the incomplete information structure of the game. If, for example, n-2 community members have the same a_i and two other community members l and j have relatively very high a_l and a_j respectively, but, individual l with his own information calculates $u_l^l < E\left(u_l^j \mid a_j\right)$ and in the same manner, individual j with his own information calculates $u_j^j < E\left(u_j^l \mid a_l\right)$, then, intuitively, all society members may decide not to offer candidacy.

[38] Of course there may be more pure Nash equilibrium strategic profiles, however we use the profile above as an analysis reference point.

Suppose that some individual $j \in N$ with a $K(j)$ leadership technology and a T_j charisma parameter was already chosen to lead.

The first order condition for non-leader individuals in the effort decision stage leads to $e_i^* = \frac{a}{2c}(a_j - \Theta d)$.[39]

The first order condition for the leader in the effort decision stage leads to $e_j^* = \frac{a}{2c}K(j)$.[40]

The total society effort in Nash equilibrium is therefore given by $E^* = \frac{a}{2c}\left[(n-1)(a_j - \Theta d) + K(j)\right]$. Thus, the ex-ante indirect utility of each non-leader individual $i \in N \setminus \{j\}$ in Nash equilibrium is given by:

$\mathrm{E}\left(u_i^j \mid a_j\right) = \frac{a^2}{2c}\left\{(n-\tfrac{1}{2})(a_j - \Theta d)^2 + \Theta(1-\Theta)d^2\right\}$ [41].

The indirect utility function of the leader j which depends on his known charisma parameter T_j is given by:

$$u_j^j = \begin{cases} \frac{a^2}{2c}\left[(n-\tfrac{1}{2})K(j)^2 + (1-\Theta)(n-1)d \cdot K(j)\right] & T_j = d \\ \frac{a^2}{2c}\left[(n-\tfrac{1}{2})K(j)^2 - (n-1)\Theta d \cdot K(j)\right] & T_j = 0 \end{cases}$$

From this setting it follows that there are two possibilities for individual j to declare candidacy for leadership. Either $T_j=d$ and then individual j will declare candidacy if and only if the condition (*) below holds for all $l \in N \setminus \{j\}$ where:

$$(*) \qquad (n-\tfrac{1}{2})\left[K(j)^2 - (a_l - \Theta d)^2\right] \geq d(1-\Theta)\left(\Theta d - (n-1)K(j)\right).$$

[39] Stem from the non leaders first order condition:
$(1-\Theta)a_j \cdot \varphi'\left(E_{-\{i,j\}} + e_j(K(j) = a_j - d) + e_i\right) + \Theta(a_j - d)\varphi'\left(E_{-\{i,j\}} + e_j(K(j) = a_j) + e_i\right) = c'(e_i)$.

[40] This stems from the leader's first order condition $c'(e_j) = K(j)\varphi'\left((n-1)e_i^* + e_j\right)$.

[41] Note that given the parameters Θ, n and d the ex-ante indirect utility of individual i where j is the leader depends only on a_j.

Or else, Tj=0 and then individual j will declare candidacy if and only if the condition (**) below holds for all $l \in N \setminus \{j\}$ where:

(**) $$\left[K(j)^2 - (a_l - \Theta d)^2\right] \geq \frac{\Theta(1-\Theta)d^2 + (n-1)\Theta d \cdot K(j)}{n - \frac{1}{2}}$$

Note that the expression $\dfrac{\Theta(1-\Theta)d^2 - d(n-1)(1-\Theta) \cdot K(j)}{n - \frac{1}{2}}$ which is the right hand side of condition (*) decreases with n, and converges to $\left[-d(1-\Theta)K(j)\right]$ when n converges to infinity. Hence, if condition (*) holds for n=2 then individual j will have an incentive to offer his candidacy for leadership, whereas a sufficient condition for individual j not to declare candidacy, is that the opposite inequality holds where n converges to ∞.

For a specific individual j we will denote $a_u = \max\{a_l\}_{l \in N \setminus \{j\}}$.

Proposition 8: For any individual j, with charisma parameter T_j=d,

I) If $K(j) = a_u$ then individual j will declare candidacy,

II) If $K(j) = a_u - d$ then individual j will not declare candidacy

III) If $K(j) = a_u - \Theta d$ then individual j will declare candidacy only if the number of the community members n is sufficiently high (such that $\frac{(n-1)}{n} > \frac{d\Theta}{a_u}$).

Proof: See the Appendix

The two extreme cases where $K(j) = a_u$ and $K(j) = a_u - d$ are straightforward and very intuitive. If $K(j) = a_u$ then individual j has the highest leadership technology and he displays the highest outer-signal. Hence, individual j can ensure himself a higher utility as a leader than as a follower. This is due to the fact that as a leader, individual j can supply the highest leadership technology and assemble higher community effort than any other potential candidate.

If $K(j) = a_u - d$ then individuals *j* and *u* have the same outer signals implying that the rest of society members do not know which one of them exhibits excess leadership abilities. Thus the rest of the society members will provide each one of them with the same level of effort have they separately been chosen to lead. Individual *j* does not know whether individual *u* has greater leadership ability (i.e. $K(u) = a_u > K(j)$) or whether he has the same leadership ability with a positive charisma parameter (i.e. $K(u) = a_u - d = K(j)$). However in the viewpoint of individual *j*, his own ex-ante expected utility when individual *u* is the leader is higher than his own utility as a leader, and therefore individual *j* will not declare candidacy.[42]

The case where $K(j) = a_u - \Theta d$ is the most interesting situation to analyze as it demonstrates that charisma may sometime be preferable over leadership abilities. This example essentially emphasizes the tradeoff between leadership ability and charisma. If $K(j) = a_u - \Theta d$ then individual *j* does not know whether individual *u* has a greater ability to lead ($K(u)=a_u$) but less charisma or less ability to lead but a positive charisma parameter that exhibit charisma ($K(u)=a_u-d$), in either cases, community members who are not informed about leadership abilities will provide individual *j* as a leader more effort than individual *u*, that may possibly compensate individual *j*'s potential lack of leadership ability compared to individual *u*. The larger the community is (or rather the larger is the number of society members who do not know the leadership technology of others), the higher is the gap between the effort that individual *j* can attract as a leader and between the effort individual *u* can attract as a leader. Hence, if the community is sufficiently large, the advantage of individual *j* in attracting communal effort overtakes the advantage of individual *u* in potential leadership ability.[43]

[42] Note that in this case where only individual *u* and *j* have the maximum outer signals a_u and a_j respectively, and where individual *u* also has a positive charisma parameter, then in the pure Nash equilibrium we demonstrated none of the society members will declare candidacy and hence, Nature will choose either *j* or *u* by a coin flipping.
[43] Later on we will demonstrate a case where even when individual *u* knows that he has better leadership ability than that of individual *j* he may still support individual *j* as a leader, since he knows that the charismatic advantage of *j* exceeds his leadership technology advantage.

4.32 Information Structure and Charisma's Superiority

We now present an example in which even when a potential leader *has full information* about the rest of the society members and he knows that *he has the highest leadership technology*, he may still prefer to support another individual with inferior leadership technology (and renounce his own candidacy option) due to the fact that the other individual advantage in deceivingly heaving social effort dominates his leadership technology advantage.

Suppose that in a community N which consists of many members, there are two individuals $l, j \in N$ such that:

1) The leadership technology of *l,j* is superior to that of all other community members, and all community members know that individuals *l* and *j* have the highest leadership technology among the rest of society members, specifically $a_l, a_j > a_i + d$ for each $i \in N \setminus \{j, l\}$

2) Individual *l* has the highest leadership technology however individual *j* displays a higher outer signal, specifically $a_j > a_l$ and $K(j) < K(l)$

   Thus

   (i) $a_j = K(j) + d$

   (ii) $a_l = K(l)$ and

   (iii) $K(j) + d > K(l)$.

3) Contrary to our previous assumptions, we now assume that both individuals *l,j* *have mutual knowledge about their leadership technologies* (that is, *j* knows *K*(*l*) and *l* knows *K*(*j*)). Furthermore, *the rest of the society members do not know l and j's leadership technologies while individuals l and j know that the rest of the society members do not know their leadership technology.*

Proposition 9: Under assumptions (1)-(3), if $\frac{K(j)+d}{K(l)} > \frac{K(l)}{K(j)}$ then for a sufficiently large community (i.e. a sufficiently large n) and for a sufficiently small probability Θ, individual l will renounce his candidacy option, and individual j will be elected to lead the society.

Proof: See the Appendix.

Proposition 9 demonstrates the fact that the effectiveness of j's charisma depends on the number of individuals who do not know his true leadership technology. The higher is the number of individuals who do not know the true j's leadership technology, the higher is the total effort they exert, and the greater is the charismatic advantage of individual j over the leadership advantage of individual l. Hence the decision of individual l whether to declare candidacy or not, significantly depends on the number of the community members who do not know individual j's leadership technology.

5. Concluding Remarks

This paper analyses the process of leadership creation together with policy decision when the power of leaders to coerce is limited. The paper can be therefore viewed as a study of social choice when formal institutional authorities are missing. Under such conditions, social choice might become multidimensional as community members take into account not only policy but also leadership abilities as well as the allocation of resources. The paper shows that leaders might credibly compromise on favorable policies in order to attain more social efforts, and at the same time society members might be willing to compromise on favorable policies in order to gain better leaders.

The study demonstrates that the ability of leaders to transform resources into social goals in itself creates incentives among society members to offer these resources. Hence, an important inference of the paper is that the talents of leaders to transmit true or false signals about their leadership abilities as well as the way these signals are perceived by society members are important factors in the allocation of social resources and can be associated to charismatic leadership. Hence, the paper can be viewed as social theory of leadership that makes a practical distinction between charisma and leadership.

The paper can be extended to several directions. One would be to address the issue of long run reputation within a dynamical framework by exploring whether charismatic leaders can reinforce their charisma by establishing reputations as successful leaders. Another extension would to explore leadership formation where societies face a variety of future public goods which only one is eventually realized as a social requirement. Under such conditions, society members might consider candidates' comparative advantages in manufacturing different public goods in different probabilities.

APPENDIX

Proof of Proposition 1:

Applying the first order condition and the envelop condition on the leader indirect utility function in Nash equilibrium (equation (9)) provides:

$$K(j)\left[v_j'(q)\varphi(E^j*(q))+v_j(q)\varphi'(E^j*(q))\frac{dE^j*}{dq}\right]-c'(e_j^j*(q))\frac{de_j^j*}{dq}=0$$

Therefore,

$$K(j)\left[v_j'(q)\varphi(E^j*(q))+v_j(q)\varphi'(E^j*(q))\sum_{\substack{l=1\\l\neq j}}^{n}\frac{de_l^j*}{dq}\right]+\left[K(j)v_j(q)\varphi'(E)-c'(e_j^j*(q))\right]\frac{de_j^j*}{dq}=0$$

But $\left[K(j)v_j(q)\varphi'(E)-c'(e_j^j*(q))\right]\frac{de_j^j*}{dq}=0$, because whenever $v_j(q)\geq 0$, the leader's optimal effort condition given in (4) implies that:

$\left[K(j)v_j(q)\varphi'(E)-c'(e_j^j*(q))\right]=0$,

Otherwise (if $v_j(q)<0$) the leader's effort is nil and $\frac{de_j^j*}{dq}=0$

Hence, $K(j)\left[v_j'(q)\varphi(E^j*(q))+v_j(q)\varphi'(E^j*(q))\sum_{\substack{l=1\\l\neq j}}^{n}\frac{de_l^j*}{dq}\right]=0$

But since $K(j)>0$ we find that, $\left[v_j'(q)\varphi(E^j*(q))+v_j(q)\varphi'(E^j*(q))\sum_{\substack{l=1\\l\neq j}}^{n}\frac{de_l^j*}{dq}\right]=0$.

Thus $-\frac{v_j'(q)}{v_j(q)}=\frac{\varphi'(E^j*(q))}{\varphi(E^j*(q))}\cdot\sum_{\substack{l=1\\l\neq j}}^{n}\frac{de_l^j*}{dq}$

Therefore $-\mathbf{e}_{v_j,q}=\mathbf{e}_{z_g,E}\cdot\mathbf{e}_{E_{-j},q}$. Of course, if $v_j(q)<0$, the leader utility equals -∞ and therefore the condition above holds whenever $v_j(q)\geq 0$. Otherwise, $q*$ is such that $v_j(q*)=0$

□.

Proof of Proposition 2: For each individual $i \in N$ we denote the set of the most agreeable candidates in the viewpoint of individual *i* by:

(I) $$A_i = \arg\max\{u_i^l(q_l^*)\}_{l \in \Im \cup \{r\}},$$

(Where *u*=0 in the case of *r*). For each candidate $l \in \Im \cup \{r\}$ we label the set of possible supporters by E_l where:

(II) $$E_l = \{\forall i \in N : l \in A_i\}.$$

By construction the set of potential leaders is:

(III) $$W(\Im \cup \{r\}, \alpha) = \left\{ \forall l \in \Im \cup \{r\} : \sum_{\{i \in N : \alpha_i = l\}} \theta_i \geq \sum_{\{j \in N : \alpha_i = k\}} \theta_j \quad \forall k \neq j \right\}$$

If $\#W(\Im \cup \{r\}, \alpha) = 1$ then a supporting vector $(\alpha_1^*, ..., \alpha_n^*)$ such that $\alpha_i^* \in A_i$ is a supporting Nash equilibrium profile. If $\#W(\Im \cup \{r\}, \alpha) > 1$ then, a possible supporting equilibrium profile is a supporting vector $(\alpha_1^*, ..., \alpha_n^*)$ such that $\alpha_i^* \in A_i$ for all $i \in N$ and where $\alpha_i^* = u$ for all $i \in E_u$ for some $u \neq r$ in the potential leaders set *PL*. Obviously, *u* is the equilibrium winning candidate, and none of the society members can profitably deviate from $(\alpha_1^*, ..., \alpha_n^*)$. Note that α_i^*, are not weakly-dominated strategies.□

Proof of Lemma 2:

Define a function $G(E, K(j)) = E - \sum_{i \in M} c'^{-1}[v_i(q)K(j)\varphi'(E)]$. From equation (6) it follows that in the fourth stage Nash equilibrium, $G(E^*(q), K(j)) = 0$. Appling the implicit function theorem on G yields:

$$\frac{dE}{d(K(j))} = -\frac{\dfrac{\partial G(E, K(j))}{\partial K(j)}}{\dfrac{\partial G(E, K(j))}{\partial E}} = -\left[\frac{-\sum_{i \in M} c'^{-1'}[v_i(q)K(j)\varphi'(E)] \cdot v_i(q)\varphi'(E)}{1 - \sum_{i \in M} c'^{-1'}[v_i(q)K(j)\varphi'(E)] \cdot v_i(q)K(j)\varphi''(E)} \right] \qquad (*)$$

Our analytical assumptions on $\varphi(e)$ and $c(e)$ ensure that the last term of (*) is non-negative, hence, $dE^*(q)/dK(j) \geq 0$. □.

Proof of Lemma 3:

Denote $\alpha_1 = n_1/n$ and $\alpha_2 = n_2/n$, and define two functions $G_1(q,\alpha_1), G_2(q,\alpha_2)$ such that: $G_1(q,\alpha_1) = n\{v_1'(q)[\alpha_1 v_1(q) + (1-\alpha_1)v_2(q)] + v_1(q)[(\alpha_1 - 1/n)v_1'(q) + (1-\alpha_1)v_2'(q)]\}$ and $G_2(q,\alpha_2) = n\{v_2'(q)[(1-\alpha_2)v_1(q) + \alpha_2 v_2(q)] + v_2(q)[(\alpha_2 - 1/n)v_2'(q) + (1-\alpha_2)v_1'(q)]\}$.

First and second order conditions imply that any potential leader $j(1)$ from subset N_1 will choose a policy $q_{j(1)}{}^*$ such that $G_1(q_{j(1)}{}^*,\alpha_1) = 0$ and $\frac{\partial}{\partial q}G_1(q_{j(1)}{}^*,\alpha_1) < 0$ and any potential leader $j(2)$ from subset N_2 will choose a policy $q_{j(2)}{}^*$ such that $G_2(q_{j(2)}{}^*,\alpha_2) = 0$ and $\frac{\partial}{\partial q}G_2(q_{j(2)}{}^*,\alpha_2) < 0$.

Note that $G_1(\bar{q}_1,\alpha_1) = (1-\alpha_1)nv_1(\bar{q}_1)v_2'(\bar{q}_1) > 0$ and $G_1(\tilde{q},\alpha_1) = (2n_1 - 1)v_1(\tilde{q})v_1'(\tilde{q}) < 0$, hence, from the intermediate value theorem there exists at least one point q^* in the interval $(\bar{q}_1,\tilde{q})$ such that $G_1(q^*,\alpha_1) = 0$ and $\frac{\partial}{\partial q}G_1(q^*,\alpha_1) < 0$. From the concavity of $v(q)$ this point is unique. Hence, for any potential leader $j(1)$ in the subset N_1, the optimal political agenda $q_{j(i)}{}^*$ must lie in the interval $(\bar{q}_1,\tilde{q})$. The same arguments apply for $G_2(q,\alpha_2)$ which yields that for any potential leader $j(2)$ in the subset N_2, the optimal political agenda $q_{j(2)}{}^*$ must lie in the interval $(\tilde{q},\bar{q}_2)$.

From the implicit function theorem we get:

$$\frac{\partial q_1{}^*}{\partial \alpha_1} = -\frac{n\{v_1'(q_1^*)[v_1(q_1^*) - v_2(q_1^*)] + v_1(q_1^*)[v_1'(q_1^*) - v_2'(q_1^*)]\}}{\partial G_1(q_1^*)/\partial \alpha_1} < 0$$

and $\frac{\partial q_2{}^*}{\partial \alpha_2} = -\frac{n\{v_2'(q)[v_2(q) - v_1(q)] + v_2(q)[v_2'(q) - v_1'(q)]\}}{\partial G_2(q_2^*,\alpha_2)/\alpha_2} > 0$.

Define two functions:

$$\delta_1(q) = \lim_{\alpha_1 \to 1} G_1(q,\alpha_1) = n(2-1/n)v_1(q)v_1'(q)$$

$$\delta_2(q) = \lim_{\alpha_2 \to 1} G_2(q,\alpha_2) = n(2-1/n)v_2(q)v_2'(q) .$$

As $\delta_1(\overline{q}_1) = 0$ and $\delta_2(\overline{q}_2) = 0$ it follows that

$q_{j(1)}* \underset{n_1/n \to 1}{\rightarrow} \overline{q}_1$ and $q_{j(2)}* \underset{n_2/n \to 1}{\rightarrow} \overline{q}_2$.□

Proof of Proposition 5: Note that the functions $B(1,\alpha_1)$ and $B(2,\alpha_1)$ are continuous in $\alpha_1 \in (0,1)$. Also note that $\lim_{\alpha_1 \to 1} B(1,\alpha_1) = \left(\xi/\eta\right)^2$ and $\lim_{\alpha_1 \to 1} B(2,\alpha_1) = \left(\xi^2/\sigma\eta\right)$

Hence, there exist a sufficiently small $\varepsilon > 0$ and a sufficiently large $0 < \hat{\alpha} < 1$ such that $B(1,\alpha) < \left(\xi/\eta\right)^2 + \varepsilon$ for all $\hat{\alpha} < \alpha < 1$. Choosing $\varepsilon(1) > 0$ and $0 < \hat{\alpha}(1) < 1$ such that $B(1,\alpha) < \left(\frac{K(j(1))}{K(j(2))}\right)^2$ for all $\hat{\alpha}(1) < \alpha < 1$ implies that all members of N_1 will choose $j(1)$ as their leader. Furthermore, $\lim_{\alpha_1 \to 1} B(2,\alpha_1) = \left(\xi^2/\sigma\eta\right)$ implies that for a sufficiently small $\varepsilon > 0$ there exists a sufficiently large $0 < \hat{\alpha}(2) < 1$ such that for any $\hat{\alpha}(2) < \alpha < 1$ $B(2,\alpha_1) < \left(\xi^2/\sigma\eta\right) + \varepsilon$.

But as the ideas functions are such that $\sigma < \xi^4/\eta^3$, if follows that $\left(\eta/\xi\right)^2 > \left(\xi^2/\sigma\eta\right)$ and thus choosing $\varepsilon(2) > 0$ and $0 < \hat{\alpha}(2) < 1$ such that $B(2,\alpha) < \left(\frac{K(j(1))}{K(j(2))}\right)^2$ for all $\hat{\alpha}(2) < \alpha < 1$ implies that all members of N_1 will choose $j(1)$. Hence, for all societies with $\hat{\alpha} < n_1/n < 1$ where $\hat{\alpha} = \max\{\alpha(1),\alpha(2)\}$ all society members will choose $j(1)$ to lead.□

Proof of Proposition 8:

I) if $K(j)=a_u$ then from our assumption that $1<d<a_l$ we get that $2\Theta da_u-\Theta^2d^2>0>\frac{d^2(1-\Theta)\Theta-d(1-\Theta)(n-1)a_u}{(n-\frac{1}{2})}$ this means that the inequality (*) holds for *n*=2.

II) If $K(j)=a_u-d$ then $K(j)^2-(a_u-\Theta d)^2<\frac{d^2(1-\Theta)\Theta-d(1-\Theta)(n-1)K(j)}{(n-\frac{1}{2})}$ for all n>2. Thus in this pure Nash equilibrium profile individual *j* will not declare his candidacy.

III) If $K(j)=a_u-\Theta d$ then it is easy to see that individual *j* will declare his candidacy if $2d\Theta<a_u$. Suppose, however, that $2d\Theta>a_u$; then individual *j* will not declare his candidacy if and only if $0\le\frac{d^2(1-\Theta)\Theta-d(1-\Theta)(n-1)(a_u-\Theta d)}{(n-\frac{1}{2})}$ if and only if $(n-1)a_u\le nd\Theta$ □.

Proof of Proposition 9: Due to imperfect information of all the other society members, each community member $i\in N\setminus\{j,l\}$ will allocate $e_i=\frac{a}{2c}(a_j-\Theta d)$ effort to the production of the public good under the leadership of *j*, and $e_i=\frac{a}{2c}(a_l-\Theta d)$ under the leadership of *l*.[44] However, *j*'s optimal effort under *l*'s leadership is $e_j=\frac{a}{2c}K(l)$ and $e_j=\frac{a}{2c}K(j)$ under his own leadership. Similarly, *l*'s optimal effort under *j*'s leadership is $e_l=\frac{a}{2c}K(j)$ and under his own leadership is $e_l=\frac{a}{2c}K(l)$.

[44] Note that as each individual $i\in N\setminus\{j,l\}$ does not know the information structure of individuals *l* and *j*, their calculation of their ex-ante indirect utility function under the leadership of *l* or *j* is similar to the calculation in the previous example of the imperfect information leadership game, namely for each $i\in N\setminus\{j,l\}$, the indirect utility function under the leadership of *j* and *l* is given by: $E^i\left(u_i^j\mid a_j\right)=\frac{a^2}{2c}\left\{(n-\frac{1}{2})(a_j-\Theta d)^2+\Theta(1-\Theta)d^2\right\}$ and $E^i\left(u_i^l\mid a_j\right)=\frac{a^2}{2c}\left\{(n-\frac{1}{2})(a_l-\Theta d)^2+\Theta(1-\Theta)d^2\right\}$.

We will now show that individual l (who has the highest leadership ability) may renounce his candidacy option.

The indirect utility function of individual l under the leadership of individual j and himself are given by:

$$u_l^j = K(j)\frac{a^2}{2c}\left[(n-2)(a_j - \Theta d) + K(j) + K(l)\right] - c\left(\frac{a}{2c}K(l)\right)^2$$
$$= K(j)\frac{a^2}{2c}\left[(n-2)(K(j) + (1-\Theta)d) + K(j) + K(l)\right] - c\left(\frac{a}{2c}K(l)\right)^2$$

and

$$u_l^l = K(l)\frac{a^2}{2c}\left[(n-2)(a_l - \Theta d) + K(j) + K(l)\right] - c\left(\frac{a}{2c}K(l)\right)^2$$
$$= K(l)\frac{a^2}{2c}\left[(n-2)(K(l) - \Theta d) + K(j) + K(l)\right] - c\left(\frac{a}{2c}K(l)\right)^2$$

respectively.

Hence, $u_l^j > u_l^l$ if and only if $\frac{K(l)}{K(j)} < \frac{\left[(n-2)(K(j) + (1-\Theta)d) + K(j) + K(l)\right]}{\left[(n-2)(K(l) - \Theta d) + K(j) + K(l)\right]}$.

Define a function $m(\Theta)$such that $m(\Theta) = \frac{\left[(n-2)(K(j) + (1-\Theta)d) + K(j) + K(l)\right]}{\left[(n-2)(K(l) - \Theta d) + K(j) + K(l)\right]}$. This function is the right hand side of the last inequality as a function of the probability parameter Θ.

Note that $m(\Theta) > 1$ for any Θ such that $0 < \Theta < 1$, and $m(0) = \frac{(K(j) + d) + \frac{K(l)-d}{(n-1)}}{(K(l)) + \frac{K(j)}{(n-1)}}$.

Hence, for a sufficiently large n, $m(0) > \frac{K(l)}{K(j)}$. Furthermore, since $m(\Theta)$ is a continuous function (for any $0 \le \Theta \le 1$) there exists a sufficiently small positive Θ such that $m(\Theta) > \frac{K(l)}{K(j)}$.

It is easy to verify that $u_j^j > u_j^l$ □

REFERENCES

Alesina Alberto "Credibility and Policy in a Two-Party System with Rational Voters" *The American Economic Review* 1988 Vol. 78 No.4. (Sep) 796-805

Alesina Alberto and Stephen E. Spear, "An Overlapping Generations Model Of Electoral Competition" *Journal of Public Economics*, December 1988, 37(3), 359-379

Bass, B.M (1990) **Bass and Stogdill's** Handbook of leadership: theory research and managerial applications (3rd ed). New-York Free press

Besley Timothy and Coate Stephen "An Economic Model of Representative Democracy" *The Quarterly Journal of Economics,* February 1997 Vol. 112, No 1 85-114

Besley Timothy and Coate Stephen "Lobbying and Welfare in a representative democracy" *Review of Economic Studies*, 68 (1), January 2001, 67-82

Bray, D.W., Campbell, R.J., and Grant, D.L. (1974) *Formative Years in Business: A Long Term AT&T Study of Managerial Lives.* New York: Wiley.

Calvert L. Randall "Robustness of the Multidimensional Voting: Candidate Motivation, Uncertainty, and Convergence" *American Journal of Political Science*, 1985 Vol. 29, No1. (Feb.,), 69-95

Caselli Francesco and Morelli Massimo, 2004. "Bad Politician" *Journal of Public Economics* Vol. 88(3-4), pages 759-782, March.

Downs, Anthony, *An Economic Theory of Democracy*, New York**:** Harper and Brothers, 1957.

Gibb, C.A (1954) Leadership In G. Lindzey (ed.), *Handbook of Social Psychology.* Cambridge, M.A: Addison-Wesley.

Hermalin, Benjamin E. "Toward an Economic Theory of leadership: Leading by Example" *The American Economic Review,* 1998 Vol.88, No 5 (Dec.,),1188-1206

House, Robert J. and Baets, Mary L. "Leadership: Some Empirical Generalizations and New Research Directions" in Barry M Staw, ed., *research in organizational behavior* Vol.1 Greenwich, CT: JAI press 1979 pp. 341-423

Howard, A. , and Bray, D.W. (1988) *Managerial Lives in Transition: Advancing Age and Changing Times.* New-York: Guilford Press

Mann, R.D (1959). "A Review of the Relationship between Personality and Performance in Small Groups" *Psychological Bulletin*, 56 , 241-270

Northouse Peter. G *Leadership: Theory and Practice* 3rd Ed 1997 Sage Publications.

Osborne, Martin J. and Al Slivinski "A Model of Political Competition with Citizen-Candidates" *The Quarterly Journal of Economics* February 1996 Vol.111, No.1, 65-96

Rotemberg, Julio J. and Saloner, Garth "Leadership Style and Incentives" *Management Science,* November 1993, 39 (11) 1299-1318

Stogdill R.M.(1948) "Personal Factors Associated with Leadership:A Survey of The Literature" *Journal of Psychology*,25, 35-71

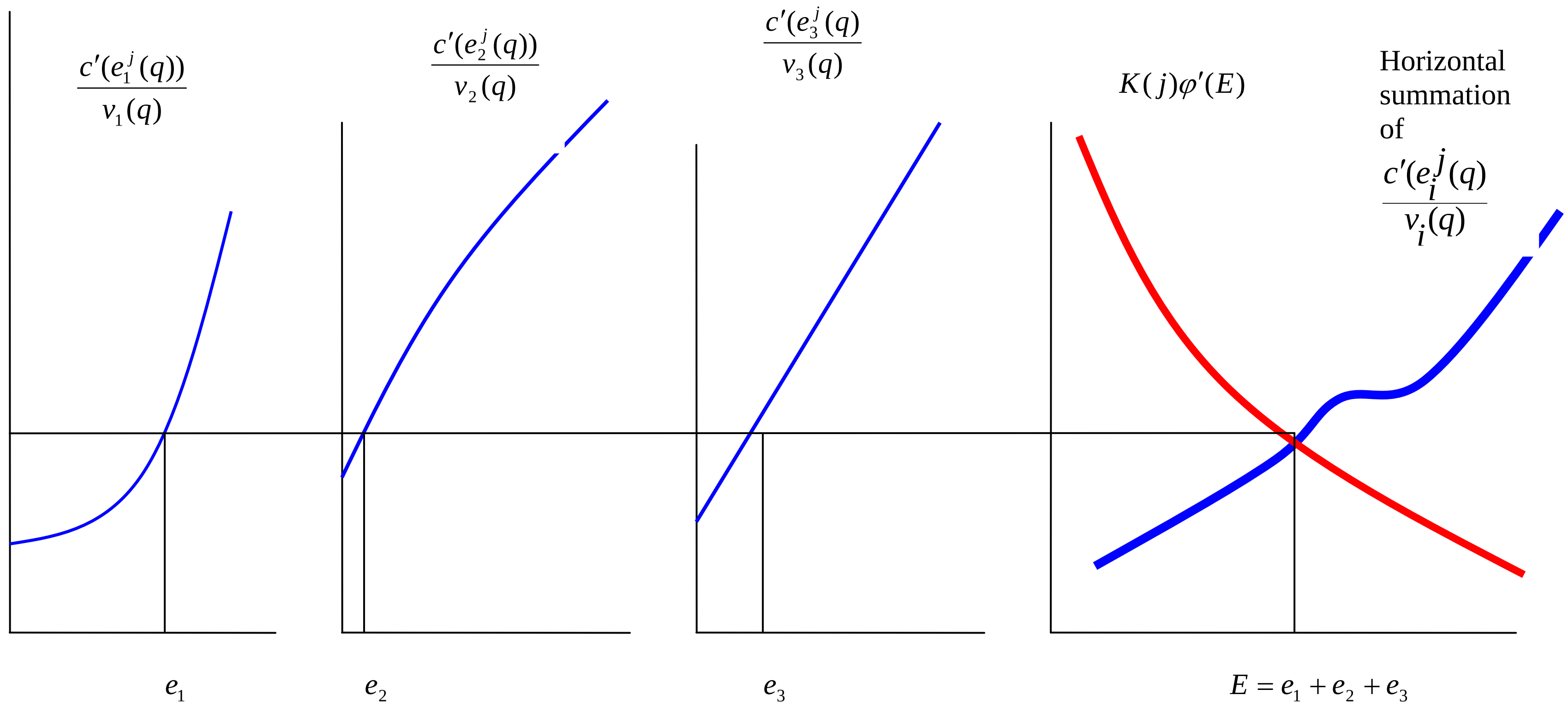
Wittman, Donald (1977) "Candidates with Policy Preference: A Dynamic Model", *Journal of Economic Theory*, Vol. 14, 180-189, February

Wittman, Donald (1983) "Candidate Motivation: A Synthesis of Alternatives", *American Political Science Review, Vol.* LXXVI, 142-157

Yukl Gary A. Leadership in Organizations, 4th Ed Prentice Hall International Editions 1998

Yukl Gary A. and Van Fleet David D. (1990) "Theory and Research on Leadership in Organizations" In M.D Dunnette and L.M Hough (eds.), *Handbook of Industrial and Organizational Psychology, Volume 3* Second edition; Palo Alto, CA: Consultion Psychologists Press, Inc,. 147-197.

Figure 1



$\frac{c'(e_1^j(q))}{v_1(q)}$
$\frac{c'(e_2^j(q))}{v_2(q)}$
$\frac{c'(e_3^j(q)}{v_3(q)}$
$K(j)\varphi'(E)$
Horizontal summation of $\frac{c'(e_i^j(q)}{v_i(q)}$
e_1
e_2
e_3
$E = e_1 + e_2 + e_3$

Figure 2

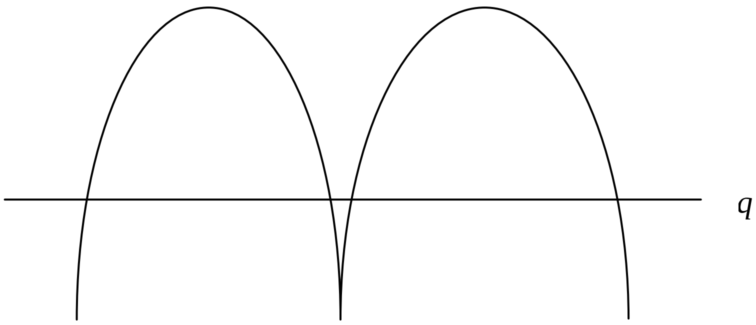


Figure 3

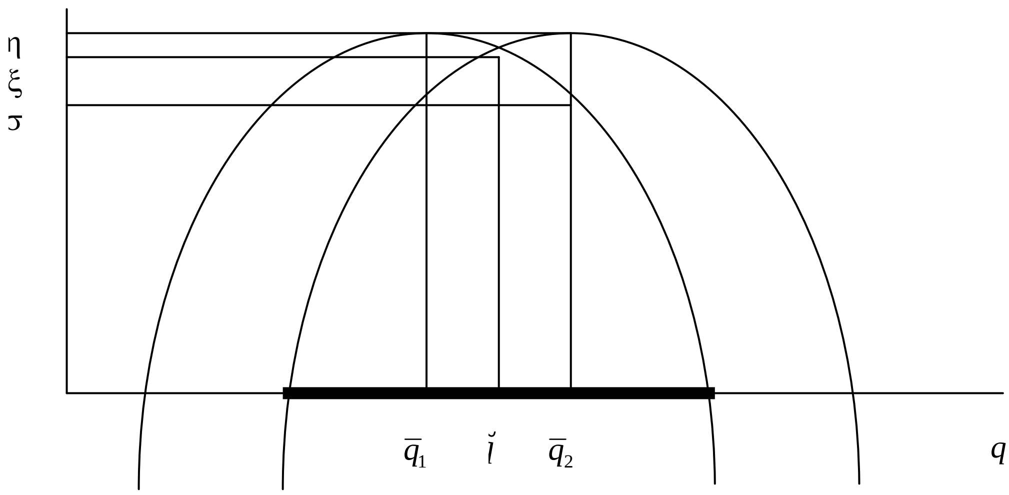


Figure 4

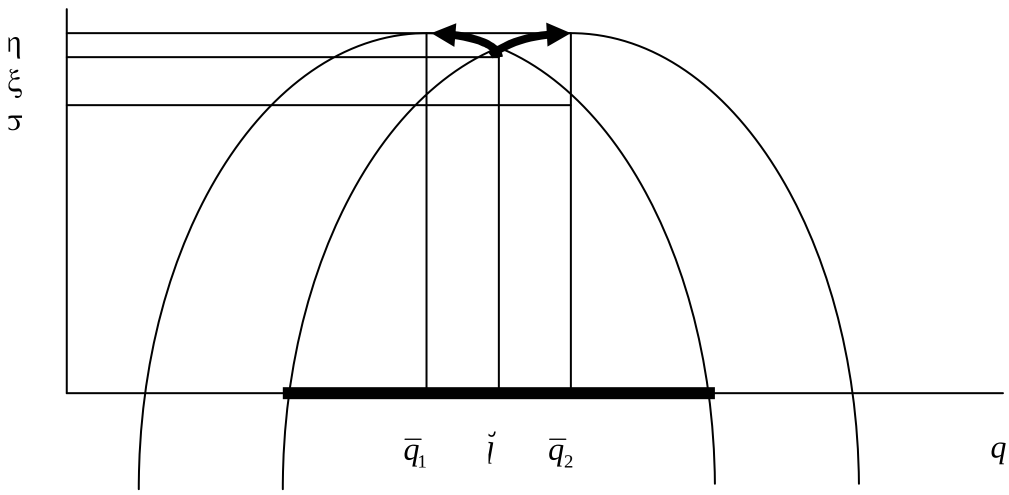


Figure 5