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by

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Abstract

The paper analyzes the options open to monopoly firms that sell software or internet service. We consider customers who have different reservation prices that are rectangularly distributed. The monopoly in general undertakes price discrimination between customers by producing two versions of the product, basic and advanced, where a zero price is charged for the lower quality product (i.e., the free version). The monopoly may also sell advertising space to increase revenues but may lose those customers that are annoyed by being exposed to compulsory advertising. We analyze the situation where the monopoly has an incentive to increase its output due to the network externality and allow free of charge basic service.

Keywords: Network Externality, Advertising, Basic and Advanced Service, Non-Homogeneous Customers

Introduction

The microcomputer software industry has been characterized in recent years as a market where two different products or services are sold at different price ranges. Some firms in this industry even advertise informational software programs and site services which are supplied to some customers for free. More advanced programs, requiring backup services, are sold to customers who are charged a monthly service fee. Some examples are given below:

(1) Search engine companies (e.g., Hotmail and Yahoo) offer either basic email services with limited storage ability (between 4 and 6 mega bites) or very basic anti-virus scanning with limited access. These services are offered for free. However, subscribing and paying a membership fee allows customers higher storage capability as well as advanced anti-virus scanning including spam protection, etc.

(2) Another example are the very popular "Dating" and "Blind Dates" sites that offer various services to customers. Usually, the site offers on-line registration for free, and in turn the agency obtains personal information such as the names of customers, their age, profession, hobbies, etc., and their various contact details. In addition, each member is usually required to fill in a questionnaire about the characteristic details of the person they would like to meet. A non-member cannot connect and communicate with anyone. He can leave a message to be called by a specific member, but he cannot initiate direct communication with the member. The member who pays a monthly fee has, in addition to the regular service, the ability to send a message to the site and can receive any information either from non-members or subscribed members. Non-members can receive messages from subscribed members only, but not from other non-members. This subject was recently researched by Bernard and Bruno (2003). They looked at internet sites offering membership with advanced services and discussed the issue of how to create and generate external effects more effectively. They bring "Dating Site" as an example, and discuss the possible policies the site's owner should consider: He can offer advanced services with more options and various types of membership in order to

attract customers to the site. On the other hand it is clear that the number of new customers willing to join is an increasing function of the number of customers that have already joined. Thus we have a difficult and complicated analytical question of determining the optimal procedure, in a way reminiscent of the ancient chicken and egg dilemma

Another source of revenue generation is from IT companies who add links to computer business home pages, where information is advertised and distributed. Adding standard information services via the internet, combined with commercials that feature information and expose willing (and often unwilling) customers to these commercials may also yield substantial revenues.

The information delivery monopoly may consider different kinds of policies to achieve its goal of profit maximization. We focus on these possible policies from the perspective of their social welfare implications and compare the social optimum solution to that of the profit maximizing monopoly.

We discuss five possible policies available to the monopoly: The first possible policy is to allow all customers to have access to and free use of all information service, both simple and advanced. This policy will of course lead to a large number of customers using the site's information. These customers are then exposed either willingly or unwillingly to advertisements sold by the site owner to other firms, who pay the site owner fees in proportion to the number of customers entering the site.

The second policy is to differentiate (or even discriminate) between customers. Some receive free basic information and service, while others pay a monthly fee for advanced high-quality information and service. All customers who use the site in either level of service are exposed to advertising.

The third policy is to differentiate between a free basic service and a membership fee for advanced service, while members can avoid receiving unwanted advertisements.

The fourth policy consists of charging for all services, whether basic or advanced high-quality, with full compulsory exposure to advertising.

The fifth possible policy of the monopoly is to charge for all services used by all customers and to add an additional periodic charge for the convenience of not being exposed to advertising.

This paper extends some earlier work of the nineties. Conner and Rumelt (1991) address the question as to whether a software publisher should pursue a strategy of software protection or allow some pirating by customers (i.e., free use of software products). The main benefit of allowing free use by customers is the advantage of increasing returns to size. By having more users (even free users) positive network externalities in the use of the software are generated.[1] The tradeoff between revenue losses by removing protection devices, thus allowing a free-for-all policy and the revenue and benefit gain as a result of network externalities is examined.

The concept of returns to scale in the network industry that supports free use of software is also discussed by Shy and Thisse (1999) in the duopoly case. This idea of increasing returns to scale that could indicate adopting a policy of free entry and use was recently discussed by Gayer and Shy (2003, 2004) who examined music piracy. They show that if network externalities exist it is worthwhile to allow free copying/recording. Compensation for revenue losses from free riders can be covered by taxing hardware and transferring the proceeds to the software industry, and to record companies and musicians. Their conclusion about optimal membership fee is in some respects similar to ours. We however have incorporated into the analysis the issue of revenues from advertising that might compensate for membership fee losses. We also address the social welfare implications for the society, i.e., the customers and the monopoly.

[1] The concept of network externalities was also discussed in the eighties in a paper by Katz and Shapiro (1985, 1986), and Farrel and Saloner (1985, 1986).

Our paper illustrates the use of various qualities of services in combination with different prices that are charged in two ways. The first is that of a direct membership fee imposed on the user. The second option is to pay indirectly by being forced to be exposed to advertising, with the option of paying a fee to reduce the degree of forced exposure. The optimal policy of the monopoly is affected by the particular circumstances of the following factors:

(a) The distribution of customers having different requirements for the site services.

(b) The conditions of increasing returns to scale that exist under the network industry including the positive externalities generated when customers benefit from other users.

In the next section we develop the model where we discuss the main five policies used by the profit-maximizing monopoly and their implications on the social welfare of the society, i.e., the customers and the monopoly.

The Model

We start with the main assumptions of the model that are related to all five cases:

1. Market A contains a given number of customers who are interested in receiving some degree of software services supplied by the monopoly. The monopoly supplies two different kinds of services: (i) a basic program with low quality levels of service and (ii) a more advanced program with high-quality service levels, including many features that are absent in the basic service.
2. The revenue of the monopoly is generated either from selling two different levels of service, or by selling commercials.
3. The monopoly should consider five different policies and opt for the one that maximizes profits.

a. To allow free and full access to the site and to all available services to the **"**A**"** customers, while selling advertising to outside firms and forcing "A" customers to view commercials.

b. To allow only basic free service to some customers and charge them a monthly fee if they desire to subscribe to the high quality services. Advertising is forced on all customers.

c. The same as (b) above, except that high quality customers have the right to make the advertising optional instead of mandatory, in exchange for an appropriate fee.

d. To supply only advanced and high-quality service in exchange for a periodic fee, with all users being subjected to advertising.

e. Same as (d) above, except that customers are permitted to avoid compulsory advertising in exchange for an additional periodic fee.

4. The heterogeneity of customers can result from differences of socio economic background such as:

   a. Different wage rates (with different time values) affect the demand for different qualities of services. For example, high wage earners desire high-quality service and may desire to avoid advertising.

b. Different talents of customer where the more qualified and talented customers can save money by buying low service levels from the company. As a result of (a) and (b) we assume that customers have heterogeneous demand functions. For simplicity we assume the monopoly faces a uniform rectangular distribution of demand functions, where the highest reservation price of the first customer who has the highest utility from the software is A. The reservation price of the second customer is A-1, etc. The last customer has reservation price of 0, thus the size of the customer population is given by A. For each customer *i* we find η_{0i}

who have a higher reservation price than customer i , and η_{Ii} customers who have a lower reservation price than customer *i*. Therefore $A = \eta_{0i} + \eta_{iI} + 1$.

The heterogeneity of the customers determines three categories of customers. The first group pays for advanced service, and of course also reaps external benefits from all other types of members. The second group consists of those who get low level service free of charge. The third and final group is composed of those who prefer not to use any kind of service, and their utility from the computer service is zero. In addition those who use the service are exposed to advertisements. For the free users the advertisements are mandatory, while for the paying customers the advertisements can be eliminated either for free or for an additional charge.

We can summarize the utility of different customers in different positions in equation (1) below where A represents as mentioned above the population size (number of customers).

$$(1) \quad U_i \overset{def}{=} \begin{cases} (A+\beta-i)q-p & \textit{Membership customers and possibility of avoiding advertisements} \\ (A+\beta-i)q-p-\psi & \textit{Membership customers and pay } \psi \textit{ for possibility of avoiding advertisements} \\ (A-i)q-p & \textit{Membership customers} \\ q & \textit{Free use customers} \\ 0 & \textit{No use customers} \end{cases}$$

q represents all the software customers (paying members as well as free users). β represents the additional benefit from not being exposed to advertisements. p is the membership fee and ψ is the fee that members are charged for the right to avoid compulsory exposure to advertisements.

The monopoly earns revenues from two sources:

(a) Fees from the sale of memberships (price p for each member).

(b) Fees from the sale of advertisements is a function of the number of customers who may be exposed to the ads. ρ_m is used to represent the price that advertisers are willing to pay for each paying member who is exposed to advertisements, and ρ_{nm} is used to represent the price that advertisers are willing to pay for each nonpaying (free user) customer. Since the paying member's exposure to advertising is voluntarily it is

safe to assume that he will be exposed to fewer ads than his nonpaying counterpart, whose exposure to ads is mandatory. Thus we can conclude that ρ_m and τ_{nm} are significantly different. In addition the fixed costs of the site are given by ϕ. The marginal cost of servicing the marginal customer is c, and u is defined as the cost of the software and manpower needed to prevent unauthorized use of the site. Based on the above we can formulate a general profit function

(2) $$\pi = (p - c - \mu)q_m - \phi + \rho_m \tau_m q_m + \rho_{nm} \tau_{nm} (q - q_m)$$

where the site is a monopoly who has to determine the optimal membership fee, P, and where ρ_m and ρ_{nm} depend on the number of users of the site.

The expected relationship between the two payments is $\rho_{nm} > \rho_m$. The reasons for the response of the site owner and paying members are as follows:

(1) The cost of exposure to commercials to the paying member is presumably higher than that of the nonpaying member, who we assume has a higher time value than that of the nonpaying member. He will therefore try to avoid unsolicited commercials. Thus advertisers will be willing to pay less per member in the case of paying members who are not compelled to view their advertising.

(2) The site owner can at his option choose not to expose paying members to commercial viewing, while simultaneously exposing non paying users to increased levels of advertising If the viewers of the commercials spend additional time looking at and researching the specific products displayed in the site's ads the advertisers would be willing to pay more for exposure to non-member viewers. Thus, we conclude that it is likely that $\rho_m < \rho_{nm}$

This general profit function can be applied to the various strategies the monopoly can choose from. Each strategy determines simultaneously the monopoly profit and the social welfare (which is assumed to be the sum of consumer surplus and monopoly profits). We

discuss below the results of five different strategies and compare their profit and social welfare.

In the model below we discuss the price strategy in the software industry when a monopoly can choose either to supply the software program (or information) for free or to prevent free use.

Assume that the monopoly supplies a program on its website Customers are heterogeneous, where type O customers are support-oriented consumers who prefer a more advanced program, and are willing to pay a membership fee for these services. Type I customers are support independent consumers who have no need for or benefit from advanced services, and thus prefer the standard free service.

Thus, each consumer in the society faces three alternatives, and must choose one out of the following three options:

1. To sign up and pay a membership fee for the advanced service
2. To receive a basic service for free
3. Not to use the service at all

q customers from the population use (either for free or by paying a fee) the services. The utility of each individual increases when more customers use the service.

This phenomenon of increasing returns to scale in the network industry is very common, e.g., exchanging information or files is more beneficial as more customers are involved in the market. This approach follows Cabral, Salant and Woroch (1999) who discuss the issue of monopoly that initiates new technology or promotes the entry of a durable good that may generate network externalities. The question is what should be the pricing policy of such a monopoly? He can adopt a strategy of either limiting sales and introducing a small original output at a very high price and over time reduce prices, or alternatively he can adopt a strategy of selling more units to more customers by initially charging a relatively low price

and then increase prices over time. According to their model they conclude that the optimal policy is to start initially with a low price free which the benefit of externalities is generated.

From the monopoly's general profit function, we can derive several pricing strategies. The optimal solution from the profit point of view is compared to the optimal solution from a social welfare perspective. The following three cases allow free entry for basic services, yet they differ with regard to receiving advanced services.

Case A - Free use of Basic Service Only without Protection Policy

In this case no protection policy exists and only basic service is supplied for free. The user is obligated be exposed to advertising, without the choice of paying a fee to avoid compulsory advertisements. In this case the utility of each customer *i*, is $Ui = q$.

Thus,

$$\sum_{i=1}^{A} U_i = q \cdot A$$

The monopoly profit is

(3) $\pi^a = \rho_{nm}\tau_{nm}A - \phi$

The consumers' surpluses are distributed uniformly as follows: (i) The surplus of the first customer with the highest reservation price is A; (ii) the second customer is A-1, and (iii) the last customer's consumer surplus is zero.

Therefore the summation of consumers' surpluses is:

(4) $CS^a = \sum_{i=1}^{A} CS_i = q \cdot q = A^2$

and the social welfare *W* is as follows:

(5) $W^a = A^2 + \rho_{nm}\tau_{nm}A - \phi$

Case B – Non-protection Policy

We discuss the case of non-protection policy with the possibility of buying membership for advanced service or receiving basic service for free but with compulsory exposure to advertising.

The basic service is distributed free, but a membership fee of *p* is imposed on those customers who prefer the advanced service. The key questions the monopoly has to consider based on the reservation price distribution are: what is the optimal fee for *p* and consequently, how many of the loyal *i* customers pay the fee of *p*, and how many of the customers only get the free basic service (i.e., (*A*-*i*) customers).

Customers *i* who are willing to pay for advanced service are those for whom the net utility (utility-membership fee) is larger than the utility *q* from the basic service, i.e., those for whom the condition below exists pay the fee $(A-i)q-p \geq q$.

Since all customers *A* get some kind of service *q*=*A*, equilibrium *p(i)* is as follows:

(6) $p(i) = (A-i-1)A$

From (6) and (2) we get the profit function

(7) $\pi(i) = \left[(A-i-1)A-c\right]i + \rho_{nm}\tau_{nm}A - \phi$

where the optimal number of customers is derived by taking the profit derivative with respect to i:

(8) $$\frac{\partial \pi(i)}{\partial i} = A^2 - 2Ai - A - c = 0$$

Thus we get the optimal i^* that maximizes profit as follows:

(9) $$i^* = \frac{A}{2} - \frac{1}{2} - \frac{c}{2A} = \frac{1}{2}\left(A - 1 - \frac{c}{A}\right)$$

and the value of i^* indeed maximizes profit as the second order condition holds: i.e., $\frac{\partial \pi^2}{\partial^2 i} = -2A < o$

Formally, we can distinguish between (i) the customers who benefit from the advanced service *for free*; and (ii) the customers who benefit from the basic service *for free*, as follows:

$$\begin{cases} \text{membership customers} & i \in (1, \frac{A}{2} - \frac{1}{2} - \frac{c}{2A}) \\ \text{free of charge cutomers} & i \in (\frac{A}{2} + \frac{1}{2} - \frac{c}{2A}, A) \end{cases} .$$

We can use the optimal paying customers i^* of (9) in order to determine in case B the optimal price, p^b, and the profit, π^b, as follows

(10) $p^b = \frac{1}{2}\left(A^2 - A + c\right)$

(10') $\pi^b = \frac{A}{4}\left(A - 1 - \frac{c}{A}\right)^2 + \rho_{nm}\tau_{nm}A - \phi$

The optimal policy is to star initially with a low price from which the benefit of externalities is generated.

Using equations (8) - (10') we can draw several conclusions that are also shown in the figures:

I. For i=0 where no fee is paid by any customer, profit is made only from advertisements.

II. If A-1 customers are members, i.e., the fee for each customer is zero, then the monopoly loses from the supply of advanced service. Because the loss value is $C(A\text{-}1)$, the monopoly will not supply advanced service at all.

III. The optimal number of paying customers is definitely less than one half of the potential customer of A.

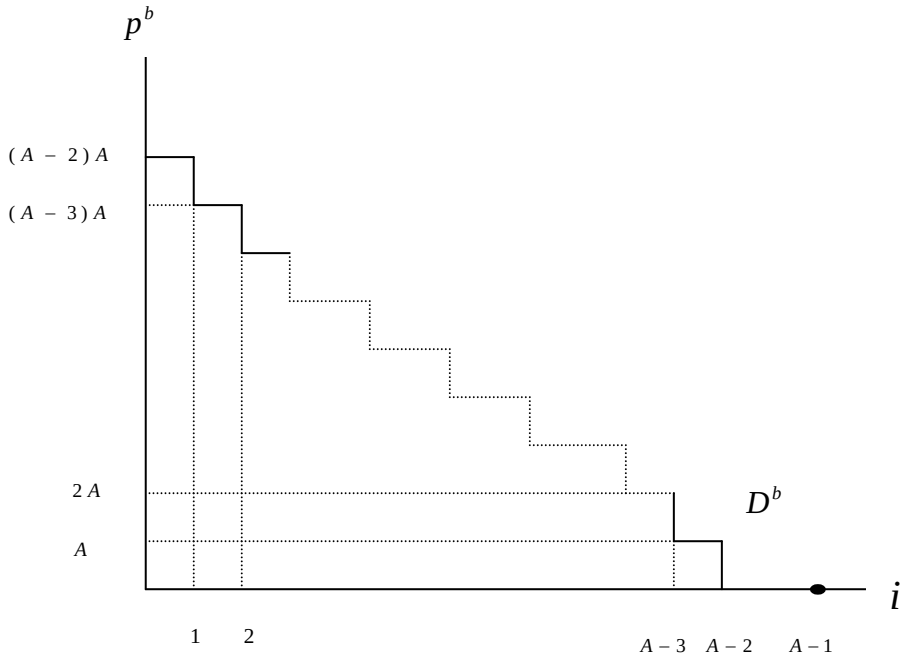


Figure 1: The quantity demand for advance service consumers.

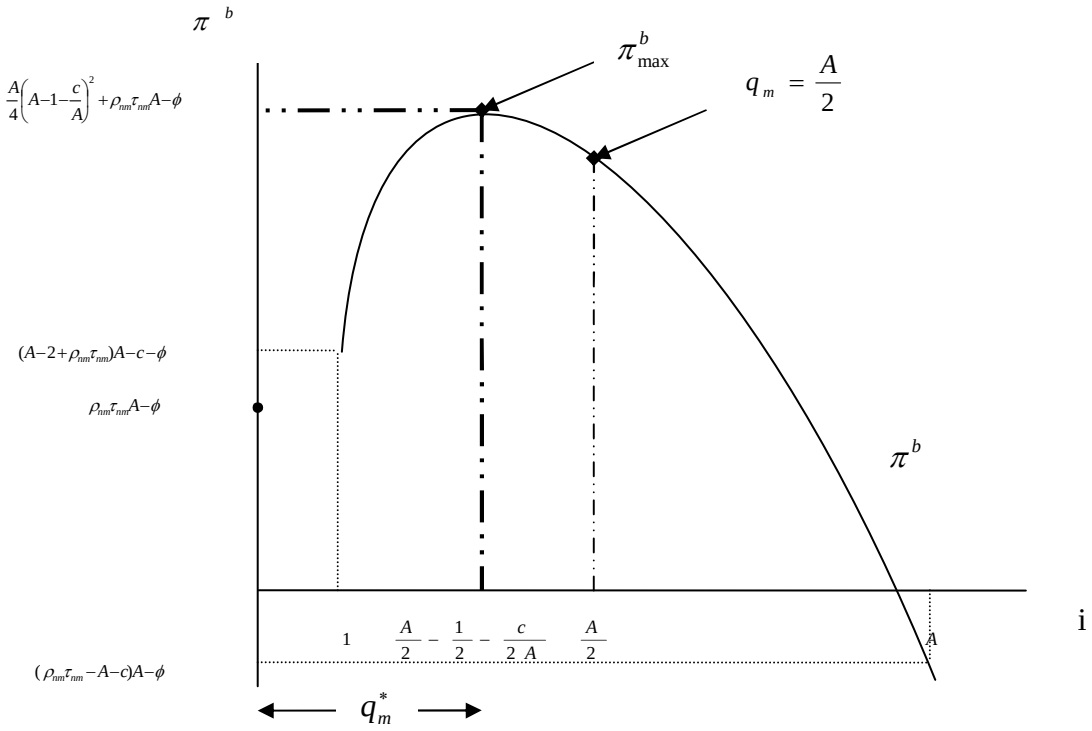


Figure 2: The profit of the monopoly as a function of the number of membership customers, *i*.

$$\pi^n(i)=\begin{cases}\rho_{nm}\tau_{nm}A-\phi & if \quad i=0\\ (A^2-A-c)i-Ai^2+\rho_{nm}\tau_{nm}A-\eta^2-c-\phi & if \;\; 1\le i\le A\end{cases}$$

The Social Welfare

$$CS^b=\sum_{i=1}^{i^*}U_{O_i}+\sum_{i=i^*}^{A}U_{I_i}=\sum_{i=1}^{i^*}\left[(A-i)A-p^b\right]+\left(A-i^*\right)A=$$

(11)
$$=\underbrace{\frac{A}{8}\left(A+1-\frac{c}{A}\right)\left(A-1-\frac{c}{A}\right)}_{oriented}+\underbrace{\frac{A}{2}\left(A+1+\frac{c}{A}\right)}_{indepandent}$$

$$W^b=\underbrace{\frac{A}{8}\left(A+1-\frac{c}{A}\right)\left(A-1-\frac{c}{A}\right)}_{oriented}+\underbrace{\frac{A}{2}\left(A+1+\frac{c}{A}\right)}_{indepandent}+\underbrace{\frac{A}{4}\left(A-1-\frac{c}{A}\right)^2+\rho_{nm}\tau_{nm}A-\phi}_{\pi^b}=$$

(12)
$$=\frac{A}{8}\left(A-1-\frac{c}{A}\right)\left(3A-1-\frac{3c}{A}\right)+\frac{A}{2}\left(A+1+\frac{c}{A}\right)+\rho_{nm}\tau_{nm}A-\phi=$$

$$=\frac{A}{2}\left[3\left(\frac{A}{2}-\frac{c}{2A}\right)^2+\frac{5}{4}+\frac{2c}{A}\right]+\rho_{nm}\tau_{nm}A-\phi$$

Case C – Free Use of Basic Service with Compulsory Exposure to Advertisements (Non-protection Policy)

In case C the monopoly allows free use of basic service combined with compulsory exposure to advertisements, with the possibility of paying for advanced service where exposure to advertisements is voluntarily (i.e. there is no extra charge for eliminating unwanted advertisements). Only *O* customers with high reservation price pay the high membership fee, while the rest of the customers benefit from the basic service free of charge. Two prices for membership can be derived:

High-fee equilibrium

1. *High-price equilibrium of membership* - the highest price, *p*, that can be charged by the monopoly is derived as follows:

   The customer buys membership if (13) $(A+\beta-i)q-p\geq q$,

   thus the $p(i)$ at equilibrium is:

   (13) $p(i)=(A+\beta-i-1)$

   From (13) and (2) we find the profit

   (14) $\pi(i)=\left[(A+\beta-i-1)A-c\right]i+\rho_m\tau_m i+\rho_{nm}\tau_{nm}(A-i)-\phi$

   This function is maximized by

   (15) $\frac{\partial\pi(i)}{\partial i}=A^2+\beta A-2Ai-A-c+\rho_m\tau_m-\rho_{nm}\tau_{nm}=0$

   which determines the optimal number of customers, i^*

   (16) $i^*=\frac{A}{2}-\frac{1}{2}-\frac{c}{2A}+\frac{\beta A+\rho_m\tau_m-\rho_{nm}\tau_{nm}}{2A}=\frac{1}{2}\left(A-1-\frac{c}{A}+\frac{\beta A+\rho_m\tau_m-\rho_{nm}\tau_{nm}}{A}\right)$

   and the value of i^* indeed maximizes profit since the second order condition holds:

   i.e., $\frac{\partial\pi^2}{\partial^2 i}=-2A<o$

   The number of loyal paying customers and the second order condition depends on the advertisement aversion of customers and formally,

$$\beta A + \rho_m \tau_m \underset{<}{\overset{>}{}} \rho_{nm} \tau_{nm}$$

When the population size, *A*, is very large then the number of paying customers approaches A/2. The distribution of paying and free customers is given as follows:

$$\left.\begin{array}{ll} i \in (1, \frac{A}{2} - \frac{1}{2} - \frac{c}{2A} + \frac{\beta A + \rho_m \tau_m - \rho_{nm} \tau_{nm}}{2A}) & membership\, customers \\ i \in (\frac{A}{2} + \frac{1}{2} - \frac{c}{2A} + \frac{\beta A + \rho_m \tau_m - \rho_{nm} \tau_{nm}}{2A}, A) & free\ of\ ch\arg e\, customers \end{array}\right\}$$

From (13), (14) and (16) we get the optimal membership price, $p^{c1,H}$, and the profit $\pi^{c1,H}$ in (17) and (17')

(17) $p^{c1,H} = \frac{1}{2}\left[A^2 + (\beta - 1)A + c + \rho_{nm}\tau_{nm} - \rho_m \tau_m\right]$

(17') $\pi^{c1,H} = \frac{A}{4}\left(A + \beta - 1 - \frac{c}{A} - \frac{\rho_{nm}\tau_{nm} - \rho_m \tau_m}{A}\right)^2 + \rho_{nm}\tau_{nm} A - \phi$

Low-fee equilibrium

2. *Low-price equilibrium of membership* - the lowest price is charged such that $i^* = A$, i.e., all customers get advanced service for free. Thus, the monopoly charges the following price: $(A + \beta - i)q - p \geq q$ or,

   (18) $p^{c2,L} = (\beta - 1)A$

   and the profit $\pi^{c2,L}$ is

   (19) $\pi^{c2,L} = \left[(\beta - 1)A - c + \rho_m \tau_m\right]A - \phi$

   The comparison of (17') and (19) leads to the immediate conclusion that possibility C1 always dominates C2, thus the monopoly charges the high equilibrium price.

The Social Welfare

-The social welfare in High-fee

The social welfare of the market is a simple summation of the consumers' surplus and the monopoly profits. The consumers' surplus is the summation of O_i members and I_{i-1} consumers who benefit only from the free service. We define $i^* \equiv z$

$$CS^{c1,H} = \sum_{i=1}^{i^*} U_{O_i} + \sum_{i=i^*}^{A} U_{I_i} = \sum_{i=1}^{i^*} \left[(A-i)A - p^b \right] + \left(A - i^* \right) A = \tag{20}$$
$$= \frac{z}{2}\left[A^2 + (\beta - 1)A - 1 - c - z - (\rho_{nm}\tau_{nm} - \rho_m \tau_m) \right] + A^2$$

thus, the social welfare is:

$$W^{c1,H} = \frac{A}{4}\left(A + \beta - 1 - \frac{c}{A} - \frac{\rho_{nm}\tau_{nm} - \rho_m \tau_m}{A} \right)^2 + \rho_{nm}\tau_{nm} A - \phi + \tag{21}$$
$$+ \frac{z}{2}\left[A^2 + (\beta - 1)A - 1 - c - z - (\rho_{nm}\tau_{nm} - \rho_m \tau_m) \right] + A^2$$

-The social welfare in Low-fee

Using the same definition of social welfare as in the high fee we get the consumers' surplus and the social welfare as follows:

$$CS^{c2,L} = \sum_{i=1}^{i^*} \left[(A + \beta - i) i^* - p^{c2,L} \right] = A^3 + \frac{A^2}{2} - \frac{A}{2} \tag{22}$$

and

$$W^{c2,L} = \left[(\beta - 1)A - c + \rho_m \tau_m \right] A - \phi + A^3 + \frac{A^2}{2} - \frac{A}{2} \tag{23}$$

Case D - Full protection against free use with mandatory exposure to advertisement when a monopoly allows only advanced paid service

We discuss two possible kinds of fees: (i) high-fee equilibrium, and (ii) low-fee equilibrium. Only customers with a reservation price above the fee registers as a paying member. Thus the monopoly should consider a high fee with few members, or reduce the fee in order to encourage the entry of more paying members. Let us discuss both equilibrium fees:

High-fee equilibrium

Only members with high reservation price (with high benefit from the advanced service) "enter the club" and are exposed to compulsory/mandatory advertisement. The monopoly can charge a fee from a customer only if $(A-i)q-p \geq 0$, thus the equilibrium price with *i* members is determined as:

(24) $p(i) = (A-i)i$

By applying equation (2) of the profit function to this case we then get:

(25) $\pi(i) = \left[(A-i)i - c - \mu\right]i + \rho_{nm}\tau_{nm}i - \phi$

In order to maximize profit we have to find the optimal number of member, *i*, by deriving the first and second order conditions for maximization:

(26) $\frac{\partial \pi(i)}{\partial i} = 2Ai - 3i^2 - c - \mu + \rho_{nm}\tau_{nm} = 0$

(26') $\frac{\partial \pi^2}{\partial^2 i} = 2A - 6i$

If we define the value: $\rho_{nm}\tau_{nm} - c - \mu \equiv k > 0$, we can find the optimal number of customers, i^*, as a function of parameter A (number of potential customers) and *k*:

(27) $$i^* = \frac{A + \sqrt{A^2 + 3k}}{3} = \frac{A}{3} + \frac{\sqrt{A^2 + 3k}}{3}$$

From (27) we find that just one positive solution is relevant (as the other is a negative value). Moreover, we can see that for any positive value *k*, the actual membership number exceeds two third of the potential member *A* and the number increases with *A* and *k*.

Formally, we can distinguish between members and non-members as follows:

$$\left.\begin{array}{ll} i \in (1, \frac{A + \sqrt{A^2 + 3k}}{3}) & membership\, customers \\ i \in (\frac{2A - \sqrt{A^2 + 3k}}{3}, A) & free\ of\ ch\arg e\, customers \end{array}\right\}$$

From (24), (25) and (27) we can find that (28) and (28') gives the optimal membership fee and profit in this case as follows:

(28) $$p^{d1,H} = \frac{1}{9}\left(A^2 + A\sqrt{A^2 + 3k} - 3k\right)$$

and

(28') $$\pi^{d1,H} = \frac{\left(2A^2 + 6k\right)\left(A + \sqrt{A^2 + 3k}\right) + 3kA}{27} - \phi$$

Low-fee equilibrium

The low fee equilibrium is for the case where i^*=*A*-1, however not all customers are willing to pay a positive fee, and hence not all customers participate (except for *A*-1).. Only if the fee charged is zero will all customers get the advanced service. Again, the customer pays the fee only if (29) holds:

(29) $(A-i)q-p\geq 0$

Thus, the low-fee equilibrium is:

(30) $p^{d2,L}=A$

and the profit is

(30') $\pi^{d2,L}=(A+\rho_{nm}\tau_{nm}-c-\mu)(A-1)-\phi$

The Social Welfare

-The social welfare in high-fee

The social welfare is the summation of consumers' samples of 0_i members, while the other I_{i-1} customers' utility is zero. Thus,

(31) $CS^{d1,H}=\sum_{i=1}^{i^*}\left[(A-i)i^*-p^{d1,H}\right]=\frac{A+\sqrt{A^2+3k}}{54}\left(13A^2-9A-5A\sqrt{A^2+3k}+2k\right)$

and the social welfare is:

(32) $W^{d1,H}=CS^{d1,H}+\pi^{d1,H}=\frac{A+\sqrt{A^2+3k}}{18}\left(4A^2+6k-3A\right)-\frac{kA}{18}-\phi$

- The social welfare in low-fee

In the case of A-1, customers pay the low fee and their consumer surplus is:

(33) $CS^{d2,L}=\sum_{i=1}^{i^*}\left[(A-i)i^*-p^{d2,L}\right]=A(A-1)\left(\frac{A}{2}-1\right)$

the social welfare is:

(34) $W^{d2,L}=CS^{d2,L}+\pi^{d2,L}=(A-1)\left(\frac{A^2}{2}+\rho_{nm}\tau_{nm}-c-\mu\right)-\phi$

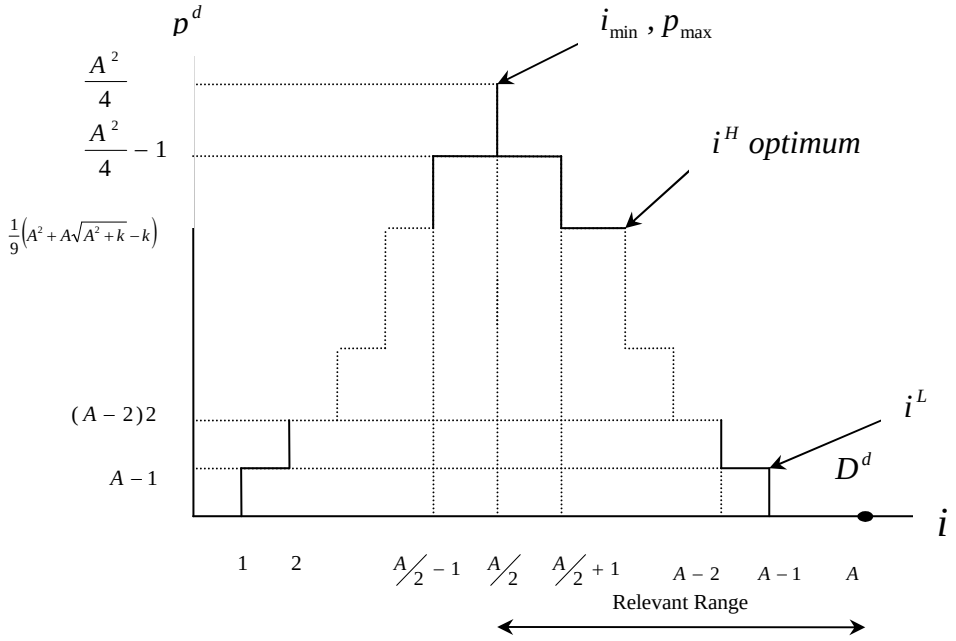


Figure 3: The quantity of advanced services demanded

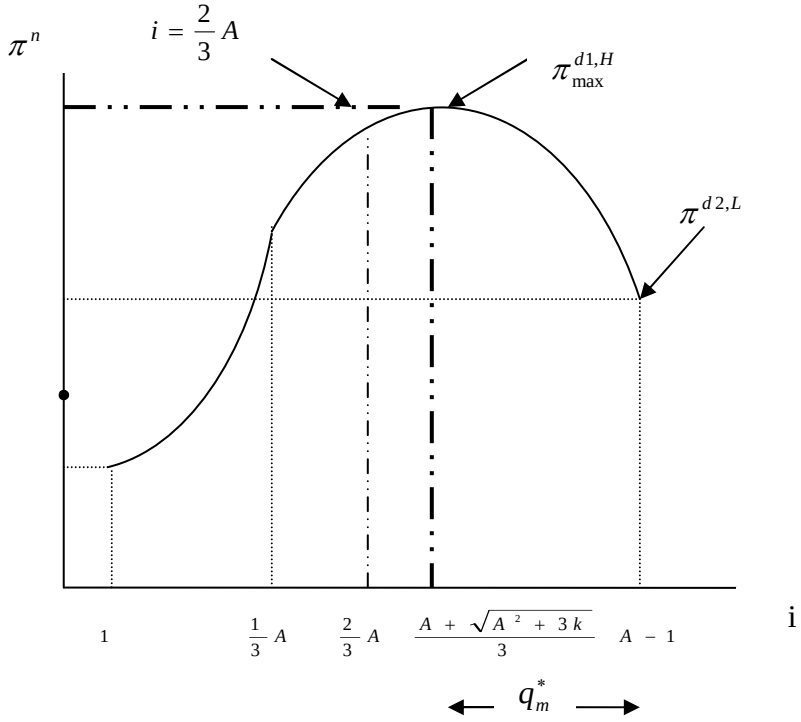


Figure 4: The profit of the monopoly as a function of the number of paying customers, *i*.

$$\pi^n(i) = \begin{cases} -\phi & if \quad i = 0 \\ Ai^2 - i^3 + (\rho_{nm}\tau_{nm} - c - \mu)i - \phi & if \quad 1 \le i \le A \end{cases}$$

The demand for advanced service in the case of full protection that prevents free use is introduced in Figure 3. Two different fees are possible for the same quantity demanded, and, of course**,** the higher fee that leads to greater profit**s** will be preferred by the monopoly.

The positive slope of the demand curve is in the range between zero and one half of the customers $\left(\frac{A}{2}\right)$. However, the monopoly prefers the equilibrium of higher quantity, i.e., $i > \frac{A}{2}$.

Figure 4 illustrates the monopoly profit with respect to the high and low fees. Profit maximization holds at the high fee. While the profit function rises at an increasing rate to the point where $i = \frac{1}{3}A$, it further continues to increase but at a diminishing rate to the optimal equilibrium fee.

Case E - Full protection with the possibility of eliminating compulsory exposure to advertisement.

In this case we change the scenario of case D by allowing involuntary exposure to advertisement for ψ. Two possible fees are again available:

High-fee equilibrium

Again only i customers join the advanced service, while all other customers are prevented from accessing the service. The monopoly can charge the fee p, only from customers for whom the following condition holds: $(A+\beta-i)q - p - \psi \geq 0$, thus, the fee as a function of i is (35):

(35) $p(i) = (A+\beta-i)i - \psi$

Using (35) in equation (2) leads to the following new profit function of the monopoly:

(36) $$\begin{aligned}\pi(i) &= \left[(A+\beta-i)i - \psi - c - \mu\right]i + (\rho_m \tau_m + \psi)i - \phi = \\ &= \left[(A+\beta-i)i - c - \mu\right]i + \rho_m \tau_m i - \phi\end{aligned}$$

The optimal number of paying customers can be derived from the first and second order conditions for maximization:

(37) $\frac{\partial \pi(i)}{\partial i} = 2Ai + 2\beta i - 3i^2 - c - \mu + \rho_m \tau_m = 0$

(37') $\frac{\partial \pi^2}{\partial^2 i} = 2(A+\beta) - 6i$

We use the notation $\rho_m \tau_m - c - \mu \equiv f > 0$ where 0<*f*<*k* (*k* in case d is larger than *f* as $\rho_m \tau_m < \rho_{nm} \tau_{nm}$)

The optimal number of paying members i^* is given by (38):

(38) $i^* = \frac{A+\beta+\sqrt{(A+\beta)^2+3f}}{3} = \frac{A+\beta}{3} + \frac{\sqrt{(A+\beta)^2+3f}}{3}$

and from the two solutions, only the one which has a positive value of *i is relevant*, such that formally we can get the following results:

$$\left.\begin{array}{ll} \text{membership customers} & i \in (1, \frac{A+\beta+\sqrt{(A+\beta)^2+3f}}{3}) \\ \text{free of charge cutomers} & i \in (\frac{2A-\beta-\sqrt{(A+\beta)^2+3f}}{3}, A) \end{array}\right\}$$

From (35) and (38) we get the optimal fee and profits:

(39) $p^{e1,H} = \frac{1}{9}\left[(A+\beta)^2 + (A+\beta)\sqrt{(A+\beta)^2+3f} - f\right] - \psi$

(39') $\pi^{e1,H} = \frac{\left[2(A+\beta)^2+6f\right]\left(A+\beta+\sqrt{(A+\beta)^2+3f}\right)+3f(A+\beta)}{27} - \phi$

Low-fee equilibrium

In this low fee equilibrium only $i^* = A$ customers are registered as members. Thus the highest price the monopoly can charge is:

(40) $p^{e2,L} = \beta A - \psi$

and the maximum profit is:

(41) $\pi^{e2,L} = [\beta A + f]A - \phi$

The Social Welfare

- The social welfare in High-fee

The consumer surplus is derived only from O_i members, while $I_{i\text{-}1,}$ non-members do not derive any benefit from any of the site's service. Thus the consumer surplus is:

(42) $$CS^{e1,H} = \sum_{i=1}^{i^*} [(A + \beta - i)i^* - p^{e1,H} - \psi] =$$
$$= \frac{A + \beta + \sqrt{(A+\beta)^2 + 3f}}{54} \left\{ [2(A+\beta) - 3]\left(A + \beta + \sqrt{(A+\beta)^2 + 3f}\right) + 9f \right\}$$

And the summation of CS and the monopoly profit represent the social welfare, *W*, as follows:

(43) $$W^{e1,H} = CS^{e1,H} + \pi^{e1,H} =$$
$$= \frac{A+\beta+\sqrt{(A+\beta)^2+3f}}{54}\left(3(A+\beta)[2(A+\beta)-1] + \sqrt{(A+\beta)^2+3f}[2(A+\beta)-3] + 21f\right) + \frac{+6f(A+\beta)}{54} - \phi$$

- The Social Welfare in Low-fee

In the case of a low membership fee the consumers' surplus increases as more customers join the advanced service. Therefore, the consumers' surplus is:

(44) $CS^{e2,L} = \sum_{i=1}^{i^*}\left[(A+\beta-i)i^* - p^{e2,L} - \psi\right] = \frac{A^3}{2} - \frac{A^2}{2}$

and the social welfare is:

(45) $W^{e2,L} = \left[\beta A + f\right]A - \phi + \frac{A^3}{2} - \frac{A^2}{2}$

From the point of view of the monopoly more profit is made by charging the high fee, thus e_1 is the preferred choice.

Table 1 below summarizes the different cases discussed above:

Table 1

cases	No. Of Customers	Consumers' Surplus	Monopoly Profit	Social Welfare
a. No service fee, no advertising protection.	0	A^2	$\rho_{nm}\tau_{nm}A - \phi$	$A^2 + \rho_{nm}\tau_{nm}A - \phi$
b. Fee for advanced service, no advertising protection.	$\frac{1}{2}\left(A-1-\frac{c}{A}\right)$	$\frac{A}{8}\left(A+1-\frac{c}{A}\right)\left(A-1-\frac{c}{A}\right) + \frac{A}{2}\left(A+1+\frac{c}{A}\right)$	$\frac{A}{4}\left(A-1-\frac{c}{A}\right)^2 + \rho_{nm}\tau_{nm}A - \phi$	$\frac{A}{2}\left[3\left(\frac{A}{2}-\frac{c}{2A}\right)^2 + \frac{5}{4} + \frac{2c}{A}\right] + \rho_{nm}\tau_{nm}A - \phi$
c1. High fee for both advanced service and advertising protection, no fee basic service.	$\frac{1}{2}\left(A+\frac{c}{A}-\frac{\beta A+\rho_{nm}\tau_{nm}-\rho_m\tau_m}{A}\right)$	$\frac{Z}{2}\left[A^2+(\beta-1)A-1-c-z-(\rho_{nm}\tau_{nm}-\rho_m\tau_m)\right]+A^2$	$\frac{A}{4}\left(A+\beta-1-\frac{c}{A}-\frac{\rho_{nm}\tau_{nm}-\rho_m\tau_m}{A}\right)^2 + \rho_{nm}\tau_{nm}A - \phi$	$\frac{A}{4}\left(A+\beta-1-\frac{c}{A}-\frac{\rho_{nm}\tau_{nm}-\rho_m\tau_m}{A}\right)^2 + \rho_{nm}\tau_{nm}A - \phi + \frac{z}{2}\left[A^2+(\beta-1)A-1-c-z-(\rho_{nm}\tau_{nm}-\rho_m\tau_m)\right]+A^2$
c2. low fee for both advanced service and advertising protection, no fee basic service.	A	$A^3 + \frac{A^2}{2} - \frac{A}{2}$	$\left[(\beta-1)A - c + \rho_m\tau_m\right]A - \phi$	$\left[(\beta-1)A - c + \rho_m\tau_m\right]A - \phi + A^3 + \frac{A^2}{2} - \frac{A}{2}$
d1. No basic service, no advertising protection, high fee for advance service.	$\frac{A}{3} + \frac{\sqrt{A^2+3k}}{3}$	$\frac{A+\sqrt{A^2+3k}}{54}\left(13A^2 - 9A - 5A\sqrt{A^2+3k} + 2k\right)$	$\frac{\left(2A^2+8k\right)\left(A+\sqrt{A^2+3k}\right)+3kA}{27} - \phi$	$\frac{A+\sqrt{A^2+3k}}{18}\left(4A^2+6k-3A\right) - \frac{kA}{18}$
d2. No basic service, no advertising protection, low fee for advance service.	A-1	$A(A-1)\left(\frac{A}{2}-1\right)$	$\left(A+\rho_{nm}\tau_{nm}-c-\mu\right)(A-1)-\phi$	$(A-1)\left(\frac{A^2}{2}+\rho_{nm}\tau_{nm}-c-\mu\right)-\phi$
e1. No basic service, extra fee for advertising protection, high price.	$\frac{A+\beta}{3} + \frac{\sqrt{(A+\beta)^2+3f}}{3}$	$\frac{2(A+\beta)^2+6f\left[A+\beta+\sqrt{(A+\beta)^2+3f}\right]+3f(A+\beta)}{27} - \phi$	$\frac{\left[2(A+\beta)^2+6f\right]\left[A+\beta+\sqrt{(A+\beta)^2+3f}\right]+3f(A+\beta)}{27} - \phi$	$\frac{A+\beta+\sqrt{(A+\beta)^2+3f}}{54}\left[3(A+\beta)(2A+\beta)-1\right]+\sqrt{(A+\beta)^2+3f}\left[2(A+\beta)-3\right]+21f\Big]+\frac{+6f(A+\beta)}{54} - \phi$
e2. No basic service, extra fee for advertising protection, low price.	A	$\frac{A^3}{2} - \frac{A^2}{2}$	$\left[\beta A + f\right]A - \phi$	$\left[\beta A + f\right]A - \phi + \frac{A^3}{2} - \frac{A^2}{2}$

Maximization Profit

Using Table 1 above we find that if no protection policy is adopted by the monopoly, then neither policy b nor policy c1 are relevant. $\pi^{c1} > \pi^{b}$ if the following condition exists:

$$(\beta+1)A + \rho_m\tau_m > \rho_{nm}\tau_{nm}$$

or,

$$\beta > \frac{\rho_{nm}\tau_{nm} - \rho_m\tau_m}{A} - 1.$$

At value β_1 the monopoly is indifferent between the two policies where

$$\beta_1 = \frac{\rho_{nm}\tau_{nm} - \rho_m\tau_m}{A} - 1$$

If the protection policy is applied by the monopoly, preventing free use of basic service, either policy d1 or e1 are relevant. A sufficient condition for $\pi^{e1} > \pi^{d1}$ is:

$$2\beta A + \beta^2 + 3\rho_m\tau_m > 3\rho_{nm}\tau_{nm}$$

or

$$\beta > \sqrt{A^2 + \rho_{nm}\tau_{nm} - \rho_m\tau_m} - A.$$

Again, we can assign the value β_2 where the monopoly is indifferent between requiring and not requiring mandatory advertisements, where $\beta_2 \equiv \sqrt{A^2 + \rho_{nm}\tau_{nm} - \rho_m\tau_m} - A$.

The next question is whether protection of the site from free users of any kind is preferable to allowing free use of basic service. This can be done by isolating the effect of advertising exposure. In the case where $(\beta = 0)$, i.e., consumers are exposed to mandatory advertisements, the profits of cases b (free use) and d1 (full protection) are considered.

The comparison of profits is not an easy task, and requires using the Solver found in Excel.

From the results we find that for positive values of the parameters in parentheses $(A, c, \mu, \rho_{nm}\tau_{nm})$, profit maximization is achieved at π^b , i.e., free use of the basic service.

However, if we compare between case e1 (where a fee for voluntary exposure to advertisements is charged) and case c1 we find that for positive values of parameters

$(A, \beta, c, \mu, \rho_{nm}\tau_{nm}, \psi)$ more profit is gained in case c1, π^{c1}, than in applying the full-protection policy.

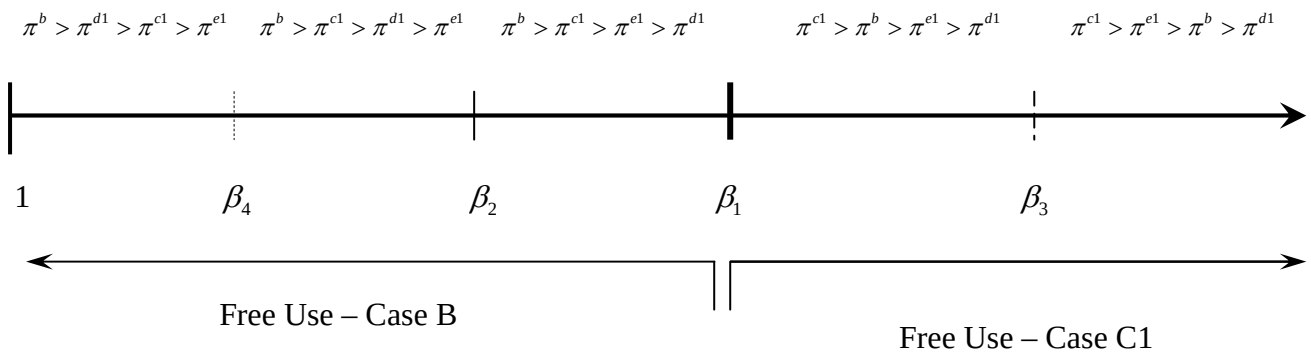
The next comparison is between free use of basic service (case b) with full protection, and a charge for avoiding compulsory advertisements. There are cases where $\pi^{e1} > \pi^{b}$ and there are other cases where the opposite holds. Using the Solver we find that if $(\beta+1)A + \rho_m\tau_m > \rho_{nm}\tau_{nm}$ then $\pi^{c1} > \pi^{e1} > \pi^{b}$. A β_3 monopoly is indifferent between $\pi^{e1} > \pi^{b}$.

The last comparison is between cases d1 and c1 (free use of basic service). In some cases $\pi^{d1} > \pi^{c1}$. In other cases the opposite holds. .

Again, by using the Solver we find that if $(\beta+1)A + \rho_m\tau_m < \rho_{nm}\tau_{nm}$, then $\pi^{b} > \pi^{d1} > \pi^{c1}$ holds.

At β_4 the monopoly is indifferent between cases c1 and d1. The above results can be summarized and demonstrated in Figure 1.

Figure 1



From the figure above we can conclude the following:

When the users of the site benefit from externalities of returns to scale and the customers exhibit diminishing reservations prices in the form of a rectangularly uniformed

distribution, then the optimal policy consists of offering the customers the basic service for free.

Conclusions

This paper discusses the monopoly's optimal strategy in the special case of a network market. The uniqueness of this market is that positive network externalities exist, a factor that encourages the monopoly to increase its customer base. This goal can be accomplished by implementing two different policies: (i) to allow free use of the basic service that increases the number of customers (members and non members); and (ii) to implement a full-protection policy against the entrance of free users, in combination with a low-membership fee thereby enabling more customers to use advanced services. Furthermore, in the case of free use of the basic service the monopoly may consider imposing an additional fee on members for the right to avoid compulsory exposure to advertising. As can be expected, a high level of disutility from compulsory advertising encourages the monopoly to charge a fee from members for their right to view only voluntary advertisements. In the case where the full-protection policy is preferred by the monopoly, the policy towards compulsory or voluntary advertisements is ambiguous. However, when the disutility from compulsory advertisements of the members is high, the monopoly prefers to eliminate the compulsory advertisements. In any event, the optimal pricing for the advanced service in the full-protection case is that of high fees, such that fewer members participate in using the protected advanced service.

An important conclusion of our paper is that free use of basic service that increases the number of customers is preferable to full-protection policy because the heterogeneity of the customers and the high benefits of members from the network externalities makes it optimal to attract some free users. As a result of attracting these free customers, the monopoly can charge much higher fees from members of the advanced service that compensates for the reduction of revenues from the nonpaying customers. In addition, further revenues can be

generated from the higher advertising fees that can be charged as more nonpaying customers are exposed to commercials. This policy is even more profitable in the case where members are exposed to mandatory advertisements and thus are ready to pay a very high fee for a membership that exposes them only to voluntary advertisements.

The social optimum policy of course differs from that of the monopoly. Social welfare requires a low membership fee that attracts many more possible members.

Another interesting conclusion that we demonstrate is related to the shape of the demand curves. In the case of free use of basic service, the demand for membership is affected negatively by the membership fee. However, in the case of full-protection policy, due to returns to scale and positive externalities, we find a backward bending demand curve. Where there is an initial small number of members, an increase in the fee will lead to an increase in consumer surplus. Customers are prepared to pay more for the service, and therefore the shape of demand curve shows a positive relationship between the fee charged and the number of customers.

Another interesting aspect of our analysis are the sources of the monopoly profits. The monopoly generates income from membership fees and from selling commercials to advertisers who pay a fee proportional to the number of people exposed to the commercial. These two sources depend on several factors such as the population size and its homogeneity, the disutility from compulsory exposure to advertising, the price of each ad, the fee for basic service, etc. By examining all cases/policies we found that as the population increases, the revenues from membership fees increase relative to the revenues from advertisements. Furthermore, as the disutility from compulsory exposure to advertising is rises, relatively more revenues are generated from membership fees. And finally, whenever a higher price for advertising can be charged, the relative share of revenue from advertising increases. This last conclusion, which differs from other research, is derived from our analysis showing that the monopoly would choose the strategy of offering free basic service to take advantage of the

network externality, rather than adopting a policy of full protection. Instead of preventing access to the less enthusiastic potential customers, the monopoly can differentiate between customers groups and charge a positive fee for advanced service while allowing free use of basic service, a policy which would yield benefits to all customers as well as to the monopoly and to the advertising companies.

References

Bernard C., and Bruno J., 2003, "Chicken & Egg: Competition Among Intermediation Service Providers", *The RAND Journal of Economics* 34: 309-328.

Conner, K., and Rumelt, R., 1991, "Software Piracy: An Analysis of Protection Strategies", *Management Science* 37: 125-139.

Farrell, J., and Salonar, G., 1986, "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation", *American Economic Review* 76: 940-955.

Farrell, J., and Salonar, G., 1985, "Standardization, Compatibility, and Innovation", *The RAND Journal of Economics* 16:70-83.

Gayer, A., and Shy O., 2003, "Copyright Protection and Hardware Taxation", *Information Economics & Policy* 15(4): 467-483.

Gayer, A., and Shy O., "Publishers, Artists, and Copyright Enforcement," Written papers 2004/01/29.

Katz, M., and Shapiro, C., 1986, "Technology Adoption in the Presence of Network Externalities", *Journal of Political Economy* 94: 822-841.

Katz, M., and Shapiro, C., 1985, "Network Externalities, Competition, and Compatibility", *American Economic Review* 75: 424-440.

Shy, O., and Thisse, T, 1999, "A Strategic Approach to Software Protection", *Journal of Economics & Management Strategy* 8: 163 - 190.