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Tomer Blumkin and David Lagziel

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Monaster Center for
Economic Research
Ben-Gurion University of the Negev
P.O. Box 653
Beer Sheva, Israel

Fax: 972-8-6472941
Tel: 972-8-6472286

Pay Secrecy in Labor Markets with Matching Frictions*

Tomer Blumkin† and David Lagziel‡

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Abstract:

This paper studies the strategic role of wage secrecy arrangements in a labor market with matching frictions. We demonstrate how firms employ pay-secrecy policies to control the dissemination of wage-related information to job applicants, via professional social networks. Consequently, wage dispersion, both within and across firms, arises in equilibrium, ensuring that firms derive positive rents. We further show that matching frictions arise endogenously in equilibrium and serve as a coordination device to support tacit collusion between competing firms.

Journal of Economic Literature classification numbers: E24, D82, J30, J31, J71.

Keywords: secrecy; wage signaling; matching friction; tacit collusion.

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†Ben-Gurion University of the Negev, Beer-Sheva 8410501, Israel. e-mail: Tomerblu@bgu.ac.il; CESifo, Poschingerstrasse 5, 81679 Munich, Germany; IZA, Schaumburg-Lippe Strasse 7/9, 53113 Bonn, Germany.

‡Ben-Gurion University of the Negev, Beer-Sheva 8410501, Israel. e-mail: davidlag@bgu.ac.il.

1 Introduction

The search and matching literature, in the labor-market context, has emphasized how informational imperfections shape the formation of jobs and affect pay. Typically, imperfectly informed job applicants engage in costly search for job-related information to improve their employment prospects, both in terms of the likelihood of getting the job and the level of compensation. In modern labor markets, professional social networks, such as LinkedIn, serve as natural platforms through which job-related information is transferred from current employees to applicants. Acknowledging that informed applicants improve their position relative to uninformed ones, employers may attempt to make this information less accessible. One tool that serves this purpose is the use of wage-secrecy arrangements.

Indeed, labor contracts often stipulate wage-secrecy clauses that require employees to maintain confidentiality regarding their pay-check. The use of such clauses is generally described as a strategic tool used by employers to: (i) mitigate the potentially demoralizing effect of pay gaps; and (ii) improve their position in wage negotiations.[1] In the current paper we address the second aspect by exploring wage secrecy at the market level, focusing on the interaction amongst firms that compete over a pool of prospective employees.

We consider a market with two identical firms and a continuum of homogeneous job applicants seeking job offers at both firms. Due to informational frictions, only a fraction of applicants is successfully matched with each firm. Each firm posts wage offers and applicants acquire wage-related information through interactions with current employees via professional social networks. In the backdrop of matching frictions, we show that firms strategically employ wage-secrecy policies, in equilibrium, to control the dissemination of wage-related information. This, in turn, results in wage dispersion *within and* (potentially) *across* firms, which allows the firms to earn positive rents. Notably, secrecy arrangements are shown to be more prevalent at high wage levels, and we further show that matching frictions arise endogenously in equilibrium. Although the presence of such frictions a-priori constraint the ability of the firms to recruit, it essentially serves as a coordination device that allows firms to *tacitly collude* through the wage-secrecy mechanism, thereby ensuring positive rents.

1.1 Related literature

Few theoretical studies examine the desirability of wage-secrecy arrangements in optimal labor contracts. The first study to formally and directly address this subject is Danziger and Katz

[1] The first aspect is the main focus of Blumkin and Lagziel (2018) which explores the micro-foundations of wage secrecy at the firm level. See the literature review in Section 1.1 below.

(1997) which demonstrates how a wage secrecy convention can facilitate risk shifting between firms and workers in response to productivity shocks. Taking a different perspective on the subject, Blumkin and Lagziel (2018) explore the micro-foundations of wage secrecy at the firm level, where ex-ante homogeneous workers exhibit other-regarding preferences with respect to the remuneration of their co-workers. Assuming that relative pay concerns induce over/under-paid workers to exert higher/lower effort levels, Blumkin and Lagziel (2018) characterize necessary and sufficient conditions for wage secrecy to be part of the optimal labor contract, and allude to the role played by the extent of complementarity exhibited by the team's production function. Cullen and Pakzad-Hurson (2018) employ a dynamic model of wage negotiations to explore the equilibrium impacts of enhanced pay-transparency along both the demand (wage-setting and hiring policies) and supply (workers' bargaining strategies) channels. In their set-up, workers stochastically learn the wages of their peers and can voluntarily choose to re-negotiate their contracts. Cullen and Pakzad-Hurson (2018) show that an increase in pay transparency induces an information externality that shifts, de-facto, the bargaining power from the workers to the firm, resulting in lower wage rates and higher hiring rates.

The current study relates to the extensive search literature in the context of labor markets (for an elaborate survey, see Rogerson et al. (2015)). One strand of this literature, that combines random matching with wage posting, focuses on pure wage dispersion, namely, exploring mechanisms via which workers with identical abilities are paid different wages in equilibrium. Typically, in these models there is a continuum of homogeneous firms with no market power, where each posts a single wage offer, so wage dispersion arises in equilibrium across firms, trade-offing higher wages with lower vacancy risks. In our set-up, instead, with both homogeneous firms and workers, wage dispersion arises in equilibrium at the firm level and, potentially, across firms. The latter is attained in a setting with only two identical firms that do possess some market power. As shown in Blumkin and Lagziel (2018) and exemplified in the current study, wage dispersion at the firm level is crucial for wage-secrecy arrangements since firms have nothing to gain from limiting the dissemination of wage-related information to current and/or future employees, in case all workers are equally remunerated.

The effect of pay transparency on workers' satisfaction and incentives is examined in several recent empirical studies (see Card et al. (2012), Perez-Truglia (2015) Rege and Solli (2015), Cullen and Perez-Truglia (2018), and Cullen and Pakzad-Hurson (2018) inter-alia). The empirical evidence alludes to the role played by pay transparency in determining labor market outcomes both along the intensive margin (effort levels/performance) and the extensive margin (recruitment/retention). For instance, Rege and Solli (2015) use the 2001 policy change in the on-line availability of Norwegian tax records to show that the information shock increased job separation for low-earning workers relative to high-earning ones. More recently, Cullen and Perez-Truglia

(2018) show, in a field experiment, that higher perceived peer salary decreases effort and output, as well as retention.

1.2 Structure of the paper

The paper is organized as follows. In Section 2 we present a reduced-form static set-up. In Section 3 we present the main results: (i) in Section 3.1 we characterize the equilibrium in the reduced-form set-up for different levels of matching frictions; (ii) in Section 3.2 we embed the one-stage problem into a dynamic set-up; (iii) in Section 3.3 we examine the optimality and robustness of the search procedure invoked in the reduced-form set-up; and (iv) in Section 3.4 we allow the matching frictions to be determined endogenously by the firms. We conclude our analysis with a brief discussion of the normative implications in Section 4.

2 The model

This section formulates a parsimonious analytical framework that demonstrates the role of wage-secrecy clauses in labor contracts. To facilitate the exposition, we first present a (reduced form) single-stage problem and later extend it to a full-fledged dynamic set up.

Consider a market comprised of two identical firms and a continuum of risk-neutral homogeneous job applicants who search for available positions in both firms. The duration of a labor contract upon formation of a successful match between a worker and a firm is normalized to a single period. Firms employ a linear production function where the productivity of each worker is denoted by $q > 0$. Without loss of generality, we set the workers' outside option, associated with either alternative job opportunities outside the market or government support programs, to zero. We further normalize the firms' reservation to zero, again, with no loss in generality.

We turn next to describe in detail the search-and-matching procedure that governs the process of job-formation in the labor market. By the normalization of both the workers' and firms' reservation to zero, it follows that the formation of a match between a firm and a typical worker is mutually beneficial given any wage level between 0 and q. However, match formation is assumed to be restricted by some exogenous friction such that only a fraction $p_i \in (0, 1]$ of applicants are successfully matched with each firm i. To determine wages, we assume a standard wage-posting protocol. Namely, each firm posts a distribution of wage offers from which applicants randomly draw once (i.e., one draw from each firm). A wage draw forms a take-it-or-leave-it offer such that an applicant can either accept the offer to work for the firm, for the specified wage offer (with the firm being the residual claimant), or remain idle, in which case the applicant collects the reservation. As, by assumption, applicants' reservation is normalized to zero, every successfully matched applicant accepts the wage offer. In case an applicant is successfully matched with both firms he opts for the higher wage offer, with a symmetric tie-breaking rule.

More formally, each firm $i \in \{1, 2\}$ dictates a distribution of wage offers given by the CDF $F_i \in \Delta\mathbb{R}_+$. Denote by $w_i \sim F_i$ a random offer from firm i's distribution. Subject to a realized offer $w_i = w$ and given F_{-i}, the expected profit of firm i is

$$\pi_i (w|F_{-i}) \;=\; p_i \left[1 - p_{-i} \left[1 - F_{-i}(w) + \tfrac{1}{2}\Pr(w_{-i} = w)\right]\right] (q - w) .$$

The term $p_{-i} \left[1 - F_{-i}(w) + \frac{1}{2}\Pr(w_{-i} = w)\right]$ is the probability that the applicant is successfully employed by firm $-i$, rather than by firm i, due to an offer of at least w. Assuming that the mass of applicants is normalized to unity and with a slight abuse of notation, the expected payoff of firm i is denoted by $\pi_i(F_i|F_{-i}) = \mathbf{E}_{F_i} [\pi_i (w|F_{-i})]$. Thus, a profile (F_1, F_2) forms an *equilibrium* if $\pi_i(F_i|F_{-i}) \geqslant \pi_i(F_i^*|F_{-i})$, for every distribution of wage offers F_i^* and firm i.

Under the search procedure described above, the distributions of wage offers (F_1, F_2) induce a realized-wage distribution G_i, for every firm i, given by the CDF

$$G_i(w) \;=\; \frac{\mathbf{E}_{F_i}\left[\frac{\pi_i(t|F_{-i})}{(q-t)}\mathbf{1}_{\{t \leqslant w\}}\right]}{\mathbf{E}_{F_i}\left[\frac{\pi_i(t|F_{-i})}{(q-t)}\right]}. \tag{1}$$

It is straightforward to verify that G_i and F_i do not necessarily coincide. This discrepancy is not a mere technical issue, but precisely the point where the firms' *wage-secrecy* policies are manifested. To see this, notice that each firm can convey the information summarized in its distribution of wage offers in various ways. One possibility is to advertise it directly to job applicants, without specifying the wage level associated with each distinct vacancy (as is commonly the case with job-ads). Another option would be to rely on professional social networks where job applicants interact with current employees to extract wage-related information. As will be formally shown in Section 3.2 below, the realized distribution of wages of each firm, in equilibrium, would first-order stochastically dominate its respective distribution of wage offers. So in any case, implementing the desired distribution of wage offers would necessitate some form of wage-secrecy arrangements, otherwise the two distributions must coincide by definition. In other words, every firm must employ some wage-secrecy arrangements to (partially or fully) control the dissemination of wage-related information to its pool of job applicants.

Before concluding this section, we briefly discuss some of the restrictive assumptions invoked to render our framework more tractable. First, notice that our simplified wage-posting set-up in which wage offers directly translate to realized wage-rates, is only a technical simplification. Given any continuous and monotonic mapping between wage offers and realized rates (which could follow, for example, from some bargaining process), each firm i can properly choose F_i in a manner that implements its desired distribution of realized rates. In this sense, the given specification is non-consequential for our qualitative results. Second, notice that we impose an exogenous search procedure where applicants are assumed to sample twice and do it horizontally (one sample from each firm) rather than vertically (drawing twice from one of the firms). Moreover, the search procedure is independent of the distributions of wage offers. These restrictive assumptions will be discussed at length in Section 3.3, where we examine the optimality and robustness of the specified search procedure. Finally, the random draws reflect the fact that applicants are familiar with the distribution of wage offers but are imperfectly informed with the wage level associated with a particular posted vacancy. This is a standard stylized manner in which we capture search frictions.

3 Main results

Our analysis consists of four parts. In the first part we analyse the previously defined single-stage game for any friction level. In the second part we embed the single-stage game into a repeated set-up, and substantiate the single-stage equilibria as stationary solutions of the dynamic problem. In the third part we study the applicants' search procedure and in particular the optimality of the applicants' choice to sample both firms, along with the ability of the firms to affectively manipulate this sampling rule. In the last part we revert from the exogenous-frictions set-up by allowing firms to strategically determine the extent of matching frictions in the labor market. Let us now review each of these parts separately.

First, in section 3.1, we maintain our assumption that frictions are exogenous and analyse the equilibria with respect to the level of matching friction in the market. An important insight from this part concerns the minimal level of friction which leads to wage dispersion and wage secrecy. As it turns out, *any* positive friction leads to wage dispersion and secrecy. That is, for every $p_i \in (0,1)$ there exists a unique equilibrium where the induced offers differ from the realized wages and both are fully supported on $[0, qp_1]$, assuming $p_1 \leqslant p_2$. Notably, and in contrast to standard search theoretic models of the labor market, wage dispersion is shown to arise at the firm level (see our discussion in the literature review).

The second part of our analysis, given in Section 3.2, embeds the one-stage game into a repeated overlapping-generations framework. We first show how our static analysis carries through to the dynamic set-up. We then show how a proper choice of secrecy policies serves to reconcile the differences between the distributions of wage offers and the realized wage distributions. In particular, we demonstrate that the realized wage distribution stochastically dominates the distribution of wage offers, which implies that wage secrecy arrangements *must be used* in order to support the distribution of wage offers in equilibrium and, plausibly, that pay secrecy is more prevalent at high wage levels.

In the third part (Section 3.3) we examine the optimality and robustness of the hitherto exogenously invoked search procedure. In particular, we show that sampling horizontally (one draw from each firm) dominates sampling vertically (drawing twice from one of the firms) whenever the distributions of wage offers and the matching frictions are identical across firms. We further show that for sufficiently low levels of matching friction, there exist no distribution of wage offers that induces workers to search vertically. In other words, we prove that firms do not have a feasible way to manipulate the applicants' decision to sample across firms.

The forth part of our analysis, presented in Section 3.4, extends the basic model by allowing every firm i to first choose its desired level of friction p_i, and later choose its distribution of wage offers, F_i. The main insight from this part concerns the firm's desire to introduce friction into the recruitment process, although the latter is a-priori counter-productive. A firm that introduces friction into its recruitment process de-facto limits its ability to attract new employees, while facilitating it for the other firm. We nevertheless prove that such losses are outweighed by the benefits associated with wage dispersion and wage secrecy, reflected in limited wage competition and consequently, lower expected wages. The combination of wage secrecy and the ability to set matching frictions serves the firms to coordinate on an equilibrium (i.e., an implicit collusion) in which they derive positive rents despite wage competition.

Remark 1. *To shorten the exposition and unless stated otherwise, the complete analysis and statements hold* almost surely (a.s.)*, i.e., with probability* 1*. This remark holds throughout the paper since zero-measure deviations do not effect the expected payoff of either firm.*

3.1 The single-stage problem

We start with the one-stage game defined in Section 2. The first observation concerns the distribution of wage offers in equilibrium. We show that, in equilibrium, both distributions have a convex support and that atoms are only possible at the end points, 0 and q. To be clear, we define an *atom* as a point $w \in [0, q]$ such that $\Pr(w_i = w) > 0$, and F_i is not left-side continuous at w.

Lemma 1. *For fixed values $p_1, p_2 \in (0, 1]$, the wage offers in equilibrium are supported on a connected set with no atoms in $(0, q)$.*

All proofs are deferred to the Appendix.

Lemma 1 is motivated by the following reasoning. An interior atom of one firm provides a profitable deviation for the other firm, via an increase in the wage levels strictly above the atom.

In response to such deviation, the first firm would have an incentive to shift the atom downwards and decrease wage offers (and wage levels, accordingly), without affecting the probability to recruit applicants. This process cascades downwards, and stops at the lower end of the range of feasible wage offers. The only case where such profitable deviations do not exist is for atoms at one of the end points, either $w = 0$ or $w = q$.

Note that an atom at the highest feasible level q guarantees a (point-wise) zero payoff. This outcome is maintained in equilibrium if and only if the firm cannot secure a positive payoff for any lower wage rate. We pursue this possibility in the following lemma, which considers a frictionless environment (i.e., $p_i = 1$ for both firms). Building on Lemma 1, it shows that the a frictionless environment leads to a *unique* equilibrium where both firms support *only* the maximal wage level.

Lemma 2. *If no frictions exist, $p_1 = p_2 = 1$, then there exists a unique equilibrium where both distributions of wage offers induce only the highest wage level q (i.e., both equal the Dirac measure δ_q).*

The competitive wage level q has specific characteristics that follow from Lemmas 1 and 2. If firm i can generate a strictly positive payoff, then it will not support wages sufficiently close to the competitive level q, since any atom at q generates a (point-wise) zero payoff. One thus concludes that the competitive wage level q is only supported by one firm if the other firm faces no friction and supports the unique wage level of q. The result in Lemma 2 is a replication of the familiar Bertrand paradoxical prediction that the outcome of a price (wage, in our case) competition between two firms coincides with the competitive equilibrium allocation. With no frictions in place, both firms engage in a "wage-war" which drives the equilibrium wage rate all the way up to its competitive level, where all rents are fully dissipated.

We now extend our analysis to account for the possibility of matching frictions. The next result establishes that in the presence of some friction, the Bertrand paradoxical result fails to hold. In particular, wage dispersion arises in equilibrium for any level of friction, which in turn implies that both firms derive positive rents. Theorem 1 also incorporates, as a special case, the frictionless scenario characterized in Lemma 2.

Theorem 1. *Given that $0 < p_1 \leqslant p_2 \leqslant 1$, the unique equilibrium is*

$$F_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(1-p_1)}{p_1(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases} \qquad F_2(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1. \end{cases}$$

Under the given equilibrium, the expected payoff of firm i is $p_i(1 - p_1)q$.

To fully grasp the economic intuition behind the expected payoff $p_i(1-p_1)q$, consider the symmetric case with frictions, and denote $p = p_1 = p_2 < 1$. The value $p(1-p)$ denotes the probability that an applicant is matched only with firm i. As, by presumption, the outside option of an applicant is normalized to zero, firm i can hire the applicant by offering him the minimal wage level, extracting the entire surplus and securing an expected payoff of $p(1-p)q$. The presence of matching frictions, thus, limits the extent of competition over the pool of applicants and allows firms to derive strictly positive rents.

Notice that under any asymmetric scenario in which, e.g., firm 2 has an advantage over firm 1, reflected in a higher probability of recruiting applicants conditional on both firms making the same wage offer (i.e., $p_2 > p_1$), it sets an atom at the minimal wage level, that is $\Pr(w_2 = 0) > 0$. This atom sustains the equilibrium by ensuring that firm 1 would find it optimal to offer the minimal wage level. For instance, when $p_2 = 1$ and in the absence of an atom, the probability that firm 1 would recruit applicants by offering $w_1 = 0$ would be zero, which clearly renders such a strategy suboptimal.

3.2 The dynamic set-up: an implementation issue

The previously studied single-stage problem should be viewed as a snapshot from a dynamic game with overlapping generations. Every individual is attached with the labor market for two consecutive periods (referred to as young and old generations, respectively): in the first period as a job applicant and in the second as an employee or unemployed, pending on match success. In any period the old generation of current employees and the young generation of job applicants overlap, and the former disseminate wage-related information to the latter. In the period which follows the old generation quits the labor market, whereas the hired young generation (from the previous period) switches roles to become the current period's old generation overlapping with the newly born young generation of applicants. We turn now to demonstrate how our results carry through as a stationary solution for the dynamic model.

We assume that each firm is maximizing its discounted sum of expected profits in the continuation game, and let the friction levels be fixed at (p_1, p_2). At the beginning of stage $t \geqslant 1$, every firm i employs a mass of employees under a wage distribution $G_{i,t-1}$, and chooses a distribution of wage offers, denoted by $F_{i,t}$. A new generation of applicants approaches both firms, and the single-stage game is played. After wages are realized and applicants are employed according to the relevant offers and probabilities, the "old" generation leaves the firms while the "new" generation of stage t becomes the "old" generation of stage $t+1$. At the beginning of stage $t+1$, each firm i employs a mass of employees with an associated wage distribution given by $G_{i,t}$, and chooses a corresponding distribution of wage offers given by $F_{i,t+1}$. This game continues indefinitely.

An immediate observation concerns the existence of a stationary sub-game perfect Nash equilibrium, which is attained by an infinite replication of the single-stage equilibrium characterized in Theorem 1. It is straightforward to verify that this indeed forms an equilibrium, since whenever one firm repeatedly plays its single-stage equilibrium strategy, replicating the optimal single-stage strategy constitutes a best response for the other firm. Note that we do not claim these equilibria are unique (as is seldom the case in repeated games), so our focus in the remainder of this section would be on the issue of implementability.

To see how the stationary equilibrium is implemented, let the t-stage wage distribution $G_{i,t-1}$ denote the state variable at stage t. As already observed, the distribution of wage offers $F_{i,t}$ typically does not coincide with the realized wage distribution $G_{i,t-1}$. To support the stationary equilibrium, each firm thus relies on wage secrecy policy, implicitly defined by a mapping from the state variable $G_{i,t-1}$ to the control variable $F_{i,t}$. In order to attain the desired distribution of wage offers each firm essentially "shuts down" a proper fraction of the informative offers. As shown in Lemma 3, the realized wage distribution G_i first-order stochastically dominates the distribution of wage offers, F_i.[2] The fact that wage secrecy arrangements turn out to be more prevalent at the higher end of the wage distribution appears to be empirically plausible, and essentially captures the role played by wage secrecy in restraining the competition between the two firms over the pool of job applicants.

To attain this "shut-down" property each firm can apply a simple binary rule such that some employees are allowed to disclose their level of remuneration, whereas others must refrain from sharing it. Alternatively, all employees could be subjected to identical non-disclosure clauses, but enforcement may vary and become stricter at the higher end of the distribution. To illustrate the role of wage secrecy to attain the desired mapping, consider the following example with a discrete support. Suppose that G supports two wage levels, 0 and q, with equal proportions. Further assume that the firm desires to induce the distribution $(2/3, 1/3)$ over $\{0, q\}$. The latter can be implemented by allowing all 0-wage employees to disclose their level of remuneration, while restricting one half of the q-wage employees from doing so.

An alternative method to implement F_i is to disallow any transfer of wage-related information from current employees. Once the transfer of information is completely restricted, each firm can resort to direct wage posting through alternative information channels, rather than through the employees. Such method requires a comprehensive secrecy policy such that current employees refrain from distorting the required distribution of offers. Either way, the stochastic dominance of G implies that some form of wage secrecy is necessary to support the distribution of wage offers, F.

[2]The time index is omitted to abbreviate notation, as we focus on a stationary equilibrium.

Notice that in a frictionless case both the distribution of wage offers and the distribution of realized wages collapse to an atom at the competitive wage rate, q. In this case, no wage secrecy is needed to support the stationary equilibrium, as the mass point at q constitutes an absorbing state. The following lemma characterizes the distributions of realized wage rates corresponding with the distributions of wage offers given in Theorem 1, and further establishes the first-order stochastic dominance of G over F.

Lemma 3. *Fix $0 < p_1 \leqslant p_2 \leqslant 1$, and consider the equilibrium distributions (F_1, F_2) from* Theorem 1. *The ex-post single-stage wage distributions of both firms is*

$$G_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(2q-w)(1-p_1)^2}{(q-w)^2(2-p_1)p_1}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases} \quad G_2(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{(p_2-p_1)(1-p_1)+\frac{w(1-p_1)^2(2q-w)}{2(q-w)^2}}{p_2(1-p_1)+\frac{p_1^2}{2}}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1. \end{cases}$$

For every firm i, the wage distribution G_i (first-order) stochastically dominates the distribution of wage offers F_i.

The proof is a straightforward computation according to Equation (1), hence omitted.

3.3 Vertical versus horizontal sampling

The extensive search literature emanating from the early seminal contributions by Stigler (1961, 1962), Diamond (1971), and Lippman and McCall (1976), emphasizes the costly nature of information acquisition by agents. The optimal search rule, typically characterized as repeated random draws from a given distribution following a cut-off (reservation) strategy, is known to depend on the properties of the distribution at stake. In two sided search models, the latter distribution is determined endogenously in equilibrium, both affecting, and being shaped by the optimal search rules. For tractability reasons and to facilitate our exposition, we have thus far invoked an exogenous search procedure adopted by all job applicants. We have therefore not examined the optimal search rule from the perspective of the applicants, nor have we tested the extent to which the search rule is affected by the firms' distribution of wage offers. In this section, we study the optimality and robustness of the invoked search procedures.

In the preceding analysis we assumed that job applicants draw one wage offer from each firm; we refer to this strategy as *horizontal* search. An alternative search strategy would be to draw twice from the same firm; we refer to this strategy as *vertical* search. The choice between horizontal and vertical sampling trades-off the likelihood of securing a job (enhanced by horizontal sampling), and the expected level of remuneration conditional on finding a job (enhanced by vertical sampling). Notice that the possibility to engage in vertical sampling hinges on a novel feature of our model, absent from standard search theoretic models of the labor market, which is the wage dispersion at the firm level. Such wage dispersion is essential for a wage-secrecy policy to be instrumental.

In this section we produce two key results. The first result establishes the dominance of horizontal sampling under the presumption of identical distributions of wage offers across firms. Indeed, by virtue of Theorem 1, the distributions of wage offers are identical in equilibrium under symmetric friction. However, every firm i's optimal choice of F_i was based on the presumption that job applicants resort to horizontal sampling, independently of the choices made by the firms. Therefore, one must consider the possibility of a profitable deviation (by at least one firm) to an alternative distribution of wage offers which induces vertical sampling from applicants. This leads us to the second result of this section.

Our second result establishes that, for sufficiently low levels of friction, the horizontal sampling is indeed invariant to the choices made by the firms, in the sense that no firm can profitably deviate from the equilibrium path by inducing applicants to switch to vertical sampling. Notably, for sufficiently high levels of friction, such profitable deviations become feasible (see the example following Theorem 2).

Before proceeding to the formal analysis we make two simplifying assumption for tractability reasons. First, we capture the costly nature of search by letting job applicants sample only twice, rather than solving endogenously for the optimal intensity of search. Second, we assume a uniform level of friction $p \in (0, 1)$. Relaxing these assumptions will not change the qualitative nature of our results.

Formally, let w_i and $\tilde{w}_i$ denote two i.i.d. wage samples from firm i. Given a uniform level of friction $p \in (0, 1)$ and an equilibrium profile (F_1, F_2), an applicant's expected payoff from sampling both firms is

$$\Pi_{12}(F_1, F_2, p) = p^2\mathbf{E}[\max\{w_1, w_2\}] + p(1-p)\mathbf{E}[w_1 + w_2] + 0 \cdot (1-p)^2,$$

while the expected payoff from double-sampling firm i is

$$\Pi_i(F_1, F_2, p) = p\mathbf{E}[\max\{w_i, \tilde{w}_i\}] + 0 \cdot (1-p).$$

To clarify, the aforementioned payoffs and strategies are derived from a two-stage game. First, the firms choose their distributions of wage offers (F_1, F_2), and then the applicants decide whether to sample vertically or horizontally. Thus, our analysis typically follows the notion of Subgame Perfect Nash Equilibrium (SPNE), where the applicants choose a sampling process and the firms optimize according to the applicants' optimal decision rule.

The following Lemma 4 establishes the optimality of horizontal sampling under the presumption of identical distributions of wage offers. Note that the statement of Lemma 4 is not confined to the equilibrium profile, but holds for any two identical distributions, which is indeed the case under a uniform level of friction according to Theorem 1.

Lemma 4. *For every friction level p and distributions of wage offers $F_1 = F_2$, it follows that horizontal sampling is a strictly dominant strategy (i.e., $\Pi_{12} > \Pi_i$).*

Next, we establish the invariance of horizontal sampling to the choices made by the two firms, assuming that the level of friction is sufficiently low.

Theorem 2. *Fix a friction level $p \in (0.62, 1)$ and assume that firm 1 follows the equilibrium strategy F_1. Then, for every distribution of wage offers F, horizontal sampling is a strictly dominant strategy (that is, $\Pi_{12}(F_1, F, p) > \Pi_2(F_1, F, p)$).*

Theorem 2 states that, for a low enough level of friction, no firm can profitably deviate from the equilibrium path characterized in Theorem 1, by inducing applicants to switch to vertical sampling. The rationale underlying the result derives from the fact that with low levels of friction, applicants extract much of the surplus, so that the deviating firm cannot offer much to its prospective employees. This situation changes markedly once friction levels are sufficiently high, as demonstrated in the following example.

Example 1. *Fix $p = 0.5$ and consider the symmetric equilibrium defined in* Theorem 1*. If firm 1 diverts to a distribution of wage offers F such that*

$$F(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q}{4(q-w)}, & \text{for } 0 \leqslant w < \frac{q}{2}, \\ \frac{1}{2}, & \text{for } \frac{q}{2} \leqslant w < q, \\ 1, & \text{for } w \geqslant q, \end{cases}$$

then the applicants' dominant strategy becomes Π_1 and firm 1 secures a payoff strictly greater than $\frac{q}{4}$, its payoff under the symmetric equilibrium defined in Theorem 1*.*

The underlying reasoning for the proposed deviation is as follows. The probability of a successful match under vertical sampling is given by p, whereas the corresponding probability under horizontal sampling is given by $[1 - (1 - p)^2] > p$. This generates a basic disadvantage for vertical sampling. To face this challenge the deviating firm needs to construct an elaborate strategy that simultaneously meets two requirements. First, it must maintain enough variation in its distribution of wage offers such that a double sample carries a significant advantage over a single sample. Second, the deviating firm must offer applicants a significantly higher expected remuneration relative to the other firm, such that the combination of the two criteria offsets the structural advantage of horizontal sampling. Both attributes are reflected in proposed distribution of wage offers given in Example 1, which is comprised of an atom at the maximal wage level q and a dispersed distribution over the support of the symmetric equilibrium defined in Theorem 1.

3.4 Strategic frictions

In the preceding sections we considered exogenous matching frictions that were shown to be essential for the emergence of wage dispersion and wage secrecy. In the current section we extend our analysis by allowing for an endogenous formation of frictions. These frictions may, for instance, be associated with the fact that applicants are partially informed about available job opportunities, say due to firms' limited advertising policies. It may also be associated with the firms' screening processes, where only a fraction of job candidates is eventually hired. Another possibility is the presence of a coordination friction in the random matching process, where several applicants may end up competing over a single open vacancy.

Regardless of the precise friction-inducing mechanism, the fact that job applicants are assumed to be homogenous raises a puzzling question: why should frictions emerge endogenously in equilibrium? That is, why should firms introduce frictions that limit their ability to recruit workers and, allegedly, bound their profits? As it turns out, frictions emerge in equilibrium as a coordination device, which restrains the competition between firms over the pool of applicants. In the spirit of the seminal study by Kreps and Scheinkman (1983), introducing a 'capacity constraint', which could take the form of posting a limited number of job vacancies, serves a firm to credibly commit to recruiting a limited number of workers. This in turn induces its rival to offer lower wage offers knowing that it would still be able to hire enough workers, and ultimately enables both firms to derive positive rents. Without introducing these frictions, a firm would induce its rival to engage in an intense wage competition á la Bertrand, resulting in full dissipation of the rents. We turn next to the formal analysis.

Consider the following two-stage game. In the first stage both firms choose simultaneously their desired level of friction (p_1, p_2), and in the second stage, firms choose their distributions of wage offers (F_1, F_2). Note that it is assumed that firms are committed to the friction levels chosen in the first stage. The solution concept we adopt is the Subgame Perfect Nash Equilibrium (SPNE). Relying on our previous analysis in Theorem 1, the following corollary characterizes the unique SPNE of the two-stage game.

Corollary 1. *In the unique* SPNE *one firm chooses a frictionless regime, while the other firm chooses a friction level of half,* $(p_i, p_{-i}) = \left(1, \frac{1}{2}\right)$*, and in the second stage both follow the distributions of wage offers given in* Theorem 1*. Under the given* SPNE*, the expected payoffs of firms* i *and* $-i$ *are* $\frac{q}{2}$ *and* $\frac{q}{4}$*, respectively.*

Two notable insights emerge from Corollary 1. The first concerns the fact that frictions do arise endogenously in equilibrium. The introduction of such frictions serves as a commitment device to restrain the competition over the pool of workers and ultimately ensure that, in equilibrium, firms derive positive rents. The patterns of equilibrium where the introduction of frictions by one firm is reciprocated by reduced wage offers by its rival, is a form of *tacit collusion* between the two firms. In the likely presence of matching frictions in the labor market (associated with reasons other than the strategic motive we describe), detecting the collusive behavior would pose a daunting challenge to any regulator.

The second, which is somewhat striking, concerns the asymmetric nature of the equilibrium, although both firms and workers are assumed to be homogeneous. The reason for this surprising result may be explained as follows. Provided that its rival is introducing some friction, a firm has a dominant strategy to refrain from introducing frictions so as to maximize its recruitment probability. Thus, a symmetric equilibrium with frictions is not feasible. An asymmetric equilibrium is however feasible, as switching to a frictionless regime would induce full dissipation of the rents for both firms (see our discussion following Theorem 1).[3]

4 Normative implications

In the current study we focused on the positive aspects of wage secrecy, so a concluding note on the normative implications is called for. Imposing a regulatory restriction that prevents firms from implementing wage-secrecy policies leads to a unique equilibrium in which all workers are remunerated according to their marginal productivity, and firms' rents are hence fully dissipated. With endogenously formed matching frictions, this would ensure that the aggregate social surplus is maximized. Thus, from an efficiency perspective, ruling out pay secrecy is socially desirable. Moreover, as wage secrecy entails wage dispersion amongst ex-ante homogeneous workers, ruling out pay secrecy may be warranted on equity grounds as well. However, as shown in Blumkin and Lagziel (2018), wage secrecy could be efficiency-enhancing at the firm level, serving to mitigate

[3]Notice that the asymmetry obtained in our set-up stands in contrast to Kreps and Scheinkman (1983) that derive a unique symmetric SPNE in a model with two identical firms engaging in a price competition á la Bertrand, after making capacity choices.

disincentives associated with relative pay/status concerns amongst workers. Thus, taking a broader perspective, one should be cautious in deriving direct policy conclusions from the current analysis.

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Appendices

Lemma 1. *For fixed values $p_1, p_2 \in (0, 1]$, the wage offers in equilibrium are supported on a connected set with no atoms in $(0, q)$.*

Proof. Fix F_2 and consider the point-wise payoff $\pi_1(w|F_2)$ of firm 1. Assume that policy F_2 produces an atom $w_0 \in (0, q)$, therefore π_1 is not continuous at w_0 such that $\lim_{w \to w_0^-} \pi_1(w|F_2) < \pi_1(w_0|F_2)$, and firm 1 would not support wage offers below and sufficiently close to w_0. Specifically, for a sufficiently small $\varepsilon > 0$, firm 1 can transfer any positive probability from $(w_0 - \varepsilon, w_0]$ to $w_0 + \varepsilon_0$ where $0 < \varepsilon_0 < \varepsilon$, and strictly increase its payoff due to the discontinuity. But if there exists an $\epsilon > 0$ such that firm 1's strategy does not support wages levels between $w_0 - \varepsilon$ and w_0, then the atom at w_0 is suboptimal. If either $F_1(w_0) > 0$ or $p_1 < 1$, then firm 2 can strictly increase its positive payoff $\pi_2(w_0|F_1) > 0$ by shifting the atom downwards, towards $w_0 - \varepsilon$; this shift reduces costs without affecting the probability to recruit since F_1 is fixed on $(w_0 - \varepsilon, w_0]$. Otherwise, if $F_1(w_0) = 0$ and $p_1 = 1$, then firm 2 cannot hire applicants at wage level w_0 and $\pi_2(w_0|F_1) = 0$. Thus, firm 2 can increase its expected payoff by shifting the atom upwards. The latter deviation provides a strict improvement unless firm 2's expected payoff is necessarily zero at any wage level, which occurs if and only if $F_1(w) = 0$ for every $w < q$. In other words, firm 2 can sustain an interior atom, in equilibrium, if and only if firm 1 follows an OP policy with a Dirac measure at q (a unique wage level of q).

Yet, by the indifference principle, any atom at q implies a zero payoff from any wage level. Namely, assume that firm 1 sustains an atom at q, while firm 2 does not employ a Dirac measure at q. There exists a wage level $w^* \in [0, q)$ such that $F_2(w^*) > 0$, and $\pi_1(w^*|F_2) > 0 = \pi_1(q|F_2)$, so firm 1 has a strictly profitable deviation from q to w^*. Therefore, no firm would support the wage level q with positive probability unless the other firm does so as well, since it assures an expected payoff of zero independently of the other firm's strategy. We conclude that no interior atoms exist, and the payoff functions are continuous on $(0, q]$, where continuity at q follows from the $(q - w)$ term of π_i.

We now prove that the distributions are supported on a connected set. Assume there exists an open interval $I = (w_-, w_+) \subset [0, q]$ such that $\Pr(w_2 \in I) = 0$, while $0 < F_2(w_-) < 1$. By the elimination of interior atoms, we can take the maximal I that sustains the above conditions. In other words, we take the maximal interval I such that for any other interval $I_0 \subset [0, q]$ where $I \subsetneq I_0$, it follows that $\Pr(w_2 \in I_0) > 0$. Since F_2 is fixed on I while w increases, it follows that π_1 is linearly decreasing on $[w_-, w_+]$ and $\pi_1(w_-|F_2) > \pi_1(w_+|F_2)$. Note that w_+ is generally not an atom of F_2 unless $w_+ = q$, which ensures a linear decrease towards zero, in any case. So for some small $\varepsilon > 0$, firm 1 would not support wage levels in $[w_+, w_+ + \varepsilon)$, as these wage levels are strictly dominated by wage levels in I, sufficiently close to w_-. However, the maximal choice of I suggests that the interval $[w_+, w_+ + \varepsilon)$ is supported by firm 2 with positive probability. This is clearly suboptimal since firm 2 has a strictly positive deviation of shifting these wage levels downwards. Therefore, we conclude that such I does not exist, and wage distributions are supported on a connected set, as needed. ■

Lemma 2. *If no frictions exist, $p_1 = p_2 = 1$, then there exists a unique equilibrium where both distributions of wage offers induce only the highest wage level q (i.e., both equal the Dirac measure δ_q).*

Proof. Assume that both firms choose an OP policy. The point-wise payoff of firm 1, given F_2, is $\pi_1(w|F_2) = \left[F_2(w) - \frac{\Pr(w_2=w)}{2}\right](q-w)$. If F_2 supports a unique wage level of q, then firm 1's weakly dominant strategy is to follow the same Dirac measure, establishing an equilibrium where both get a zero expected payoff. Any other strategy of firm 1 would provide a profitable deviation to firm 2, so there exists no other equilibrium where $\Pr(w_i = q) = 1$. Moreover, the indifference principle suggests that, in equilibrium, an atom at q exists only if the maximal expected payoff is zero, thus no other equilibrium exists such that $\Pr(w_i = q) > 0$.

We move on to prove uniqueness under the assumption that $\Pr(w_i = q) = 0$ for both firms. First, we eliminate the possibility of having an atom at 0. Assume that $\Pr(w_2 = 0) > 0$. If $\Pr(w_1 = 0) > 0$, then either firm can shift the atom upwards an profit by the increased probability of recruiting. Moreover, if only one firm supports an atom at 0, there is a zero probability to recruit applicants at this level, and the point-wise payoff is zero. Again, the indifference principle suggests that the maximal expected payoff at any wage level would also be zero, which leads to a unique atom at q, and the above-mentioned equilibrium.

Thus far we have established that any alternative equilibrium has no atoms, so the continuous payoff functions are given by $\pi_i(w|F_{-i}) = F_{-i}(w)(q-w)$. One can easily verify that F_1 and F_2 have the same support a.s., similarly to the reasoning presented in the proof of Lemma 1. Denote the support by I_0, and assume there exists a wage level $w \in I_0$ such that $0 < F_2(w) < 1$. This implies that the point-wise payoff at w and the expected payoff $\mathbf{E}[\pi_1(w_1|F_2)]$ of firm 1 are strictly positive. However, the fact $F_2(\inf I_0) = 0$ suggests that $\pi_1(\inf I_0|F_2) = 0$. By continuity, one can take a small $\varepsilon > 0$ such that $\pi_1(w|F_2) < \mathbf{E}[\pi_1(w_1|F_2)]$ for every $w \in I_1 = [\inf I_0, \inf I_0 + \varepsilon)$. This implies that the wage levels in I_1 are suboptimal for firm 1, but $\Pr(w_1 \in I_1) > 0$. A contradiction. We conclude that no alternative equilibrium exists, as stated. ■

Theorem 1. *Given that* $0 < p_1 \leqslant p_2 \leqslant 1$*, the unique equilibrium is*

$$F_1(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{w(1-p_1)}{p_1(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1, \end{cases} \qquad F_2(w) = \begin{cases} 0, & \text{for } w < 0, \\ \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}, & \text{for } 0 \leqslant w < qp_1, \\ 1, & \text{for } w \geqslant qp_1. \end{cases}$$

Under the given equilibrium, the expected payoff of firm i *is* $p_i(1-p_1)q$*.*

Proof. We first compute the point-wise and expected payoffs of both firms to establish an equilibrium, and later prove uniqueness. Note that the given strategies are well-defined as CDFs, both supported on $[0, qp_1]$, where F_1 is non-atomic and F_2 potentially has an atom of size $1 - \frac{p_1}{p_2}$ at $w = 0$. Given (F_1, F_2), the point-wise payoff functions are

$$\begin{aligned} \pi_1(w|F_2) &= p_1 \left[1 - p_2 \left[1 - F_2(w) + \tfrac{1}{2}\Pr(w_2 = w)\right]\right](q-w), \\ \pi_2(w|F_1) &= p_2 \left[1 - p_1 \left[1 - F_1(w)\right]\right](q-w). \end{aligned}$$

For $w \in (0, qp_1]$, the point-wise payoff of firm 1 is

$$\begin{aligned} \pi_1(w|F_2) &= p_1 \left[1 - p_2 \left[1 - F_2(w)\right]\right](q-w) \\ &= p_1 \left[1 - p_2 \left[1 - \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}\right]\right](q-w) \\ &= p_1 \left[(1-p_2)(q-w) + q(p_2-p_1) + w(1-p_2)\right] \\ &= p_1 (1-p_1) q, \end{aligned}$$

and the payoff is independent of w, establishing the indifference principle for any positive-measure set of valuations in $[o, qp_1]$. A similar computation for $w = 0$ would show $\pi_1(0|F_2) < qp_1(1-p_1)$. The latter inequality does not contradict the equilibrium statement since $\Pr(w_1 = 0) = 0$ and zero-measure suboptimal outcomes do no effect the expected payoff. Also, any wage offer above qp_1 is suboptimal, since it leads to higher wage levels without increasing the probability of recruiting an employee (by the fact that $F_i(pq_1) = 1$).

Similarly, for every $w \in [0, qp_1]$, the point-wise payoff of firm 2 is

$$\begin{aligned} \pi_2(w|F_1) &= p_2 \left[1 - p_1 \left[1 - F_1(w)\right]\right](q-w) \\ &= p_2 \left[1 - p_1 \left[1 - \frac{w(1-p_1)}{p_1(q-w)}\right]\right](q-w) \\ &= p_2 \left[(1-p_1)(q-w) + w(1-p_1)\right] \\ &= p_2 (1-p_1) q. \end{aligned}$$

Again, the payoff is independent of w, and similar arguments (as noted for firm 1) hold for firm 2.

We move on to prove uniqueness. In case $p_1 = 1$, we revert back to Lemma 2. The statement of Lemma 2 is embedded in the current one, so we can assume that $p_1 < 1$. Assume, to the contrary, that a different equilibrium (F_1, F_2) exists. We know from Lemma 1 that the distributions have no atoms at $(0, q)$ and the supports are connected sets.

We first focus on the least upper bound of the supports. Firm 2 can secure an expected payoff of at least $p_2(1 - p_1)q$ by fixing a Dirac measure at $w = 0$ (denote this measure δ_0). Therefore it will not support an atom at $w = q$, which produces a point-wise zero payoff. Using left-side continuity and the fact the support is connected, firm 2 will not support any wage levels close to q, thus firm 1 cannot support these wage levels as well. That is, wage levels close to q produce a point-wise payoff close to 0, while a strictly positive payoff for both firms can be secured by taking wage levels bounded away from q. We conclude that both firms have a strictly positive expected payoff, in equilibrium, while the least upper bound is strictly below q.

Let us now show that both distributions are supported on the same set of valuations.[4] Denote the support of F_i by I_i such that $\inf I_i = \underline{w_i}$ and $\sup I_i = \overline{w_i}$. If either $\underline{w_1} \neq \underline{w_2}$ or $\overline{w_2} \neq \overline{w_1}$, then one firm has a strictly decreasing payoff function at the high or low wage levels (the probability to recruit applicants remains fixed while wages increase). By Lemma 1 we know that both distributions are supported on a connected (positive-measure) set of valuations, so the latter conjecture yields a suboptimal expected payoff. We deduce that both distributions have the same support.

Denote $\underline{w} = \inf I_i$ and $\overline{w} = \sup I_i$, and let us prove that $\underline{w} = 0$. Assume that $\underline{w} > 0$. In that case, $\underline{w}$ is not an atom (by Lemma 1) and $F_i(\underline{w}) = 0$. Using left-side continuity, we get $\lim_{w \to \underline{w}^+} \pi_2(w|F_1) = p_2(1-p_1)(q-\underline{w})$, which is strictly less than $p_2(1-p_1)q$ that firm 2 can secure with δ_0. Hence, both distributions are necessarily supported on $\underline{w} = 0 < \overline{w} < q$. In addition, note that the profile of strategies where both firms support an atom at 0 cannot be an equilibrium, since each firm would revert to an infinitesimal increase, due to the discontinuity of the payoff function. So, we need to analyse the remaining possibilities of either no atoms, or a single atom for only one firm.

Consider the case where firm 1 does not have an atom at 0. We can employ the indifference principle for firm 2 over connected positive-measure sets, subject to F_1. The payoff function of firm 2 is continuous and point-wise equals $\pi_2(0|F_1) = p_2(1 - p_1)q$. The fact there are no atoms above $w = 0$ implies left-side continuity of the payoff function. Along with the indifference principle, it follows that the same point-wise payoff must hold throughout the support of F_2, specifically for

[4] We remind the reader that all statements hold almost surely, with probability 1.

$w \to \overline{w}^+$. Therefore,

$$\pi_2(\overline{w}|F_1) = p_2\left[1 - p_1\left[1 - F_1(\overline{w}) + \tfrac{1}{2}\Pr(w_1 = \overline{w})\right]\right](q - \overline{w}) = p_2(q - \overline{w}) = p_2(1 - p_1)q,$$

and $\overline{w} = qp_1$. Similarly, for every $0 \leqslant w \leqslant qp_1$, we get

$$\begin{aligned} p_2(1 - p_1)q &= \pi_2(w|F_1) \\ &= p_2\left[1 - p_1\left[1 - F_1(w) + \tfrac{1}{2}\Pr(w_1 = w)\right]\right](q - w) \\ &= p_2\left[1 - p_1 + p_1F_1(w)\right](q - w), \end{aligned}$$

which leads to $F_1(w) = \frac{w(1-p_1)}{p_1(q-w)}$, as already stated.

Now take $F_2(w)$ and the maximal wage level $\overline{w} = qp_1$. Left-side continuity and the indifference principle yield

$$\pi_1(\overline{w}|F_2) = p_1\left[1 - p_2\left[1 - F_2(\overline{w}) + \tfrac{1}{2}\Pr(w_2 = \overline{w})\right]\right](q - \overline{w}) = p_1(q - qp_1) = p_1(1 - p_1)q.$$

Applying the same reasoning as before, the point-wise payoff $p_1(1 - p_1)q$ must holds throughout the support of F_1. The latter statement holds up to a zero-measure set w.r.t. F_1 (which does not have an atom at $w = 0$ by assumption), so there is no problem with the evident discontinuity at $w = 0$, generated by the symmetric tie-breaking rule. Specifically, for every $0 < w \leqslant pq$, we get

$$\begin{aligned} p_1(1 - p_1)q &= \pi_1(w|F_2) \\ &= p_1\left[1 - p_2\left[1 - F_1(w) + \tfrac{1}{2}\Pr(w_2 = w)\right]\right](q - w) \\ &= p_1\left[1 - p_2 + p_2F_2(w)\right](q - w), \end{aligned}$$

which leads to $F_2(w) = \frac{q(p_2-p_1)+w(1-p_2)}{p_2(q-w)}$. Note that $\Pr(w_2 = 0) = 1 - \frac{p_1}{p_2} \geqslant 0$, so $p_1 < p_2$ leads to an atom of F_2 at $w = 0$.

We should now consider the other possibility where firm 1 supports an atom at $w = 0$. Denote $a = \Pr(w_1 = 0) > 0$. Since both firms cannot simultaneously have an atom at $w = 0$, we can use the continuity of π_1 and the indifference principle on connected positive-measure sets to compare $\pi_1(0|F_2)$ and $\pi_1(\overline{w}|F_2)$. Namely,

$$\begin{aligned} p_1(q - \overline{w}) &= \pi_1(\overline{w}|F_2) \\ &= \pi_1(0|F_2) \\ &= p_1\left[1 - p_2\left[1 - F_2(0) + \tfrac{1}{2}\Pr(w_2 = 0)\right]\right](q - 0) \\ &= p_1\left[1 - p_2\left[1 - 0\right]\right]q, \end{aligned}$$

which yields $\overline{w} = qp_2$. A similar comparison of $\lim_{w\to 0^+} \pi_2(w|F_1)$ and $\pi_2(\overline{w}|F_1)$, which follows from right-side continuity at $w = 0$, and left-side continuity at $w = \overline{w}$, yields

$$\begin{aligned} p_2(q-\overline{w}) &= \pi_2(\overline{w}|F_1) \\ &= \lim_{w\to 0^+} \pi_2(w|F_1) \\ &= \lim_{w\to 0^+} p_2\left[1 - p_1\left[1 - F_1(w) + \tfrac{1}{2}\Pr(w_1 = w)\right]\right](q-w) \\ &= p_2\left[1 - p_1\left[1 - F_1(0)\right]\right](q-0) \\ &= p_2\left[1 - p_1\left[1 - a\right]\right]q. \end{aligned}$$

Thus, $\overline{w} = qp_1(1-a)$. Since both distributions have the same support, we get $qp_1(1-a) = \overline{w} = qp_2$, and $p_2 = p_1(1-a) < p_1$. A contradiction to the initial condition of $p_1 \leqslant p_2$. In conclusion, F_1 is non-atomic whenever $p_1 \leqslant p_2$, and uniqueness follows. ■

Lemma 4. *For every friction level p and distributions of wage offers $F_1 = F_2$, it follows that horizontal sampling is a strictly dominant strategy (i.e., $\Pi_{12} > \Pi_i$).*

Proof. Fix $p \in (0,1)$ and $F_1 = F_2 = F$ supported on some set I of positive wage levels. We need to show that $\Pi_{12}(F,F,p) - \Pi_1(F,F,p) > 0$. That is,

$$\begin{aligned} \Pi_{12} - \Pi_i &= p^2\mathbf{E}[\max\{w_1, w_2\}] + p(1-p)\mathbf{E}[w_1 + w_2] - p\mathbf{E}[\max\{w_1, \tilde{w}_1\}] \\ &= p^2\mathbf{E}[\max\{w_1, \tilde{w}_1\}] + 2p(1-p)\mathbf{E}[w_1] - p\mathbf{E}[\max\{w_1, \tilde{w}_1\}] \\ &= (p^2 - p)\mathbf{E}[\max\{w_1, \tilde{w}_1\}] + 2p(1-p)\mathbf{E}[w_1] \\ &= p(1-p)\left[2\mathbf{E}[w_1] - \mathbf{E}[\max\{w_1, \tilde{w}_1\}]\right] \end{aligned}$$

where the second equality follows from the fact that all distribution are identical. To compute the expected value, note that $\mathbf{E}[X] = \int_{\mathbb{R}_+}[1 - F_X(t)]dt$. Hence,

$$\begin{aligned} \frac{\Pi_{12} - \Pi_i}{p(1-p)} &= 2\mathbf{E}[w_1] - \mathbf{E}[\max\{w_1, \tilde{w}_1\}] \\ &= 2\int_{\mathbb{R}_+}[1 - F(t)]dt - \int_{\mathbb{R}_+}[1 - F^2(t)]dt \\ &= \int_{\mathbb{R}_+}\{2[1 - F(t)] - [1 - F(t)][1 + F^2(t)]\}dt \\ &= \int_{\mathbb{R}_+}[1 - F(t)]\{2 - [1 + F(t)]\}dt \\ &= \int_{\mathbb{R}_+}(1 - F(t))^2 dt > 0, \end{aligned}$$

as needed. ■

Theorem 2. *Fix a friction level $p \in (0.62, 1)$ and assume that firm 1 follows the equilibrium strategy F_1. Then, for every distribution of wage offers F, horizontal sampling is a strictly dominant strategy (that is, $\Pi_{12}(F_1, F, p) > \Pi_2(F_1, F, p)$).*

Proof. Fix $p > 0.62$, the equilibrium strategy F_1, and a distribution of wage offers F supported on $[0, q]$. Let w_f and $\tilde{w}_f$ denote the two i.i.d. wage samples from F. We need to show that $\Pi_{12}(F_1, F, p) > \Pi_2(F_1, F, p)$. That is,

$$\begin{aligned}
\frac{\Pi_{12} - \Pi_2}{p} &= p\mathbf{E}[\max\{w_1, w_f\}] + (1-p)\mathbf{E}[w_1 + w_f] - \mathbf{E}[\max\{w_f, \tilde{w}_f\}] \\
&= \mathbf{E}[p\max\{w_1, w_f\} + (1-p)[w_1 + w_f] - \max\{w_f, \tilde{w}_f\}] \\
&= \int_{[0,q]} \{p\,[1 - F_1(t)F(t)] + (1-p)\,[1 - F_1(t) + 1 - F(t)] - \left[1 - F^2(t)\right]\}dt \\
&= \int_{[0,q]} \{F^2(t) + F(t)\,[-pF_1(t) - 1 + p] + (1-p)\,[1 - F_1(t)]\}dt \\
&= \int_{[0,qp]} \{F^2(t) + F(t)\,[-pF_1(t) - 1 + p] + (1-p)\,[1 - F_1(t)]\}dt + \int_{[qp,q]} \{F^2(t) - F(t)\}dt \\
&\geqslant \int_{[0,qp]} \{F^2(t) - F(t)\,[1 - p(1 - F_1(t))] + (1-p)\,[1 - F_1(t)]\}dt - \frac{q(1-p)}{4},
\end{aligned}$$

where the last inequality follows from a point-wise minimization of $F^2(t) - F(t)$. We can follow a similar point-wise minimization for the function in first integral w.r.t. $F(t)$, and get

$$\frac{\Pi_{12} - \Pi_2}{p} \geqslant \int_{[0,qp]} \{(1-p)\,[1 - F_1(t)] - \tfrac{1}{4}\,[1 - p(1 - F_1(t))]^2\}dt - \frac{q(1-p)}{4}.$$

Using F_1 explicitly, one gets $1 - F_1(t) = \frac{pq-t}{p(q-t)}$ and $1 - p(1 - F_1(t)) = \frac{q(1-p)}{q-t}$. Hence, the previous inequality translates to

$$\begin{aligned}
\frac{\Pi_{12} - \Pi_2}{p} &\geqslant \left[\frac{1-p}{p}\right] \int_{[0,qp]} \frac{pq-t}{q-t}dt - \left[\frac{q^2(1-p)^2}{4}\right] \int_{[0,qp]} \frac{1}{(q-t)^2}dt - \frac{q(1-p)}{4} \\
&= \frac{1-p}{p} \int_{[0,qp]} \frac{pq-t}{q-t}dt - \frac{q(1-p)p}{4} - \frac{q(1-p)}{4}.
\end{aligned}$$

A straightforward computation of the first integral yields $\int_{[0,qp]} \frac{pq-t}{q-t}dt = pq + q(1-p)\ln(1-p)$. Therefore,

$$\begin{aligned}
\frac{\Pi_{12} - \Pi_2}{p(1-p)q} &\geqslant \frac{p + (1-p)\ln(1-p)}{p} - \frac{1+p}{4} \\
&= \frac{3-p}{4} + \frac{(1-p)\ln(1-p)}{p},
\end{aligned}$$

and one can verify that the last function is strictly positive for $p \in (0.62, 1)$. ∎

Corollary 1. *In the unique* SPNE *one firm chooses a frictionless regime, while the other firm chooses a friction level of half,* $(p_i, p_{-i}) = \left(1, \frac{1}{2}\right)$*, and in the second stage both follow the distributions of wage offers given in* Theorem 1*. Under the given* SPNE*, the expected payoffs of firms* i *and* $-i$ *are* $\frac{q}{2}$ *and* $\frac{q}{4}$*, respectively.*

Proof. For every friction profile (p_1, p_2), Theorem 1 states that firm i's unique equilibrium expected payoff is $p_i(1-\min\{p_1, p_2\})q$. By this uniqueness outcome and the use of a SGPE, we can restrict the analysis to the preliminary stage of choosing the friction levels. Hence, we consider an axillary one-stage game where firms simultaneously choose friction levels (p_1, p_2) and firm i's payoff is $p_i(1-\min\{p_1, p_2\})$. Given $p_{-i} \leqslant 1$, the best response of firm i is either to play $p_i = 1 \geqslant p_{-i}$, which generates a payoff of $1 - p_{-i}$, or to choose some value $p_i < p_{-i}$, which yields a payoff of $p_i(1 - p_i)$. So, for $p_2 = 1$ the best response of firm 1 is $p_1 = 0.5$, and symmetry suggests that the best response of firm 2 is $p_2 = 1$, which establishes an equilibrium. Now fix a profile $(p_1, p_2) \neq (0.5, 1)$. Clearly $p_1 = p_2 = 1$ is not an equilibrium so we can ignore this possibility. Assume, w.l.o.g., that $p_1 \leqslant p_2$. Again, the best response of firm 2 is $p_2 = 1$, and then firm 1 would deviate to $p_1 = 0.5$. We revert back to the only possibility where one firm chooses a friction of half and the other chooses a frictionless regime, thus concluding the proof. ■