TRANSFERABLE DEPOSITS AS A SCREENING MECHANISM

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Transferable Deposits as a Screening Mechanism*

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Abstract:

We design incentives schemes for portfolio managers that screen low-skill managers: only the best portfolio managers, in terms of expected payoffs, agree to participate in the single-period investment. The results hold in general financial markets, where uninformed investors face managers of different capabilities, and can only observe their one-shot realized returns. Our model is robust and accounts for general screening problems.

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1 Introduction

One's ability to identify and harness top-level experts becomes a crucial requirement in an environment of growing complexity. The gravity of this problem is even more significant when entire markets and economies are at stake. The 2007–2009 financial crisis offers some insights on how ill-motivated experts can produce poorly-designed, risky contracts that, in return, unleash a dramatic chain reaction throughout the global economy.[1] In this paper, we tackle the problem of experts testing in such a set-up, i.e., in the delegated portfolio-managers market.

Our model deals with a single-period investment where an investor, faced with managers of different abilities, can only observe their realized returns. Every manager has a set of investment strategies such that high-skill managers produce excess expected returns relative to low-skill ones. The problem becomes significant by the potential discrepancy between realized and expected values. Given that low-skill manager use mimicking strategies, the investor cannot distinguish between different types until funds are lost due to poor investments. In this framework we wish to design incentives schemes such that *only the best* portfolio managers agree to participate in the single-period investment.

This goal is ambitious in several respects. First, experts testing typically involves some form of repetition.[2] The consecutive sampling enables the investor to gain additional information about the managers' subjective capabilities. However, such learning cannot take place in a single-stage model as ours. Moreover, we do not settle for an ex-post detection, but aim to deter low-skill managers from entering the market. Second, our model follows the strict assumptions of previous impossibility studies: the investor is subjected to a limited-liability constraint with no superior monitoring capabilities, while managers have no liquidation boundaries. Thus, we do not restrict low-skill managers from using mimicking strategies. Third, we assume that the investor has no prior information over the managers' individual capabilities, therefore she cannot use biased schemes and must treat all managers equally.

To face these challenges, we devise a constant-sum competition between managers. Out solution dictates that managers' collaterals are first pooled together, such that the devised contract (ex-post) compensates managers for a-priori depositing funds, according to their relative performance. Doing so, we bypass previous impossibility results where the deposits and returns of one manager do not balance off with the ones of other managers. In other words, we generate a trade-off between managers that incentivizes high-skill managers to participate in the investment, and others to refrain.[3]

[1]See, e.g., Hansen (2009); Fligstein and Goldstein (2010); Simpson (2011). For a general survey on the wrong incentives that led to the financial crisis of 2007–2009, see Fligstein and Roehrkasse (2013).

[2]See, e.g., Al-Najjar and Weinstein (2008) and Feinberg and Stewart (2008).

[3]We do emphasize that, for applicative purposes, deposits need not actually be transferred. The notion of transferable deposits could be applied through a simple reimbursement rule by the investor.

Our analysis also includes an impossibility result. In case potential-profits exceed 100%, we show that two managers of different abilities that interact with each other through risky binary options cannot be separated by any feasible contract. That is, in case one of the managers can more-than-double his funds while the other losses everything, then no contract, based solely on their realized returns, can distinguish between the two.[4] This double-or-nothing condition is not a mere technical issue: it goes to the core of our solution, and our understanding of the screening problem. It stems from the inability to compensate a manager's potential loss due to the personal collateral, with another deposit of the same amount. Therefore, the same condition is also crucial for our positive results.

One could find an interesting (and rather entertaining) precedent to our transferable-endowments notion in Warren Buffett's million-dollar bet with the money management firm, Protégé Partners.[5] The "Oracle of Omaha" made a 10-year bet, which expired on the last day of 2017, that the market performance of an index fund of his choice can beat the average performance of five hedge funds carefully chosen by Protégé. Ten years later, Buffett won with a compounded annual average return of 7.1% versus 2.2% of Protégé Partners. Though it is merely a friendly wager, the bet holds the same idea of *implementing a direct competition between two funds*, and granting the entire deposit of a suboptimal contender to the superior one (and, in this case, his charity of choice). This example also shows that our solution is not limited to the portfolio-managers setting. In fact, every competition that includes an entering fee, ranging from best-photography contests to submitting academic papers to top journals, could employ our reward scheme to attract only high-skill competitors, and thereby set a high-level competition.

1.1 A wide range of failing mechanisms and uniqueness

Though we advise the diligent reader to consider alternative solutions to the problem presented in this paper, we wish to emphasize several key components making this task practically impossible. In other words, our proposed solution is, in many respects, the unique screening mechanism for this problem.

Assume, for the sake of simplicity, there are two managers, 1 and 2, such that the former produces excess expected returns relative to the latter. The investor's goal is to devise a mechanism that allocates the entire fund to Manager 1, while excluding Manager 2 from participating in the investment. However, the investor has no prior information regarding their subjective abilities and she is restricted by a limited-liability constraint, so payments must be non-negative. Thus, the mechanism must

[4]The interaction between managers carries some resemblance to the more recent trend of studies concerning optimal contracting jointly with asset prices. See, e.g., Qiu (2009); Buffa et al. (2014); Malamud and Petrov (2014); and Garleanu and Pedersen (2015).

[5]Fortune, "Why Buffett's Million-Dollar Bet Against Hedge Funds Was a Slam Dunk" on May 11, 2016.

incorporate an entry fee (i.e., an opportunity cost) as inactive managers receive no payoff. In other words, the mechanism must induce self-screening on the side of Manager 2.

Now, consider the mimicking strategy proposed by Foster and Young (2010). They proved that low-skill managers can use options to match, with positive probability, the realized returns of high-skill managers. Their main result states that *any individual contract* that generates a positive expected payoff to Manager 1, will also generate a positive expected payoff to Manager 2. Their result also accounts for penalties (through entry fees), showing that personal collaterals are more problematic for high-skill managers, as they can produce excess expected returns for every dollar held in escrow

Therein lays the problem. Assume that Manager 2 invests according to this mimicking strategy, and otherwise matches any action (message, bid, commitment, and so on - depending on the mechanism) of Manager 1. As long as payments to one manager are independent of the other's return, Manager 2 will be willing to participate in the investment, assuming that Manager 1 does. A preliminary messaging system to determine the payoffs (as, e.g., a second-price auction where managers bid over the expected returns) cannot screen Manager 2, since any individual contract fails to reach an ex-ante screening, *independently of the method by which that contract was devised.*

Moreover, Lagziel and Lehrer (2018) showed that linearity w.r.t. realized returns is necessary to prevent any distortion of the managers' investment strategies. Therefore, benchmarking against a fixed expected return or a dynamic one, say the S&P 500, either fails to meet the basic condition of Lagziel and Lehrer (2018) as well as previous impossibility results, or fails to meet the limited-liability constraint. To summarize, combining the above conclusions with the use of personal collaterals suggests that linear monetary transfers between managers are a necessary condition for screening. Thus, we arrive to our proposed solution.

Note that we do not remain naive to the possible implications of our proposed contracts. Personal deposits fix an opportunity cost with potentially significant repercussions. The accumulated practical and theoretical experience teaches us that many suggested reforms, such as postponing bonuses and instituting clawback provisions, do not effectively limit the gaming ability of financial managers. Therefore, our theoretical analysis is aimed to what could be done relative to previous impossibility results, rather than what should be done.

1.2 Related literature and main contribution

For several decades, the portfolio-managers problem has been the focus of many empirical and theoretical studies.[6] A significant part of these studies derive impossibility results, showing that an investor

[6]See, among many others, Sharpe (1981); Barry and Starks (1984); Starks (1987); Scharfstein and Stein (1990); Chevalier and Ellison (1997); Carpenter (2000); Lo (2001); Hodder and Jackwerth (2007); Goetzmann et al. (2007); Van

cannot separate the skilled from the unskilled managers.[7] The reason is clear: it is quite difficult to detect low-skill managers, or even charlatans, in a noisy risky market with little to no prior information. As both fields are well-studied, we can only address several key papers that strongly relate to our work, starting with the work by Bhattacharya and Pfleiderer (1985).

Similarly to our work, Bhattacharya and Pfleiderer (1985) consider a one-period setting with no-learning involved. They showed that it is quite difficult, yet sometimes possible, to screen managers below a certain skill level. However, their results depend greatly on some form of risk-aversion, as well as additional restrictions over the distributions of the available assets. Normally-distributed returns are also a basic requirement in Admati and Pfleiderer (1997), which studies the design of a benchmark-adjusted schemes to infer the managers' skills from ex-post returns. Their analysis shows that the use of exogenous benchmarks in linear contracts is, generally, suboptimal with respect to risk sharing, exerting effort, and choosing the optimal portfolio for the investor. We solve this problem by generating a competition between managers such that each manager is compared to all other managers. In other words, our benchmark is endogenously derived from the induced-competition equilibrium, thus enabling screening. Bhattacharya and Pfleiderer (1985) and Admati and Pfleiderer (1997) also assume that managers can invest either in a risk free asset, or in a unique asset of normally-distributed returns. In such a set-up, the difference between managers depends solely on their private signals, while mimicking strategies are not considered.

The idea to use competition for screening instead of individual contracts is not new. Al-Najjar and Weinstein (2008) and Feinberg and Stewart (2008) proved, as do we, that competition-based screening can solve the impossibility result reached through individual testing. Yet, there are two complementing differences between our results and theirs. First, we study a single-stage environment while they study a repeated one. In accordance, we require ex-ante deterrence while they seek ex-post detection. In our context, ex-post detection can occur only after all funds are lost, thus cannot ideally solve the screening problem.

Foster and Young (2010) present one of the broadest impossibility result so far. In a risk-neutral environment, they explore the profitable ability of low-skill managers to mimic the investment strategies of high-skill ones. They also show that penalties either deter all managers, or deter none. In many respects, our positive result responds to their impossibility result with a single condition that returns are bounded by the double-or-nothing range. That is, we assume that managers cannot produce profits that exceeds 100% in a single time period. Though this assumption could be weakened and is broadly

Binsbergen et al. (2008); Dasgupta et al. (2011); Chassang (2013).

[7] The manipulation abilities of low-skill managers were proven to exist in many papers, such as Foster and Vohra (1998); Fudenberg and Levine (1999); Lehrer (2001); Sandroni et al. (2003); Sandroni (2003); Shmaya (2008); and Olszewski and Sandroni (2008).

discussed in Section 2.0.1, we stress that the length of a single time period is not limited in any way. Thus, for practical purposes, this restriction is quite weak.

Bounded assets are also vital for the positive result of He et al. (2015), showing that an investor with superior monitoring and regulating capabilities can screen low-skill managers. Their result depends on the ability to apply a liquidation boundary and prevent managers from exercising risky investment strategies. Recently, Carroll (2015) showed that linear contracts are optimal in a wide range of principle-agent settings. This outcome slightly resembles the results of Lagziel and Lehrer (2018) and the current work. However, Carroll's optimality relates to the worst-case outcome and individual contracts, thus falls short of the current goal.

In light of previous studies, we can underline several key contributions of the current work. First and most importantly, we produce a scheme that screens low-skill managers in a single-period setting, with no prior information and without distorting the participating managers' incentives. In other words, our solution eliminates the agency problem by achieving an optimal screening outcome. Note that this was considered unachievable even under the use of deposits and penalties.[8]

Our set-up is general and robust, resembling the work of Carroll (2015), but with a stricter goal and less information on the side of the investor. This generality enables us to minimize the cost of applying our solution, i.e., minimize the size of the required deposits. In addition, our results support dynamic settings, and would produce the same outcomes once used repeatedly.

1.3 Structure of the paper

The paper is organized as follows. In Section 2 we described a two-manager model. Section 3 presents the proposed scheme along with the first main result such that low-skill manager is screened from investing. In Section 4 we extend the model and previous results to numerous portfolio managers. Our impossibility result with several concluding remarks and comments are given in Section 5.

2 The model - a two-manager framework

Consider an investor and two fund managers, all risk neutral. Every Manager i can invest in any portfolio from a set A of available financial assets, and has an *optimal* portfolio Y_i which maximizes expected returns.[9] Denote $v_i = \mathbf{E}[Y_i]$, and assume Manager 1 has superior trading abilities than Manager 2 in the sense that $v_1 > v_2 \geqslant 1$. The excess abilities of Manager 1 may stem from several

[8] see, e.g., Stracca (2006) for a general survey of the screening problem along with previously-mentioned impossibility results).

[9] The return on an investment is its realized value, whereas the profit relates to the surplus. E.g., a 5% profit, turning \$100 into \$105, is referred to as a 1.05 return on the investment.

sources, such as a superior understanding of the market, lower transaction costs, a technological trading hedge, and so on. The investor has an initial amount of $2w$ that she wishes to invest using the managers. For that purpose, she introduces a mechanism and allows managers to decide whether to participate in the investment. Ideally, the investor lures only Manager 1 to invest her funds and only in Y_1.

Formally, the problem begins when the investor publicly commits to a mechanism (C, f), which consists of a *scheme* $f : \mathbb{R}_+^2 \to \mathbb{R}_+$ and a deposit $C \geqslant 0$. The scheme is a payoff function and the deposit is an entry fee for participating. The mechanism induces an *investment game*, denoted G_f, which is a 2-stage game. In the first stage, every Manager i decides whether to participate (i.e., become *active*) or decline. This decision is based on their available information. Namely, we assume that $\{v_i\}_{i=1,2}$ are common knowledge among the managers.[10] In the second stage, the problem splits into two complementing scenarios depending on the number of active managers.

In case both managers are active, they get an equal share of w, and individually decide on investments. Let $\sigma = (\sigma_1, \sigma_2)$ be their investment strategies, where σ_i is Manager i's portfolio. The expected payoff of Manager i is $\pi_i = \mathbf{E}[f(\sigma_i, \sigma_{-i})] - Cv_i$, and the expected gross return on the investor's portfolio is $w\mathbf{E}[\sigma_1 + \sigma_2]$. Note that f defines the payoff of both managers, assuming both are active, while Cv_i is the individual opportunity cost for participating. However, if only Manager i decides to be active, then he receives the entire amount of $2w$ to invest. In this case, we assume that no deposit is needed and Manager i's payoff is fixed to be a share $\lambda \in (0, 1)$ of the overall funds after the investment is realized. Namely, if Manager i is the only active manager, then his dominating action is the optimal portfolio Y_i and his expected payoff is $\pi_i = 2w\lambda\mathbf{E}[Y_i]$, whereas the investor's expected net profit is $2w\mathbf{E}[(1 - \lambda)Y_i - 1]$. In any case, the payoff of an inactive manager is set to zero.

The mechanisms we consider satisfy, by definition, two essential conditions: *limited liability* and *no prior information*. Limited liability follows from the non-negativity of f, such that any penalty for bad performance is limited to the managers' personal uninvested funds (deposit C), previously held in escrow in some risk-free asset. This requirement is presumably the toughest obstacle in designing a screening mechanism, since it significantly limits the investor's ability to impose penalties on under-preforming managers (ex post). Moreover, the funds held in escrow generate a significant loss to high-performance managers, relative to low-performance managers, thus resulting in a difficulty to effectively deter low-skill managers without deterring the better ones. Using invested funds to penalize managers is not considered feasible, since such funds are possibly lost due to bad investments. The second condition of no-prior-information implies that f cannot depend on private values, $\{v_i\}_{i=1,2}$.

[10]This assumption is discussed more broadly in Section 3.1, and in Section 4.3 we describe how it could be omitted completely in a more general framework.

The considered (C, f)-mechanisms may seem arbitrary, but they go to the core of the problem in question. As mentioned in Subsection 1.1, one can consider various mechanisms ranging from auctions to cheap talk. However, the outcome of all these procedures is some payoff function, a contract. Once a limited-liability constraint is placed, the payoff plan must include deposits, to enable an expected payoff below the zero-profit of screened managers. Thus, the (C, f) mechanisms enable us to abstract from all preliminary stages and focus on the actual payoff functions, independently of the methods by which these functions were generated.

A mechanism is called a *screening mechanism* if, in every equilibrium, only Manager 1 is active and invests only in Y_1. We should state that our solution also accommodates for optimal returns, in a first-best sense, with respect to the investor's expected net profit (see Remark 1 after Theorem 1).

2.0.1 Main assumptions

There are two main assumptions in our model that we wish to clarify: (i) distribution of available portfolios, and (ii) risk neutrality. Starting with the former, we assume that all managers can produce returns ranging from losing the entire fund to doubling it. That is, $\Pr(0 \leqslant \sigma_i \leqslant 2) = 1$ for every portfolio σ_i, and every Manager i. The reason for these bounds is found in our transferable-deposit notion. The upper bound limits the realized opportunity cost of superior managers from a deposit of C. A return above 2 implies a theoretical loss of more than 100% due to a deposit of C. This loss cannot be matched by another deposit of the same amount.

From a practical perspective, this restriction is weak, as one can shorten the basic time frame such that the probability of more than doubling the fund is sufficiently close to zero. For example, in the Buffett-Protégé wager neither party exceeded this threshold. Otherwise, we make no additional assumptions over the distributions of portfolios. Our results are robust in the sense that we condition neither on the types of distributions, nor on possible correlations or dependence. For the sake of simplicity, we fix the risk-free return rate to zero. However, one could assume that the returns generated by the portfolios are relative to some non-zero risk-free rate.

The second assumption we wish to clarify is the managers' risk neutrality. If managers are risk averse, the investor can impose simple restrictions to eliminate low-skill managers. For example, the investor may require that active managers invest their own private funds in the same portfolio, and by doing so he can eliminate managers that do not generate excess returns relative to a risk-free asset. For this reason, the assumption that managers are risk neutral complicates the screening problem. Moreover, the fact that $v_i \geqslant 1$, for every Manager i, implies that personal investments cannot deter risk-neutral managers, augmenting the complications of the screening problem.

3 Screening mechanisms

The design of a screening mechanism must be a based on a combinations of penalties and positive rewards, carrots and sticks. Any mechanism based solely on penalties will not attract Manager 1, just as strictly positive payoffs will not deter Manager 2. Our proposed mechanism (C, f) is based on a deposit $C > 0$, which will be later determined, and our screening scheme defined as follows. Assume that Manager i produces a realized returns of r_i. The *Transferable-Deposits* (TD) scheme f states that

$$f(r_i, r_{-i}) \quad = \quad \lambda w r_i + C\left[1 + \frac{r_i - r_{-i}}{2}\right].$$

In words, every Manger i is paid a share $\lambda \in (0, 1)$ of his portfolio's realized value wr_i, and receive a portion $\left[1 + \frac{r_i - r_{-i}}{2}\right]$ of the deposits which depends on the difference between realized returns. Note that the realized returns (r_1, r_2) are public, therefore the TD scheme is well-defined (i.e., sustains limited-liability and no-prior-information assumptions).

An important property of the TD scheme is the relation between the rewards and deposits, given by the second term of the scheme (a term similar to the General Reward Scheme presented in Lagziel and Lehrer (2018)). The potential loss of Manager 1 due to the opportunity cost, is matched to the loss of Manager 2 by the expected reimbursement. The reimbursement bypasses the main problem raised by previous impossibility results. Moreover, the scheme's linearity suggests that the unique *dominant-strategy* equilibrium of the induced investment-game is $\sigma = (Y_1, Y_2)$. Thus, the expected payoff of an active Manager i, including the opportunity cost, is $\pi_i = \lambda w v_i + C\left[1 - \frac{v_i + v_{-i}}{2}\right]$.

The following theorem specifics the values of C such that the scheme induces a screening mechanism. (All proofs are deferred to the Appendix.)

Theorem 1. *If* $\frac{C}{2\lambda w} \in \left(\frac{v_2}{v_1+v_2-2,}, \frac{v_1}{v_1+v_2-2}\right)$*, then the mechanism induced by the proposed scheme is a screening mechanism.*

Note that the deposit of every active manager is proportional to his basic fee of λw. This is necessary since a screening mechanism must ensure that, in every equilibrium, the expected payoff of a low-skill manager is negative, while maintaining a positive expected payoff for high-skill managers. In addition, any reduction in the asset-under-management fees λ immediately reduces the required deposits. Interestingly, one of the crucial aspects in Buffet's million-dollar bet is the high fees of actively-managed funds. Warren Buffet claimed (and, probably, still does) that the fees consume much of the investors' profits, thus supporting low-fees investment strategies. Our solution supports this goal through the deposits-fees relation. This fees-cutback idea also motivates the following observation regarding the screening notion used in Theorem 1 and throughout the paper.

Remark 1. *The main goal throughout the paper is to screen low-skill managers. However, our mechanism also solves the agency problem in a first-best sense. From a theoretical perspective, taking* λ *to be infinitesimally small and applying our solution scheme would not change the result of* Theorem 1*, since deposits and payoffs are proportional to* λ*. A minor technical problem is the inability to take* $\lambda = 0$*, since positive payments must be made. Thus, the set of feasible solutions is open allowing only for an* ϵ*-optimal solution.*

Before extending Theorem 1, let us consider a concrete environment. Fix the fee λ to 0.5% and assume that the optimal portfolio of Manager 1 produces 5% w.p. 1, whereas Manager 2's optimal portfolio produces the same rate w.p. $\frac{1}{1.05}$, and -100% otherwise. Using the condition of Theorem 1, take $C = 0.205w$. Specifically, an active manager must deposit approximately 20¢ for every dollar bestowed in his hands to invest. When doing so, Theorem 1 shows that Manager 1 would be the only active manager. A priori, a 20¢ on-the-dollar deposit may seem excessive compared, e.g., to the Basel III regulatory framework that requires less than roughly 12% in high-quality liquid assets.[11] However, this assessment is quite naive as we do not limit the time frame for evaluation. Namely, we could consider shorter or longer time frames that do not violate the double-or-nothing range, and potentially reduce the effective size of the deposit, relative to the potential gains.

3.1 Determining C

A problem arising from Theorem 1 is the difficulty to accurately determine C with no information regarding the managers' potential portfolios. For this problem we offer several solutions. First, the implementation of our scheme requires only the average of optimal expected returns, since one can fix $C = \lambda w \left[1 - \frac{2}{v_1 + v_2}\right]^{-1}$ according to the given bounds. Second, the result holds for a range of values, hence some error in C is acceptable. However, the most important part is the ability to generate an endogenous C using a *Dutch auction*.

Consider the following Dutch auction for C starting with a sufficiently high value. The fee is lowered until a manager agrees to deposit the stated amount. Next, the entry fee is fixed accordingly and both managers are offered to become active. In such a set-up, there exists an equilibrium where an always-active Manager 1 bids within the relevant range of Theorem 1. In such a case, Manager 2 cannot gain from overbidding and will never become active, maintaining the screening solution.

[11] See Atkinson and Blundell-Wignall (2010); Slovik and Cournède (2011); Financial Times, "Europes banks face tougher demands" on July 15, 2012; Wall Street Journal, "Basel III Is Simpler and Stronger" on October 14, 2012.

4 A managers market

In this section, we show how our solution could be used with multiple managers. Formally, consider $k \geqslant 2$ portfolio managers, and recall that Manager i's optimal portfolio is Y_i where $\mathbf{E}[Y_i] \geqslant 1$. Denote $v_i = \mathbf{E}[Y_i]$ and assume that the managers' indices are aligned according to their skill level, such that a lower index implies a higher optimal expected return (i.e., $v_i > v_j$ for every $i < j$).

The k-manager set-up evolves similarly to the 2-manager one. The investor (potentially a market designer of some kind) determines the mechanism (C, f) where $C \geqslant 0$ is the deposit for entering the market, and f is a k-manager scheme, which defines the payoffs for every set of active managers. That is, for every set of active managers of size $2 \leqslant l \leqslant k$, the vector $r \in \mathbb{R}^l_+$ states their returns, and $f(r_i, r_{-i})$ defines the payoff of an active Manager i. Every manager, who is willing to be active, pays the deposit C and receives an initial amount of w to invest.[12] In case only a single manager is active, assume that no deposit is needed and the manager gets a share of the realized value of the managed funds.

In this section we assume that the yields of all portfolios are bounded by a $\pm 100R\%$ profit rate, where $0 < R \leqslant 0.5$. That is, $\Pr(1 - R \leqslant Y \leqslant 1 + R) = 1$ for every portfolio Y. Though this assumption is stricter than the assumption given in the previous section, we still do not limit the time frame by which the managers are assessed. Therefore, one can take a sufficiently-short time frame such that this assumption holds for practical purposes.

4.1 A general screening scheme

Define the *General Scheme* (GS) for every active Manager i with a realized return r_i by

$$f(r_i, r_{-i}) = \lambda w r_i + C\left[1 + \frac{r_i - \tilde{r}_{-i}}{2R}\right],$$

where $\tilde{r}_{-i}$ is the average realized return of all active managers excluding Manager i. In words, every active Manager i with a realized return of r_i, is paid a fee of $\lambda w r_i$ along with a compensation $C\left[1 + \frac{r_i - \tilde{r}_{-i}}{2R}\right]$ based on Manager i's return relative to the others' average. This offset is managed by the investor using the initial deposits of all active managers. Again, the GS admits limited-liability, no-prior information, and depends linearly on returns. Thus, the dominant strategy of every active Manager i remains Y_i.

An important observation regarding the GS is the relation between the decision to become active and managers' abilities. Specifically, if C is fixed such that a low-skill manager gains from entering

[12] To simplify the computation and without loss of generality, we assume that w is uniform among active managers. Since all payoffs are linear in w, one could change this assumption by fixing w according to the number of active managers.

the market, the same decision holds for any manager of higher skill. This is proved in Lemma 1.

Lemma 1. *In every equilibrium induced by the* GS *with more than one active Manager j, every superior Manager $i < j$ is also active.*

The more-than-one condition prevents an absurdly high C from being fixed, such that no manager can gain from entering the market along with another manager. Otherwise, the only equilibria include a single (arbitrary) active manager with no deposit and no competition. One can also replace this condition by an upper bound on C, as indicated in Subsection 4.2 that follows.

The monotonicity effect presented in Lemma 1 has a complementary aspect: high-skill managers can completely drive out low-skill managers from the market. Namely, when high-skill mangers become active, they increase the market's average, thus increasing the benchmark of all other active managers. The increase leads to higher losses for below-average managers, that could result in an elimination from the market. In some respects, the GS is designed to create *an inverse effect to the adverse-selection problem* á la Akerlof (1970), as high-skill managers drive out low-skill managers form the market through the direct competition and the high-level induced benchmark.

4.2 Relatively superior managers

Before we present the main result of this section, a few notations are needed. For every Manager i, let $S_i = \{1, \ldots, i\}$ be the set of managers that are at least as good as Manager i in terms of optimal expected returns. For every $i > 1$, denote $\tilde{v}_{-i} = \frac{1}{i-1}\sum_{j=1}^{i-1} v_j$ to be the average expected return of managers superior to Manager i and define

$$\alpha_i = \frac{v_i}{\frac{1}{2R}[\tilde{v}_{-i} - v_i(1-2R) - 2R]}$$

to be the ratio between the expected return of Manager i and the expected loss for depositing C (including the induced compensation) in case only more skilled managers are active. Since $\tilde{v}_{-1}$ is not defined, fix $\alpha_1 = \frac{v_1}{v_1 - 1}$. These α_i-ratios play a crucial part in the ability to screen out inferior traders.

We say that Manager i is *relatively superior* (RS) if $\alpha_i > \alpha_j$ for every $j > i$. That is, Manager i is RS if its expected optimal return, relative to the potential loss, due to superior active managers, is higher than the same ratio of every inferior Manager j, where $j > i$. A straightforward examination shows that the RS condition is equivalent to the monotonicity of $v_i/(\tilde{v}_{-i} - 2R)$ w.r.t. i. Specifically, Manager i is RS if

$$\frac{v_i}{\tilde{v}_{-i} - 2R} > \frac{v_j}{\tilde{v}_{-j} - 2R}, \tag{1}$$

for every $j > i$.

One can verify that the condition given by Ineq.(1) becomes weaker as R decrease. In other words, a manager remains RS when limiting the support to $R' < R$ instead of R. In addition, Ineq. (1) is sensitive to changes in R. For example, taking the maximal $R = 0.5$, meaning that the portfolios realizations range between -50% and $+50\%$, implies that the condition does not hold whenever the expected optimal returns change linearly. That is, assuming that $v_i = a - (i-1)b$ for some $a > 1$ and $b > 0$, there exists no relatively-superior manager. However, if $R = 0.25$ and given the same linearity, then every manager is RS. In fact, if $R = 0.25$ and the expected returns are decreasing according to some concave function, then every manager is RS.

The necessity of the RS condition follows from the ability of low-skill managers to enter the market and decrease the average expected returns, thus reducing the penalty for poor performance. Once the penalties are reduced, the deposit C must be sufficiently high to deter low-skill managers. However, if low-skill managers choose not to participate, then the average expected return remains high, and combined with a high deposit, might also deter high-skill managers from actively investing. Hence, the expected optimal return of inactive managers must be bounded away from the lowest-active manager's return, such that the investor has a sufficient margin to keep the deposit C as low as possible, while deterring low-skill managers.

From a technical preceptive, the decision of a low-skill manager to enter the market causes a discontinuity in $\tilde{r}_{-i}$, and the margin produced by the relatively-superior condition enables the investor to deal with such discontinuity. The following theorem shows that the relative superiority is a necessary and sufficient condition to screen out managers below a certain skill level.

Theorem 2. *Consider the* GS*-induced game. Manager i is relatively-superior if and only if there exists $C < \lambda w \alpha_i$ such that, in every equilibrium, only managers from S_i are active. In addition, Manager i is relatively-superior to Manager $i+1$ (i.e., $\alpha_i > \alpha_{i+1}$) if and only if there exists $C < \lambda w \alpha_i$ and an equilibrium where the set of active managers is S_i.*

Note that the first statement of Theorem 2 holds in every equilibrium, given that Manager i is RS. To compare, the second statement requires a much weaker condition where Manager i is RS to Manager $i+1$. The latter condition becomes even weaker in case the number of managers is large, such that the influence of every manager over the market is negligible. Similar redundancy could arise in case managers are allowed to enter the market with different volumes of trade. Another possible extension for future research is to condition C on the number of active managers. Such extension might eliminate the RS condition completely.

4.3 Private valuations and an endogenous C

The transition from two managers to numerous managers caries some advantages concerning the investor's available information. The screening of a low-skill manager among two possible candidates leaves the investor with no benchmark to asses the performance of the, allegedly, high-skill manager. Thus, the investor cannot re-evaluate the performance of such manager and reward or penalize him, in accordance. This lack of information also affects the investor's ability to accurately, and a priori, fix the entry fee C. On the other hand, the possibility to maintain more than one manager (post screening) enables the investor to use the different active managers to evaluate each other in a competitive set-up. In addition, it also enables the investor to endogenize C to the point where he needs no information over the possible expected returns in the market.

Specifically, consider a mechanism similar to the one described in Subsection 3.1 where the entry fee C is fixed through an auction among the managers. To simplify the auction, assume that C is proportional to λw, i.e., proportional to the active managers' share of the managed funds. Now, assume that the investor runs a first-price auction for C when the highest value defines the entry fee for all managers. Contrarily to the Dutch auction mentioned in Subsection 3.1, the investor also establishes a *competition-only* rule where no funds are distributed when only a single manager agrees to pay the entry fee. That is, the investment is annulled when only one manager agrees to participate.

The competition-only rule eliminates the possibility of an over-bidding always-active manager, as described in Subsection 3.1. That is, no manager can gain from overbidding C and eliminating all other managers from the market. Thus, the conditions of Lemma 1 are met and high-skill managers can fix C (through their bidding strategy) to eliminate low-skill managers such that at least two managers remain in the competition. In return, the investor needs no prior information about the ability of the available managers, other than the R-bound which could be manipulated based on the time frame in question. For practical purposes, high-skill managers have a strong incentive to eliminate some of the competition, as their expected payoff is proportional to the amount of funds they manage. Therefore, we conclude that the investor can use the market to induce a self-sustainable competition, that inherently generate the required information for its existence.

A more important extension (under the general framework and the competition-only rule) relates to managers' common-knowledge of others' abilities. Previous results are based on the assumption that the abilities of managers are common-knowledge among managers, so one must wonder what happens in case this assumption is violated. Namely, assume that the values $\{v_i\}_{1\leqslant i\leqslant k}$ are independently drawn according to a common distribution F over $[1, 1+R]$, such that every v_i is private information of Manager i, while F is common knowledge among managers. Therefore, managers have noisy assessments of others' abilities, and their strategies depend on these assessments.

Now, the investor can use an English auction where C is set higher at any given stage. Conditional on others' strategies and C, every manager can derive his expected benchmark. That is, managers update their common belief over others' abilities and their expected benchmark, and the competition-only rule suggests that at least two managers must remain in the competition. If Lemma 1 remains valid, then low-level managers will leave the competition prior to high-level ones. Though the complete analysis is more extensive and complicated (thus left for future research), we conjecture that this process will eventually yield and equilibrium where only top-level managers participate in the investment, while managers' abilities remain private.

5 Discussion

5.1 The impossible competition

A natural question regarding the bounds over returns is whether the result of Theorem 1 is extendible beyond the double-or-nothing condition. Though the answer might be positive in some cases, we can also find situations where the answer is negative for any symmetric and feasible scheme. We describe here one possible scenario where two managers of different capabilities interact in a risky binary investment, making them inseparable from an uninformed investor's perspective.

Consider the two managers case described in Section 2, and assume that both managers interact with each other only via the options market, similarly to the cash-or-nothing puts-based strategies of Foster and Young (2010). Specifically, let (Y_1, Y_2) agree with the joint distribution given in Table 1, where $a > 2$ and $\mathbf{E}[Y_1] > \mathbf{E}[Y_2] \geqslant 1$. That is, the managers are entangled in an investment where

$Y_1 \backslash Y_2$	0	a
0	0	$1-p$
a	p	0

Table 1: The joint distribution of (Y_1, Y_2).

one generates a return of $100a\%$ (with probability $p > 0.5$ in favor of Manager 1), whereas the other's return is -100%. The following lemma shows that a symmetric and feasible scheme f cannot screen Manager 2.

Lemma 2. *Let f be a feasible scheme (i.e., sustains limited-liability and no prior information). For every $C \geqslant 0$ there exists an equilibrium where Manager 2 is active.*

Proof. Fix a feasible scheme f with a deposit $C \geqslant 0$. Assuming both managers are active, their expected payoffs are

$$\begin{aligned}\pi_1 = \mathbf{E}[f(Y_1,Y_2)] - C\mathbf{E}[Y_1] &= pf(a,0) + (1-p)f(0,a) - Cap,\\ \pi_2 = \mathbf{E}[f(Y_2,Y_1)] - C\mathbf{E}[Y_2] &= (1-p)f(a,0) + pf(0,a) - Ca(1-p).\end{aligned}$$

Denote $C_+ = \frac{(1-p)f(a,0)+pf(0,a)}{(1-p)a}$ and $C_- = \frac{pf(a,0)+(1-p)f(0,a)}{pa}$. A straightforward examination shows that $p > 0.5$ yields $C_- \leqslant C_+$. One can verify that a deposit $C \leqslant C_+$ ensures that $\pi_2 \geqslant 0$, thus Manager 2 would become active. Otherwise, a deposit $C > C_+$ produces negative expected payoffs for both managers in case both are active. Thus, there exist two equilibria, where in each a different Manager is active, and this concludes the proof. ■

We could further extend Lemma 2 to cases where both managers lose their funds at the same time with positive probability, i.e., to cases where $\Pr(Y_1 = Y_2 = 0) > 0$. However, the fact that both managers cannot produce profitable investments simultaneously is a necessary condition. Specifically, for every arbitrarily small, yet positive, probability $\Pr(Y_1 = Y_2 = a)$ there exists a scheme that screens out only Manager 2. The reason for this zero-probability constraint lays in the limited-liability condition. Once this probability is positive, the opportunity cost of both managers increase by the same term of $aC\Pr(Y_1 = Y_2 = a)$, thus allowing the investor to enlarge both the required deposit and the compensation for the highest earning manager. Though this increase affects both managers, it is (relativity) more significant for Manager 2 since his opportunity cost, as well as his probability of being the highest earning manager, are originally lower than the ones of Manager 1.

5.2 Concluding remarks

In this paper we presented a method for, a-priori, screening low-skill managers, using liability contracts. Our design shows that personal collaterals could be effective, once used to compensate high-skill managers for their potential losses. These contracts apply in either small, or large scale environments, while using traditional share-the-profits incentives. However, the practical decision to use deposits in the portfolio-managers market should not be taken in a light-headed manner. The implications of such restrictions (in a relatively open market) bears a lot of risk towards the managers and financial institutions that may surpass the advantages. Therefore, our theoretical analysis should be taken with prudence, since it mainly establishes what can be done, rather than what should be done.

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6 Appendix

Theorem 1. *If* $\frac{C}{2\lambda w} \in \left(\frac{v_2}{v_1+v_2-2}, \frac{v_1}{v_1+v_2-2}\right)$*, then the mechanism induced by the proposed scheme is a screening mechanism.*

Proof. Given that both managers are active, the linearity of f_i w.r.t. the return of Manager i implies that the dominant-strategy equilibrium is $\sigma = (Y_1, Y_2)$. Hence, their expected payoffs are

$$\begin{aligned}
\pi_2 &= \lambda w v_2 + C\left[1 - \frac{v_2 + v_1}{2}\right] \\
&< \lambda w v_2 + \left(\frac{2\lambda w v_2}{v_1 + v_2 - 2}\right)\left[1 - \frac{v_2 + v_1}{2}\right] \\
&= 0 \\
&= \lambda w v_1 + \left(\frac{2\lambda w v_1}{v_1 + v_2 - 2}\right)\left[1 - \frac{v_2 + v_1}{2}\right] \\
&< \lambda w v_1 + C\left[1 - \frac{v_1 + v_2}{2}\right] \\
&= \pi_1,
\end{aligned}$$

where the first and second inequalities follow from the lower and upper bound on $\frac{C}{2\lambda w}$ respectively, and from the assumption that $v_1 + v_2 > 2$. Thus, Manager 2 cannot gain from being active along with Manager 1 (since $\pi_2 < 0$), while the latter's dominating strategy is to always be active (since $\pi_1 > 0$). In this case, Manager 1 is the only active manager and no deposit is required, by assumption. He invests the entire amount of $2w$ in Y_1 in order to maximize his expected λ-share of the portfolio, and the result follows. ■

Lemma 1. *In every equilibrium induced by the* GS *with more than one active Manager j, every superior Manager $i < j$ is also active.*

Proof. Assume, by contradiction, there exists an equilibrium σ where Manager j participates, but Manager $i < j$ does not. Without loss of generality, assume that Manager j is the lowest-skill active manager (w.r.t. expected optimal returns). The GS is linearly increasing in an active manager's return and linearly decreasing in the other active-managers' average return. Thus, if Manager i decides to participate, his expected payoff will be higher then Manager j's payoff for two reasons. First, Manager i produces excess expected return relative to Manager j as $v_i > v_j$. Second, Manager i also gains from the reduced benchmark, which consists of the returns of other active managers (including Manager j), compared to Manager j's benchmark in the assumed equilibrium σ (that does not include Manager j). Since Manager j's expected payoff is positive given σ, Manager i must enter the market as well, contradicting the equilibrium assumption. ■

Theorem 2. *Consider the* GS*-induced game. Manager i is relatively-superior if and only if there exists $C < \lambda w \alpha_i$ such that, in every equilibrium, only managers from S_i are active. In addition, Manager i is relatively-superior to Manager $i+1$ (i.e., $\alpha_i > \alpha_{i+1}$) if and only if there exists $C < \lambda w \alpha_i$ and an equilibrium where the set of active managers is S_i.*

Proof. Fix a relatively-superior Manager i. In case $i = k$, then one can fix $C = 0$ to show that all managers are active. Otherwise, consider $i^* = \text{argmax}_{j>i}\alpha_j$ and fix $\frac{C}{\lambda w} \in (\alpha_{i^*}, \alpha_i)$. We start by proving the existence of an equilibrium with active managers S_i.

Consider the strategy profile $\sigma = (Y_1, \ldots, Y_i)$. For every active Manager $j \leqslant i$, the GS f_j is linearly increasing in r_j, thus Y_j is still a dominant strategy of Manager j. If $i = 1$, then the expected payoff of Manager i given σ is strictly positive as no deposit is needed. Otherwise, the expected payoff of Manager $1 < i < k$ given σ is

$$\begin{aligned}
\pi_i &= \mathbf{E}[f_i(\sigma)] - C\mathbf{E}[Y_i] \\
&= \lambda w v_i + C\left[1 + \frac{v_i}{2R} - \frac{1}{2R(i-1)}\sum_{l<i} v_l\right] - Cv_i \\
&= \lambda w v_i + C\left[1 + \frac{v_i(1-2R) - \tilde{v}_{-i}}{2R}\right] \\
&= \lambda w v_i - C\frac{v_i}{\alpha_i} > 0, \qquad (2)
\end{aligned}$$

where the inequality follows from the chosen C. By the proof of Lemma 1, the expected payoff of every active Manager $j < i$ given σ is higher than $\mathbf{E}[f_i(\sigma)]$ due to the reduced benchmark and the excess expected return relative to Manager i. Thus, no active Manager $j \in S_i$ can gain from becoming inactive.

On the other hand, in case an inactive Manager $j \notin S_i$ becomes active and invests in Y_j, then

$$\begin{aligned}
\pi_j &= \mathbf{E}[f_j(\sigma, Y_j)] - C\mathbf{E}[Y_j] \\
&= \lambda w v_j + C\left[1 + \frac{v_j}{2R} - \frac{1}{2Ri}\sum_{l\in S_i} v_l\right] - Cv_j \\
&\leqslant \lambda w v_j + C\left[1 + \frac{v_j(1-2R) - \tilde{v}_{-j}}{2R}\right] \\
&= \lambda w v_j - C\frac{v_j}{\alpha_j} < 0, \qquad (3)
\end{aligned}$$

where the first inequality follows from the reduced benchmark, and the second inequality follows from the chosen C and the relatively-superior condition. Hence, no Manager has a profitable deviation from σ, establishing the existence of an equilibrium.

The last inequality also assures that the set of active managers in a subset of S_i. Lemma 1 proves that, in every equilibrium with an active Manager $j > i$,[13] all the managers of higher ability than Manager j must be active as well. If j is potentially the highest-index active manager, then

[13]The trivial case of a single active Manager $j > i$ is impossible, since C is bounded by $\lambda w \alpha_i$.

his expected payoff would be $\lambda w v_j - C\frac{v_j}{\alpha_j} < 0$, and therefore actively investing in Y_j cannot be an equilibrium strategy. This concludes the first part of the proof.

For the second part of the proof, fix $C < \lambda w \alpha_i$ such that the stated condition holds. Assume, by contradiction, that Manager i is not relatively-superior. Hence, there exists $j^* > i$ such that $\alpha_{j^*} \geqslant \alpha_i > \frac{C}{\lambda w}$. Fix the profile of optimal portfolios $\sigma' = (Y_1, \ldots, Y_{j^*})$, and note that

$$\mathbf{E}[f_{j^*}(\sigma')] - C\mathbf{E}[Y_{j^*}] = \lambda w v_{j^*} - C\frac{v_{j^*}}{\alpha_{j^*}} > 0,$$

where the inequality follows from $\alpha_{j^*} > \frac{C}{\lambda w}$. By the proof of Lemma 1 and given σ', we know that the expected payoff of every manager superior to j^* is also positive. Therefore, there exists an equilibrium with managers not only from S_i.

Note that σ' is not necessarily an equilibrium as additional managers of lower abilities may join the investment. However, as already mentioned, once low-skill managers become active, superior managers have a stronger incentive to invest, thus establishing an equilibrium with active managers beyond S_i. A Contradiction which concludes the first statement of the theorem.

We now prove the equivalence between $\alpha_i > \alpha_{i+1}$ and the existence of an equilibrium $\sigma = (Y_1, \ldots, Y_i)$. Assume $\alpha_i > \alpha_{i+1}$ and fix $\frac{C}{\lambda w} \in (\alpha_{i+1}, \alpha_i)$. Relying on the previous part of the proof and given σ, we know that no active manager can gain from deviating and becoming inactive. Also, Ineq. (3) still holds when taking $j = i + 1$. Thus, we only need to prove that every inactive Manager $j > i + 1$ cannot gain from deviating. In case an inactive Manager $j > i + 1$ becomes active,

$$\begin{aligned}
\mathbf{E}[f_j(\sigma, Y_j)] - C\mathbf{E}[Y_j] &= \lambda w v_j + C\left[1 + \frac{v_j}{2R} - \frac{1}{2Ri}\sum_{l \in S_i} v_l\right] - Cv_j \\
&= \lambda w v_j + C\left[1 + \frac{v_j(1 - 2R) - \tilde{v}_{i+1}}{2R}\right] \\
&< \lambda w v_{i+1} + C\left[1 + \frac{v_{i+1}(1 - 2R) - \tilde{v}_{i+1}}{2R}\right] \\
&= \mathbf{E}[f_{i+1}(\sigma, Y_{i+1})] < 0,
\end{aligned}$$

where the first inequality follows from $v_j < v_{i+1}$ as $j > i + 1$. Therefore, σ is indeed an equilibrium.

Now assume, by contradiction, that σ is an equilibrium and $\alpha_{i+1} > \alpha_i$. By Ineq. (2) we know that $\alpha_{i+1} \geqslant \alpha_i > \frac{C}{\lambda w}$. Hence, if Manager $i + 1$ becomes active, then by a similar computation to Ineq. (2) we get

$$\mathbf{E}[f_{i+1}(\sigma, Y_{i+1})] - C\mathbf{E}[Y_{i+1}] = \lambda w v_{i+1} - C\frac{v_{i+1}}{\alpha_{i+1}} > 0.$$

Thus establishing that σ is not an equilibrium and concluding the proof. ■