FINANCIAL INTEGRATION AND CYCICALITY OF MONETARY POLICY IN SMALL OPEN ECONOMIES

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Financial Integration and Cyclicality of Monetary Policy in Small Open Economies*

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Abstract

Should countries follow counter-cyclical or pro-cyclical monetary policies? This paper documents that in contrast to developed economies, developing countries tend to follow pro-cyclical monetary policies. The paper then constructs a New-Keynesian small open economy model with wage rigidity and solves for the optimal monetary policy under different levels of integration in the international financial markets. The model suggests that as economies gain access to the international financial markets the optimal monetary policy shifts from pro-cyclical to counter-cyclical. Also, when economies are denied access to financial markets the optimal policy partially offset exchange rate movements which may be perceived as “fear of floating”. Results are robust to a wide range of parameter values and utility specifications.

JEL classification: E58, F30, F41

Keywords: Financial integration, Monetary policy, Wage rigidity, Small open economy.

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1 Introduction

Conventional wisdom recommends that countries follow counter-cyclical monetary policies. This recommendation dates back to at least Wicksell (1907) and has proven to be robust to many developments in monetary theory during the past century.\(^1\) This paper documents that while developed economies typically follow this advice, developing countries do not. Specifically, the paper finds that monetary authorities in developed economies tend to raise their interest rate in times of economic boom and reduce it during slumps, while in developing countries the opposite is true.\(^2\) This finding raises the question whether policymakers in developing countries should adopt policies similar to the ones applied in the developed world, or whether the special conditions under which they operate call for a different policy prescription.

Developing countries have limited access to the international financial markets relative to developed economies. This motivates an analysis that studies how the level of financial integration affects the optimal monetary policy. To that end we employ a standard New-Keynesian small open economy model with wage rigidity and productivity shocks.\(^3\) The model suggests that under full financial integration, i.e. when the economy can borrow and lend freely in the international financial markets, the optimal policy is counter-cyclical, while under financial autarky, i.e. when the economy is denied access to the international financial markets, the optimal policy is pro-cyclical. The paper also demonstrates that the transition from pro-cyclical to counter-cyclical policies is monotonic in the level of financial integration.\(^4\) The main force behind the results is that greater integration stabilizes the

\(^1\)Counter-cyclical monetary policy was advocated by the “Chicago Plan”; see for example, Mints (1946) and Friedman (1948). The Keynesian IS-LM model, Hicks (1937), also supports counter-cyclical monetary policies. Fischer (1977) and Phelps and Taylor (1977) reestablished the optimality of activist monetary stabilization in a rational expectations framework. Taylor (1993) and much of the New-Keynesian literature that followed also support counter-cyclical monetary policies; see for example, Woodford (2001) and Giannoni and Woodford (2002).

\(^2\)Calvo and Reinhart (2002), Calderón et al. (2004), and Kaminsky et al. (2004) also document this phenomenon.

\(^3\)In the model the productivity shocks can also be interpreted as terms of trade movements.

\(^4\)The level of financial integration is captured in the model by an adjustment cost to the portfolio of
exchange rate, which, in turn, tampers its effectiveness as an endogenous shock absorber and therefore calls for a counter-cyclical policy to step in. Lack of integration, on the other hand, induces excess volatility of the exchange rate and the attempt to moderate its movement results in a pro-cyclical policy. The results are robust to a wide range of parameter values and utility specifications, to introducing demand shocks, and to the introduction of a non-tradable sector.

It should be stressed that the research question in this paper is a normative one rather than positive. That is, we do not interpret the results as necessarily providing an explanation for the observed difference in monetary conduct between developing and developed economies; instead, the results provide a normative benchmark for monetary policy under different levels of financial integration.

The model incorporates two market imperfections: monopolistic competition in the labor market and nominal wage rigidity. The monopolistic competition distorts the steady state equilibrium; hence, in order to motivate monetary policy by cyclical considerations alone, we allow for a constant labor subsidy that restores the first best allocation in steady state.5 As productivity fluctuates, the role of the monetary policy is therefore to minimize the distortion introduced by wage rigidity. In this framework the monetary authority manipulates the nominal interest rate so as to replicate the flexible wages equilibrium.

To gain intuition for the cyclical stance of the optimal policy it is useful to review two polar cases: full financial integration and financial autarky.

Under full financial integration the optimal monetary policy is counter-cyclical. In this case the exchange rate is governed by interest rate differentials. Therefore, in the absence of a policy response, an improvement in productivity has no effect on the exchange rate. A stable exchange rate combined with rigid nominal wages implies that real wages are stable as well. However, if nominal wages were flexible, a positive productivity shock would result

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5This approach is common in the New-Keynesian literature. See for example, Rotemberg and Woodford (1999), Erceg et al. (2000), and Galí and Monacelli (2005).
in higher real wages as it increases labor demand. Therefore, in order to restore the first best allocation, the monetary policy aims to appreciate the domestic currency so as to increase real wages. This is achieved by raising the nominal interest rate. That is, when the economy is fully integrated in the world’s financial markets, the optimal monetary policy is counter-cyclical.

Under financial autarky the optimal monetary policy is pro-cyclical. In this case households cannot use the international financial markets to smooth consumption; as a result, a positive productivity shock increases current consumption and hence reduces the domestic real interest rate. In absence of a policy response, the fall in the real interest rate comes about through an immediate appreciation of the exchange rate (and expectations for depreciation in subsequent periods). The appreciation increases real wages and hence endogenously counteracts the expansionary effect of productivity. The magnitude of this effect depends on the value of the intertemporal elasticity of substitution, or the curvature of the utility function; a lower elasticity, i.e. greater curvature, implies that marginal utilities are more sensitive to movements in consumption and therefore a greater adjustment to the real interest rate is required in order to restore equilibrium. Hence, when the economy enjoys a positive productivity shock, a lower elasticity implies a greater appreciation of the exchange rate and therefore higher real wages. In particular, for elasticity lower than unity (which is the empirically relevant case) the increase in real wages is greater than the one implied by the flexible wages equilibrium. Therefore, the optimal policy is to lower the nominal interest rate so as to depreciate the currency and reduce real wages. That is, under financial autarky the optimal monetary policy is pro-cyclical.

This paper is part of the vast New-Keynesian open-economy literature. Typically, contributions in this literature characterize monetary policies either in terms of an exchange rate regime or by a policy target. Although the cyclical stance of the policy can often be

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inferred, it is not the focus of attention and is rarely discussed explicitly.

There is also a large body of research that studies monetary policy under financial constraints. However, this literature is typically concerned with the response to sudden stops rather than the conduct of monetary policy in normal times. For example, Christiano et al (2004) and Braggion et al (2007) conclude that in face of a sudden stop it is optimal is to raise interest rate, i.e. to follow a pro-cyclical policy. Their results are driven by binding collateral constraints, a balance sheet currency mismatch, and a real friction in the tradable sector. In the absence of collateral constraints these models reproduces the traditional monetary transmission mechanism and policy recommendations. Similarly, Devereux and Poon (2004), in a model of currency mismatch and nominal wage rigidity, argue that under binding collateral constraints the optimal policy is pro-cyclical since a monetary contraction in this situation helps to relax the collateral constraint. When the constraint slack the optimal policy is counter-cyclical. Caballero and Krishnamurthy (2005) provide a different view. They develop a model where firms face both domestic and foreign collateral constraints. In their model the standard credit channel holds during normal times and the optimal monetary policy is counter-cyclical, but in face of a sudden stop the expansionary effect of monetary policy vanishes. However, due to precautionary externalities firms under-accumulate international liquidity if they expect the central bank to protect the exchange rate during crises. This insurance incentive calls for counter-cyclical policies with a flexible exchange rate system.

The rest of the paper is organized as follows. Section 2 presents evidence regarding the different cyclical stance of monetary policies in developing versus developed economies. Section 3 presents the model. For simplicity, this section focuses only on two polar cases of financial integration. Section 4 provides closed form solutions to the optimal monetary policies. Section 5 extends the model to include intermediate levels of integration and conducts sensitivity analysis. As additional robustness checks, section 6 introduces demand shocks and a non-tradable sector to the model. Section 7 concludes.
2 Cyclicality of Monetary Policy: Stylized Facts

This section documents that developed economies tend to follow counter-cyclical monetary policies while developing countries do the opposite.

The empirical analysis is mainly influenced by Kaminsky Reinhart and Végh (2004), KRV hereafter. KRV have documented that OECD countries tend to follow counter-cyclical monetary policies while non-OECD countries typically follow pro-cyclical policies. Their dataset consists of a panel of 104 countries in annual frequency. Although informative, we wish to refine their results along two dimensions. First, monetary policy is often evaluated on a monthly basis and therefore it is interesting to reestablish their findings using data of higher frequency. Second, and more importantly for our purpose, KRV’s analysis does not discriminate across exchange rate regimes, while the theoretical model in this paper suggests that the pro-cyclicality of monetary policy hinges on movements in the exchange rate. This result directs our empirical investigation to focus on countries that let their currency fluctuate. These considerations guide our criteria for sample selection.

2.1 Sample Selection

We use quarterly data for the period 1974-2004 and restrict the sample to include countries with floating or dirty-floating exchange rate regimes with at least 20 consecutive observations. In addition, we remove observations during periods of annual CPI inflation greater than 100 percent.\footnote{This criterion is not very restrictive. It only removes the high inflation periods in Peru and Israel and eliminates Belarus from the sample. It does not, however, change any of the conclusions.}

Exchange rate regimes are determined using the classification of Levy-Yeyati and Sturzenegger (2005). Based on actual fluctuations in the exchange rate, its rate of change, and international reserves, their work provides a classification of de-facto exchange rate regimes for IMF-reporting countries. We use data from the International Financial Statistics (IFS) database, and use the IMF’s classification of advanced economies and developing countries...
as reported in the World Economic Outlook.\textsuperscript{8} The data appendix provides more details on country-specific sample periods and selection criteria of exchange rate regimes.

Our selection criteria leave us with 15 developed economies and 15 developing countries with varying sample periods. The developed economies are: Australia, Canada, France, Germany, Greece, Israel, Italy, Japan, Korea, Singapore, Spain, Switzerland, United Kingdom, United States, and the Euro Area. The developing countries are: Chile, Colombia, Croatia, Czech Republic, Ecuador, Georgia, Indonesia, Kyrgyz Republic, Mexico, Peru, Philippines, Poland, South Africa, Thailand, and Turkey.

2.2 Methodology and Results

We use the short-term nominal interest rate as the policy instrument. Specifically, we use either the central bank’s discount rate or the interbank short-term market rate, depending on data availability. To determine the cyclical stance of the monetary policy we measure the correlation between cyclical movements in the interest rate and output; in addition, we estimate interest rate rules and evaluate the sign of the coefficients on output. Positive correlations and positive regression coefficients indicate counter-cyclical monetary policies, while negative values indicate pro-cyclical policies. In what follows we report averages for each country group; country-specific values are presented in the data appendix.

2.2.1 Interest-Output Correlation

We calculate the correlation coefficient in each country between the cyclical components of the interest rate and the natural logarithm of real GDP; these are measured by removing Hodrick-Prescott (HP hereafter) trend from the series.

Panel A in Table 1 summarizes the results. On average the correlations are positive in developed economies (0.26) and negative in developing countries (-0.18). The distribution of the correlations within each group of countries provides further support to the difference

\textsuperscript{8}“Advanced economies” is the World Economic Outlook’s terminology for developed economies.
in monetary policies. In developed economies 13 of the 15 correlations are positive, and 7 of them are significantly different from zero while none of the negative correlations is significant.\(^9\) In developing countries 10 of the 15 correlations are negative, with 5 significantly different from zero and none of the positive correlations is significant. Figure 1 illustrates these findings graphically.

### 2.2.2 Interest Rate Rules

We estimate an interest rate rule for each country. Our specification takes the form:

\[
\tilde{R}_t = \beta_\pi \tilde{\pi}_t + \beta_y \tilde{Y}_t + \beta_\sigma \Delta \tilde{\sigma}_t + \varepsilon_t
\]

where \(\tilde{R}_t\), \(\tilde{\pi}_t\), \(\tilde{Y}_t\), and \(\Delta \tilde{\sigma}_t\) are the cyclical components (measured using HP filter) of the nominal interest rate, CPI inflation in the past 4 quarters, the logarithm of real GDP, and the rate of change in the nominal exchange rate,\(^10\) respectively. For \(\beta_\sigma = 0\), this specification is similar to the one in Taylor (1993).

Some authors argue that in open economies the monetary authority should react to exchange rate movements in addition to inflation and output,\(^11\) for that reason we include it in (1). It should be noted, however, that others argue against the inclusion of the exchange rate in the policy rule. Taylor (2001), for example, argues that policy rules like (1) do not perform much better, and sometimes even worse, than rules that impose \(\beta_\sigma = 0\). The reason is that fluctuations in the exchange rate affect both inflation and output; as a result, the interest rate reacts indirectly to exchange rate fluctuations even when it is not included explicitly in the policy rule. A direct reaction to the exchange rate brings only minor improvement, if any.\(^12\)

We first impose \(\beta_\sigma = 0\) and estimate (1) by OLS for each country separately. We also use a GLS panel regression where we restrict output coefficients to be equal for all countries.

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\(^9\)Unless otherwise stated, we use a 5 percent significance level.

\(^10\)We use effective exchange rates whenever available, otherwise we use the exchange rate against the US dollar. See the data appendix for details.

\(^11\)See Ball (1999) and Benigno and Benigno (2001).

\(^12\)Clarida et al. (2001) and Leitner and Söderström (2005) make similar arguments.
within each group.\textsuperscript{13} Panel B in Table 1 summarizes the results. Similarly to the indication of the correlation coefficients, on average $\beta_y$ is positive for developed economies (0.22) and negative for developing countries (-0.33). In developed economies 11 of the 15 coefficients are positive, and 7 of them are significantly different from zero. In developing countries 12 of the 15 coefficients are negative, and 5 of them are significant. The panel regression also produces a positive coefficient for developed economies (0.18) and a negative coefficient for developing countries (-0.11), with both significantly different from zero.

Next, we add the exchange rate to the regression. We estimate (1) by TSLS and use the lagged interest rate, $\tilde{R}_{t-1}$, and the lagged change in the exchange rate, $\Delta \tilde{e}_{t-1}$, as instruments for $\Delta \tilde{e}_t$. Panel C summarizes the results. As before, output coefficients are positive, on average, in developed economies (0.13) and negative in developing countries (-0.14); however, significance levels deteriorate in these estimations. The panel regression seems to result in more accurate estimates as it pools more observations for the output coefficients. For developed economies $\beta_y$ is positive (0.27) and significant, and for developing countries the coefficient is negative (-0.17) and significant at 8 percent significance level.

In sum, our analysis supports KRV’s findings. That is, we also find that developed economies tend to follow counter-cyclical monetary policies while developing countries typically follow pro-cyclical policies.

3 The Model: Two Polar Cases of Financial Integration

This section lays down the model. Here we start by analyzing two polar cases of financial integration, complete markets and financial autarky, since these can be easily solved with closed form solutions (after taking log-linear approximation). Later, in Section 5, we extend

\textsuperscript{13}The estimation allows for country-specific inflation coefficients and variances. We do not use fixed effects since all intercepts are zero by construction. Also, endogeneity problems are probably minor when $\beta_p = 0$ since typically the transmission mechanism from monetary policy to output and prices works with lags.
the model to include intermediate levels of integration.

3.1 The environment

Consider a small open economy that is perfectly integrated in the world’s goods markets. Agents in the economy produce and consume one tradable good. Production technology is subject to productivity shocks which, for now, is the only source of aggregate uncertainty in the model.\textsuperscript{14} We consider two structures of international financial markets: complete markets and financial autarky.

The economy consists of households, employment agencies, firms, insurance companies, and a government. Households consume the single good and are endowed with differentiated labor skills. They act as monopolists in the labor market as each household sets the nominal wage of its own labor skill. Nominal wages are sticky à la Calvo (1983); that is, households can adjust their nominal wage only when they receive a random idiosyncratic signal that allows them to do so. The insurance companies provide households with insurance against these shocks. The employment agencies rent labor services from the households. They aggregate differentiated skills into labor inputs which, in turn, are supplied to the firms. The firms use labor inputs to produce the good. The government sets the monetary policy by controlling the nominal interest rate. Following Woodford (2003), money, or more precisely - cash, serves only as a unit of account. We find this approach appropriate to the question of this paper since it allows abstracting from considerations that motivate the Friedman rule as optimal; as a result, we are able to set the steady state nominal interest rate at any arbitrary level and focus the analysis only on the cyclical properties of the monetary policy.\textsuperscript{15}

\textsuperscript{14}It should be noted that the model economy is equivalent to one where firms produce an exportable good and households consume an importable good. In that case productivity can be interpreted as terms of trade (or the product of terms of trade and productivity).

\textsuperscript{15}An earlier version of this paper used money in the utility function. When solving for the optimal monetary policy it is common to assume that the utility from liquidity services is arbitrarily small. See, for example, Obstfeld and Rogoff (1995, 1998, 2000), Devereux and Engel (2003), and Corsetti and Pesenti (2005). This assumption allows abstracting from considerations that motivate the Friedman rule. Here we
3.1.1 Prices

Let $P_t$ denote the domestic currency price of the good, and $S_t$ the nominal exchange rate (the price of one unit of foreign currency in terms of the domestic currency). Normalizing the foreign currency price of the good to 1 and assuming that the law of one price holds, we get:

$$P_t = S_t$$

From now on we will use the exchange rate in place of the price level.

3.1.2 Shocks

Productivity, $A_t$, is the only source of aggregate uncertainty. We assume that it follows a stationary $AR(1)$ process in logs:

$$\log (A_t) = (1 - \rho_A) \mu_A + \rho_A \log (A_{t-1}) + \varepsilon_{A,t} \quad 0 < \rho_A < 1$$

where:

$$\varepsilon_{A,t} \sim iid N(0, \sigma_A^2)$$

In addition, there are household-specific shocks. Households are indexed by $h$, $h \in [0, 1]$, and they receive binary idiosyncratic shocks, $I_t(h)$, that either get the value 0 or 1. Whenever $I_t(h) = 1$ household $h$ is allowed to freely adjust its nominal wage for period $t$, otherwise $I_t(h) = 0$ and the nominal wage is set to a predetermined level. The shocks are iid over time and across households, and they are independent of the aggregate shock $A_t$. Specifically we assume that the idiosyncratic shocks follow a Bernoulli distribution:

$$I_t(h) \sim B(1 - \xi_w)$$

Where $1 - \xi_w$ is the probability of a household to receive a signal that allows it to change its wage.

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adopt a cashless economy model as in Woodford (2003). The two modeling strategies result in an identical system of equations except, of course, for money demand. The cashless economy, however, avoids the need to resort to conflicting assumptions between motives for holding money and policy considerations.
We denote by \( s_t \) the realization of aggregate events in date \( t \). \( s^t \) denotes the history of aggregate events from date zero to date \( t \), that is \( s^t = (s_0, s_1, \ldots, s_t) \). We assume that the economy starts from steady state and therefore there is no uncertainty with respect to \( s_0 \).

### 3.1.3 Financial Assets

A full set of contingent financial assets, denoted by \( B^*_t (s^t, s_{t+1}) \), is available. Each unit of \( B^*_t (s^t, s_{t+1}) \) pays one unit of the foreign currency if \((s^t, s_{t+1})\) realizes, and is traded in date \( t \) after history \( s^t \). Each unit of \( B^*_t (s^t, s_{t+1}) \) costs \( Q^*_t (s^t, s_{t+1}) \) units of the foreign currency.

The stochastic discount factor in terms of domestic currency, \( Q_t (s^t, s_{t+1}) \), is determined by the no arbitrage condition:

\[
Q_t (s^t, s_{t+1}) = \frac{S_t (s^t)}{S_{t+1} (s^t, s_{t+1})} Q^*_t (s^t, s_{t+1})
\]

The risk-free foreign interest factor, \( R^*_t \), and its domestic counterpart, \( R_t \), are given by:

\[
\frac{1}{R^*_t} = \int_{s_{t+1}} Q^*_t (s^t, s_{t+1}) \, ds_{t+1}
\]

\[
\frac{1}{R_t} = \int_{s_{t+1}} Q_t (s^t, s_{t+1}) \, ds_{t+1}
\]

Under full financial integration asset prices, \( Q^*_t (s^t, s_{t+1}) \), are taken exogenously; we assume, however, that these are actuarially fair, that is:

\[
\frac{Q^*_t (s^t, \tilde{s}_{t+1})}{Q^*_t (s^t, \hat{s}_{t+1})} = \frac{Pr (s^t, \tilde{s}_{t+1} \mid s^t)}{Pr (s^t, \hat{s}_{t+1} \mid s^t)} \forall \tilde{s}_{t+1}, \hat{s}_{t+1}
\]

where \( Pr (s^t, s_{t+1} \mid s^t) \) is the probability of event \( s_{t+1} \) to realize conditional on history \( s^t \).

This implies:

\[
\frac{1}{R^*_t} = \frac{Q^*_t (s^t, s_{t+1})}{Pr (s^t, s_{t+1} \mid s^t)} \forall (s^t, s_{t+1})
\]

We assume that in the international financial markets \( R^*_t \) is constant, such that:

\[
R^*_t = \beta^{-1} \quad \forall t
\]
where $\beta$ is the subjective discount factor of the households. When the economy is in financial autarky foreign assets are restricted to zero and $R^*_t$ becomes endogenous and hence no longer equal $\beta^{-1}$.\footnote{In that case $Q^*_t(s', s_{t+1})$ is endogenous. Its equilibrium level brings the holdings of $B^*$ to zero. Note that although households are faced with idiosyncratic shocks, the existence of insurance companies (discussed in section 3.2.1) guarantees that all households choose the same portfolio of $B^*$.}

Finally, there are also domestic government bonds, denoted by $B_t$, that yield a safe return of $R_t$.

3.2 Agents

3.2.1 Insurance Companies

In every period households and insurance companies meet to sign state-contingent insurance contracts against next period’s idiosyncratic shocks. These contracts provide insurance against the inability to control nominal wages in the next period. Under each contract it is agreed that if $I_{t+1}(s^t, s_{t+1}, h) = 1$ then household $h$ pays the insurance company one unit of the domestic currency, while if $I_{t+1}(s^t, s_{t+1}, h) = 0$ the household receives $q$ units.

Let $b_t(s^t, s_{t+1}, h)$ denote the quantity of such contracts. Notice that any possible future aggregate state, $s_{t+1}$, characterizes a type of insurance contract. Zero profits for every type of contract requires:

$$Z \sum_{v^w_{t+1}} L_{w+1} (v^w > v^w_{t+1} > k) \cdot e_{w} (v^w > v^w_{t+1} > k) = t Z \sum_{v^w_{t+1}} (1 - I_{t+1}(s^t, s_{t+1}, h)) b_t(s^t, s_{t+1}, h) dh$$

Taking expectations conditional on date $t$ information gives:

$$q = \frac{1 - \xi_{w}}{\xi_{w}}$$

which reflects actuarially fair pricing in the insurance market.
3.2.2 Employment Agencies

Employment agencies are price takers. They construct labor inputs, $L_t$, by aggregating differentiated labor skills, $l_t (h) h \in [0, 1]$, in a Dixit-Stiglitz aggregator:

$$L_t = \left[ \int_0^1 l_t (h)^{\theta - 1} dh \right]^{\frac{\theta}{\theta - 1}} \quad \theta > 1$$

$L_t$ is then supplied to the firms. Competition among employment agencies generates the following labor demand for $l_t (h)$:

$$l_t (h) = \left( \frac{w_t (h)}{W_t} \right)^{-\theta} L_t$$

where $w_t (h)$ is the nominal wage of labor skill type $h$, and $W_t$ is the aggregate wage index (the price of $L_t$), which is given by:

$$W_t = \left[ \int_0^1 w_t (h) 1 - \theta dh \right]^{\frac{1}{1 - \theta}}$$

3.2.3 Firms

There is a large number of identical competitive firms. The firms produce a tradable good with labor as the only factor of production. Output, $Y_t$, is given by:

$$Y_t = A_t L_t^\alpha \quad 0 < \alpha < 1$$

Given $W_t$, $S_t$, and $A_t$, firms choose labor to maximize profits. This results in the following labor demand:

$$\frac{W_t}{S_t} = \alpha A_t L_t^{\alpha - 1}$$

3.2.4 Government

The government issues nominal domestic risk-free bonds, $B_t$, and provides lump sum transfers, $T_t$, to the households. It also subsidizes labor at rate $\tau_w$ in order to offset the distortionary effect of the monopolistic competition in the labor market. The government budget
constraint is given by:

\[ B_t = R_{t-1}B_{t-1} + T_t + \tau_w \int_0^1 w_t(h) l_t(h) dh \]

The monetary policy is carried out by controlling \( R_t \).

### 3.2.5 Households

Households consume the good and trade in financial assets. They also set their wage, \( w_t(h) \), whenever they receive a signal that allows them to do so, i.e. when \( I_t(h) = 1 \). If the household does not receive a signal, then the nominal wage is automatically updated by the steady state inflation rate, \( \pi_{ss} \), as in Erceg et al. (2000), that is: \( w_t(h) = (1 + \pi_{ss}) w_{t-1}(h) \).

Given the nominal wage, households supply any level of labor so as to satisfy labor demand (4).

It proves convenient to formulate the households’ problem recursively:

\[
V_1 \left[ s^t, I_t(h) = 1, B_{t-1}(h), B^*_{t-1}(s^t, h), b_{t-1}(s^t, h) \right] =
\]

\[
\max_{c_t(h), B_t(h), w_t(h), B^*_1(s^t+1, h), b_1(s^t+1, h)} U \left[ c_t(h), l_t(h) \right]
\]

\[
+ (1 - \xi_w) \beta E \left\{ V_1 (\cdot)/ s^t, I_{t+1}(h) = 1 \right\}
\]

\[
+ \xi_w \beta E \left\{ V_0 (\cdot)/ s^t, I_{t+1}(h) = 0 \right\}
\]

s.t.

\[
l_t(h) = \left[ \frac{w_t(h)}{W_t} \right]^{-\theta} L_t
\]

\[
w_{t+1}(h) = (1 + \pi_{ss}) w_t(h)
\]

\[
\int_{s_{t+1}} Q^*_t(s^t, s_{t+1}) B^*_t(s^t, s_{t+1}, h) ds_{t+1} =
\]

\[
(1 + \tau_w) \frac{w_t(h)}{S_t} l_t(h) + B^*_{t-1}(s^t, h) - \frac{B_t(h)}{S_t}
\]

\[
+ R_{t-1}B_{t-1}(h) + \Pi_t + T_t - b_{t-1}(s^t, h) - c_t(h)
\]

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and:

\[
V_0 [s^t, I_t (h) = 0, B_{t-1} (h), B_0^* (s^t, h) , b_{t-1} (s^t, h), \overline{w}_t (h)] =
\]

\[
\max_{c_t (h), B_t (h), B_0^* (s^{t+1}, h), b_t (s^{t+1}, h)} U [c_t (h), l_t (h)]
+ (1 - \xi_w) \beta E \{ V_1 (\cdot) / s^t, I_{t+1} (h) = 1 \}
+ \xi_w \beta E \{ V_0 (\cdot) / s^t, I_{t+1} (h) = 0 \}
\]

\[s.t.\]

\[
l_t (h) = \left[ \frac{\overline{w}_t (h)}{W_t} \right]^{-\theta} L_t
\]

\[
\overline{w}_{t+1} (h) = (1 + \pi_{ss}) \overline{w}_t (h)
\]

\[
\int_{s_{t+1}} Q_t^* (s^t, s_{t+1}) B_0^* (s^t, s_{t+1}, h) ds_{t+1} =
(1 + \tau_w) \frac{\overline{w}_t (h)}{S_t} l_t (h) + B_{t-1}^* (s^t, h) - \frac{B_t (h)}{S_t}
+ \frac{R_{t-1} B_t-1 (h) + \Pi_t + T_t + g b_{t-1} (s^t, h)}{S_t} - c_t (h)
\]

where \( U (\cdot) \) is the periodical utility function. It is twice continuously differentiable with \( U_c > 0, U_l < 0, U_{cc}, U_{ll} < 0, \) and \( U_{cc}U_{ll} - U_{cl}^2 > 0 \). We also assume that both consumption and leisure are not inferior goods, which implies that \( U_c U_{ll} - U_l U_{cl} \leq 0 \) and \( U_c U_{le} - U_l U_{ce} \leq 0. \) \( \Pi_t \) is profits, and \( \overline{w}_t (h) \) is the wage whenever \( I_t (h) = 0 \). Notice that the difference between \( V_t \) and \( V_0 \) is that under former the wage rate is a choice variable, while under the latter it is taken as given and it is part of the state variables. In addition, the budget constraints differ in the payment to/from the insurance companies.

**Optimality Conditions** Optimal choice of wage insurance contracts together with their equilibrium price (3) gives:

\[
U_{c_t} (s^t, I_t (h) = 1) = U_{c_t} (s^t, I_t (h) = 0)
\]

That is, the marginal utility of consumption is perfectly smoothed across idiosyncratic states.
Access to the world financial markets allows for smoothing the marginal utilities over time and across aggregate states of nature:

\[
U_{ci} \left( s^t, h \right) = \begin{cases} 
U_{ci+1} \left( s^{t+1}, h \right) & \text{under full financial integration} \\
\beta \frac{Pr(s^{t+1} | s^t)}{Q_t(s_t, s^{t+1})} U_{ci+1} \left( s^{t+1}, h \right) & \text{under financial autarky}
\end{cases}
\]

(8)

Note that under financial autarky \( Q_t(s_t, s^{t+1}) \) is endogenous, as assets are traded domestically with zero net supply.

Households optimality condition with respect to domestic bonds gives:

\[
\frac{U_{ci} \left( h \right)}{S_t} = \beta R_t E_t \left\{ \frac{U_{ci+1} \left( h \right)}{S_{t+1}} \right\}
\]

(9)

We assume symmetry across households. Specifically, endowment of assets in period zero is such that initially all households have the same marginal utility of consumption. This assumption coupled with the optimality conditions above implies that marginal utilities are equated across households at all dates and states, that is:

\[
U_{ci} \left( s^t, \tilde{h} \right) = U_{ci} \left( s^t, \check{h} \right) \quad \forall s^t, \tilde{h}, \check{h}
\]

(10)

Therefore, from this point on we will drop the index \( h \) from the marginal utility of consumption.

Optimal wage setting gives:

\[
w_t \left( h \right) = -\frac{1}{1 + \tau_w} \frac{\theta}{\varsigma} \sum_{s=0}^{\infty} E_{t,0} \left\{ \lambda_{t+s} \frac{S_{t+s}}{(1 + \pi_{ss})^s} \frac{U_{c_{t+s}} \left( h \right)}{S_{t+s}} \right\}
\]

(11)

where:

\[
\lambda_{t+s} = \frac{\xi_s \beta^s (1 + \pi_{ss})^{(1-\theta)s} U_{c_{t+s}} \left( h \right)}{\varsigma_s \sum_{s=0}^{\infty} \xi_s \beta^s (1 + \pi_{ss})^{(1-\theta)s}} \frac{S_{t+s}}{S_{t+s}} \frac{W_{t+s}^{\theta} L_{t+s}}{W_{t+s}^{\theta} L_{t+s}}
\]

where \( E_{t,0} \) denotes the mathematical expectation conditional on history \( s^t, I_t \left( h \right) = 1 \), and \( I_{t+s} \left( h \right) = 0 \) for all \( s > 0 \).

Generally (11) does not characterize a solution for \( w_t \left( h \right) \) in terms of aggregate quantities and market prices. This is because the marginal utility of labor, \( U_l \), may differ across households as it depends on household specific consumption and labor levels. In order to
overcome this problem previous studies have resorted to additively separable utility in $c$ and $l$. In that case, equation (4) suggests that $U_t$ only depends on $w_t(h), W_t$ and $L_t$; hence, given prices and aggregate quantities (11) characterizes a solution for $w_t(h)$. Furthermore, this argument also suggests that in each period all households that are allowed to adjust their wage choose the same level. Therefore, in order to solve for aggregate quantities it is sufficient, in the additively separable case, to keep track of new and average wages only.

This paper extends the analysis to non-separable utility functions. The main argument is that although consumption may now differ across households, their marginal utility, $U_c$, does not. As a result, households with the same wage also have the same level of consumption. Therefore, the result of the additively separable case remains; in each period, all households that are allowed to adjust their wage choose the same level. The difference is that now one should keep track of new and average consumptions in addition to wages.\textsuperscript{17}

From this argument together with labor demand (4) we have:

\begin{align*}
  w_t(\hat{h}) &= w_t(\tilde{h}) \quad \forall \hat{h}, \tilde{h} \in [0, 1] \quad s.t. \ I_t(\hat{h}) = I_t(\tilde{h}) = 1 \\
  l_t(\hat{h}) &= l_t(\tilde{h}) \quad \forall \hat{h}, \tilde{h} \in [0, 1] \quad s.t. \ I_t(\hat{h}) = I_t(\tilde{h}) = 1 \\
  c_t(\hat{h}) &= c_t(\tilde{h}) \quad \forall \hat{h}, \tilde{h} \in [0, 1] \quad s.t. \ I_t(\hat{h}) = I_t(\tilde{h}) = 1
\end{align*}

Notice that households $\hat{h}$ and $\tilde{h}$ will continue having the same wage, labor effort, and consumption as long as both cannot readjust their wage. This result leads us to characterize households by cohorts rather than their index $h$.

\textbf{Notation 1} Let $w^t_i$ and $c^t_i$ denote date $t$ wage and consumption, respectively, of households such that $I_t(h) = 1$.

To gain intuition for the optimal wage setting condition, equation (11), notice that under flexible wages (i.e. $\xi_w = 0$) it becomes:

\[
\frac{w^t_i}{S_t} = -\frac{\theta}{\theta - 1 + \frac{1}{\tau_w} \frac{U_{l_t}}{U_{c_i}}}
\]

\textsuperscript{17}See Appendix A in Yakhin (2007) for a derivation.
which simply suggests that households charge a constant markup, \( \frac{\theta}{\pi} \), over labor supply.
The supply function is given by the marginal rate of substitution between consumption and labor, \(-\frac{L}{K} \), divided by \(1 + \tau_w\). The subsidy simply shifts the position of the supply curve. Under sticky wages, i.e. \( \xi_w > 0 \), the markup is over a weighted average of future labor supplies for histories in which the household would not be able to readjust its wage. The weights are given by \( \lambda_{t+s} \).

### 3.3 Aggregate Consumption

As mentioned, in order to solve for the macroeconomic equilibrium we need to keep track of consumption of households with \( I_t(h) = 1 \) and the average consumption level in the economy. To that end define:

\[
C_t^O \equiv \frac{1}{\xi_w} \int_{h \notin \Upsilon} c_t(h) \, dh
\]

where \( \Upsilon = \{h : I_t(h) = 1\} \).

That is, \( C_t^O \) is date \( t \) average consumption of households that cannot adjust their wage. Using this notation aggregate consumption in the economy, \( C_t \), is given by:

\[
C_t = (1 - \xi_w) c_t + \xi_w C_t^O \tag{12}
\]

### 3.4 The Resource Constraint

Combine the households’ budget constraint with the government budget constraint, firms’ profits, and the zero profit condition in the insurance sector to get the resource constraint of the economy:

\[
\frac{1}{R^*} E_t \left[ B_t^* (s^t, s_{t+1}) \right] = B_{t-1}^* (s^t) + Y_t - C_t \tag{13}
\]

Under financial autarky we will impose \( B_t^* = 0 \) for all \( t \).
3.5 Model’s Solution

We solve the model by taking a first order approximation around the deterministic steady state. This results in a simple linear system that allows us to derive closed form solutions.

Before approximating the system we must impose stationarity on the nominal variables; to that end, we deflate them by the steady state inflation rate and define:

\[ \Omega_t \equiv \frac{W_t}{(1 + \pi_{ss})^t} \]
\[ \omega^t \equiv \frac{w_t}{(1 + \pi_{ss})^t} \]
\[ \sigma_t \equiv \frac{S_t}{(1 + \pi_{ss})^t} \]

3.5.1 System of Equations

Under full financial integration the steady state level of foreign assets is exogenous; therefore, for consistency with the case of financial autarky, we assume \( B^*_ss = 0 \). Log-linearization of equations (2), and (5) through (13), results in:

\[ \tilde{A}_t \approx \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t} \]  
\[ \tilde{\Omega}_t \approx (1 - \xi_w) \tilde{\omega}^t + \xi_w \tilde{\Omega}_{t-1} \]  
\[ \tilde{Y}_t \approx \tilde{A}_t + \alpha \tilde{L}_t \]  
\[ \tilde{\Omega}_t - \tilde{\sigma}_t \approx \tilde{A}_t - (1 - \alpha) \tilde{L}_t \]

\[ \gamma_c \tilde{C}_t + \gamma_d \tilde{L}_t \approx \begin{cases} 
\gamma_c \tilde{C}_{t+1} + \gamma_d \tilde{L}_{t+1} & \text{under full financial integration} \\
\tilde{R}_t^* + E_t \left( \gamma_c \tilde{C}_{t+1} + \gamma_d \tilde{L}_{t+1} \right) & \text{under financial autarky} 
\end{cases} \]  
\[ \tilde{R}_t \approx E_t (\tilde{\sigma}_{t+1}) - \tilde{\sigma}_t + \gamma_c \tilde{C}_t + \gamma_d \tilde{L}_t - E_t \left( \gamma_c \tilde{C}_{t+1} + \gamma_d \tilde{L}_{t+1} \right) \]  
\[ \gamma_c \tilde{c}_t^t - \gamma_d \theta \tilde{\omega}^t \approx \gamma_c \tilde{C}_t^t - \gamma_d \theta \tilde{\Omega}_{t-1} \]  
\[ \left\{ \frac{1 + \xi_w/\beta \Gamma \theta}{1 - \xi_w/\beta} \right\} \tilde{\omega}^t \approx (\gamma_{lc} - \gamma_c) \tilde{c}_t^t + (\gamma_l - \gamma_d) \left[ \theta \tilde{\Omega}_t + \tilde{L}_t \right] + \tilde{\sigma}_t \]  
\[ + \frac{\xi_w/\beta}{1 - \xi_w/\beta} (1 + \Gamma \theta) E_t \{ \tilde{\omega}_{t+1} \} \]
\[ \tilde{C}_t \cong (1 - \xi_w) \tilde{c}_t + \xi_w \tilde{C}_t^O \]  
\[ E_t (B_t^*) \cong \frac{1}{\beta} B_t^* \beta (\tilde{Y}_t - \tilde{C}_t) \quad \text{where } B_t^* = 0 \text{ under financial autarky.} \] 

where tilde variables denote log deviations from steady state, that is \( \tilde{x}_t \equiv \log(x_t) - \log(x_{ss}) \), also:

\[ \gamma_C \equiv \frac{U_{c(ss)}}{U_{c}} < 0, \quad \gamma_l \equiv \frac{U_{l_{ss}}}{U_{l}} > 0 \]

\[ \gamma_{CL} \equiv \frac{U_{c,ss}}{U_{c}} L_{ss}, \quad \gamma_{LC} \equiv \frac{U_{l_{ss}}}{U_{l}} C_{ss} \]

\[ \Gamma \equiv \frac{\gamma_c \gamma_l - \gamma_{lc} \gamma_d}{\gamma_c} > 0 \quad \text{by concavity} \]

This system gives us 10 equations in 11 variables. Under full financial integration the variables are: \( \tilde{L}_t, \tilde{Y}_t, \tilde{\Omega}_t, \tilde{\omega}_t, \tilde{\theta}_t, \tilde{C}_t, \tilde{c}_t, \tilde{C}_t^O, B_t^*, \tilde{R}_t, \) and \( \tilde{A}_t \), while under financial autarky \( \tilde{R}_t^* \) replaces \( B_t^* \). The model is closed by letting the monetary authority choose \( \tilde{R}_t \).

## 4 Optimal Monetary Policy

This section solves for the first best allocation and then finds a monetary policy that recovers it as an equilibrium outcome. Clearly, if such a policy exists, it is optimal. We conduct this exercise for both structures of financial markets and then analyze the cyclical properties of the optimal policies. It should be noted that for simplicity we discuss only non-sunspot equilibria. Appendix 1 shows how the policy rules presented in this section can be modified in order to establish uniqueness without affecting any of the results.

### 4.1 Full Financial Integration

Under full financial integration the first best allocation is found by solving:

\[
\text{Max}_{C_t, L_t, B_t^*(s^{t+1})} \sum_{t=0}^{\infty} \beta^t E_0 [U(C_t, L_t)]
\]

s.t. \[ \int_{s_{t+1}} Q_t^* (s^t, s_{t+1}) B_t^* (s^t, s_{t+1}) d s_{t+1} = B_{t-1}^* (s^t) + A_t L_t^0 - C_t \]
For simplicity, this formulation already imposes a symmetric allocation across households and therefore we only need to find the aggregate quantities.\textsuperscript{18} The first best allocation is characterized by:

\begin{align}
U_{C_l} &= U_{C_{l+1}} \\
-U_{L_t} &= \alpha A_t L_t^{\alpha-1} \\
\frac{1}{R^z} E_t \left[ B_t^* \left( s^t, s_{t+1} \right) \right] &= B_{t-1}^* \left( s^t \right) + A_t L_t^\alpha - C_t
\end{align}

(24) \hspace{1cm} (25) \hspace{1cm} (26)

Notice that by (7), (8), (11), and (13), the economy with staggered wages has the same steady state as the first best allocation provided that:

\[ \tau_w = \frac{1}{\theta - 1} \]

That is, the optimal labor subsidy exactly offsets the distortion of the monopolistic competition in the labor market. As in Rotemberg and Woodford (1999), the fiscal policy in the model is focused on restoring the optimal level of economic activity and is independent of the monetary policy. The role of the monetary policy is therefore to restore optimal fluctuations.

\subsection*{4.1.1 Optimal Monetary Policy}

The first best allocation clearly differs from the staggered wages equilibrium in its labor market condition.\textsuperscript{19} Recall, however, that in characterizing the equilibrium we had one degree of freedom left as we derived 10 equations in 11 unknowns. We can therefore impose efficiency in the labor market as suggested by (25), and then solve for the implied monetary policy.

After complementing the equilibrium conditions with the log linearized version of (25),

\textsuperscript{18} Symmetry follows by concavity of preferences and equal welfare weights across households.

\textsuperscript{19} Under flexible wages, i.e. $\xi_w = 0$, the economy achieves the first best regardless of the monetary policy (assuming $\tau_w = \frac{1}{\theta - 1}$).
the implied interest rate rule is given by:

\[
\tilde{R}_t \approx \frac{\Gamma}{1 - \alpha + \Gamma} (1 - \rho_A) \tilde{A}_t
\]  

(27)

and in terms of output:

\[
\tilde{R}_t \approx \frac{\Gamma}{1 + \Gamma} (1 - \rho_A) \tilde{Y}_t
\]  

(28)

By concavity of \( U(\cdot) \) we have \( \Gamma > 0 \), which suggests that the optimal monetary policy is counter-cyclical since the interest rate increases with output.

Interestingly, we find that under the optimal policy nominal wages always take their steady state value, that is:

\[
\tilde{\Omega}_t \equiv 0
\]

Therefore, the optimal monetary policy targets wage inflation. The reason is that this policy exactly offsets the distortion that sticky wages introduce. The monetary authority manipulates the interest rate in a way that households who can adjust their wage choose to change it by exactly the steady state inflation rate. As a result, the relative price of any two labor skills is constant at unity and all households enjoy the optimal wage level that would have prevailed if wages were flexible.

### 4.2 Financial Autarky

Under financial autarky the first best allocation is found by solving:

\[
\begin{align*}
\max_{C_t, L_t} & \quad \sum_{t=0}^{\infty} \beta^t E_0 [U(C_t, L_t)] \\
\text{s.t.} & \quad C_t = A_t L_t^{\alpha}
\end{align*}
\]

As discussed in chapter 4 of Woodford (2003), an interest rate rule that depends only on exogenous shocks may lead to sunspots equilibria and therefore may result in undesirable outcomes. Woodford (2003) also shows that uniqueness may be restored by modifying the interest rate rule to react to endogenous variables. Appendix 1 finds optimal rules that react to both the productivity and the exchange rate such that equilibrium is unique and all results discussed in the text remain.
The first best allocation is now characterized by:

\[- \frac{U_{L_t}}{U_{C_t}} = \alpha A_t L_t^{\alpha-1} \tag{29}\]
\[C_t = A_t L_t^\alpha \tag{30}\]

Again, in steady state, this allocation coincides with the staggered wages equilibrium provided that:

\[\tau_w = \frac{1}{\theta - 1}\]

4.2.1 Optimal Monetary Policy

As in the case of full integration, the first best allocation differs from the staggered wages economy only in its labor market condition. Taking the same approach as before, we complement the equilibrium conditions of the staggered wages economy by imposing efficiency in the labor market and then find the implied interest rate rule.

The solution for the optimal rule in this case is given by:\footnote{The comment in the previous footnote applies to equation (31) as well.}

\[\tilde{R}_t \approx \frac{\gamma_c \gamma_l - \gamma c \gamma l + \gamma l + \gamma l}{1 - \alpha (1 + \gamma_c - \gamma_l) + \gamma_l - \gamma c} (1 - \rho_A) \tilde{A}_t \tag{31}\]

and in terms of output:

\[\tilde{R}_t \approx \frac{\gamma_c \gamma_l - \gamma c \gamma l + \gamma l + \gamma l}{1 + \gamma_l - \gamma c} (1 - \rho_A) \tilde{Y}_t \tag{32}\]

Also, as in the case of full integration we have:

\[\tilde{\Omega}_t = 0\]

That is, the monetary authority completely offsets the distortion that sticky wages introduce.

We now turn to analyzing the cyclical properties of the optimal monetary policy. The assumption of normality in consumption and leisure implies that \(\gamma_l - \gamma c > 0\), which suggests...
that the denominator in (32) is positive; therefore, the cyclical stance is determined by the sign of the numerator. Specifically the optimal monetary policy is pro-cyclical if:

\[
\gamma_c \gamma_l - \gamma_{cd} \gamma_{lc} < - (\gamma_l + \gamma_{lc})
\]

\[
\Leftrightarrow \quad U_{wss} U_{lss} - U_{dss} U_{lcss} > - \frac{U_{wss} U_{lss}}{C_{ss} L_{ss}} (\gamma_l + \gamma_{lc})
\]

The left hand side is the determinant of the Hessian matrix of the utility function and it is positive by assumption of concavity. The right hand side can be either positive or negative. This suggests that under financial autarky the optimal policy is pro-cyclical provided that the utility function exhibits sufficient curvature as measured by the determinant of its Hessian matrix.

In order to assess the empirical plausibility of this condition we continue by experimenting with different types of utility functions and evaluate the parameter requirements for generating pro-cyclical policies. In what follows we experiment with an additively separable utility in consumption and labor, a Cobb-Douglas utility function, and preferences that exhibit no wealth effect in labor supply as in Greenwood Hercowitz and Huffman (1988), GHH preferences hereafter.

**Additively Separable Utility Function** For additively separable utility function we have \( \gamma_{cd} = \gamma_{lc} = 0 \). In this case The optimal monetary policy is given by:

\[
\tilde{R}_t \cong (1 + \gamma_c) \frac{\gamma_l}{1 + \gamma_l} (1 - \rho_A) \bar{Y}_t
\]

which suggests that the policy is pro-cyclical if \( \gamma_c < -1 \). Recall that \( \gamma_c = \frac{U_{wss}}{U_{css}} C_{ss} < 0 \) and its absolute value is the relative risk aversion coefficient, or the inverse of the intertemporal elasticity of substitution (IES). Notably, relative risk aversion coefficients greater than unity, or IES below unity, lie well within empirical estimates.\(^{22}\)

\(^{22}\)Van Wincoop (1999) argues that a relative risk aversion of 3 represents an average of estimates in the literature, while Reinhart and Végh (1995) argue that estimates for the IES are typically well below unity.
Cobb-Douglas Utility Function  Let:

\[ U (C_t, L_t) = \frac{[C_t^{\mu} (1 - L_t)^{1-\mu}]^{1-\gamma} - 1}{1-\gamma} \]

where 0 < \mu < 1 and \gamma > 0. \gamma is a curvature parameter and in this case the IES is given by \frac{1}{1-\mu(1-\gamma)}. Under these preferences the optimal policy becomes:

\[ \tilde{R}_t \cong \mu (1 - \gamma) (1 - \rho_A) \tilde{Y}_t \]

This suggests that the policy is pro-cyclical if \gamma > 1, which also implies an IES below unity.

GHH Preferences  Let:

\[ U (C_t, L_t) = \frac{(C_t - \psi L_t^v)^{1-\gamma} - 1}{1-\gamma} \]  \hspace{1cm} (33)

where \psi, \gamma > 0 and \nu > 1. Again, \gamma is a curvature parameter and in this case the IES is given by \frac{1}{\gamma}. Under these preferences the optimal policy becomes:

\[ \tilde{R}_t \cong \frac{\nu-1}{\nu^2} \frac{-\gamma}{(1 - \rho_A)} \tilde{Y}_t \]

This suggests that the policy is pro-cyclical if \gamma > \frac{\nu-1}{\nu}. Therefore, an IES below unity is sufficient for pro-cyclicality.

To conclude, under financial autarky the optimal monetary policy is pro-cyclical provided that preferences are ”sufficiently concave”. By experimenting with different types of utility functions it seems that preferences with IES below unity satisfy this requirement.

4.3 Intuition

We have just shown the main result of the paper: if the economy is fully integrated in the world’s financial markets then the optimal monetary policy is counter-cyclical, while under financial autarky the optimal policy is pro-cyclical provided that utility is sufficiently concave.
The key to understanding this result is the behavior of the exchange rate and its effect on real wages. In order to gain some intuition we analyze the reaction of the economy to an increase in $\tilde{A}_t$. We will first hold the interest rate constant and then ask in what direction it should move in order to restore the flexible wages equilibrium.

For simplicity we make two assumptions. First, assume that $\xi_w$ is arbitrarily close to 1, hence the nominal wage index remains unchanged throughout the analysis. Second, we assume that labor supply is fixed throughout the analysis, which is equivalent to assuming GHH preferences. Of course, neither of the assumptions are needed for the results to hold.

**Full Financial Integration** As $\tilde{A}_t$ increases, labor demand shifts outward, while labor supply remains unchanged. This is illustrated in Figure 2.

Also, notice that since households perfectly smooth their marginal utility of consumption the Euler equation (19) coincides in this case with the interest parity condition:

$$\tilde{R}_t = E_t (\tilde{\sigma}_{t+1}) - \tilde{\sigma}_t$$

If the interest rate is unchanged then the exchange rate is constant, which in turn leaves real wages unchanged. As a result the economy moves from point $A$ to point $B_{CM}$ in the figure. Recall that the first best allocation is achieved by the flexible wages equilibrium, which is represented by point $C$. It is now clear that the optimal policy in this case aims to appreciate the domestic currency so as to increase the real wage and move the economy to point $C$. This is achieved by an increase in the interest rate and therefore the optimal monetary policy is counter-cyclical.

**Financial Autarky** As before, an improvement in productivity shifts labor demand outward, while labor supply remains unchanged. However, under financial autarky the economic expansion reduces the current marginal utility of consumption as households can no longer use the international financial markets to smooth it.
The economic expansion is expected to die out over time; therefore, the fall in marginal utilities reduces the real interest rate. This can be seen from the Euler equation (19). Since we hold the nominal interest rate fixed, a fall in the real interest rate can come about only through an appreciation of the exchange rate, this in turn raises real wages and endogenously counteracts the initial expansion.

We know that the optimal policy attempts to manipulate the exchange rate in a way that brings the economy to point \( C \). Therefore, in order to evaluate the direction in which the monetary authority should change the interest rate, we need to determine the magnitude of the increase in real wages. If the appreciation of the exchange rate leads the economy to a point below \( C \) (a point like \( B_{FA,1} \) in Figure 2), then the optimal response is to increase the interest rate; while if the economy reaches a point above \( C \) (such as \( B_{FA,2} \)) then the optimal response is to reduce the interest rate.

We can now see how the curvature of the utility function plays a role in determining the cyclical stance of the optimal policy. From (19) it is clear that the greater the movement in marginal utilities the greater is the movement in the real interest rate that is required in order to restore equilibrium. Therefore, a greater curvature implies a greater movement in the exchange rate, which in turn makes it more likely to bring the economy to a point like \( B_{FA,2} \) where the optimal policy is pro-cyclical, that is to reduce the interest rate in response to a positive productivity shock.

Notice that this policy suggests that developing economies should pursue policies that actively stabilize their exchange rate. Calvo and Reinhart (2002) label this type of policies as "fear of floating", and while they criticize them our model supports these policies as optimal.

5 Different Levels of Financial Integration

The analysis so far has focused on two polar cases: full financial integration and financial autarky. This section allows for intermediate levels of integration and demonstrates that
the optimal monetary policy shifts monotonically from pro-cyclical to counter-cyclical as the economy becomes more integrated in the international financial markets.

5.1 Model’s Alteration

Now assume that domestic residents can only trade risk-free bonds in the international markets. Let $B_t^*$ denote the number of foreign bonds bought in date $t$. Each unit of $B_t^*$ pays $R_t^*$ units of the foreign currency in $t + 1$ with certainty.

Financial integration is modeled by introducing a convex adjustment cost to the portfolio of foreign assets, $k(B_t^*)$, where the function $k(\cdot)$ satisfies $k(B_{ss}^*) = 0$, $k'(B_{ss}^*) = 0$, and $k''(\cdot) \geq 0$. This cost represents various barriers that prevent domestic residents from trading freely in the international financial markets; such barriers may include taxes, transaction costs, costs that arise from exchanging currencies, country risk, and any form of capital controls. Clearly, lower cost implies greater financial integration.

Note that since the model is linearized, and due to our assumptions regarding the cost function, only its second derivative evaluated at steady state, $k''(B_{ss}^*)$, matters for the dynamics of the model. Therefore, all functions that satisfy the assumptions above are equivalent for our purpose; for simplicity of exposition we will employ a quadratic cost function:

$$k(B_t^*) = \frac{\kappa}{2}(B_t^* - B_{ss}^*)^2 \quad \kappa \geq 0$$

Varying $\kappa$ allows us to study the effect of financial integration. When $\kappa = 0$ the economy can borrow and lend freely at the world’s interest rate and therefore this case represents a developed economy.\(^{23}\) In addition, when $\kappa$ is negligibly small the dynamics of the model are very similar to those of the complete markets case analyzed earlier.\(^{24}\) As $\kappa$ increases the...
country faces greater barriers to adjusting its foreign portfolio, and as $\kappa \to \infty$ the country is in financial autarky. We can therefore move continuously from full financial integration to financial autarky by varying $\kappa$.

This modification to the model only alters the Euler equation for foreign assets and the resource constraint of the economy. The Euler equation is now given by:

$$U_{C_t} [1 + \kappa (B_t^* - B_{ss}^*)] = \beta R_t^* E_t (U_{C_{t+1}})$$

(34)

where again $R_t^* = \beta^{-1}$, and the resource constraint takes the form:

$$B_t^* + \frac{\kappa}{2} (B_t^* - B_{ss}^*)^2 = R_{t-1}^* B_t^* + Y_t - C_t$$

(35)

Equations (34) and (35) replace (8) and (13), respectively. As before, the optimal monetary policy is found by imposing efficiency in the labor market.

The log-linearized system of equations is solved numerically using the method of Sims (2002). The model is then simulated in order to study the cyclical properties of the monetary policy for different values of $\kappa$. The choice of parameter values and utility function is described below.

### 5.2 Calibration

In all simulations we use GHH preferences as specified by (33). These preferences are commonly used in small open economy models, see for example Mendoza (1991), Correia et al (1995), and Neumeyer and Perri (2005). It should be noted that similar results are obtained from using a Cobb-Douglas utility function and additively separable preferences.

Parameter values are mainly adopted from other studies in the literature. Table 2 reports the baseline parameterization.  

small open economy models with risk-free bonds behave almost identically to their complete markets counterparts. Baxter and Crucini (1995) and Heathcote and Perri (2002) find similar results in two-country models.
A period in the model corresponds to one quarter. The discount factor, $\beta$, is set to 0.99, which suggests a world real interest rate of 4 percent. Labor income share, $\alpha$, is set to 0.7.

We set the curvature parameter, $\gamma$, to 2. Mendoza (1991) experiments with two values 1 and 2, Correia et al (1995) choose 2, Neumeyer and Perri (2005) follow the recommendation of Reinhart and Végh (1995) regarding the intertemporal elasticity of substitution in developing countries and set $\gamma$ to 5. However, since the analysis of the case of financial autarky suggests that the cyclical pattern of the monetary policy may crucially depend on this parameter, we will experiment with other values as well.

The elasticity of labor supply is given by $\frac{1}{\psi}$. We follow Neumeyer and Perri (2005) and set $\psi$ to 1.6 (an elasticity of 1.67); Mendoza (1991) choose $\psi = 1.455$ (an elasticity of 2.2) and Correia et al (1995) choose a value of 1.7 (an elasticity of 1.43).

The parameter $\psi$ affects steady state values of macroeconomic aggregates, however it does not appear in any of the log-linearized equations and therefore it is irrelevant for the calibration.

The elasticity of substitution between differentiated labor skills, $\theta$, takes a value of 6. This implies a markup of 20 percent ($\frac{\theta}{\frac{\theta}{1+\theta}}$) in the labor market. For comparison, Erceg et al (2000) take $\theta = 4$ (a markup of 33 percent), Chari et al (2002) use $\theta = 7.67$ (a markup of 15 percent), Huang and Liu (2002) use values of $\theta = 2, 4, 6$ (markups of 100, 33, and 20 percent, respectively).

The probability to receive a wage adjustment signal, $1 - \xi_w$, is set to 0.25. This corresponds to an average wage duration of 4 quarters ($\frac{1}{1-\xi_w}$). This value is consistent with Erceg et al (2000), Chari et al (2002), and Huang and Liu (2002).

The autocorrelation coefficient of productivity, $\rho_A$, is set to 0.9. This is somewhat lower relative to other values in the literature; however, as mentioned earlier, productivity shocks in the model can be equivalently interpreted as terms of trade movements, and in
our sample the latter display lower autocorrelations.\footnote{With the exception of Singapore, which displays an extremely low autocorrelation, the coefficient for developed economies varies from 0.84 to 0.99 with an average of 0.91. For developing counties the autocorrelation varies from 0.51 to 0.94, with an average of 0.70. Due to limitation of data availability these statistics are based on 12 developed economies and 6 developing countries. See the data appendix for details.}

The standard deviation of the innovation to productivity, $\sigma_A$, is set arbitrarily to 0.01. Naturally, this parameter affects the volatility of economic aggregates; however, since the model is linearized and is driven by only one shock, the volatility of the error term has no effect on correlations, and specifically it has no effect on the correlation between output and the nominal interest rate, which is our object of interest.

Finally, the parameter $\kappa$ allows us to move from the case of full integration to financial autarky. We will therefore experiment with a wide range of values.

5.3 Results

We use the baseline parameterization to simulate the model under different levels of $\kappa$. Figure 3 presents the correlation between the nominal interest rate, $\tilde{R}_t$, and output, $\tilde{Y}_t$, as a function of $\kappa$ using the baseline parameterization. Each data point in the figure is the average correlation over 10,000 simulations each 100 periods long. The solid line corresponds to this version of the model. As expected, the correlation falls as $\kappa$ increases, it is positive for small $\kappa$’s and becomes negative for large $\kappa$’s. That is, as the economy becomes less integrated in the world’s financial markets the optimal monetary policy shifts from counter-cyclical to pro-cyclical.

It should be noted that results are robust to the specification of the utility function, specifically the use of a Cobb-Douglas utility function or additively separable preferences generate similar result.
5.4 Sensitivity Analysis

The choice of parameter values is largely based on studies that focused on developed economies. We therefore wish to reevaluate the implications for the cyclical pattern of monetary policy using other values as well.

The closed form solution for the case of financial autarky, equation (32), indicates that the results crucially depend on the curvature of the utility function. Specifically, under GHH preferences if $\gamma < \frac{v-1}{v}$ then the optimal policy is counter-cyclical even when the economy is cutoff from the international financial markets. Under our parameterization the cutoff value is $\frac{v-1}{v}$ = 0.375; however, since $v > 1$ the cutoff value cannot exceed unity.

In order to evaluate the robustness of the results we experiment with different values of $\gamma$. Panel A of Figure 4 depicts the correlation between output, $\tilde{Y}_t$, and the nominal interest rate, $\tilde{R}_t$, for different values of $\gamma$. The figure reveals, as one may expect, that when $\gamma > \frac{v-1}{v}$ the correlation falls monotonically as the economy becomes less integrated in the international financial markets. On the other hand, when $\gamma < \frac{v-1}{v}$ the model displays high correlations for any level of financial integration.

The rest of the panels in Figure 4 study the effect of other parameters on the cyclical stance of the monetary policy. Practically none of the parameters seems to have any important quantitative effect on the results.

6 Robustness

This section conducts two robustness experiments. The first introduces demand shocks to the model and the second adds a non-tradable sector. We show that the results are largely unaffected by these modifications.
6.1 Demand Shocks

Typically, counter-cyclical monetary policies are recommended under the premise that the business cycle is driven by demand shocks. In contrast, the business cycle in the model is driven by supply shocks. We therefore wish to reevaluate the results after introducing demand shocks to the model.

6.1.1 Model’s Alteration

We alter the model only by adding a stochastic preference parameter. The periodical utility function is now given by:

\[ \eta_t U (C_t, L_t) \]

where:

\[ \log (\eta_t) = (1 - \rho_{\eta}) \mu_{\eta} + \rho_{\eta} \log (\eta_{t-1}) + \varepsilon_{\eta,t} \quad 0 < \rho_{\eta} < 1 \]

\[ \varepsilon_{\eta,t} \sim \text{N} (0, \sigma_{\eta}^2) \quad ; \quad \text{COV} (\varepsilon_{\eta,t}, \varepsilon_{A,t}) = 0 \]

That is, \( \eta_t \) follows a stationary AR(1) process (in logs) and it is uncorrelated with the supply shock. Other than that, the model is identical to the one presented earlier.

The preference shock affects the intertemporal decisions of the households. A rise in \( \eta_t \) increases current marginal utility of consumption relative to its expected value in future periods, this in turn stimulates current demand and causes a consumption boom.

The introduction of preference shocks only affects the Euler equation for foreign assets (34), the Euler equation for domestic bonds (9), and the optimal wage setting (11). The only difference is that all marginal utilities, \( U_{ct} \) and \( U_{lt} \), are now multiplied by \( \eta_t \). As before, the optimal monetary policy seeks to restore efficiency in the labor market.

6.1.2 Parameter Values

It is hard to evaluate the relative importance of preferences shocks relative to supply shocks, and unfortunately the literature provides little guidance in this respect. Parkin (1988) con-
cludes that fluctuations in preference shocks are tiny, while Hall (1997) finds that they play a dominant role in driving the business cycle and specifically are more volatile than productivity shocks. Baxter and King (1991) find that both shocks display similar persistence while the variance of preference shocks is about 36 percent greater than that of productivity shocks. Stockman and Tesar (1995) report difficulties in estimating a stochastic process for preference shocks. Depending on the procedure they employ, their estimates of the variance of the preference parameter varies from 0.01 to 2.5 times the variance of productivity.

In absence of clear guidance from the literature we experiment with a wide range of relative volatilities. We regard the case where demand shocks follow the same time series process as productivity as the baseline scenario, that is $\rho_\eta = 0.9$ and $\sigma_\eta = 0.01$, but also let the ratio $\sigma_\eta/\sigma_A$ take different values between 0 and 5. We do that by fixing $\sigma_A$ at its baseline level and letting $\sigma_\eta$ vary. It should be stressed that the absolute value of the variances have no effect on the interest-output correlation, only their ratio matters.

### 6.1.3 Results

Figure 5 depicts the optimal interest-output correlation as a function of $\kappa$ in the different simulations. In all cases the correlation increases as $\kappa$ falls. That is, greater financial integration implies that the monetary authority should pursue counter-cyclical policies more aggressively. As expected, greater $\sigma_\eta/\sigma_A$ pushes the optimal policy towards the counter-cyclical end; however, the figure also demonstrates that the model still supports pro-cyclical policies even when demand shocks are overwhelmingly more volatile than supply shocks, provided that $\kappa$ is large enough.

### 6.2 Non-Tradable Sector

The monetary authority in the model manipulates the interest rate in order to influence real wages. It does so through its effect on the exchange rate. The model with only tradable goods assumes a complete pass-through from the exchange rate into domestic prices; as a
result, fluctuations in the exchange rate translate directly into fluctuations in real wages. Therefore, at first glance, the assumption of a complete pass-through seems to be important for generating pro-cyclical policies. We now relax this assumption by introducing a non-tradable sector and reevaluate the results.

The main conclusions of this section are twofold. First, the introduction of a non-tradable sector tends to shift the optimal monetary policy toward pro-cyclicality; and second, the main message remains; greater financial integration pushes the optimal monetary policy from pro-cyclical to counter-cyclical.

6.2.1 Model’s Alteration

Consumption is now a composite of tradable and non-tradable goods, specifically:

\[ C_t = C_{N,t}^{\omega} C_{T,t}^{1-\omega} \]

where \( C_{N,t} \) is consumption of non-tradables and \( C_{T,t} \) is consumption of tradables. Firms in the two sectors use the following production technologies:

\[ Y_{N,t} = A_{N,t} L_{N,t}^{\alpha_N} \]
\[ Y_{T,t} = A_{T,t} L_{T,t}^{\alpha_T} \]

where \( L_{N,t} \) and \( L_{T,t} \) are labor inputs in the non-tradable sector and the tradable sector, respectively. Similarly, \( A_{N,t} \) and \( A_{T,t} \) are total factor productivity in the two sectors; these follow a stationary AR(1) process:

\[ \log(A_{N,t}) = (1 - \rho_N) \mu_N + \rho_N \log(A_{N,t-1}) + \varepsilon_{N,t} \]
\[ \log(A_{T,t}) = (1 - \rho_T) \mu_T + \rho_T \log(A_{T,t-1}) + \varepsilon_{T,t} \]

where:

\[
\begin{bmatrix}
\varepsilon_{N,t} \\
\varepsilon_{T,t}
\end{bmatrix} 
\sim iid N\left(0, \begin{bmatrix}
\sigma^2_N & \sigma_{N,T} \\
\sigma_{N,T} & \sigma^2_T
\end{bmatrix}\right)
\]
Market clearing in the two sectors requires:

\[ C_{N,t} = Y_{N,t} \]

\[ \frac{1}{R_t}E_t(B_t) = B_t^* + Y_{T,t} - C_{T,t} \]

Finally, total output is measured in units of the composite consumption good:

\[ Y_t = \frac{P_{N,t}Y_{N,t} + S_tY_{T,t}}{P_t} \]

where \( P_{N,t} \) is the domestic currency price of the non-tradable good, and \( P_t \) is the consumption price index, which is given by:

\[ P_t = \left( \frac{P_{N,t}}{\omega} \right) ^{\omega} \left( \frac{S_t}{1 - \omega} \right) ^{1 - \omega} \]

The introduction of a non-tradable sector does not distort any of the equilibrium conditions; hence, it is still the case that the flexible wages equilibrium achieves the first-best allocation. Therefore, our solution strategy remains; we simply complement the system with the efficiency condition in the labor market.

6.2.2 Parameter Values

In steady state the share of spending on non-tradable goods is given by \( \omega \); we set it to 0.5. This value is consistent with Stockman and Tesar (1995), and with evidence presented in Burstein et al (2005) regarding the share of non-tradable goods in the CPI of various countries.\(^{26}\)

The non-tradable sector is typically more labor-intensive relative to the tradable sector, this implies that \( \alpha^N > \alpha^T \). We leave \( \alpha^T \) at its previous value, 0.7, and set \( \alpha^N \) to 0.8.

Productivity in the non-tradable has no terms of trade component, we therefore let it display a somewhat greater degree of persistence than productivity in the tradable sector. We set \( \rho_N = 0.95 \) and let \( \rho_T \) take its previous value of 0.9. Similarly to Stockman and

\(^{26}\)It should be noted that Burstein et al (2005) argue that the share of non-tradables is actually higher since many tradable goods have non-traded components.
Tesar (1995) we assume that the correlation of innovations across sectors is 0.5. Finally, we assign the innovations the same standard deviation, $\sigma_N = \sigma_T = 0.01$.

### 6.2.3 Results

We start by simulating the model without demand shocks. Figure 3 presents simulation results for the baseline parameterization (line with triangle markers). The introduction of a non-tradable sector reduces the interest-output correlation for any level of $\kappa$, suggesting greater pro-cyclicality. The sensitivity analysis with respect to the non-tradable share, $\omega$, yields similar results (See figure 6). The reason is that the non-tradable sector is similar to the one-sector economy under financial autarky, and for $\omega = 1$ the two are in fact identical. Therefore, as $\omega$ increases the optimal policy becomes more similar to the one under financial autarky.

Finally, we add demand shocks with time series parameters as reported in Table 2. Figure 3 demonstrates that, as in the one-sector model, demand shocks push the optimal policy toward counter-cyclicality (line with square markers).

The results are therefore robust to the introduction of a non-tradable sector even though the passthrough from exchange rate movement to prices is now incomplete; as a country enjoys greater financial integration the optimal monetary policy shifts from pro-cyclical to counter-cyclical.

### 7 Conclusion

This paper has documented that, in contrast to developed economies, developing countries tend to follow pro-cyclical monetary policies. The paper then showed that a fairly standard New-Keynesian model supports this behavior as optimal, provided that developing countries face greater barriers to the international financial markets relative to developed economies. Specifically, the model suggests that under full financial integration the optimal monetary policy is counter-cyclical, while if the economy is excluded from the inter-
national financial markets then the optimal policy is pro-cyclical. The model also suggests that the transition from pro-cyclical to counter-cyclical policies is monotonic in the level of integration. That is, greater integration pushes the optimal monetary policy toward counter-cyclicality. These results are robust to a wide range of parameter values, utility specifications, and to the introduction of demand shocks and a non-tradable sector.

Furthermore, in the model pro-cyclicality is coupled with policies that actively stabilize the exchange rate. Often, countries are criticized for following such policies; nevertheless, our model supports them as optimal.

Finally, an important insight from the paper is that the cyclical pattern alone may not serve as an indicator for evaluating policies. It may, however, serve as an indicator for the economic conditions under which policymakers operate. In terms of the model, a pro-cyclical monetary policy might be a symptom of the inability to borrow and lend in the international financial markets. Therefore, advising countries to follow counter-cyclical policies merely treats the symptom and may result in undesirable economic outcomes. Instead, attention should be focused on measures for promoting integration in the international financial markets. Once integrated, policies would endogenously shift toward counter-cyclicality.

8 Appendix 1: Determinacy of Equilibria

The results in the text ignore determinacy issues as they were derived by assuming a non-sunspot equilibrium. This appendix shows that uniqueness can be achieved by including the exchange rate in the interest rate rule with an arbitrary positive coefficient, and that such modification does not affect any of the results discussed in the text.
8.1 Full Financial Integration

8.1.1 Determinacy of the Exchange Rate

Under full integration marginal utilities are perfectly smoothed, that is $\gamma_c \tilde{c}_t + \gamma_d \tilde{L}_t \cong 0$. Therefore, the interest rate rule (27) and the Euler equation (19) give:

$$\frac{\Gamma}{1 - \alpha + \Gamma} (1 - \rho_A) \tilde{A}_t \cong E_t (\tilde{\sigma}_{t+1}) - \tilde{\sigma}_t$$

which characterizes the solution for the exchange rate. Under the non-sunspot equilibrium we have:

$$\tilde{\sigma}_t \cong -\frac{\Gamma}{1 - \alpha + \Gamma} \tilde{A}_t$$

(36)

However, the Euler equation clearly has a unit root which opens the possibility for multiple solutions. Specifically:

$$\tilde{\sigma}_t \cong -\frac{\Gamma}{1 - \alpha + \Gamma} \tilde{A}_t + \chi_t$$

is also a solution, where $\chi_t$ is some non-fundamental noise such that $\chi_t = E_t (\chi_{t+1})$.

Following Woodford (2003) we now show that by including the exchange rate with a positive coefficient in the interest rate rule, the monetary policy can support a unique equilibrium. Consider the following rule:

$$\tilde{R}_t \cong a \tilde{A}_t + b \tilde{\sigma}_t \quad b > 0$$

(37)

Substituting $\tilde{R}_t$ into (19), and imposing $\gamma_c \tilde{c}_t + \gamma_d \tilde{L}_t \cong 0$, gives:

$$E_t (\tilde{\sigma}_{t+1}) \cong a \tilde{A}_t + (1 + b) \tilde{\sigma}_t$$

Since the exchange rate is a non-predetermined endogenous variable, and since $b > 0$ this equation has a unique solution. One can easily check that this solution is given by:

$$\tilde{\sigma}_t \cong -\frac{a}{b + 1 - \rho_A} \tilde{A}_t$$

In order to recover the non-sunspot solution as given by equation (36), we must impose:

$$a = \frac{\Gamma}{1 - \alpha + \Gamma} (b + 1 - \rho_A) \quad b > 0$$

39
Therefore, any policy rule that satisfies this condition is able to determine the exchange rate at the level of the non-sunspot equilibrium. Furthermore, after substituting the exchange rate into the policy rule (37) and using the restrictions on \( a \) and \( b \) we recover the expression for the interest rate, equation (27), as a reduced form solution. We now turn to show that given (36) the rest of the system is uniquely determined, and that its solution is the same as the non-sunspot equilibrium.

8.1.2 The Rest of the System

By using (36) to substitute for \( \tilde{\sigma}_t \), and by manipulating equations (15), (17), (20), (21), and (22), we get:

\[
\begin{bmatrix}
E_t \left( \frac{\tilde{\Omega}_{t+1}}{\tilde{\Omega}_t} \right)
\end{bmatrix} = \Lambda \begin{bmatrix}
\frac{\tilde{\Omega}_t}{\tilde{\Omega}_{t-1}}
\end{bmatrix}
\]

where

\[
\Lambda \equiv \begin{bmatrix}
\frac{1+\beta}{\beta} + \frac{(1-\xi_w \beta)(1-\xi_w)}{\xi_w \beta} & \frac{1+\alpha+\Gamma}{1+\Gamma \theta(1-\alpha)} & -\frac{1}{\beta} \\
\end{bmatrix}
\]

This system has a unique solution if and only if \( \Lambda \) has one eigenvalue inside the unit circle and the other outside the unit circle. It is sufficient to show that:

\[
\det (\Lambda) = \lambda_1 \lambda_2 > 1
\]

\[
\det (\Lambda) - TR (\Lambda) + 1 = (\lambda_1 - 1) (\lambda_2 - 1) < 0
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the eigenvalues of \( \Lambda \). Notice that:

\[
\det (\Lambda) = \frac{1}{\beta} > 1
\]

and therefore both eigenvalues have the same sign, and at least one of them lies outside the unit circle. Also:

\[
\det (\Lambda) - TR (\Lambda) + 1 = \frac{(1-\xi_w \beta)(1-\xi_w)}{\xi_w \beta} \frac{1+\alpha+\Gamma}{(1+\Gamma \theta)(1-\alpha)} < 0
\]

This rules out the possibility that both eigenvalues lie outside the unit circle since we have already established that they both have the same sign. We can therefore conclude that the solution is unique.
Clearly, $\tilde{w}_t = 0$ solves the difference equation, and using (15), we also get $\tilde{w}_t = 0$. These results indicate that this policy recovers the flexible wages equilibrium by targeting wage inflation, as suggested in the text. Given the solution for wages and the exchange rate the rest of the variables, except for $B_t^*$, are determined by a set of static relationships as suggested by the non-sunspot solution. Finally, after solving for $\tilde{C}_t$ and $\tilde{Y}_t$, foreign assets, $B_t^*$, are uniquely determined from the resource constraint since the coefficient on $B_{t-1}^*$ is greater than unity.

We therefore conclude that the monetary authority is able to recover the non-sunspot allocation as a unique equilibrium while maintaining the results presented in the text.

8.2 Financial Autarky

We now show that under financial autarky there exist interest rate rules of the form of (37) that impose determinacy and recover the non-sunspot equilibrium. By substituting equations (15), (16), (17), (20), (22), (23), and (37) into (19) and (21) the system becomes:

$$
\begin{bmatrix}
E_t(\tilde{\Omega}_{t+1}) \\
E_t(\tilde{\sigma}_{t+1}) \\
\tilde{\sigma}_t
\end{bmatrix} = \Lambda_1
\begin{bmatrix}
\tilde{\Omega}_t \\
\tilde{\sigma}_t
\end{bmatrix} + \Lambda_2 \tilde{A}_t
$$

where:

$$
\Lambda_1 \equiv \begin{bmatrix}
1 + \frac{1}{\beta} + \Xi & -\Xi & -\frac{1}{\beta} \\
- \frac{\alpha_{\gamma e} + \gamma_{cd}}{1 - \alpha - \alpha_{\gamma e} - \gamma_{cd}} \left( \frac{1}{\beta} + \Xi \right) & \Xi \frac{\alpha_{\gamma e} + \gamma_{cd}}{1 - \alpha - \alpha_{\gamma e} - \gamma_{cd}} + 1 & \frac{1 - \alpha}{\beta} \frac{\alpha_{\gamma e} + \gamma_{cd}}{1 - \alpha - \alpha_{\gamma e} - \gamma_{cd}} \\
0 & 0 & 0
\end{bmatrix}
$$

$$
\Lambda_2 \equiv \begin{bmatrix}
0 & \Xi - \frac{\alpha_{\gamma e} + \gamma_{cd}}{1 - \alpha - \alpha_{\gamma e} - \gamma_{cd}} \gamma_{\gamma e} + \gamma_{\gamma c} + \alpha (\gamma_{\gamma c} - \gamma_{\gamma e}) & \Xi - \frac{\alpha_{\gamma e} + \gamma_{cd}}{1 - \alpha - \alpha_{\gamma e} - \gamma_{cd}} \gamma_{\gamma e} + \gamma_{\gamma c} + \alpha (\gamma_{\gamma c} - \gamma_{\gamma e}) + \frac{1 - \alpha}{1 - \alpha - \alpha_{\gamma e} - \gamma_{cd}} [ a - \frac{\gamma_{\gamma e} + \gamma_{cd}}{1 - \alpha} (1 - \rho_A)] \\
0 & 0 & 0
\end{bmatrix}
$$

$$
\Xi \equiv \frac{(1 - \xi_w) (1 - \xi_{w,\beta})}{\xi_{w,\beta}} \frac{1 - \alpha + \gamma_{\gamma e} - \gamma_{\gamma c} + \alpha (\gamma_{\gamma c} - \gamma_{\gamma e})}{(1 - \alpha) (1 + \Gamma \theta)} > 0
$$

This system has a unique solution if and only if $\Lambda_1$ has one eigenvalue inside the unit circle and two outside. The characteristic polynomial of $\Lambda_1$ is given by:

$$
P(\lambda) = \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0$$
where: 
\[ A_0 = -\frac{1}{\beta} \left[ 1 + \frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} b \right] \]
\[ A_1 = \frac{1}{\beta} + \bar{\varepsilon} \frac{\alpha \gamma_e + \gamma_cl}{1 - \alpha - \alpha \gamma_e - \gamma_cl} + \left( \frac{1 + \beta}{\beta} + \bar{\varepsilon} \right) \left( 1 + \frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} b \right) \]
\[ A_2 = -\bar{\varepsilon} \frac{\alpha \gamma_e + \gamma_cl}{1 - \alpha - \alpha \gamma_e - \gamma_cl} - \left( 1 + \frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} b \right) - \left( \frac{1 + \beta}{\beta} + \bar{\varepsilon} \right) \]

For uniqueness it is *sufficient* to show that either:  
\[ 1 + A_2 + A_1 + A_0 > 0 \]
\[-1 + A_2 - A_1 + A_0 < 0 \]
\[ |A_2| > 3 \]

or:

\[ 1 + A_2 + A_1 + A_0 < 0 \]
\[-1 + A_2 - A_1 + A_0 > 0 \]

Given the parameters above:
\[ 1 + A_2 + A_1 + A_0 = \bar{\varepsilon} \frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} b \]
\[-1 + A_2 - A_1 + A_0 = -4 \frac{1 + \beta}{\beta} - \left( \bar{\varepsilon} + 2 \frac{1 + \beta}{\beta} b + 2 \bar{\varepsilon} \right) \frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} \]
\[ A_2 = -\frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} (\bar{\varepsilon} + b) - 2 - \frac{1}{\beta} \]

Case 1:
\[ \alpha \gamma_e + \gamma_cl < 1 - \alpha \]

In this case:
\[ \frac{1 - \alpha}{1 - \alpha - \alpha \gamma_e - \gamma_cl} > 0 \]

\(^{27}\)See appendix C in Woodford (2003).
and therefore it is immediate to show that:

\[
1 + A_2 + A_1 + A_0 > 0 \\
-1 + A_2 - A_1 + A_0 < 0 \\
|A_2| > 3
\]

for any \( b > 0 \).

Case 2:

\[
\alpha \gamma_c + \gamma_{cl} > 1 - \alpha
\]

In this case:

\[
\frac{1 - \alpha}{1 - \alpha - \alpha \gamma_c - \gamma_{cl}} < 0
\]

Therefore:

\[
1 + A_2 + A_1 + A_0 < 0
\]

and:

\[
-1 + A_2 - A_1 + A_0 > 0
\]

provided that \( b > -\frac{\alpha 1 + \beta \frac{1 - \alpha - \alpha \gamma_c - \gamma_{cl}}{1 - \alpha} + 2 \Xi}{\Xi + 2 1 + \beta} \)

That is, we can always find a \( b \) large enough such that policy rule (37) imposes uniqueness on the economic system.

As in the case of full financial integration we need to put restrictions on the relationship between \( a \) and \( b \) in order for the system to recover the non-sunspot equilibrium. One can check that for:

\[
a = \left[ 1 - \alpha - \alpha \gamma_c - \gamma_{cl} (1 - \rho_A) + b \right] \frac{\gamma_l - \gamma_{cl} + \gamma_{lc} - \gamma_c}{1 - \alpha + \gamma_l - \gamma_{cl} + \alpha (\gamma_{lc} - \gamma_c)} + \frac{\gamma_e + \gamma_{cl}}{1 - \alpha} (1 - \rho_A)
\]

(38)

The solution is given by:

\[
\tilde{\Omega}_t \cong 0 \\
\tilde{\sigma}_t \cong -\frac{\gamma_l - \gamma_{cl} + \gamma_{lc} - \gamma_c}{1 - \alpha + \gamma_l - \gamma_{cl} + \alpha (\gamma_{lc} - \gamma_c)} \tilde{A}_t
\]
Given the solution for wages and the exchange rate the remaining variables are determined by a set of static relationships as suggested by the non-sunspot solution.

Finally, after substituting the solution for the exchange rate into the policy rule (37) and using the restrictions on $a$ and $b$ as given by (38), we recover the expression for the interest rate, equation (31), as a reduced form solution. We therefore conclude that the monetary authority is able to recover the non-sunspot allocation as a unique equilibrium while maintaining the results presented in the text.

9 Appendix 2: Data

The data source for all series is the IFS. The paper uses quarterly data for the period 1974.1-2004.4 on short-term nominal interest rate, real GDP, CPI, the nominal exchange rate, and the terms of trade.

For nominal interest rate we use either the central bank rate, line 60, or the interbank market rate, line 60B, depending on data availability (see Table 3). For quarterly data we average monthly observations in each quarter.

For real GDP we use volume indices, line 99BVR (seasonally adjusted) or 99BVP (not seasonally adjusted), depending on data availability. For countries with no seasonal correction we adjust the series using X12 multiplicative method.

Inflation is measured using CPI, line 64. For quarterly data we average monthly observations. We then calculate inflation by the CPI growth rate during the last four quarters.

For the nominal exchange rates we use the nominal effective exchange rate, line NEC or NEU. For countries with no data on the effective rates we use the nominal exchange rate against the US Dollar, line AH. See Table 3 for the choice of series for each country. For the estimation of equation (1), the rate of change of the exchange rate was calculated by log differences.

For terms of trade series we divide export unit value, line 74, by import unit value, line 75. These were used for evaluating the persistence of the terms of trade. See Table 3 for
the autocorrelations of the terms of trade.

For the cyclical components we remove HP trends from the interest rate, log GDP, inflation, the exchange rate, and its rate of change. We use a smoothing parameter of 1600 in all cases.

We restrict our sample to include countries with floating or dirty-floating exchange rate regimes with at least 20 consecutive observations. In addition, we remove observations in periods of annual CPI inflation of more than 100 percent.

The classification of exchange rate regimes is based on Levy-Yeyati and Sturzenegger (2005). They assign a five-way index to each country-year observation. The value 1 indicates an inconclusive regime, 2 is float, 3 is dirty float, 4 is either dirty float or a crawling peg, and 5 indicates a fixed exchange rate regime. We include observations that take the values 2 or 3; however, we also include observations that take the value 1 or 4 if they follow directly after periods of floating or dirty-floating (i.e. indexed by 2 or 3) and return to these regimes immediately afterwards. In other words, observations indexed by 1 or 4 are allowed to the extent that they do not open or close the sample period.

Table 3 reports the resulting sample periods for each country. The table also complements Table 1 by reporting country specific interest-output correlations and output coefficients from the estimation of the interest rate rules.

References


Figure 1: Country Correlations Between Cyclical Components of GDP and Short-Term Interest Rate
Figure 2: The Labor Market – A Positive Productivity Shock*

* Labor supply is parameterized by GHH preferences.
Figure 3: Interest-Output Correlation As a Function of $\kappa$, Baseline Parameterization*

* Each data point is the average correlation over 10,000 simulations, each 100 periods long.
Figure 4: Interest-Output Correlation As a Function of $\kappa$, Sensitivity Analysis*

(A) Curvature of Preferences, $\gamma$

(B) Curvature of Disutility from Labor, $v$

(C) Labor Income Share, $\alpha$

(D) Probability of Wage Change, $1 - \xi_{w}$

(E) Elasticity of Substitution of Labor Skills, $\theta$

(F) Auto Correlation, $\rho_{A}$

* Each data point is the average correlation over 10,000 simulations, each 100 periods long.
Figure 5: Interest-Output Correlation As a Function of $\kappa$, A Model with Demand Shocks ($\rho_\eta = 0.9$)*

* Each data point is the average correlation over 10,000 simulations, each 100 periods long.
Figure 6: Interest-Output Correlation As a Function of $\kappa$,
A Two-Sector Model (no demand shocks)*

* Each data point is the average correlation over 10,000 simulations, each 100 periods long.
<table>
<thead>
<tr>
<th>Country Group</th>
<th>Sig. Positive</th>
<th>Sig. Negative</th>
<th>Sig. Positive**</th>
<th>Sig. Negative**</th>
<th>Panel Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Interest-Output Correlation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Developed Economies</td>
<td>0.26</td>
<td>13</td>
<td>2</td>
<td>7</td>
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<tr>
<td>Developing Countries</td>
<td>-0.18</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>5</td>
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<table>
<thead>
<tr>
<th><strong>Panel B: Output Coefficient in Interest Rule Estimation, ( \beta_o = 0 ), OLS</strong></th>
<th>Panel Regression</th>
</tr>
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<tbody>
<tr>
<td>Developed Economies</td>
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<td>Developing Countries</td>
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<table>
<thead>
<tr>
<th><strong>Panel C: Output Coefficient in Interest Rule Estimation, ( \beta_o \neq 0 ), TSLS</strong></th>
<th>Panel Regression</th>
</tr>
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<tbody>
<tr>
<td>Developed Economies</td>
<td>0.13</td>
</tr>
<tr>
<td>Developing Countries</td>
<td>-0.14</td>
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</tbody>
</table>

* Output refers to real GDP and interest rates are either the central bank’s rate or the interbank market rate, depending on data availability. All statistics are calculated using the cyclical components of the variables; these are measured by the deviation of the variables from HP trend.

** 5 percent significance level.
Table 2: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences:</td>
<td>Discount factor, $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Curvature, $\gamma$</td>
<td>2</td>
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<td></td>
<td>Elasticity of labor supply, $\frac{1}{v-1}$</td>
<td>1.67</td>
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<tr>
<td>Labor Market:</td>
<td>Elasticity of substitution, $\theta$</td>
<td>6</td>
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<tr>
<td></td>
<td>Wage rigidity, $\xi_w$</td>
<td>0.75</td>
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<tr>
<td>Technology:</td>
<td>Labor income share, $\alpha$</td>
<td>0.7</td>
</tr>
<tr>
<td>Productivity:</td>
<td>Autocorrelation, $\rho_A$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>Std. of innovations, $\sigma_A$</td>
<td>0.01</td>
</tr>
<tr>
<td>Portfolio adj. cost:</td>
<td>Second derivative at SS, $\kappa$</td>
<td>varies</td>
</tr>
<tr>
<td></td>
<td>SS assets, $B_{ss}^*$</td>
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<tr>
<td>Demand shocks:</td>
<td>Autocorrelation, $\rho_\eta$</td>
<td>0.9</td>
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<tr>
<td></td>
<td>Std. of innovations, $\sigma_\eta$</td>
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<tr>
<td>Non-tradable sector:</td>
<td>Non-tradable share, $\omega$</td>
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<tr>
<td></td>
<td>Autocorrelation, $\rho_N$</td>
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<tr>
<td></td>
<td>Std. of innovations, $\sigma_N$</td>
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</tr>
<tr>
<td></td>
<td>Correlation of innovations, $\rho_{NT}$</td>
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</tr>
<tr>
<td></td>
<td>Labor income share, $\alpha^N$</td>
<td>0.8</td>
</tr>
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</table>
### Table 3: Complementary Table

<table>
<thead>
<tr>
<th></th>
<th>Developed Economies</th>
<th>Developing Countries</th>
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</thead>
<tbody>
<tr>
<td><strong>OLS</strong></td>
<td><strong>TSLS</strong></td>
<td><strong>Interest</strong></td>
</tr>
<tr>
<td>$\rho_{\tilde{Y}}$</td>
<td>P-val.</td>
<td>$\beta_y$</td>
</tr>
<tr>
<td>Australia</td>
<td>0.45</td>
<td>0.000</td>
</tr>
<tr>
<td>Canada</td>
<td>0.52</td>
<td>0.000</td>
</tr>
<tr>
<td>France</td>
<td>0.34</td>
<td>0.139</td>
</tr>
<tr>
<td>Germany***</td>
<td>-0.13</td>
<td>0.485</td>
</tr>
<tr>
<td>Greece</td>
<td>-0.34</td>
<td>0.055</td>
</tr>
<tr>
<td>Israel</td>
<td>0.11</td>
<td>0.339</td>
</tr>
<tr>
<td>Italy</td>
<td>0.08</td>
<td>0.542</td>
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<td>Japan</td>
<td>0.33</td>
<td>0.000</td>
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<td>Korea</td>
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<td>0.263</td>
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<tr>
<td>Singapore</td>
<td>0.02</td>
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<td>Spain</td>
<td>0.17</td>
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<td>Switzerland</td>
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<td>0.000</td>
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<td>UK</td>
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<td>0.000</td>
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<tr>
<td>US</td>
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<td>0.000</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.78</td>
<td>0.000</td>
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<tr>
<td>Chile</td>
<td>0.16</td>
<td>0.280</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.24</td>
<td>0.110</td>
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<tr>
<td>Croatia</td>
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<td>0.009</td>
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<tr>
<td>Czech Rep.</td>
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<td>Ecuador</td>
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<td>Georgia</td>
<td>-0.26</td>
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<td>Indonesia</td>
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<td>Kyrgyz Rep.</td>
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<tr>
<td>Peru</td>
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<td>0.001</td>
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<tr>
<td>Poland</td>
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<tr>
<td>Thailand</td>
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<td>0.279</td>
</tr>
<tr>
<td>Turkey</td>
<td>-0.69</td>
<td>0.000</td>
</tr>
</tbody>
</table>

* Line 60 is the central bank discount rate, line 60B is the interbank market rate.

** Lines NEC and NEU are nominal effective rates. Line AH is the nominal rate against the US Dollar.

*** Data for Germany start in 1991.1, however the calculation of annual inflation takes 4 observations and therefore the estimation of the interest rate rules starts in 92.1.