ESTIMATING STOCK MARKET VOLATILITY USING ASYMMETRIC GARCH MODELS

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Abstract

A comprehensive empirical analysis of the return and conditional variance of Tel Aviv Stock Exchange (TASE) indices is performed using GARCH models. The prediction performance of these conditional changing variance models is compared to newer asymmetric GJR and APARCH models. We also quantify the day-of-the-week effect and the leverage effect and test for asymmetric volatility. Our results show that the EGARCH model using a skewed Student-t distribution is the most successful in forecasting the TASE indices.

Keywords: GARCH, Leverage Effect, Day-of-Week Effect, Market Volatility.
Estimating Stock Market Volatility Using Asymmetric Changing Variance Models

1. Introduction

Volatility clustering and leptokurtosis are common observations in financial time series (Mandelbrot (1963)). Another phenomenon often encountered is the so-called “leverage effect” (Black (1976)), which occurs when stock prices changes are negatively correlated with changes in volatility. Observations of this type in financial time series have led to the use of various changing variance models.

In his seminal paper, Engle (1982) proposed to model time-varying conditional variance with Auto-Regressive Conditional Heteroskedasticity (ARCH) processes using lagged disturbances. Early empirical evidence shows that a high ARCH order is needed to capture the dynamic behavior of conditional variance. The Generalized ARCH (GARCH) model of Bollerslev (1986) fulfills this requirement as it is based on an infinite ARCH specification which reduces the number of estimated parameters from infinity to two. Both models capture volatility clustering and leptokurtosis, but as their distribution is symmetric, they fail to model the “leverage effect.” To address this problem, many nonlinear extensions of GARCH have been proposed, such as the Exponential GARCH (EGARCH) model by Nelson (1991), the so-called GJR model by Glosten, Jagannathan, and Runkle (1993) and the Asymmetric Power ARCH (APARCH) model by Ding, Granger, and Engle (1993).

Another problem encountered when using GARCH models is that they do not always fully embrace the thick tails property of high frequency financial time-series. To overcome this drawback Bollerslev (1987), Baillie and Bollerslev (1989), Kaiser (1996) and Beine, Laurent, and Lecourt (2000) have used the Student's t-distribution. Similarly to capture skewness Liu and Brorsen (1995) use an asymmetric stable density. But the variance of such a distribution rarely exists. For modelling both skewness and kurtosis Fernandez and Steel (1998) used the skewed Student's t-distribution that was later extended to the GARCH framework by Lambert and Laurent (2000, 2001).

Forecasting conditional variance with asymmetric GARCH models has been comprehensively studied by Pagan and Schwert (1990), Brailsford and Faff (1996), Franses, Neele, and Van Dijk (1998) and Loudon, Watt, and Yadav (2000). A

The purpose of this paper is to characterize a volatility model by its ability to forecast and capture commonly held stylized facts about conditional volatility, such as persistence of volatility, mean reverting behavior, and asymmetric impacts of negative versus positive return innovations. We investigate the forecasting performance of GARCH, EGARCH, GJR and APARCH models together with the different density functions: normal distribution, Student's t-distribution, and asymmetric Student's t-distribution. We also compare between symmetric and asymmetric distributions using the three different density functions.

We forecast two major Tel-Aviv Stock Exchange (TASE) indices: TA100 and TA25. To compare the results, we use several standard performance measurements. Our results suggest that one can improve overall estimation by using the asymmetric GARCH model with fat-tailed densities for measuring conditional variance. Moreover, we find that the asymmetric EGARCH model is a better predictor than the asymmetric GARCH, GJR and APARCH models.

The paper is structured as follows. Section 2 presents the data. In Section 3, we present the methodology and the GARCH models used in the paper. In Section 4, we describe the estimation procedures and present the forecasting results.
2. Data

The data consist of 3058 daily observations of the TA25\(^1\) index from 20/10/1992 to 31/5/2005 and 1911 daily observations of the TA100\(^2\) index from 02/07/1997 to 31/5/2005. To estimate and forecast these indices, we created many calibrating programs\(^3\) and use G@RCH 2.0 by Laurent and Peters (2001), a package whose purpose is to estimate and forecast GARCH models and many of its extensions. The code written by Doornik (1999) in the Ox programming language provides a dialog-oriented interface with features that are not available in standard econometric software.

Parameters were estimated using the QML technique by Bollerslev and Wooldridge (1992). The optimization algorithm used is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton method.

---

\(^1\) The TA25 Index is a value-weighted index of 25 stocks traded on the Tel Aviv Stock Exchange (TASE).

\(^2\) The TA100 Index is a value-weighted index of 100 stocks traded on the TASE.

\(^3\) Coded with Visual Basic Applications
3. Methodology

Early empirical evidence has shown that to capture conditional variance dynamics one needs to select a high ARCH order. The Bollerslev (1986) Generalized ARCH (GARCH) model, which is based on infinite ARCH specifications, allows us to reduce the number of estimated parameters by imposing non-linear restrictions. The GARCH \((p, q)\) model expresses the variance as:

\[
\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
\]

Using the lag operator \(L\), the variance becomes:

\[
\sigma_t^2 = w + \alpha(L) \varepsilon_t^2 + \beta(L) \sigma_t^2
\]

with \(\alpha(L) = \sum_{i=1}^{q} \alpha_i L^i\) and \(\beta(L) = \sum_{j=1}^{p} \beta_j L^j\).

If all the roots of the polynomial \(|1 - \beta(L)| = 0\) lie outside the unit circle, we have:

\[
\sigma_t^2 = w [1 - \beta(L)]^{-1} + \alpha(L) [1 - \beta(L)]^{-1} \varepsilon_t^2.
\]

This may be envisaged as an ARCH \((\infty)\) process since the conditional variance depends linearly on all previous squared residuals. As such, the conditional variance of \(y_t\) can become larger than the unconditional variance. Then, if past realizations of \(\varepsilon_t^2\) are larger than \(\sigma_t^2\) it is given by:

\[
\sigma^2 = E(\varepsilon_t^2) = \frac{w}{1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j}
\]

Like ARCH, some restrictions are needed to ensure that \(\sigma_t^2\) is positive for all \(t\). Bollerslev (1986) shows that imposing \(w > 0, \alpha_i \geq 0\) \((\text{for } i = 1, \ldots, q)\) and \(\beta_j \geq 0\) \((\text{for } j = 1, \ldots, p)\) is sufficient for the conditional variance to be positive.

To capture the asymmetry observed in the data, a new class of ARCH models was introduced: the GJR, the exponential GARCH, and the EGARCH \((p,q)\), models:

\[
\ln(\sigma_t^2) = a_0 + \sum_{i=1}^{q} \left(a_i |\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}^2\right) + \sum_{j=1}^{p} b_j \ln(\sigma_{t-j}^2), \text{ where } z_{t-i} = \frac{\varepsilon_{t-i}}{\sigma_{t-i}}
\]

Mean equation: \(y_t = E\left(y_t | \Omega_{t-1}\right) + \varepsilon_t\), where \(\Omega_{t-1}\) is the information set at time \(t-1\)
The parameters allow us to capture the asymmetric effects. For example, if $\gamma_i = 0$ a positive surprise $\varepsilon_i > 0$ has the same effect on volatility than a negative surprise $\varepsilon_i < 0$. The presence of a leverage effect can be investigated by testing the hypothesis that $\gamma_i < 0$.

Engle's (1982) ARCH model uses the normal distribution of normalized residuals $z_i$. Bollerslev (1987), on the other hand, proposed a standardized Student's t-distribution with $v > 2$ degrees of freedom whose density is given by:

$$D(z_i; v) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \left(1 + \frac{z_i^2}{v-2}\right)^{-\frac{v+1}{2}} ,$$

where $\Gamma(v) = \int_0^\infty e^{-x}x^{v-1}dx$ is the gamma function and $v$ is the parameter measuring the tail thickness. The Student's t-distribution is symmetric around mean zero. For $v > 4$, the conditional kurtosis equals $3(v-2)/(v-4)$, which exceeds the normal value of 3.

The common methodology for estimating ARCH is by maximum likelihood assuming i.i.d. innovations. For $D(z_i; v)$, the log-likelihood function of $\{y_i(\theta)\}$ for the Student's t-distribution is given by:

$$L_T(\{y_i; \theta\}) = T \left( \ln \Gamma\left(\frac{v+1}{2}\right) - \ln\left(\frac{v}{2}\right) - \frac{1}{2} \ln\left(\pi(v-2)\right) \right) - \frac{1}{2} \sum_{t=1}^T \left( \ln(\sigma_i^2) + (1+v) \ln\left(1 + \frac{z_i^2}{v-2}\right) \right).$$

where $\theta$ is the vector of parameters to be estimated for the conditional mean, the conditional variance and, the density function. When $v \to \infty$ we have a normal distribution, so that the lower $v$ is, the fatter are the tails. Recently, Lambert and Laurent (2000, 2001) extended the skewed Student's t-distribution proposed by Fernandez and Steel (1998) to the GARCH framework. Using $D(z_i; v)$, the log-likelihood function of $\{y_i(\theta)\}$ for the skewed Student's t-distribution is given by:

$$L_T(\{y_i; \theta\}) = T \left( \ln \Gamma\left(\frac{v+1}{2}\right) - \ln\left(\frac{v}{2}\right) - \frac{1}{2} \ln\left(\pi(v-2)\right) + \ln\left(\frac{2}{\xi + \frac{1}{\xi}}\right) + \ln(s) \right)$$

$$- \frac{1}{2} \sum_{t=1}^T \left( \ln(\sigma_i^2) + (1+v) \ln\left(1 + \left(\frac{z_i^2 + m}{v-2}\right)\right) \right).$$
where $\xi$ is the asymmetry parameter, $\nu$ the degree of freedom of the distribution and

$$I_t = \begin{cases} 
1, & \text{if } z_t \geq -\frac{m}{s} \\
-1, & \text{if } z_t < -\frac{m}{s} 
\end{cases}
$$

$$m = -\frac{\Gamma\left(\frac{\nu+1}{2}\right) \sqrt{\nu-2} \left(\frac{\xi - 1}{\xi}\right)}{\sqrt{\pi \Gamma\left(\frac{\nu}{2}\right)}}, \text{ and } s = \sqrt{\frac{\xi^2 + 1}{\xi^2 - 1} - m^2}
$$

(See Lambert and Laurent (2001) for more details.)

Maximum likelihood estimates of parameters are usually obtained using the BFGS numerical maximization procedure. In our work instead, we use the quasi-maximum likelihood estimator (QMLE). According to Bollerslev and Wooldridge (1992) this estimator is generally consistent, has a normal limiting distribution, and provides asymptotic standard errors that are valid under non-normality.
4. Estimation Results

4.1 Descriptive Statistics and the Stationarity Constraint

To obtain a stationary series, we use returns $r_t = 100(\log(P_t) - \log(P_{t-1}))$ where $P_t$ is the closing value of the index at date $t$. The samples for TA25 and TA100 have means of 0.0433 and 0.0452; standard deviations of 1.4895 and 1.3198; skewness of -0.2106 and -0.4546; and kurtosis of 3.4024 and 5.0898. The sample kurtosis is greater than 3, meaning that return distributions have excess kurtosis for both indices. Excess skewness is also observed, leading to high Jarque-Bera statistics indicating non-normality.

**Table 1: Descriptive Statistics for Logarithm Differences** $100 \cdot [\ln(P_t) - \ln(P_{t-1})]$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TA25</td>
<td>0.0433</td>
<td>-10.1555</td>
<td>7.1408</td>
<td>1.4895</td>
<td>3.4024</td>
<td>-0.2106</td>
<td>43.24</td>
</tr>
<tr>
<td>TA100</td>
<td>0.0452</td>
<td>-10.3816</td>
<td>7.6922</td>
<td>1.3198</td>
<td>5.0898</td>
<td>-0.4546</td>
<td>413.56</td>
</tr>
</tbody>
</table>

As daily stock returns may be correlated with the day-of-the-week effect, we avoid the potential calendar effect on the volatility analysis by filtering the daily means and variances using the following two regressions:

1. $r_t = \alpha_1 \text{SUN}_t + \alpha_2 \text{MON}_t + \alpha_3 \text{TUE}_t + \alpha_4 \text{WED}_t + \alpha_5 \text{THU}_t + \delta_t$,

2. $(r_t - \hat{r}_t)^2 = \beta_1 \text{SUN}_t + \beta_2 \text{MON}_t + \beta_3 \text{TUE}_t + \beta_4 \text{WED}_t + \beta_5 \text{THU}_t + \epsilon_t$,

where $\text{SUN}_t$, $\text{MON}_t$, $\text{TUE}_t$, $\text{WED}_t$, and $\text{THU}_t$ are the dummy variables for Sunday, Monday, Tuesday, Wednesday and Thursday; and $\hat{r}_t$ is the ordinary least squares (OLS) fitted value of $r_t$ from regression (1) at date $t$.

**Table 2: Regression Coefficients for Day-of-the-Week Effect – TA25**

<table>
<thead>
<tr>
<th>Regression</th>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (1)</td>
<td>0.12**</td>
<td>0.020</td>
<td>0.041</td>
<td>-0.0395</td>
<td>0.074</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Variance (2)</td>
<td>3.41**</td>
<td>1.45**</td>
<td>2.19**</td>
<td>1.91**</td>
<td>2.12**</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.205</td>
<td>0.205</td>
<td>0.207</td>
<td>0.208</td>
<td>0.207</td>
</tr>
</tbody>
</table>
Table 3: Regression Coefficients for Day-of-the-Week Effect – TA100

<table>
<thead>
<tr>
<th>Regression</th>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (1)</td>
<td>0.178**</td>
<td>0.089</td>
<td>-0.040</td>
<td>-0.097</td>
<td>0.090</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0663</td>
<td>0.0663</td>
<td>0.0673</td>
<td>0.0673</td>
<td>0.0671</td>
</tr>
<tr>
<td>Variance (2)</td>
<td>2.732**</td>
<td>0.878**</td>
<td>1.658**</td>
<td>1.757**</td>
<td>1.626**</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.231</td>
<td>0.231</td>
<td>0.235</td>
<td>0.235</td>
<td>0.235</td>
</tr>
</tbody>
</table>

The OLS estimates of the two regressions on Tables 2 and 3 show that the TA25 and TA100 indices have significantly positive daily means on Sunday and significant daily variations for Sunday through Thursday\(^5\).

To eliminate the daily effects, we “standardize” the daily returns using

\[ y_t = \frac{r_t - \hat{r}_t}{\sqrt{\hat{\sigma}_t^2}}, \]

where \( \hat{r}_t \) is the fitted value of \( (r_t - \hat{r}_t)^2 \) from regression (2) at date \( t \). We now substitute the original daily return \( r_t \) with \( y_t \) and referred to it as the daily return at date \( t \).

Table 4: Descriptive Statistics for "Standardized" Returns \( y_t \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TA25</td>
<td>0</td>
<td>-6.7131</td>
<td>5.30431</td>
<td>0.9999</td>
<td>3.1528</td>
<td>-0.2179</td>
<td>27.17</td>
</tr>
<tr>
<td>TA100</td>
<td>0</td>
<td>-8.0305</td>
<td>5.8756</td>
<td>1.00026</td>
<td>4.569</td>
<td>-0.4561</td>
<td>262.27</td>
</tr>
</tbody>
</table>

Returns \( y_t \) are thus normalized to zero mean and unit variance. The sample skewness and kurtosis of \( y_t \)’s are -0.2179 and 3.1528;-0.4561 and 4.569 for the two indices.

4.2 Choosing a Volatility Model

For the TA25 index, convergence could not be reached with the EGARCH model and a Student's t-distribution. Therefore we turn to the other three models where all asymmetric coefficients are significant at standard levels. Moreover, the Akaike information criteria (AIC) and the log-likelihood values indicate that the EGARCH, APARCH or GJR models better estimate the series than traditional GARCH.

These models are estimated by the approximate quasi-maximum likelihood estimator assuming normal, Student-t or skewed Student-t errors. Note that it is quite

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\(^5\) * and ** - means significance at 5% and 1% levels, respectively.
evident that the recursive evaluation of maximum likelihood is conditional on unobserved values and therefore the estimation cannot be considered to be perfectly exact. To solve the problem of unobserved values, we set these quantities to their unconditional expected values.

When we analyzed the densities we found that the two Student's t-distributions (symmetric and skewed) clearly outperform the normal distribution. Indeed, the log-likelihood function increases when using the skewed Student's t-distribution, leading to AIC criteria of 2.701 and 2.730 for the normal density versus 2.665 and 2.697 for the non normal densities, for the TA 100 and the TA 25 respectively.

Table 5: Comparison between the Models for the TA100

<table>
<thead>
<tr>
<th>TA100</th>
<th>Normal</th>
<th>Student's t</th>
<th>Skewed t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>APARCH</td>
<td>APARCH</td>
<td>EGARCH</td>
</tr>
<tr>
<td>Q(20)</td>
<td>20.739</td>
<td>20.731</td>
<td>20.313</td>
</tr>
<tr>
<td>$Q^2(20)$</td>
<td>24.051</td>
<td>27.762</td>
<td>24.630</td>
</tr>
<tr>
<td>P(50)</td>
<td>72.909</td>
<td>48.472</td>
<td>46.326</td>
</tr>
<tr>
<td>Prob[1]</td>
<td>0.015</td>
<td>0.494</td>
<td>0.582</td>
</tr>
<tr>
<td>Prob[2]</td>
<td>0.002</td>
<td>0.168</td>
<td>0.196</td>
</tr>
<tr>
<td>AIC</td>
<td>2.730</td>
<td>2.697</td>
<td>2.697</td>
</tr>
<tr>
<td>Log-Lik</td>
<td>-2600.690</td>
<td>-2568.060</td>
<td>-2566.940</td>
</tr>
</tbody>
</table>

The skewed Student's t-distribution shows results that are superior to the symmetric Student-t distribution when modeling the TA 25 and TA100. A possible explanation for this result is that, if skewness is significant in both series, its magnitude will be inferior in both indices. It may therefore be necessary to add two asymmetric parameters (asymmetric GARCH + asymmetric distribution).

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6 In Tables 5 and 6 $Q(20)$ and $Q^2(20)$ are respectively the Box-Pierce statistics at lag 20 of the standardized and squared standardized residuals. $P(50)$ is the Pearson goodness-of-fit with 50 cells. $AIC$ is the Akaike information criterion. Log-Lik is the log-likelihood value.
### Table 6: Comparison between the Models for the TA25

<table>
<thead>
<tr>
<th>TA25</th>
<th>Normal</th>
<th>Student’s t</th>
<th>Skewed t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GJR</td>
<td>APARCH</td>
<td>GJR</td>
</tr>
<tr>
<td>Q(20)</td>
<td>32.290</td>
<td>32.363</td>
<td>32.080</td>
</tr>
<tr>
<td>Q^2(20)</td>
<td>14.875</td>
<td>17.177</td>
<td>18.705</td>
</tr>
<tr>
<td>P(50)</td>
<td>58.710</td>
<td>65.806</td>
<td>46.055</td>
</tr>
<tr>
<td>Prob[1]</td>
<td>0.161</td>
<td>0.055</td>
<td>0.593</td>
</tr>
<tr>
<td>Prob[2]</td>
<td>0.045</td>
<td>0.008</td>
<td>0.271</td>
</tr>
<tr>
<td>AIC</td>
<td>2.701</td>
<td>2.699</td>
<td>2.669</td>
</tr>
<tr>
<td>Log-Lik</td>
<td>-4122.670</td>
<td>-4118.770</td>
<td>-4073.290</td>
</tr>
</tbody>
</table>

All the models describing the dynamics of the first two moments of the series are shown by Box-Pierce statistics for residuals and squared residuals. All are non-significant at the 5% level. The stationary constraints are observed for every model and for every density. The values (ranging from 0.831 to 0.982) suggest long persistence of the volatility for the indices.
4.3 Forecasting

The forecasting ability of GARCH models has been comprehensively discussed by Poon and Granger (2001). However, Andersen and Bollerslev (1997) pointed out that the squared daily returns may not be the proper measure to assess the forecasting performance of the different GARCH models for conditional variance. Thus, we consider the following five measures to assess forecasting ability:

1. Mean squared error (MSE):

\[
MSE = \frac{1}{h+1} \sum_{t=S}^{S+h} \left( \hat{\sigma}_t^2 - \sigma_t^2 \right)^2
\]


\[
MedSE = \text{Inv}(f_{Med}(e_i)), \text{ where } e_i = \left( \hat{\sigma}_i^2 - \sigma_i^2 \right)^2 \text{ and } t \in [S, S + h]
\]

3. Mean absolute error (MAE):

\[
MAE = \frac{1}{h+1} \sum_{t=S}^{S+h} |\hat{\sigma}_t^2 - \sigma_t^2|
\]

4. Adjusted mean absolute percentage error (AMAPE):

\[
AMAPE = \frac{1}{h+1} \sum_{t=S}^{S+h} \left| \frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2 + \sigma_t^2} \right|
\]

where \( h \) is the number of lead steps, \( S \) is the sample size, \( \hat{\sigma}_t^2 \) is the forecasted variance and \( \sigma_t^2 \) is the “actual” variance.

5. Theil’s inequality coefficient (TIC):\(^7\)

\[
TIC = \sqrt{\frac{1}{h+1} \sum_{t=S}^{S+h} (\hat{y}_t - y_t)^2} \sqrt{\frac{1}{h+1} \sum_{t=S}^{S+h} (y_t)^2}
\]

\[
- \sqrt{\frac{1}{h+1} \sum_{t=S}^{S+h} (\hat{y}_t)^2} \sqrt{1 - \frac{1}{h+1} \sum_{t=S}^{S+h} (y_t)^2}
\]

The forecasting ability is reported by ranking the different models with respect to the five measures. This is done in tables 7 and 8 where we compare the distributions for the TA 25 and TA 100 indexes.

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\(^7\) The Theil inequality coefficient is a scale invariant measure that lies always between zero and one, where zero indicates a perfect fit.
Table 7: Forecasting Analysis for the TA25 Index: Comparing between Densities

<table>
<thead>
<tr>
<th></th>
<th>GARCH</th>
<th>EGARCH</th>
<th>GJR</th>
<th>APARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Student-t</td>
<td>Skewed-t</td>
<td>Skewed-t</td>
<td>Student-t</td>
</tr>
<tr>
<td>MSE(1)</td>
<td>0.187</td>
<td>0.187</td>
<td>0.188</td>
<td>0.187</td>
</tr>
<tr>
<td>MSE(2)</td>
<td>0.344</td>
<td>0.343</td>
<td><strong>0.269</strong></td>
<td>0.407</td>
</tr>
<tr>
<td>MedSE(1)</td>
<td>0.029</td>
<td>0.029</td>
<td>0.033</td>
<td>0.033</td>
</tr>
<tr>
<td>MedSE(2)</td>
<td>0.223</td>
<td>0.227</td>
<td><strong>0.128</strong></td>
<td>0.309</td>
</tr>
<tr>
<td>MAE(1)</td>
<td>0.274</td>
<td>0.274</td>
<td>0.272</td>
<td>0.273</td>
</tr>
<tr>
<td>MAE(2)</td>
<td>0.500</td>
<td>0.499</td>
<td><strong>0.397</strong></td>
<td>0.568</td>
</tr>
<tr>
<td>RMSE(1)</td>
<td>0.432</td>
<td>0.432</td>
<td><strong>0.397</strong></td>
<td>0.568</td>
</tr>
<tr>
<td>RMSE(2)</td>
<td>0.587</td>
<td>0.586</td>
<td><strong>0.519</strong></td>
<td>0.638</td>
</tr>
<tr>
<td>AMAPE(2)</td>
<td>0.782</td>
<td>0.782</td>
<td><strong>0.758</strong></td>
<td>0.795</td>
</tr>
<tr>
<td>TIC(1)</td>
<td>0.938</td>
<td>0.944</td>
<td>0.983</td>
<td>0.968</td>
</tr>
<tr>
<td>TIC(2)</td>
<td>0.559</td>
<td>0.559</td>
<td>0.565</td>
<td>0.565</td>
</tr>
</tbody>
</table>

(1)- Mean Equation, (2)-Variance Equation

For the TA25 index shown on Table 7 the results support the use of the asymmetric EGARCH model. For most measures in the variance equation, the EGARCH model outperforms the APARCH model. The GARCH model provides much less satisfactory results and the GJR model provides the poorest forecasts.

For the TA 100 index shown on Table 8, the EGARCH model gives better forecasts than the GARCH model while the APARCH and GJR models give the poorest forecasts. The skewed Student-t distribution is the most successful in forecasting the TA100 conditional variance, contrary to the TA25 where the results conflict. Therefore we were unable to draw a general conclusion. The skewed Student-t distribution seems to be the best for forecasting series showing higher skewness. In fact Lambert and Laurent (2001) found that the skewed Student-t density is more appropriate for modeling the NASDAQ index than symmetric densities.
Table 8: Forecasting Analysis for the TA100 Index: Comparing between Densities

<table>
<thead>
<tr>
<th>TA100</th>
<th>GARCH</th>
<th>EGARCH</th>
<th>GJR</th>
<th>APARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Skewed-t</td>
<td>Skewed-t</td>
<td>Student-t</td>
<td>Student-t</td>
</tr>
<tr>
<td>MSE(1)</td>
<td>0.218</td>
<td>0.220</td>
<td>0.219</td>
<td>0.220</td>
</tr>
<tr>
<td>MSE(2)</td>
<td>0.366</td>
<td>0.275</td>
<td>0.444</td>
<td>0.393</td>
</tr>
<tr>
<td>MedSE(1)</td>
<td>0.093</td>
<td>0.092</td>
<td>0.093</td>
<td>0.092</td>
</tr>
<tr>
<td>MedSE(2)</td>
<td>0.376</td>
<td>0.292</td>
<td>0.466</td>
<td>0.409</td>
</tr>
<tr>
<td>MAE(1)</td>
<td>0.379</td>
<td>0.381</td>
<td>0.38</td>
<td>0.381</td>
</tr>
<tr>
<td>MAE(2)</td>
<td>0.562</td>
<td>0.480</td>
<td>0.624</td>
<td>0.586</td>
</tr>
<tr>
<td>RMSE(1)</td>
<td>0.466</td>
<td>0.469</td>
<td>0.468</td>
<td>0.469</td>
</tr>
<tr>
<td>RMSE(2)</td>
<td>0.605</td>
<td>0.525</td>
<td>0.666</td>
<td>0.627</td>
</tr>
<tr>
<td>AMAPE(2)</td>
<td>0.648</td>
<td>0.621</td>
<td>0.664</td>
<td>0.654</td>
</tr>
<tr>
<td>TIC(1)</td>
<td>0.954</td>
<td>0.969</td>
<td>0.965</td>
<td>0.970</td>
</tr>
<tr>
<td>TIC(2)</td>
<td>0.538</td>
<td>0.509</td>
<td>0.561</td>
<td>0.547</td>
</tr>
</tbody>
</table>

(1)- Mean Equation, (2)-Variance Equation

5. Conclusion

We compared the forecasting performance of several GARCH models using different distributions for two Tel Aviv stock index returns. We found that the EGARCH skewed Student-t model is the most promising for characterizing the dynamic behavior of these returns as it reflects their underlying process in terms of serial correlation, asymmetric volatility clustering, and leptokurtic innovation. The results also show that asymmetric GARCH models improve the forecasting performance. Among the tested models, the EGARCH skewed Student-t model outperformed GARCH, GJR and APARCH models. This result further implies that the EGARCH model might be more useful than the other three models when applying risk management strategies for Tel Aviv stock index returns.
References


