Minimum Acceleration with Constraints of Center of Mass: A Unified Model for Arm Movements and Object Manipulation

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Introduction: Many daily tasks involve controlling and manipulating objects. The goal in such hand-object movements is to affect a certain motion of the object as well as the hand while reaching the same destination at complete rest. Previous studies investigated the strategies used by humans to control one degree of freedom objects which was simulated by mass attached to a spring. While considering the new mechanical setup of the environment, that differ from unconstrained reaching movements, new computational models were suggested to account for subject's trajectories. Using Euler-Poisson equation, models such as the minimum object crackle [1], minimum hand jerk [2] or the minimum hand driving force change [3] were created to account for hand trajectory. Although providing logical solution for object manipulation, for some mass and spring values the models don't fit well to experimental data. Moreover, some of the previous models are specific to the mass-on-spring task and are not well extended to multiple masses, while others encounter calculation complexity, trying to fit reaching movements, resulting only in numeric solutions using iterations, which in some cases predict unstable solution.

Knowing the limitations of the current criteria, it may be relevant to consider other optimal solutions for reaching movements which can also account for object manipulation tasks. A recent study suggested a model for point-to-point reaching which minimizes the square of acceleration using Pontryagin's maximum principle [4]. This model uses a constrained control signal to generate movements with interesting predictions about intermittence control approach used by the motor system.

Here we demonstrate that the minimum acceleration criteria with constrains can be expended to account for both reaching movements and object manipulation task without the need for two separate models. We show that the extension of the model accounts for all the previous results (including results which are unaccountable by the previous models) and the only model accounting for new experimental results with parameters carefully designed to test its predictions.

Theoretical and Experimental Results: We suggest that both point-to-point reaching movements and object manipulation can be explained by minimizing square acceleration of the system's center of mass while using the solution of this optimization problem as suggested in [4]. According to the solution the center of mass trajectory is divided into three segments which depend on the jerk constraint u. Considering the hand as a point mass, as illustrated in Figure 1a, the motion equation for the mass is:

$$k(x_h - x_o) = m_o \ddot{x}_o \tag{1}$$

Where the center of mass definition is:

$$x_{cm} = (m_h x_h + m_o x_o)/(m_h + m_o) \tag{2}$$

From equation (2) the object position can be isolated and substituted into equation (1) to get a differential equation linking between the hand position and the center of mass position (3):

$$\ddot{x}_h + \beta x_h = \alpha \ddot{x}_{cm} + \beta x_{cm}, \qquad \beta = k\left(\frac{m_o + m_h}{m_h m_o}\right) = \omega^2, \alpha = \left(1 + \frac{m_o}{m_h}\right) \tag{3}$$

We have derived the solution for the hand position using standard methods of finding the homogenous solution by solving the characteristic equation and then for each time interval, using the method of undetermined coefficient to find particular solution. Since the center of mass position is divided to three segments, the hand position will also be divided to three segments:

$$x_h(t) = \begin{cases} A_1^1 \cos(\omega t) + A_2^1 \sin(\omega t) + \frac{1}{6} \cdot {}^1c_0 t^3 - \frac{1}{2} \cdot {}^1c_1 t^2 + \frac{{}^1c_2 \beta + {}^1c_0(\alpha - 1)}{\beta} t + \frac{{}^1c_3 \beta - {}^1c_1(\alpha - 1)}{\beta}, & 0 \le t \le t_1 \\ A_1^2 \cos(\omega t) + A_2^2 \sin(\omega t) + \frac{1}{6} \cdot {}^2c_0 t^3 - \frac{1}{2} \cdot {}^2c_1 t^2 + \frac{{}^2c_2 \beta + {}^2c_0(\alpha - 1)}{\beta} t + \frac{{}^2c_3 \beta - {}^2c_1(\alpha - 1)}{\beta}, & t_1 \le t \le t_2 \\ A_1^3 \cos(\omega t) + A_2^3 \sin(\omega t) + \frac{1}{6} \cdot {}^3c_0 t^3 - \frac{1}{2} \cdot {}^3c_1 t^2 + \frac{{}^3c_2 \beta + {}^3c_0(\alpha - 1)}{\beta} t + \frac{{}^3c_3 \beta - {}^3c_1(\alpha - 1)}{\beta}, & t_2 \le t \le T \end{cases} \tag{4a}$$

Where:

$$A_1^1 = 0, \quad A_2^1 = -\frac{u \cdot (\alpha - 1)}{\beta\omega}, \qquad A_1^2 = \frac{(a-1)\cdot\left(u \cdot \sin(\omega T - \omega t_2)\cdot \sin(\omega t_1) + u \cdot \sin(\omega t_2)\cdot \sin(\omega t_1)\right)}{\beta\omega \sin(\omega(t_1 - t_2))}$$

$$A_2^2 = \frac{(\cos(\omega t_2)\cdot \sin(\omega t_1) + \sin(\omega T)\cdot \cos(\omega t_1)\cdot \cos(\omega t_2) - \cos(\omega T)\cdot \cos(\omega t_1)\cdot \sin(\omega t_2))\cdot u \cdot (1-a)}{\beta\omega \sin(\omega(t_1 - t_2))} \tag{4b}$$

$$A_1^3 = \frac{u \cdot \sin(\omega T)\cdot(\alpha - 1)}{\beta\omega}, \qquad A_2^3 = -\frac{u \cdot \cos(\omega T)\cdot(\alpha - 1)}{\beta\omega}$$

T – Movement duration

The solution for hand position (4a) is a function of ${}^2c_0, {}^2c_1, {}^2c_2, {}^2c_3, {}^3c_1, {}^3c_2, {}^3c_3, t_1, t_2$ which expressions can be found in [4] and are deterministic function of movement length and duration i.e. were not fitted in any way. The only free parameters are the movement length and duration which can be extracted from the trajectory and are consistent for all models.

We have reproduced the results reported in previous experiments and found that our model fit well all the previous results including results unaccountable by the previous models (e.g. Fig 5 in [1]). To further test our model ability to fit experimental data compared to previous models we perform an experiment where subjects were asked to bring both hand and the attached virtual mass-on-spring from rest at the initial point to rest at the target point. An example for trajectory and models fitting are shown in figure 1b.

From the given example it is clearly shown that the minimum acceleration with constraints criteria best predict subject's trajectory. This result, along with the ability of predicting trajectory for reaching movements, may be additional example to the use of intermittence control. Altogether, we present a simple unified model accounting for simple reaching movement as well as object manipulation.

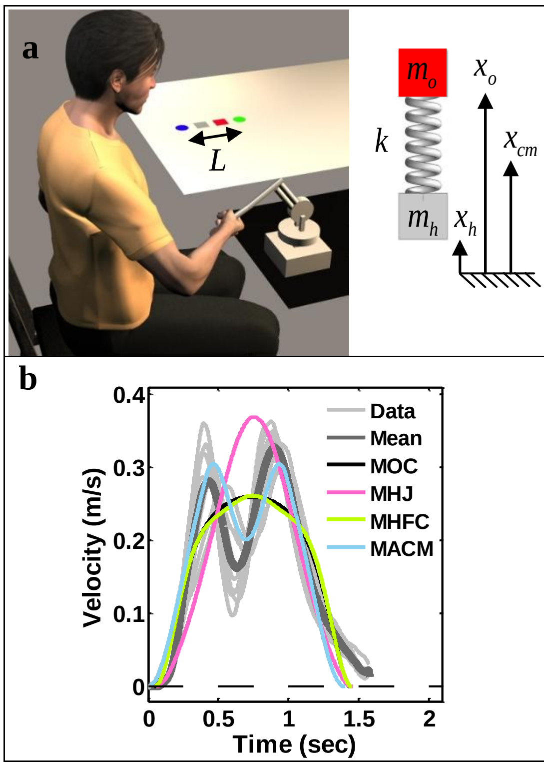

Figure 1: a) Diagram of the hand as a point mass, m_h , and the simulated mass, m_o , attached (virtually) to the hand using a spring with constant k . Hand position is denoted by x_h and object position is denoted by x_o . The center of mass (x_{cm}) is derived as a weighted sum of the hand position and the object position. b) Example for hand velocity profile performed while attached to the mass-on-spring where mass value was set to be $m_o = 3.0$ kg and spring constant $k = 120$ N/m. The subject moved between two targets 25 cm apart i.e. $L = 0.25$ m where desired movement duration was 1.4 sec. Thin light gray lines represent individual trials where thick gray lines represent the average of these trials. Optimal trajectories predicted by the models denoted by black line (minimum object crackle-MOC), pink line (minimum hand jerk-MHJ), green line (minimum hand force change-MHFC) and blue line (minimum acceleration of center of mass-MACM). This profile has two phases as predicted by the MACM. Interesting to notice the resemblance of our experiment data to the data of the same experiment reported in [1].

References: [1] Dingwell J. B. et al. J Neurophysiol. 2004; 91(3): 1158-1170. [2] Svinin M. et al. J Robotic Syst. 2005; 22:661-676. [3] Svinin M. et al. Robotics, IEEE Trans. 2006; 22:724-739 [4] Ben-Itzhak S and Karniel A. N Computation. 2008; 20: 779-812.