Human Motor Control: Learning to Control a Time-Varying, Nonlinear, Many-to-One System

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Abstract—Human motor control has always presented a great challenge to both scientists and engineers. It has presented most of the problems they have found difficult to handle and manipulate, which is a consequence of it being a distributed, nonlinear, time-varying system with multiple degrees of freedom that include redundancy on many levels. In recent years, the fast development of computers and the emergence of the new scientific field of neural computation have enabled consideration of complex, adaptive, parallel architectures in the modeling of human motor-control performance. In this paper, some of the models that have been used in the study of motor control are reviewed, and some open questions are formalized and discussed. The main topics are adaptive and artificial neural-networks control, parameters estimation, nonlinear properties of the muscles, and parallelism and redundancy.

Index Terms—Adaptive control, inverse problems, muscles, neurocontrollers, parameter estimation, redundant systems.

I. INTRODUCTION

THERE IS a wide engineering and mathematical foundation for modeling linear, time-invariant systems (see [1]–[3]). Some man-made machines satisfy these conditions, but biological systems do not. The biological system presents enormous plasticity, which means that the system is a time-varying one (e.g., in processes like regeneration and fatigue). It also generally demonstrates nonlinear behavior such as logarithmic relations, thresholds, hysteresis, saturation, and cutoff (i.e., minimum and maximum bounds). In some cases, a linear, time-invariant approximation can be made, but only for small signals and for short durations.

One of the salient characteristics of the biological motor-control system is its apparent redundancy [4], [5]. The human arm consists of seven kinematic degrees of freedom, which is more than is needed to obtain a particular position or configuration of the hand in the workspace. Most of the joints are surrounded by more muscles than needed to produce any desired movement. The muscles themselves are composed of many motor units that enable many possibilities of producing the same force at the tendon. These apparent redundancies lead the controller (i.e., the nervous system) to act on a many-to-one (MTO) system and to choose one of the many possible actions in order to obtain the same desired target.

Keeping all of the above issues in mind, our challenge is to solve the problem of learning to control a time-varying, non-linear, MTO system. Nature has found wonderful solutions, and our goal is to try and learn them. A common procedure to help us understand biological systems and mechanisms is to try and formulate their behavior mathematically, fit the parameters of the mathematical model to the experimental results, and then study the generalization power of the model. This is obviously an iterative procedure that, when successful, can elucidate silent features of the system, teach us more about that system, and suggest ways for new experiments and experimental procedures. It sometimes assists us in ultimately imitating these ingenious solutions of nature to the design of artificial systems, to build artificial organs to replace natural ones, and design aids to assist patients in need.

This paper describes the use of mathematical and engineering techniques as applied to the area of human movement control. It is organized as follows: Section II describes the main architectures that describe the biological motor control. In Section III, the problem of parameter estimation and system identification is described and demonstrated for a linear-muscle model. In Section IV, a nonlinear model of the muscle is proved to be not only biologically plausible but also a means to reduce the complexity of movement control. Section V presents a few aspects of the parallelism and of the degrees of freedom problem, which is how to invert an MTO system. Finally, Section VI concludes this challenge of modeling the biological motor-control system.

II. FROM FEEDBACK TO ADAPTATION

A salient feature in any biological system is the ability to adapt to the environment and to its own internal changes. The revolution in the beginning of modern cybernetics was the incorporation of feedback to artificial machines. In this section, we will describe the main architectures from simple feedback through adaptive control and to artificial neural-network control.

A. Feedback Control

Feedback control is based on using the outcome of the process, or the controlled system, which is usually called the "plant," in order to control it. In other words, we use the error between the desired output y_d and the actual output y in order to reduce it.

The analogy of the feedback scheme (Fig. 1) to motor control is the following. The plant corresponds to the muscles, the bones, and the dynamics of the environment, the feedback corresponds to the output of the sensory systems, and the controller corresponds to the nervous system. The control problem is how to design a controller that suits the desired performance. In the linear case, we can use the Laplace transform and describe each

Fig. 1.   Feedback control.

Fig. 2.   Adaptive control.
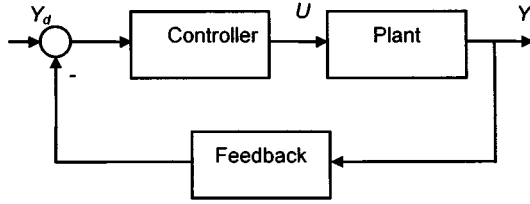
block with a transfer function (1) where k is called the gain, z_i are the zeros, and p_i are the poles

$$H(s) = \frac{\text{OUT}(s)}{\text{IN}(s)} = k \cdot \frac{\Pi(s - z_i)}{\Pi(s - p_j)}. \tag{1}$$

Let us denote the transfer functions of the blocks in Fig. 1 as follows: P for the plant, C for the controller, and F for the feedback (i.e., the sensory system). In the Laplace domain, we can write the output as a function of the input in Fig. 1 in terms of the block's transfer functions (the Laplace variable s is avoided for simplicity)

$$\frac{Y}{Y_d} = \frac{P \cdot C \cdot F}{P \cdot C \cdot F + 1}. \tag{2}$$

One major advantage of the feedback control scheme is the reduced sensitivity to changes in the parameters of the plant and to changes in the environment. The sensitivity of system H to changes in the parameter k is defined as follows:

$$S_H^k \equiv \left| \frac{\partial H}{\partial k} \cdot \frac{k}{H} \right|. \tag{3}$$

When the value of the sensitivity function is zero, the system is insensitive to changes in the parameters. Let us look at the system without feedback, where H is the transfer function and k is a gain parameter, as in (1). The system in an open loop is $H = k \cdot P$, and the sensitivity of the system would be

$$S_H^k \equiv \left| \frac{\partial H}{\partial k} \cdot \frac{k}{H} \right| = P \cdot \frac{k}{k \cdot P} = 1. \tag{4}$$

The system with feedback, i.e., in a closed loop, is $H = (k \cdot P)/(k \cdot P \cdot F + 1)$ and the sensitivity will be

$$\begin{aligned} S_H^k &\equiv \left| \frac{\partial H}{\partial k} \cdot \frac{k}{H} \right| \\ &= \frac{P \cdot (k \cdot P \cdot F + 1) - k \cdot P^2 \cdot F}{(k \cdot P \cdot F + 1)^2} \cdot \frac{k \cdot P \cdot F + 1}{P} \\ &= \frac{1}{k \cdot P \cdot F + 1} < 1. \end{aligned} \tag{5}$$

So when the loop-gain k is high, the sensitivity to changes is low.

There is a vast literature on the stability of such systems and on methods to choose a controller when the specifications of the desired performances are given [6], [7].

The first problem in using this simple feedback-to-model biological systems occurs when one tries to measure the loop-gain. In the biological system, one finds a very low loop-gain in the
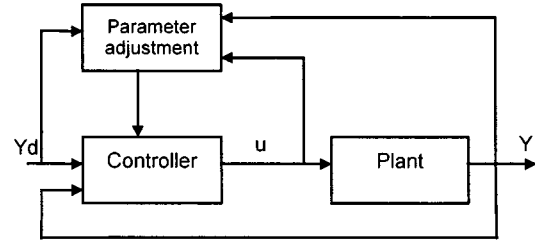
order of one. Therefore, the above reason for reduced sensitivity to changes in the parameters is not valid for the biological system. The second problem is the delays in the biological system, which can cause instability and oscillation in such a simple feedback control. So we can conclude that the simple feedback control, despite its advantages, is insufficient to describe the biological motor control. Let us continue with the development of modern control to a more complicated control scheme, the adaptive control.

B. Adaptive Control

Adaptive control can be regarded as an extension of feedback control. In adaptive control, the controller can adapt itself to changes in the plant or even learn to control a new plant. A simple description of the adaptive control scheme is given in Fig. 2. In the case where the controller is changing, the linear description is not valid even if the plant and the controller are linear. Generally, the mathematical questions in such a control scheme are related to the ability of the plant to learn and to the convergence properties of the adaptation algorithm. For a short summary of adaptive control, see [8], for analysis of the algorithms, see [9], and for the first attempt to model a biological system with an adaptive-control scheme, see [10].

The literature separates between direct and indirect adaptive control. In the first case, the adaptation algorithm operates on the controller, and in the second case there are two phases. The first is identification of the plant, and the second is adaptation of the controller. The second case raises the problem of identification or learning of the plant, and if we have a parametric group of possible plants, it brings us to the problem of parameter estimation, which is the subject of Section III. A difficult problem with adaptive controllers is the requirement for persistent excitation of the plant, which is needed in order to continuously estimate its time-varying parameters. Without excitation, the estimated value of the parameters tends to drift. This can be stopped in engineering systems by turning off the adaptation algorithm when the system operates with fixed control signals.

C. Feed-Forward Control and the Inverse-Controller Problem

The opposite of feedback or closed-loop control is the feedforward or open-loop control where the sensory information is not used during the control and execution of the task. As we mentioned before, the delays in the biological system are relatively large, and in such cases, a feed-forward control must be considered. This control scheme was first suggested and analyzed by Inbar and Yafe [11], where the term "signal adaptation" was used to describe the inverse controller's act of learning in
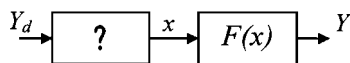
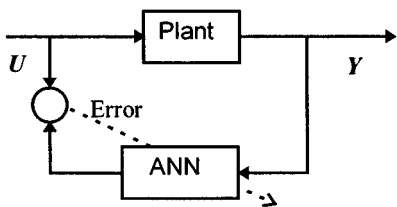Fig. 3.   Feed-forward control and the inverse-controller problem.

Fig. 4.   Direct learning of an inverse model with ANN.

Fig. 5.   ANN control with distal teacher.

Fig. 6.   ANN feedback-learning control scheme.

order to generate the desired control signals. Fig. 3 describes the controller and the plant in a feed-forward control. The control problem is: "Given a desired goal Y_d, how does one generate x such that $F(x)$ will be close to Y_d?"

This is a simple inverse problem, but the solutions to inverse problems are not simple in many cases. One of the problems is redundancy (i.e., the plant having MTO function). In Section V, this problem of redundancy is described with some suggested solutions. Another problem is to learn the inverse of the system. Most biological systems are time-varying systems, and therefore, the inverse of the system must be learned from examples and adjusted to changes in the system parameters as they occur. The field of artificial neural networks has grown rapidly in the last decade, and it provides many solutions to this problem.

D. Artificial Neural-Network (ANN) Control

The artificial neural network (ANN) is a field that draws its ideas from the structure of the biological neural networks and as such can be described separately as an example for using mathematical tools in biology. For a comprehensive review of ANN, see [12], and for a review of the use of ANN for control, see [13] and [14] and a collection of articles in [15]. In our context, we look at an ANN as a black box that operates as a function approximator with a learning algorithm that can change the structure of the ANN in order to reduce some error between the network output and the desired one, given by training examples. Three results in the study of ANN paved the way to implement this architecture in the control area. First, the introduction of the backpropagation algorithm, which is a gradient-based learning algorithm enabled the use of multilayered ANN [16]. Second, the representation theorem shows the ability of a one-hidden-layer ANN to approximate continuous functions as presented and proofed by numerous researchers (e.g., [17]). Third, the ability of a two-hidden-layer ANN to approximate an inverse of continuous functions [18].

Let us use the ANN as a controller in order to solve the control problem in Fig. 3. The simplest solution is to train an ANN to become an inverse of the plant (see [19]). This solution is described in Fig. 4, where the training phase is described.

After the training is done, one can put the ANN before the plant, and it will function as a controller. Such a control scheme has a few drawbacks. First, there is a need for two phases, and therefore, two copies of the ANN (one connected as a learner
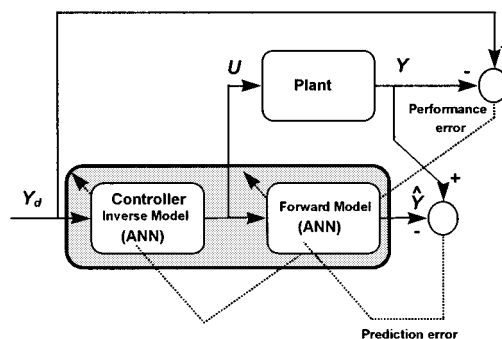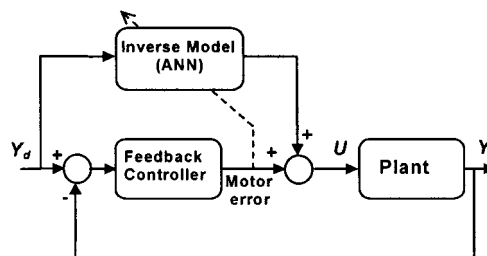
and the other as a controller). There is no evidence in the biological nervous system for such a dual system. However, lack of existence in the biological system is not the only drawback of the direct-learning scheme. Another problem appears in trying to learn the inverse of a redundant system (i.e., the problem of mapping an MTO system). Most of the learning algorithm will converge to the average of all the possible solutions, but the average of correct solutions is not always a correct solution (see [20]). A second problem in inverting an MTO system is how to represent all the solutions and which solution to choose. This problem is dealt with in Section V. One major problem in trying to train a controller that is attached to the controlled system is how to transform the error from the output coordinates of the system to the coordinates of the control signal. Following are two solutions to the training problem that were proposed as models to the biological motor control. The first is the distal teacher (as in Fig. 5) that was proposed by Jordan and Rumelhart [21], and the second is the feedback learning in Fig. 6 that was proposed by Kawato and Gomi [22].

In the distal teacher approach (Fig. 5), the problem of transforming the error from the output of the process to the output of the controller (i.e., the output of the ANN) is solved by an ANN-forward model of the plant and by using the learning algorithm in order to propagate the error through the model to the ANN controller. The forward model is trained by the prediction error, and the controller is trained by the performance error that is propagated through the forward model. In the feedback learning scheme (see Fig. 6), the error is transformed by a simple feedback controller, and the ANN is trained by the motor error. In this way, the control is reasonable even in case of big changes in the plant, since in such a case, there is an error (and then the feedback controller adds its contribution to the control signal in the correct direction).

Kawato *et al.* [23] showed that in some simple cases, the distal-teacher approach converges as the steepest decent method, and the feedback-error method converges like Newton's method. This analysis is correct only if the controller has the correct parameters. However, for nonlinear systems, the ANN has more capability to approximate the systems gradient than a conventional feedback controller has to approximate the systems Jacobian. Which architecture is better and which is more biologically plausible are still open questions.

E. Summary: The Hierarchy of Learning and Adaptation

The importance of and the need for sensory information in order to improve the controller performance cannot be over emphasized. The previous sections demonstrate this in various architectures and techniques. The terminology used in this field seem to be mixed and unclear, so as a summary, let us suggest a hierarchy in learning and adaptation. In this hierarchy, there is a tradeoff between two factors. On the one hand is the speed of adaptation, and on the other is the extent of plasticity. Feedback is the most rapid, but its changes do not last beyond the timescale of the process, while evolution is the slowest, and its changes last for generations to come.

1) Feedback: This is the process of change in the control signals during the execution of the action, accomplished by means of a sensory-feedback signal from the plant output. This change is valid only for the current execution, and its effects are "forgotten" rapidly. The feedback scheme is used for tracking or regulation, and its advantage is in noise rejection and low sensitivity to changes in the systems parameters.

2) Adaptation: This is the process of change in the controller parameters, accomplished by means of sensory information from the plant output, which is valid for the next execution of similar actions. This change is valid for a longer period, as long as the plant parameters do not change. The adaptation rate is normally much slower than the time constants of the plant but much faster than the rate of change in the plant parameters. Therefore, it may take a few repetitions of the action in order to achieve a significant adaptation and change in the controller parameters. The adaptive control scheme is used, in which the controlled system is time variant, and the controller can adjust to changes in the system.

3) Learning: This is the process of change in the control in order to produce new control signals for a new situation or for a new target. This change can involve a major change in the architecture, and it may take many repetitions of the execution in order to achieve the desired change. The results can stay permanently in the same human, animal, or machine. Learning takes place when a major change in the system occurs, or when a new task is given. In humans, there are types of learning that are strict to specific time windows in the development (e.g., in children).

4) Evolution: This is the process of change in the architecture or basic property of the control by mutation and natural selection through a long period of trial and error and optimization. This change occurs only when a new species or a new version of the machine is evolved. This change is the result of many trials of many actions and controllers (by survival of the fittest), and it may last for generations.

III. PARAMETERS ESTIMATION

In many control schemes and biological modeling, there is a strong need for a model of the system (see [24]–[27]). A few examples for such a need were demonstrated in the previous section in the context of adaptive-control schemes [8], [10], [21], [22]. A parametric model is a model that belongs to a family of models with a finite number of parameters. The modeler's task is first to choose a proper family of models and then to estimate the values of the parameters. In this section, we describe the estimation problem and then concentrate on a commonly used family of models: the linear models. We give an example for building a linear model of the muscle and estimating its parameters, and we discuss the problem of choosing the order of the model. In Section IV, we give an example for a nonlinear model and its possible advantage.

A. Estimation Problem

The general problem of parameters estimation can be formalized as follows. Let $\Theta(x, a)$ be a family of parametric functions. That is, for each parameter vector, a_0, $y = \Theta(x, a_0)$ is a static input/output function or a transfer function in the Laplace domain, where x is the input and y is the output. Suppose that we have an unknown system $F(x)$ that belongs to the above functions family. That is, $F(x) \in \Theta(x, a_0)$ for a specific but unknown parameter vector a_0. As a result of an experiment on this unknown system, we acquired a group of measurements of input/output pairs $\{x_i, y_i\}$ that naturally satisfies $y_i = F(x_i)$. In the presence of measurement noise or uncertainty in the generating function (that is, if we are not positive about the assumption that the unknown system belongs to the family of parametric functions), we can relax the requirements from the data to $|y_i - F(x_i)| < n$, where n represents the noise or the uncertainty in the fitness of the model to the system. The problem is to find the vector of parameters a that will best fit the measurements pairs according to a given criterion. If one uses the least-squares criterion, the problem is to solve the following minimization:

$$\hat{a} = \arg \min_a \sum_i (y_i - \Theta(x_i, a))^2. \qquad (6)$$

There are many methods to solve this problem and to formalize parametric groups of functions, (see [28]). In the next section, we will concentrate on the linear group of functions and on an example from the field of human motor control.

B. Linear Models

By linear models, we mean a system that can be described by a linear differential equation and therefore be transformed to the Laplace domain as a transfer function in the form of (1). In the discrete case, the same can be done with a difference equation and the Z transform. Since this formalization is easy to analyze, there are many names for all kinds of such systems. If there are only poles, the system is called autorecursive (AR), if there are only zeros, the system is called moving average (MA), and the general case is called autorecursive moving average (ARMA) system. For a comprehensive description of adaptive control, see [9]. For an example of parameter identification of discrete nonlinear muscle dynamics see [25].
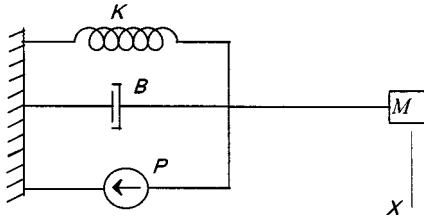
Fig. 7. Second-order mechanical model of the muscles and the joint.

Fig. 8. Length of the muscle as a linear function of the excitation and the previous lengths.

C. Parameters Estimation of a Linear-Muscle Model

There is a great interest in modeling muscles and joints dynamics. Let us identify two main reasons. One is that the muscles are the main output of the nervous system, and therefore, they are the window to investigate the operation of the nervous system. The other reason lies in building prosthetic and artificial limbs and in the external excitation of muscles in paralyzed patients, which is called functional neuromuscular stimulation (FNS) (see [26]). All the fields above require the construction of a model for the system. In this section, we will show a simple example to illustrate the main procedures and problems in parameters identification.

Let us look at the mechanical, second-order model in Fig. 7. This is a linear-lumped model, which is an approximation of the muscle behavior for a small signal [24]. In this model, P represents the internal force in the muscle that is the result of the neural excitation. K and B are the elastic and the viscose-damping elements that represent the mechanical properties of the muscle tissue, and M is the mass of the muscles and the joint.

The output to input transfer function of this model is

$$\frac{X(s)}{P(s)} = \frac{-1}{M \cdot s^2 + B \cdot s + K}. \tag{7}$$

One can derive a similar relation for external force and its relation to the position, or any other desired relationship, for the muscle model. Our goal is to estimate the value of the model parameters (M, B, and K in the above case). Since we usually use a computer and discrete measurements, our first step will be discretization. There are different procedures for discretization, just as there are many procedures for numerical integration. Let us use here the simplest method, the Euler's forward method, which is to move to the Z transform by replacing each s by $(z-1)/T$, where z is the Z transform variable, and T is the sampling interval

$$\frac{X(z)}{P(z)}$$
$$= \frac{-1}{M \cdot s^2 + B \cdot s + K}\bigg|_{s=(z-1)/T}$$
$$= \frac{-T^2}{M \cdot z^2 + (B \cdot T - 2 \cdot M) \cdot z + M - B \cdot T + K \cdot T^2}. \tag{8}$$
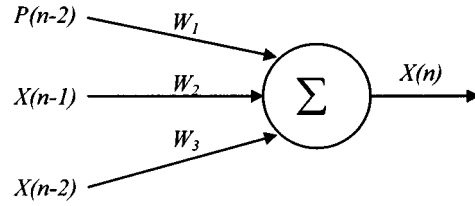
See [29] for more details about discretization methods.

From the Z transform we can move directly to discrete time

$$X(n) = -\frac{T^2}{M} \cdot P(n-2) - \frac{(B \cdot T - 2 \cdot M)}{M} \cdot X(n-1)$$
$$- \frac{(M - B \cdot T + K \cdot T^2)}{M} \cdot X(n-2). \tag{9}$$

Equation (9) can be formalized as follows:

$$X(n) = w_1 \cdot P(n-2) + w_2 \cdot X(n-1) + w_3 \cdot X(n-2) \tag{10}$$

where w_i are the parameters. If the sampling interval T is given, it is equivalent to know w_i or M, B, K. The system in (10) is described graphically in Fig. 8.

The description in Fig. 8 reminds us of a very large family of parametric models that can be presented by an artificial neural network (ANN). We use this description because these models were inspired by the structure of the biological nervous system, and because this description prepares the ground for handling much more complex nonlinear models. Let us leave this general case and continue with our special case, which is the linear model. We have presented our model as a weighted sum of the inputs, and our problem is to find the optimal weights. We can combine our input components $X(n-2)$, $X(n-1)$, $P(n-2)$ to form an input vector U and denote the output vector, which in our case has just one element $X(n)$ by the letter Y. Now, if we use the minimum mean-square error (MMSE) criterion, we can use the optimal solution, which is

$$W_{OPT} = \Phi^{-1} \cdot P$$
$$P \equiv E[Y \cdot U]$$
$$\Phi \equiv E[U \cdot U^T]. \tag{11}$$

where E stands for expectation (in practice, numerical average over the measurements is used). For the origin and proof of (11), see any textbook on linear-parameter estimation (e.g., [2]).

Let us illustrate this estimation scheme with a simulation example. A random sequence of P was chosen [normal distributed noise with standard deviation (STD) ot equal one and zero mean], and X was calculated according to (9) with the following nominal value of the parameters: $M = 5$, $B = 3$, $K = 2$, and $T = 0.1$. That is, $W1 = -0.002$, $W2 = 1.94$, and $W3 = -0.944$, according to (10). Fig. 9 shows the results of the simulation. The first graph is the random input P, and the second is the calculated X. An additional random noise was added to simulate measurement noise or uncertainties in the model (normal distributed noise with STD $= 0.01$ and zero mean), and this sequence appears in the third graph. Then, the optimal parameters (11) were calculated, and the result was $W_1 = -0.0035$, $W_2 = 2.088$, and
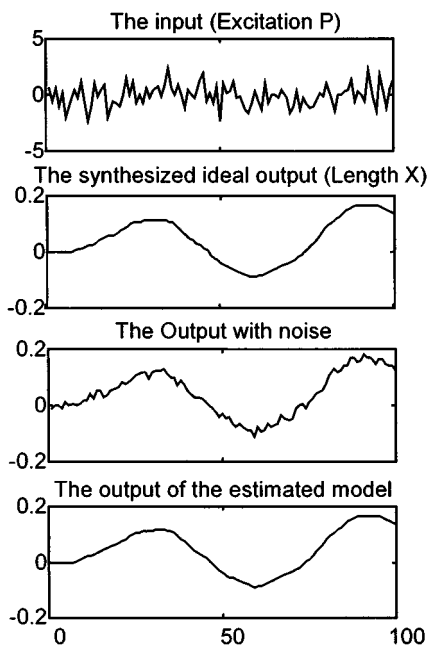
Fig. 9.   Example of parameter estimation of the linear (ARMA) muscle model.
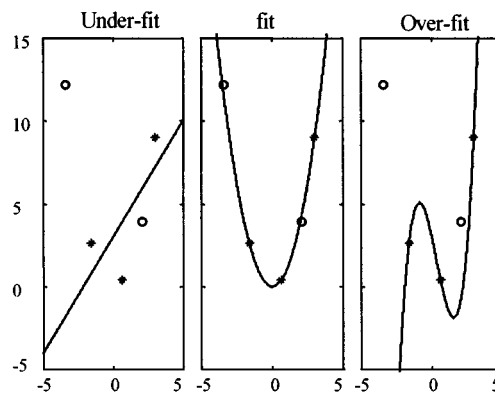

Fig. 10.   Fitting a model to data. In this illustration, the three stars are the data taken from an underlying, unknown function. On the left, a linear function was fitted to the data. In the middle, a quadratic function was fitted, and on the right, a third-order polynomial function was fitted. After the fitting was completed, two more examples were taken from the same underlying function (the two circles). One can see that the left model is too simple (i.e., it under-fits the data), while the right model is too complex (i.e., it over-fits the data). Unfortunately, it does not fit the underlying system.

$W_3 = -1.091$, which is, as expected, close to the nominal parameters. Finally, the output of the estimated model was simulated, and in the forth graph, it appears to be similar to the second graph, which is the actual model output. The above example is synthetic and has many drawbacks that will help us illustrate the problems in parameter-estimation procedures. This example regards the discrete data as a set of independent examples of a static model, and the optimal model is checked for each couple of input/output independently. In practice, the error combines from one time step to the other, since the model may use its own output to estimate the next time step and not the real-system outputs. This problem can be severe when the system has some unstable poles. Then, the error might grow very fast. The estimated parameters should be checked on a new data set and not only on the data that was used for the parameter estimation. This check is called a generalization check and can assist in avoiding overfitting the data. We discuss this method of validation in the following Section III-D. One should remember that the biological system is generally a time-varying system. For example, since the muscles can change their properties due to fatigue, the duration of the experiment must be short in order to justify the assumption that the system is a time-invariant system. We must mention here that the simple optimal-parameter calculation (11) is not always stable numerically, and there are many improvements and practical methods that can be found in modern numerical software [30].

D. Order of the Model

In the example above, the structure of the model was known and the only problem was to estimate the parameters, but in most biological cases, the model is unknown. For the sake of simplicity, and since there are many algorithms and software tools for linear systems, we restrict our discussion here to linear models. The general structure of a linear model is the ARMA model, which can be written in the discrete form as follows:

$$y(n) = \sum_{i=0}^{N} a_i \cdot x(n-i) - \sum_{j=1}^{M} b_j \cdot y(n-j). \qquad (12)$$

As an immediate example of an ARMA model, we can mention the last example (10).

In order to estimate the parameters, we first need to establish the order of the model. In the ARMA case, this involves choosing N and M in (12) above. At a first look, one can suggest that the more parameters the model has, the better it will fit the actual system. However, in actuality this is not the case. Too many parameters are not only a computational burden, but they may cause errors in the model (see [31] for extensive treatment of the model-order selection problem for the case of surface electromyography). Let us describe the pitfalls in choosing the wrong number of parameters. One can be wrong by choosing either too many or too few parameters.

1) Under-Fit: The situation where the model is less complex than the actual system. In this case, the model is unable to fit the data (see Fig. 10 on the left).

2) Over-Fit: The situation where the model is more complex than the actual system. In this case, the model will fit the observations, but if there is a noise or insufficient observations (i.e., less independent observations then the number of parameters), the model will not fit the actual system, and it may fail to predict the outcome of the system in the validation process. In the validation process, we check the generalization (the ability of the identified model to deal with cases that were not seen before; see Fig. 10 on the right).

Many approaches have been suggested for choosing the proper order. For linear models, a commonly used approach is the Akaike information criterion (AIC), which is based on a discrepancy measure. For the ARMA model, it will take the following form:

$$N^{-1} \cdot \mathrm{AIC}(n, m) = \hat{\sigma}_x^2 + \frac{2 \cdot (n + m + 1)}{N}. \qquad (13)$$
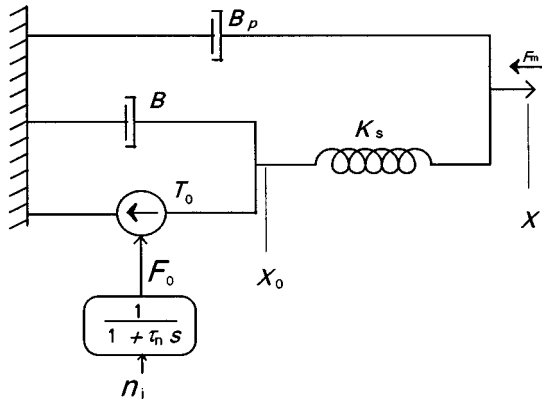
Fig. 11. Mechanical model of the muscle. n_i is the neural input. The first-order filter represents the activation–contraction coupling. T_o is the hypothetical force in the muscle. B represents the relation between force and velocity from Hill's model. The other elements represent the mechanical properties of the tendon and other connective tissues around the joint.

Since the first term, the estimation of the error $\hat{\sigma}_x^2$ is monotonically decreasing with increasing model size, and the second term is increasing. One can find an optimal model size by finding the minimal value of the AIC. Another way to choose the order of the model is through empirical validation. This method is commonly used in pattern recognition and classification in which part of the data is kept from the learning phase (in our case, this will be the fitting phase), and then the model is chosen for its generalization capabilities checked on the kept data. For more information about parameters estimation and system identification, see [2], [28], and [32].

IV. NONLINEAR MUSCLE MODEL

We have seen the advantage of linear models from the engineering and mathematical point of view in terms of simplicity and tractability. It is simple to analyze linear systems, and there exists a vast literature and many algorithms for estimating the parameters of a linear model. However, biological systems are seldom linear, and the linear analysis holds only for a small region around the working point. In this section, we describe an example that shows how nonlinearity might be exploited advantageously by nature.

Let us look at the Hill-type mechanical model of the muscle in Fig. 11. This model is taken from [33] with minor changes (see [3] for an introduction to the use of electrical and mechanical models and [34] for a detailed description of this model and this example).

The following are the differential equations of this mechanical model:

$$
\begin{aligned}
\dot{F}_0 &= \frac{1}{\tau_n} \cdot (n_i - F_0) \\
T_0 &= F_0 \cdot F_{\max} \\
\dot{X}_0 &= \frac{(K_s \cdot (X - X_0) - T_0)}{B} \\
F_m &= B_P \cdot \dot{X} + K_s \cdot (X - X_0).
\end{aligned}
\tag{14}
$$

This model was derived from the Hill model [35], and in the Hill model, the value of the viscose-damping element B de-
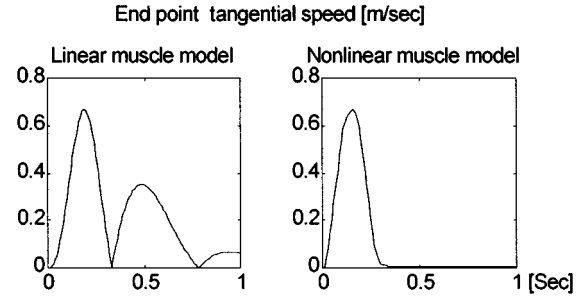
End point tangential speed [m/sec]

Fig. 12. Comparison between the speed profile of the end point of a two-degrees-of-freedom anthropomorphic arm with a linear muscle model (left) and with a nonlinear muscle model (right) in response to typical, rectangular pulse activation of the muscles. Only the nonlinear muscle model yields a bell-shaped speed profile with a smooth stop (for more details, see [34]).

pends on the internal force and on the contraction velocity, as seen in (15)

$$
B = \begin{cases} (a \cdot T_0)/(b + v), & v \geq 0 \\ a' \cdot T_0, & v < 0. \end{cases}
\tag{15}
$$

For the sake of simplicity, the value of B was taken as a constant in several models in order to get a linear model of the muscle (see [36] and [37]). This linear model is underdamped and therefore overshot, and oscillations are most likely to appear in the controlled movement. This problem is avoided by the use of the nonlinear model, and we demonstrate this for a very basic movement: the reaching movement.

The reaching movement is a fast movement of the arm toward a given target. This movement was studied experimentally and its main characteristics are roughly a straight-line path and a bell-shaped speed profile. This speed profile was found to be consistent with minimization of the jerk during the movement, i.e., minimization of the following cost function [38]:

$$
C = \frac{1}{2} \cdot \int_0^{t_f} \left(\left(\frac{d^3 x}{dt^3} \right)^2 + \left(\frac{d^3 y}{dt^3} \right)^2 \right) dt.
\tag{16}
$$

This minimization was assumed to be carried out by the nervous system in several models [22], [36]. These two models were described in the first section, and they are a complex control scheme containing a forward and/or an inverse model of the system. Both have to deal with the time throughout the trajectory. However, when the nonlinear model is incorporated, it can be shown that the desired performance (a bell-shaped speed profile with a smooth stop), can be achieved with a simple control strategy and without the need for a complex-optimization procedure.

The performance of an anthropomorphic arm with a linear and nonlinear muscle model in response to rectangular control pulses is shown in Fig. 12. For the linear muscle model, the value of the viscose-damping element B is constant instead of the Hill-type relation in (15).

It can be seen that the arm with the linear muscle model, in response to pulses at the input, does not stop when the target is reached and has an overshoot and an oscillatory behavior at the end of the movement. Under the same conditions, the nonlinear muscle can evoke a fast movement with a smooth stop. This is only a demonstration, however. This example is representa-

tive of the improved arm performance achieved with a nonlinear muscle model under the assumed conditions. For a detailed description of the model, see [34]. The reason for this phenomenon can be explained by observing the behavior of a simple second-order system in its standard form. The transfer function in the Laplace transform domain is

$$a \cdot \frac{w_n^2}{s^2 + 2 \cdot \xi \cdot w_n \cdot s + w_n^2}. \tag{17}$$

All the characteristics are known for such a system. W_n is the natural frequency, ξ is the damping coefficient, and a is the gain. Let us look at the overshoot and the time to reach the maximum $t_{\max}$

$$t_{\max} = \pi \left/ \left(w_n \cdot \sqrt{1 - \xi^2} \right) \right.$$
$$\text{O.S.} = \text{EXP} \left(-\pi \cdot \xi / \sqrt{1 - \xi^2} \right). \tag{18}$$

For the human arm, the system is underdamped (i.e., $\xi < 1$; see [39]). It can be seen from (18) that as ξ gets smaller, the movement becomes faster, but the overshoot is bigger. In a linear system, a tradeoff exists between small overshoot and fast movement. In a nonlinear system, the parameter can change during the movement to achieve a fast movement without any overshoot, and we suggest that this is what happens in the muscles. If we try to examine a simple model such as the second-order model in Fig. 7 and its transfer function (7), the parameters of the transfer function in the standard form (17) are

$$a = -\frac{1}{K} \qquad w_n = \sqrt{\frac{K}{M}} \qquad \xi = \frac{B}{\sqrt{4 \cdot M \cdot K}}. \tag{19}$$

In trying to find the parameters of such a model, it was shown that K changes during the movement, and it has a positive relation to the activation of the muscle [39], [40]. Changes in K have the same qualitative effect as the changes in B in the Hill-type model above [34]. Another nonlinearity that can produce a stop without oscillation is the one-fifth power law in the viscosity, which was found in studies of human wrist movement [41]. The inverse relation between the damping force and the contraction velocity in the model of Hill, and the increasing stiffness at the end of the movement is also in line with arm movements measurements by Gomi and Kawato (see [42] and [43]). These measurements were recently described by another nonlinear, six-muscles model, in which rather simple control signals were needed (see Gribble *et al.* [44]). The main conclusion from these examples is that the nonlinear properties of the muscle may have a functional role in simplifying the control strategy. The nonlinearities make system analysis difficult. However, they can be used by the brain to its advantage, especially since both the muscles and the nervous system developed together.

V. PARALLELISM AND REDUNDANCY

This section is about two salient biological characteristics that sometimes combine to make our understanding of these systems difficult. Parallelism relates to the architecture of the biological system, which frequently includes many pathways that execute similar tasks. For example, nerve fibers contain many axons that transport similar information, a muscle contains many parallel,
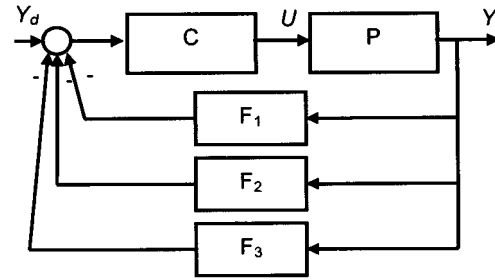

Fig. 13. Multiple-feedback loop.

similar fibers, there are many muscle spindles and Golgi tendons in each muscle that measure length, speed and force in the muscles, and so forth. Redundancy relates to the function of the biological system or to a specific task that can be executed in more than one possible way (see [5] and [45]). In many cases, the parallel architecture implies redundancy, but redundancy can exist without parallelism, as in the inverse kinematics problem. Parallelism also can exist without redundancy, as in some distributed systems (see multiple model control [27], [46]). In the following sections, we describe three aspects of these phenomena The first is the issue of multiple feedback loops, which is most common in biological systems and which can jeopardize classical attempts to measure the loop-gain. Second, we discuss one possible function of the parallelism in nerve fibers: the spatial filtering. Third, we describe a new approach to invert a redundant system in order to control an MTO system.

A. Multiple Feedback Loops

In Section II, where feedback control was discussed (see Fig. 1), the importance of the loop-gain in reducing the sensitivity to parameter change was mentioned. The loop-gain also can be a major factor in establishing the stability of the system. In order to measure the loop-gain, one should brake the loop, introduce an input in one place, and measure the output. But in the biological system, there are typically multiple feedback loops in parallel (see Fig. 13) since there are many sensory systems that work together for the same purpose. For example, in the temperature-regulation system, there are sensors in the skin, in the core of the body, and in the hypothalamus, and they all influence the temperature-regulation mechanisms (see [47]). In movement control, there are feedback loops from sensors in the muscles, joints, and skin (i.e., muscle spindles, Golgi tendon organs, joint-pressure transducers, etc.), and there are many of each type of sensor all operating in parallel. The first advantage of such a multiple loop and of any redundancy is in robustness (i.e., if one subsystem fails there are other options to activate the system). More interesting advantages are in spatial filtering and in the flexibility given to the system in the ability to choose different solutions in different situations, as will be described in the following sections.

There is a great danger in trying to estimate the loop-gain in such a system, because there may be loops that we cannot open, or of whose existence we may be unaware. In such a case, we may underestimate the loop-gain. For example, if we open the first two loops in Fig. 13 and leave F_3 connected, the transfer function from Y_d to Y will be $P \cdot C / (P \cdot C \cdot F_3 + 1)$ instead

of $P \cdot C$ when there is no additional loop. Because of this, one should be aware of these multiple loops.

B. Spatial Filtering

Another interesting property of parallelism in the biological system is the spatial filtering in a multipath-transmission mechanism. Milgram and Inbar [48] demonstrate that the fidelity of transmission of certain types of information through single neurons acting alone is comparatively poor within typical physiological ranges of operation, and that the quality of reception of these messages increases in proportion to the number of neural channels involved in transmitting them simultaneously or in parallel. The main idea behind these observations is that there is a distortion in the neuronal code, which adds high frequencies to the transmitted information. The parallel transmission produces low-pass filtering (which in this case is spatial filtering) and therefore improves the transmission fidelity by eliminating the distortion components. In order to explain this spatial filtering, let us examine a multichannel transmission line in which each channel has different delay. The input to all the channels is $f(t)$ and the output will be

$$P_o(t) = \sum_{k=0}^{M} f(t - a_k). \qquad (20)$$

If one assumes a large number of channels, and if the distribution of the delays a_k is given as a density function $g(a)$, the output can be estimated as the following integral:

$$P_o(t) \cong M \cdot \int_0^\infty f(t-a) \cdot g(a) \cdot da. \qquad (21)$$

Let us use the Laplace transform on both sides and then remember that the input is M times $f(t)$, which is $M \cdot F(s)$ in the Laplace domain

$$P_o(s) \cong M \cdot \int_0^\infty F(s) \cdot e^{-s \cdot a} \cdot g(a) \cdot da$$
$$= P_i(s) \cdot \int_0^\infty e^{-s \cdot a} \cdot g(a) \cdot da. \qquad (22)$$

Now one can see that the transfer function between the input and the output is the Laplace transform of the delays distribution

$$\frac{P_o(s)}{P_i(s)} \cong \int_0^\infty e^{-s \cdot a} \cdot g(a) \cdot da = G(s). \qquad (23)$$

This result means that as the range of delays is larger, the filter is narrower. In the extreme case, when all the channels are equal, there is no filtering and there is no advantage over a single transmission line.

There are other possible ways to achieve similar spatial filtering, such as distributed thresholds instead of delays. The latter option is more physiologically plausible, but it is less analyzable, and therefore, it is demonstrated in simulations (see [48]).

C. Learning to Invert MTO Mappings

In motor control, as mentioned in the first two sections, one is looking for an inverse of the controlled system. In the presence of redundancy, the inverse problem is an ill-posed one. The controller has to act on an MTO system and has to choose one of the many possible actions to obtain the same desired
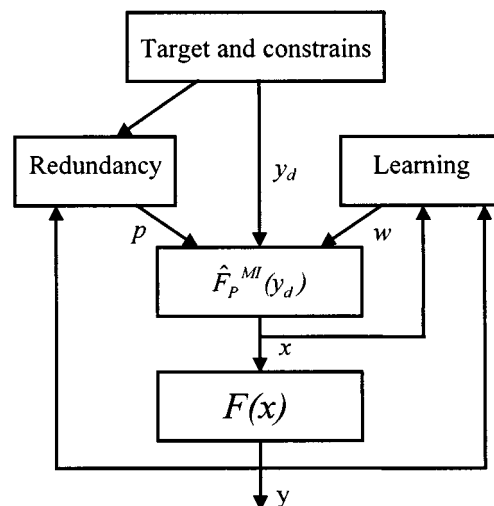

Fig. 14. Proposed scheme for learning the inverse control of an MTO system.

target. Most of the solutions to this problem use a single criterion, such as smoothness of the movement or minimum energy, and find a single optimal solution (see [49]). However, the biological system exhibits the ability to use different solutions at different occasions. The nervous system is known to be modular, and there is enough room to have many solutions ready and choose the desired solution in real time. The solution is chosen according to an appropriate criterion that can change under different circumstances. As mentioned before, the biological system is an adaptive one, and therefore, it must learn and track the environmental parameters and its own internal parameters from examples. This problem was investigated for a robot's redundancy by DeMers [50] and is under current investigation by Karniel *et al.* [51], [52]. The inverse-control problem was shown graphically in Fig. 3, and the proposed solution for the problem, which deals with both redundancy and learning, is described in Fig. 14.

Suppose that we have an unknown system $F(x)$ that is not necessarily an injective, and a serious of input and output vectors $\{x^l, y^l\}$ that are input/output pairs of the unknown system. The problem is to construct a multiple inverse function $\hat{F}_P^{\mathrm{MI}}(y_d)$ where MI stands for multiple inverse and the parameter p determines which of the many possible solutions is chosen. The formal requirement is that for any given accuracy value ε, one can construct $\hat{F}_P^{\mathrm{MI}}$ such that for any value of y_d and for any value of the parameter p, the following inequality will hold:

$$\left| F\left(\hat{F}_P^{\mathrm{MI}}(y_d) \right) - y_d \right| < \varepsilon. \qquad (24)$$

The questions that may be addressed as a consequence of such a problem are as follows.

1) What architecture can be appropriate for representing the inverse of the system? See the box in the center of Fig. 14.
2) How does one train the architecture, and what learning algorithm should be used? See the box in the right-hand side of Fig. 14.
3) How does one choose the parameter p for a specific action, and what criterion is to be used to regulate the multiple-inverse problem? See the box in the left-hand side of Fig. 14.

These questions are under current research and preliminary results for the first two questions are given in [51], where a new architecture, the polyhedral mixture of linear experts (PMLE) is proposed. This architecture is based on the mixture-of-experts architecture [53] and the hinging-hyperplanes algorithm [54]. It produces a piecewise linear approximation of the system. Each area is governed by a linear function, called an expert, and one can invert each expert and get the multiple inverse.

The PMLE architecture is one possible solution to the second question, which was about how to learn and implement the multiple inverse function $\hat{F}_P^{\mathrm{MI}}(y_d)$ in Fig. 14. The third question of choosing the proper criterion to reduce the redundancy is an open one with many possible solutions that most probably are task dependent (see [49] and [55]).

VI. CONCLUSION

Biological motor control is a great challenge for scientists, engineers, and physicians. Simple tasks such as walking or playing with a ball are much more complex for artificial machines (and therefore much less understood) than many "complex" and "intelligent" tasks, such as numerical calculations and playing the game of Chess. In this paper, some problems of motor control were introduced. The issue of learning and adapting in the motor-control framework was described and related to problems of parameter estimation. An example of the possible role of the mechanical nonlinear properties of the muscles was demonstrated, and a few aspects of redundancy and parallelism were described. The classical engineering and mathematical tools are appropriate for linear, time-invariant, injective systems. The biological system does not comply with these qualifiers, and therefore, there is a place and a need for new mathematical tools and models in order to describe and analyze the biological system. A few examples for the required tools were described in this paper and others have yet to be developed. With the aid of such mathematical tools, the scientific community can continue to strive for a better understanding of not only biological motor control, but of the mysteries of the human mind and how it operates.

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